Remarks on the Baryon Mass Spectrum

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We review, modify slightly, and generalize earlier attempts to classify negative-parity baryon states in the intermediate energy range 1 to 2.2 BeV. The mass spectrum is analyzed in terms of (i) the basic $\mathbf{70}^-$ and $\mathbf{20}^-$ multiplets of $SU(6)$ and (ii) the $\mathbf{70}$ $(L=1)$ of $SU(6)\times O(3)$. Connection between these possibilities and a fundamental structure composed of quarks and pseudoquarks for case (i) and quarks with 'orbital' excitation $L=1$ for case (ii), underlying these mass regularities, are briefly discussed.

I. INTRODUCTION

THE growth of the hadron mass spectrum in the form of resonances has yielded a remarkably rich structure of states in the energy range 1 to 3 BeV. It is nevertheless clear that these states exhibit a pattern of regularity which is quite striking, suggesting that an underlying classification scheme may be appropriate for them.

In the present study, we review, modify slightly, and generalize earlier attempts[1,2] to classify *negative-parity* baryon states in the intermediate energy range 1 to 2.2 BeV. We shall assume that the small baryon representations **70** and **20** first introduced by Pais[3] for static $SU(6)$ theory are *useful* when evaluated in conjunction with the Bég-Singh mass formulas.[4,5] The original $SU(6)$ group[6,7] contains the intrinsic spin group $SU(2)_J$ and the internal symmetry group $SU(3)$. The theory is thus naturally a nonrelativistic one. The basic group of a nonrelativistic theory (as contrasted with a Galilean or a Lorentz group) is the Newtonian group of space and time translations and rotations in three-dimensional space. It is thus natural to ask also what type of classifications are possible if we combine the static $SU(6)$ structure with this Newtonian group. As the first step in this direction, we can consider classification under $SU(6)\times O(3)$,[8] where $O(3)$ is a group of rotations in the three-dimensional space, independent of the spin group $SU(2)_J$ contained in $SU(6)$.

We have phrased the above procedure without talking of quarks.[9] The formal question is then to discuss classification of supermultiplets which correspond to the group $SU(6)$ and $SU(6)\times O(3)$, respectively. An alternative way of looking at this is to consider the quark system qqq with "higher excited L states" first proposed by Gell-Mann,[10] Greenberg,[11] and Freund and Lee,[12] and analyzed in detail for meson- and baryon-mass spectra by Dalitz more recently.[13] We consider the $\mathbf{56}^+$ of static $SU(6)$ to be lowest $(L=0)$ state of the qqq system and imagine that this system has "higher excited states" with $L>0$. Take for instance the **70**, whose $SU(3)$ content is

$$\mathbf{70}=(1,2)+(8,2)+(8,4)+(10,2) \tag{1}$$

and boost with orbital angular momentum $L=1$. We have for the **70** $(L=1)$, $\mathbf{70}\times 3=210$ states, where 3 is the dimension of the $L=1$ representation of $O(3)$. Coupling L to J, one gets from (1)

$$\mathbf{70}\otimes\mathbf{3}=(8;6)+(10+8+8+1;4)+(10+8+8+1;2), \tag{2}$$

where the number following the semicolon is $2j+1$, j being the *total*-angular momentum of the system.

In the simplest model in which the baryons and mesons are composite objects made up of a single triplet of quarks,[9] the **70** $(L=1)$ has *negative parity*, and hence the same is true for the nine $SU(3)$ multiplets on the right-hand side of (2). It is possible to understand also the negative parity for the **70** and **20** [essentially **70** $(L=0)$ and **20** $(L=0)$] as originally proposed[1–3] in the framework of the quark model. We shall need to introduce an additional triplet of *pseudoquarks*[14] q' of opposite intrinsic parity to q ($\gamma_5\phi$ transformation

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[1] I. P. Gyuk and S. F. Tuan, Phys. Rev. Letters **14**, 121 (1965).

[2] I. P. Gyuk and S. F. Tuan, Phys. Rev. **140**, B164 (1965).

[3] A. Pais, Phys. Rev. Letters **13**, 175 (1964).

[4] M. A. B. Bég and V. Singh, Phys. Rev. Letters **13**, 418 (1964).

[5] M. A. B. Bég and V. Singh, Phys. Rev. Letters **13**, 509 (1964).

[6] F. Gürsey and L. A. Radicati, Phys. Rev. Letters **13**, 173 (1964); F. Gürsey, A. Pais, and L. A. Radicati, *ibid.* **13**, 299 (1964).

[7] B. Sakita, Phys. Rev. **136**, 1756 (1964).

[8] K. T. Mahanthappa and E. C. G. Sudarshan, Phys. Rev. Letters **14**, 163 (1965).

[9] M. Gell-Mann, Phys. Letters **8**, 214 (1964); G. Zweig, CERN Report, 1964 (unpublished).

[10] M. Gell-Mann, Phys. Rev. Letters **14**, 334 (1965).

[11] O. W. Greenberg, Phys. Rev. Letters **13**, 598 (1964).

[12] P. G. O. Freund and B. W. Lee, Phys. Rev. Letters **13**, 592 (1964).

[13] R. H. Dalitz, in *Proceedings of the Oxford International Conference on Elementary Particles, September 1965* (Rutherford High Energy Laboratory, Harwell, England, 1966).

[14] R. Dashen and M. Gell-Mann (private communication).

property where ϕ is the quark field) but with the same charge, spin, and baryon number. A **70**$^-$ can then be regarded as a composite system of $q'q'q'$ (or perhaps qqq') with $L=0$. Greenberg[11] has put forward the attractive proposal that quarks should satisfy para-Fermi statistics, thus allowing the n-quark system to saturate for $n=3$. On this basis, the **70**$^-$ ($L=0$) will have the (mixed) orbital configuration s^1p^2, s^2d^1, assuming that the pseudoquarks also satisfy parastatistics. The **20**$^-$ ($L=0$) will have (mixed) configuration p^3, $s^1p^1d^1$ when composed of three parafermion quarks qqq. The fact that in one multiplet, the **70**, we deal with pseudoquarks q', while in the other multiplet, the **20** we deal with quarks q, may acocunt for the lack of supermixing between the appropriate $SU(3)$ contents of these multiplets (see Sec. II).

Let us digress briefly to examine and review the baryon states of negative parity as we know them experimentally. There is good evidence that the Λ' (1405) state[15] and $Y_0^*(1520)$ are $SU(3)$ singlets (1,2) and (1,4) with spin-parity assignments $J=\frac{1}{2}^-$ and $J=\frac{3}{2}^-$, respectively. The situation with respect to the (8,4) γ octet remains unclear, though assignment of $N_\gamma(1512)$, $\Sigma_\gamma(1660)$, and $\Xi_\gamma(1817)$ as $J=\frac{3}{2}^-$ members appears to be not unreasonable; it is evidently exceedingly desirable to find out whether or not there exists a $\Lambda_\gamma(1666)$ computed on the basis of an unmixed (i.e. unmixed with respect to (1,4) and other possible $T=Y=0$ members of yet-to-be-discovered $\frac{3}{2}^-$ octets) Gell-Mann–Okubo mass formula. The experimental basis for the above-mentioned states has been thoroughly reviewed by Dalitz.[13]

The postulated existence of an η octet of $J=\frac{1}{2}^-$ baryon states[1,2] associated with the $\eta+B$ ($B=N,\Lambda,\Sigma,\Xi$) thresholds has been the object of fairly extensive study. Detailed phenomenological and theoretical analyses[16–18] of η production and pion-nucleon phase shifts near threshold tend to support the existence of a $T=\frac{1}{2}$, $J=\frac{1}{2}^-$ baryon state $\tilde{N}$ with mass 1490 ± 50 MeV. The behavior near the $\eta+\Lambda^0$ threshold has likewise been interpreted as either a virtual bound state[19] or as an S-wave "resonance" slightly above threshold[20]; the mass for $\tilde{\Lambda}$ is in the range 1660–1675 MeV. There are promising indications[21] that the sharp rise and fall in η production near threshold is also prominent in the $\eta+\Sigma$ channel, suggesting that an $\tilde{\Sigma}$ S-wave baryon state may be present here as well. To summarize, an $SU(3)$ octet $(8,2)^-$ describing these physical phenomena is not unreasonable.

Several additional $J=\frac{1}{2}^-$ and $J=\frac{5}{2}^-$ baryon states have been uncovered recently. Of particular significance is the existence of an S_{31} baryon state deduced from the pion-nucleon phase-shift analysis[22]; such a state, of necessity, must belong to the $(10,2)^-$ sector of Eqs. (1) or (2). The mass quoted for this $J=\frac{1}{2}^-$ resonance $\tilde{N}^*$ is around 1700 MeV. Further, the phase-shift analysis of Bareyre *et al.*[22] indicates the presence of another S_{11} state (in addition to the $\tilde{N}$ of the η octet) in the neighborhood of 1700 MeV. This $N_{1/2}^*(1700)$ will be a member of a further $(\frac{1}{2}^-)$ octet. Finally the pion-nucleon scattering data of Duke *et al.*[23] have shown rather clear indications for the existence of a $T=\frac{1}{2}$, $D_{5/2}$ resonant state lying at almost the same mass value as the $T=\frac{1}{2}$, $F_{5/2}$ resonance at 1688 MeV. This $\frac{5}{2}^-$ $N_{1/2}^*(1680)$ state, coupled with the $Y_1^*(1765)$ for which the spin-parity $\frac{5}{2}^-$ has now become firm,[24] appears to form part of a $\frac{5}{2}^-$ octet or antidecuplet.

In Sec. II we analyze the experimental spectrum on the basis of the general mass formula[4,5]

$$M=a+bC_2^{(3)}+cJ(J+1)+dY +e[2S(S+1)-C_2^{(4)}+(1/4)Y^2] +f[N(N+1)-S(S+1)] +g[I(I+1)-(1/4)Y^2], \quad (3)$$

where the coefficients depend only on the Casimir operators of $SU(6)$. Equation (3) represents the general mass operator that can be derived by considering tensor operators transforming according to real representations of dimension less than 1000, which can contribute to the **70** ($L=0$) mass spectrum. These representations are the **35**, **189**, and **405**. For the **20** ($L=0$), the mass formulas are expected to be relatively simple in any case, since as with the **56**$^+$ there is no mixing possible amongst members of its $SU(3)$ contents (1,4)+(8,2).

In Sec. III we make an *exploratory* study of classification under $SU(6)\times O(3)$ for the **70** ($L=1$) states of Eq. (2). The procedure here is necessarily much more *ad hoc*, and hence the conclusions are not without ambiguity. In particular we propose a simple generalization of Eq. (3) by introducing an *additive* spin-orbit

[15] Unexplained notations are the same as those in Refs. 1 and 2; otherwise we follow the nomenclature of A. H. Rosenfeld, *et al.*, Rev. Mod. Phys. **37**, 633 (1965).
[16] A. W. Hendry and R. G. Moorhouse, Phys. Letters **18**, 171 (1965).
[17] P. N. Dobson, Jr., Phys. Rev. **146**, 1022 (1966).
[18] J. S. Ball, Phys. Rev. **149**, 1191 (1966).
[19] S. F. Tuan, Phys. Rev. **139**, B1393 (1965).
[20] D. Berley, *et al.*, Phys. Rev. Letters **15**, 641 (1965).
[21] D. B. Cline and Martin G. Olsson, Bull. Am. Phys. Soc. **11**, 76 (1966); also R. D. Tripp (private communication).
[22] P. Bareyre, C. Bricman, A. V. Stirling, and G. Villet, Phys. Letters **18**, 342 (1965); A. Donnachie, A. T. Lea, and C. Lovelace, Phys. Letters **19**, 146 (1965).
[23] P. J. Duke, *et al.*, Phys. Rev. Letters **15**, 468 (1965).
[24] R. Armenteros *et al.*, in *Proceedings of the Oxford International Conference on Elementary Particles, September 1965* (Rutherford High Energy Laboratory, Harwell, England, 1966); R. P. Uhlig, University of Maryland Technical Report No. 545, 1966 (unpublished); R. B. Bell, R. W. Birge, Y. L. Pan, and R. T. Pu, Phys. Rev. Letters **16**, 203 (1966).

term.[25] We have then

$$M_{(L)}=M+h\mathbf{L}\cdot\mathbf{J}+kL(L+1)\,, \tag{4}$$

where M is as in Eq. (3); the numerical coefficients a, b, $\cdots$ h, and k have in general to be evaluated for the $L>0$ case at hand. The total spin j is given by

$$\mathbf{j}=\mathbf{L}+\mathbf{J}$$
$$\mathbf{L}\cdot\mathbf{J}=\tfrac{1}{2}[j(j+1)-J(J+1)-L(L+1)]. \tag{5}$$

We conclude this study with some summary comments in Sec. IV.

Remarks

We recognize that the relativistic background behind $SU(6)$ theory, especially in terms of its group structure,[26] has not yet been properly delineated. Ne'eman[27] has proposed a tentative way to understand the existence of these supermultiplets in the algebraic approach, in terms of spectrum-generating algebras.

A more conservative outlook would be to regard the successes of $SU(6)$ classifications as purely phenomenological in nature. We recollect that the Bég–Singh mass formulas took advantage of the representations of $SU(6)$ to enumerate the possible types of mass splitting, and hence in a certain sense can be divorced from the stronger assumption of $SU(6)$ symmetry. The mass formulas are then useful tools for probing strong-interaction physics, even though the underlying $SU(6)$ theory (similar to a symmetry theory in nuclear physics) need be neither exact nor relativistically invariant.[28] Comparison of predictions of mass rules with empirical data can lead to fresh insights, especially in areas where small discrepancies occur in an otherwise nearly perfect agreement. Such has been our experience at least with the corresponding situations that arise in nuclear physics.

It appears physically plausible that the mass regularities we see are actual manifestations of the existence of basic heavy particles (for instance, the quarks) with a higher mass scale and strength of interaction [invariant under $SU(3)$], so that the mass formulas of the known hadrons can be usefully described as a *small perturbation* on the fundamental entities. This viewpoint gains strength from the very existence of an S-wave baryon octet (such as the η octet) following rather well the Gell-Mann–Okubo mass rule. It has long been recognized[29] that in terms of a program of understanding involving just the known hadrons and long-range forces, the motion of S-matrix poles will be substantially shifted by changes in thresholds as we move from perfect symmetry to broken $SU(3)$ for *S-wave resonances*, because of the lack of an orbital barrier amongst other considerations. Hence the logical basis for the existence of S-wave unitary multiplets is far from clear. On the other hand, as emphasized by Dalitz,[13] if these states are all considered to exist in consequence of the forces between quarks, then the long-range forces usually considered in the dynamical discussions attempted in the literature for the resonant states are essentially irrelevant to the existence of these states. Morpurgo[30] has given qualitative reasons, based on a realistic quark model, why nonrelativistic $SU(6)$ results can arise.[31] We are, of course, aware that the question of strong binding of quarks versus its relativistic (or nonrelativistic) internal motion remains an open one.[32]

II. $L=0$ CLASSIFICATION OF NEGATIVE-PARITY BARYON STATES

In Table I we have listed, for convenience of reference, solutions (a) and (b) of Gyuk and Tuan,[2] with the appropriate input stated. Agreement with the empirical data mentioned in Sec. I, when such information is available, appears to be quite reasonable and generally

TABLE I. Possible solutions (a) and (b) of the **70**⁻ mass formulas with appropriate inputs.

Input:	$\Lambda'=1405$; $(N_\gamma,\Sigma_\gamma,\Xi_\gamma)=(1512, 1660, 1817)$
	$(\tilde{N},\tilde{\Lambda})=(1488, 1663)$
	(a)
	$\Lambda_\gamma=1666$, $(\tilde{\Sigma},\tilde{\Xi})=(1693, 1844)$
	$(\tilde{N}^*,\tilde{Y}_1^*,\tilde{\Xi}^*,\tilde{\Omega}^-)=(1788, 1939, 2090, 2241)$
Input:	$\Lambda'=1405$; $(N_\gamma,\Sigma_\gamma,\Xi_\gamma)=(1512, 1660, 1817)$
	$(\tilde{N},\tilde{\Lambda})=(1455, 1660)$
	(b)
	$\Lambda_\gamma=1666$, $(\tilde{\Sigma},\tilde{\Xi})=(1706, 1857)$
	$(\tilde{N}^*,\tilde{Y}_1^*,\tilde{\Xi}^*,\tilde{\Omega}^-)=(1768, 1919, 2070, 2221)$

[25] Such a mass formula is a simple generalization of well-known mass formulas of nuclear physics (cf Ref. 12). A more detailed mass formula can certainly be derived using straightforward tensor calculus along the lines of Bég and Singh (Refs. 4 and 5). Our expectations are that, if the mixing is not unusually strong, Eq. (4) should be adequate *provided* $SU(6)\times O(3)$ *proves to be a useful basis for classification*. We thank Dr. P. Freund for a valuable communication on this point.

[26] See for instance P. G. O. Freund, Phys. Rev. **149**, 1257 (1966). A comprehensive review of $SU(6)$ theory is given by A. Pais, Rev. Mod. Phys. 38, 215 (1966).

[27] Y. Ne'eman, in *Lecture Notes of the Hawaiian Summer School of Physics*, edited by M. J. Moravezik (Gordon and Breach Science Publishers, Inc., New York, to be published).

[28] F. J. Dyson, *Symmetry Groups in Nuclear and Particle Physics* (W. A. Benjamin, Inc., New York, 1966).

[29] R. H. Dalitz, Proc. Roy. Soc. (London) **A288**, 183 (1965); G. Rajasekaran, Nuovo Cimento **37**, 1004 (1965).

[30] G. Morpurgo, Physics **2**, 95 (1965).

[31] We are not yet clear on the exact correspondence between the full treatment of the Bég-Singh mass formulas and those derived from a *realistic* quark model. The interconnection for the **35** dominant mass formulas (or the first-order Bég-Singh mass formula, cf. Ref. 2) has been established by P. Babu. We thank Professor B. Udgaonkar for a communication on this point.

[32] See for instance, O. W. Greenberg, Phys. Rev. **147**, 1077 (1966).

accurate to about 5%.[33] In particular the relative placement of the various $SU(3)$ contents of the **70**$^-$ seems to be in accord with experimental trend. The experimental evidence for an S_{31} state $\tilde{N}^*$ at around 1700 MeV[22] provides an independent check on the validity of the γ-octet assignment (without prejudice as to whether other assignments are correct or not); to wit, the linear Bég-Singh equation[5]

$$2(\tilde{\Omega}-\tilde{N}^*)=2(N_\gamma+\Xi_\gamma)-6N_\gamma$$

now gives an unambiguous determination for an unstable $\tilde{\Omega}^-((T=0),\ Y=-2,\ J=\frac{1}{2}^-)$ at a mass of 2.140 BeV, provided our γ octet is correctly assigned.

As emphasized in the introduction, the negative parity of the **70** can be understood, if we postulate the existence of pseudoquarks q' such that these baryon states are composites of $q'q'q'$ with $L=0$. Assuming that the interquark forces between q, q and q', q' are comparable, one might speculate that the mass $m(q')$ of the pseudoquark is a couple of hundred MeV greater than the mass $m(q)$ of the quark—corresponding to the relative displacement of the **70**$^-$ with respect to the **56**$^+$. They are thus nearly mass-degenerate. It is not clear, however, whether the introduction of pseudoquarks will not raise additional theoretical problems.

We turn now to states which are known experimentally, but not explained by the basic **70**$^-$. The existence of a second S_{11} state at around 1700 MeV[22] suggests that this $N_{1/2}^*(1700)$ (a member of a further $(\frac{1}{2}^-)$ octet) be combined with the $J=\frac{3}{2}^-$ singlet $Y_0^*(1520)$ to form a **20**$^-$ ($L=0$) $SU(6)$ multiplet. It is important to recognize that we cannot arbitrarily assign this higher S_{11} state to the **70**$^-$. For example, removal of members of the η octet as input and substituting instead $N_{1/2}^*(1700)$ and the S_{31} state into the **70** mass formulas will lead to predictions in violent disagreement with known empirical data.

The existence of the $J=\frac{5}{2}^-$ states[23,24] $N_{1/2}^*(1680)$ and $Y_1^*(1765)$ poses a more severe problem concerning the adequacy of an interpretation in terms of just **70**$^-$ and **20**$^-$. While they do not mix with $SU(3)$ contents of either of these $L=0$ supermultiplets (and hence in this particular respect will not modify the accuracy of **70**$^-$ predictions), their very location in the same general mass region suggests that they cannot be interpreted as Regge recurrences of, say, the η octet.[34] Their accommodation will require the presence of supermultiplets **700**$^-$ (or perhaps **1134**$^-$) for which the quark structure with $L=0$ will be $qqqq\bar{q}$. It is rather surprising to find members of these huge multiplets at the moderate energy we are considering. In this connection it will be of great interest to establish whether a $T=\frac{5}{2}$ resonance (another possible candidate for the **700**) exists at the 1580-MeV region, as once suggested.[35]

Consideration of mass formulas (or the two-point function) naturally leads one to ask about decay properties (or the three-point function) in $SU(6)$. In the context of the **70**$^-$, one can calculate the D/F ratio for decay of the γ octet into the baryon octet (of **56**$^+$) and the octet of pseudoscalar mesons (of **35**$^-$). The result is

$$D/F=-3 \quad\text{or}\quad \alpha=F(F+D)=-0.5. \tag{6}$$

From experiment, however, one has α in the range 0.3 to 0.5.[36] The discrepancy between theory and experiment is thus rather large—particularly when one compares this result with the case of the **56**$^+$ where agreement is excellent. One must point out, though, that experimental decay rates of the γ octet in $SU(3)$ are not at all well predicted even with $\alpha\approx 0.4$, unlike some other cases of decay rates calculated by $SU(3)$. While it is possible that experimental values may change, or that a better phase space can be used in the estimate, it seems to be more probable that the dynamical situation is simply too complex to be amenable to a straightforward interpretation in terms of $SU(3)$ or $SU(6)$ decay calculations. The true significance of $\alpha=-0.5$ (theory) versus $\alpha\approx 0.4$ (experiment) remains to be seen.

Actually, decay properties of **70**$^-$ and **20**$^-$ in $SU(6)$ have never been easy to understand in *conventional* terms whereby the **70** is reduced out of **35**$\times$**56**, while the **20** is a baryon–two-meson state. The decay of $\tilde{N}$ into $N+\eta$ is forbidden[2,37] in "unbroken" $SU(6)$; likewise the $Y_0^*(1520)$ of **20**$^-$ is experimentally known[15] to have a substantially larger proportion of decays into two-body channels like $\bar{K}N$ and $\pi\Sigma$ than into baryon–two-meson states like $\pi+\pi+\Lambda^0$. We reiterate again here Remark 3 of the introduction, namely, if the mass regularities represented by the mass formulas owe their existence solely to the forces between quarks rather

[33] It is amusing to note that according to R. K. Böck *et al.* [Phys. Letters **17**, 166 (1965)], some bubble-chamber data at CERN indicate a $Y_1^*(1942)$. This is in remarkable agreement with the position predicted for Y_1^* of solutions (a) and (b). The experimental state may, however, be a Regge recurrence $(J=\frac{5}{2}^+)$ of the $\Sigma(1190)$. See R. L. Cool, *et al.*, Phys. Rev. Letters **16**, 1228 (1966).

[34] Indeed, there is evidence of a $J=\frac{7}{2}^-$ $N(2190)$ [A. Yokosawa, S. Suwa, R. E. Hiu, R. J. Esterling, and N. E. Booth, Phys. Rev. Letters **16**, 714 (1966)] which may be the Regge recurrence of $N_\gamma(1512)$. We expect the Regge recurrences of the $(8,2)^-$ states to be in this same energy range $\sim$(2100 to 2300 MeV).

[35] See for instance G. Alexander, *et al.*, Phys. Rev. Letters **15**, 207 (1965); H. Harari, D. Horn, M. Kugler, H. J. Lipkin, and S. Meshkov [Phys. Rev. **140**, B431 (1965)] have suggested that such a state may have negative parity from the extreme $SU(12)$ outlook. See, however Sec. IV for an alternative interpretation.

[36] M. Goldberg, J. Leitner, R. Musto, and L. O'Raifeartaigh (to be published).

[37] Note that the η octet of Refs. 1 and 2 cannot have satisfactory decay properties even if re-assigned to the **56**$^-$. In **56**$^-\rightarrow$ **56**$^+$+**35**$^-$, we deal with S-wave decay where the appropriate coupling $\underset{\sim}{\psi}^\dagger{}_{(8)}\psi_{(8)}\phi^{(8,1)}$ in $SU(6)$ is pure F. The decays $\tilde{\Sigma}\rightarrow\Sigma+\eta$ and $\tilde{\Lambda}\rightarrow\Lambda+\eta$ are then forbidden since the couplings $\tilde{\Sigma}\Sigma\eta$ and $\tilde{\Lambda}\Lambda\eta$ are pure D even in $SU(3)$. We thank Dr. S. Pakvasa for elaborating on these points, which were first emphasized by Hendry and Moorhouse (Ref. 16).

than to long-range forces between the known hadrons, the decay properties of low-lying hadrons into other low-lying hadrons need not be a meaningful consideration[38] one way or the other.

III. 70^- WITH $L=1$

The advantage afforded by $\mathbf{70}^-$ with $L=1$ is that Eq. (2) allows us to accommodate all the negative parity baryon states between 1 and 2 BeV in one supermultiplet of 210 states. There is a natural niche for the $J=\frac{5}{2}^-$ states $N_{1/2}^*(1680)$ and $Y_1^*(1765)$ as an (8,6) $SU(3)$ octet, *without* the need to invoke either Regge recurrence or an additional supermultiplet; these states belong now to the *same* supermultiplet ($\mathbf{70}$, $L=1$) and hence their comparable masses with the other $J=\frac{1}{2}^-$ and $J=\frac{3}{2}^-$ states can be readily understood. On a more fundamental level, a single triplet of quarks qqq with $L=1$ can account for these 210 states. Despite such attractive features, an examination of the $SU(3)$ contents of Eq. (2) and the empirical data (cf. Introduction) suggests that we have no evidence yet for the additional $SU(3)$ multiplets $(8,4)^-$ and $(10,4)^-$ required in the same general mass range.

Our exploratory study will take advantage of certain known regularities present in the original $\mathbf{70}^-$ of $SU(6)$. The hope here is that the $\mathbf{70}^-$ ($L=1$) should retain at least the successful features of the original $\mathbf{70}$. These are (i) very little mixing between members of the $SU(3)$ contents of the $\mathbf{70}$ with the same quantum numbers[39] and (ii) the sum rules proposed by Freund and others,[40] namely

$$\Xi-\Sigma=\Xi_\gamma-\Sigma_\gamma,$$
$$Y_1^*+\Sigma-N-\Xi=\tilde{N}^*+N_\gamma-2\tilde{N},$$
$$\Sigma-\Lambda=4(\tilde{N}^*-\tilde{N})-2(\tilde{Y}_1^*+\tilde{\Sigma}-\Lambda'-\tilde{\Lambda}). \quad (7)$$

These sum rules are obtained by equating the mass-splitting parameters b, c, d, e, f of Bég and Singh[4,5] for the $\mathbf{56}^+$ and the $\mathbf{70}^-$. While the theoretical implication of the sum rules in terms of higher symmetry[26] is now open to question, they appear reasonably well satisfied as heuristic formulas. In addition Kuo and Radicati[41] have emphasized that in terms of simple composite models of baryon states the splitting of the $\mathbf{70}$ (and $\mathbf{20}$) is determined by the $\mathbf{56}^+$. We shall see that the mass formula (4) for $\mathbf{70}^-$ ($L=1$) is in fact compatible with both (i)[25] and (ii).

If h and k of (4) are both constant parameters, the term $kL(L+1)$ is a trivial additive scale factor for $L=1$. For convenience we shall take $k=\frac{1}{2}h$, hence

$$h\mathbf{L}\cdot\mathbf{J}+kL(L+1)=(h/2)[j(j+1)-J(J+1)]. \quad (8)$$

Take now a member (e.g. $\tilde{N}$ of $\mathbf{70}^-$) of $SU(6)$. It has $J=\frac{1}{2}$. Upon going to $L=1$ of $O(3)$ it will produce two states $\tilde{N}_{1/2}$ and $\tilde{N}_{3/2}$ with spin $j=\frac{1}{2}$ and $j=\frac{3}{2}$, respectively. We call $\tilde{N}$ the "mother" of $\tilde{N}_{1/2}$ and $\tilde{N}_{3/2}$. Similarly Σ_γ is the mother of $\Sigma_{\gamma 1/2}$, $\Sigma_{\gamma 3/2}$, $\Sigma_{\gamma 5/2}$, etc. For $j=J$, the additional terms (8) of Eq. (4) vanish; the corresponding states here are then essentially the same as those determined from the original $\mathbf{70}$ of Eq. (3). It is then readily evident that the sum rules (7) are preserved for those members of the $\mathbf{70}^-$ ($L=1$) for which $j=J$, as first emphasized by Bardakci *et al.*[40]

We must emphasize, however, that the current experimental data do not offer unambiguous support for the particular form of mass formula (4) we have chosen. For instance, it appears reasonable to determine the coefficient h of the spin-orbit term from the mass splitting between the (1,4) and (1,2) members of $\mathbf{70}$ ($L=1$). The natural assignments here are the $Y_0^*(1520)$ and the $\Lambda'(1405)$, which determine h to be of order 77 MeV. This in turn requires that the original central mass of the γ octet M_γ should now be split into

$$M_{\gamma 1/2}\approx 1545 \text{ MeV};\quad M_{\gamma 3/2}=1660 \text{ MeV};$$
$$M_{\gamma 5/2}\approx 1850 \text{ MeV}.$$

While $M_{\gamma 5/2}$ is perhaps compatible with the position of the $J=\frac{5}{2}^-$ octet, there is no evidence for an additional $j=\frac{1}{2}^-$ octet at 1545 distinct from the η octet (which belongs to $\tilde{N}_{1/2}$ in the present framework).

It appears that h should show some dependence on the various $SU(3)$ multiplets involved, at least if the currently known experimental picture is to hold. The linear $\mathbf{L}\cdot\mathbf{J}$ term is probably too simple an assumption. A somewhat different classification has been proposed by Dalitz[13] in a specific quark model; there is also evidence that the spin-orbit contribution differs from multiplet to multiplet.

IV. DISCUSSION

We have seen in Sec. II that a great deal of the mass spectrum for negative-parity baryon states of intermediate energy can be understood in terms of the $\mathbf{70}^-$ and $\mathbf{20}^-$. The existence of $\frac{5}{2}^-$ states of comparable mass *cannot* be understood on this basis,[42] suggesting that a larger supermultiplet may co-exist. There is an alternative interpretation, namely that *the* $J=\frac{5}{2}^-$ *states* $N_{1/2}^*(1680)$ *and* $Y_1^*(1765)$ *belong to an anti-decuplet* (10*, 6) *in* $SU(3)$. *The equal-spacing law then requires the remaining members to be a* $\Xi_{3/2}^*(1850)$ *with* $T=\frac{3}{2}$ *and a* K^+n *resonance in the* $T=0$ *channel at* 1595 MeV. While the

[38] Likewise, the argument of Belinfante and Cutkosky [Phys. Rev. Letters **14**, 33 (1965)], that bootstrap of meson-baryon systems in the static limit for $SU(6)$ symmetry would yield crossing matrices which are *repulsive* in the **70** representation, will not be particularly relevant to the existence (or nonexistence) of a 70-plet.

[39] The lack of mixing refers specifically to the physical states as represented by solutions (a) and (b) and discussed in some detail in Refs. 1 and 2. They *do not* refer to states of the U chain and P chain of Bég and Singh, which in fact have a mixing angle of 45° (see Ref. 5).

[40] K. Bardakci, J. M. Cornwall, P. G. O. Freund, and B. W. Lee, Phys. Letters **15**, 79 (1965). See also Ref. 2.

[41] T. K. Kuo and L. A. Radicati, Phys. Rev. **139**, B746, (1965).

[42] The situation here is reminiscent of the curious role played by the P_{11} state $N_{1/2}^*(1450)$ (cf. Ref. 13) in relation to the $\mathbf{56}^+$.

anti-decuplet represents an economical accommodation of these states, it is not clear what role it plays in the $SU(6)$ classification, since neither the **700**$^-$ nor the **1134**$^-$ contains $(10^*,6)$.

With regard to the **70**$^-$ $(L=1)$, the $SU(3)$ contents are sufficiently rich to allow accommodation of all known states. We do not yet have an adequate mass formula to interpret the results quantitatively without introducing some ambiguity.

Several other classification schemes have been suggested. Altarelli *et al.*[43] proposed that the negative-parity baryon resonances be assigned to the **20**$^-$ $(L=1)$ with content

$$(1;6)+(8+1;4)+(8+1;2). \tag{9}$$

This proposal does not allow place for the known $\frac{5}{2}^-$ resonances, nor for the S_{31} state or the possible existence of a second octet of $J=\frac{1}{2}^-$ states. Dothan *et al.*[44] have suggested that **700**$^-$ and **56**$^-$ are the next sequence of baryonic states following the **56**$^+$; the contents of these supermultiplets do not contain, however, the well-known $SU(3)$ singlet $\Lambda'(1405)$.

We have suggested that the mass regularities of $SU(3)$ and $SU(6)$ may be due to an underlying elementary-particle stuff of quarks and pseudoquarks. In particular the illustration is carried out in terms of single triplets q and q' with fractional charges. This picture need not raise conceptual difficulty about the leptons[45] with integral charges since $SU(3)$ symmetry has essentially nothing to say about these leptons with no strong interaction—other than the trivial statement that they are singlets under $SU(3)$.

The accuracy of the mass formulas gives us some semiquantitative indications about the higher mass scale M of the basic heavy triplet. It seems reasonable for, say, the baryon octet that the mass formula[46] is linear in the perturbation parameter $\lambda=m/M\ll 1$, where m can be of the order of the nucleon mass. The accuracy of the octet formula is of the order of 1%. Hence the correction term (ignored in the first-order mass formula) is $(m/M)^2\sim 1\%$ or $(m/M)\sim\frac{1}{10}$. Alternatively Gürsey *et al.*[47] have pointed out that the ratio of the Gamow-Teller-coupling constant G_A to the Fermi-coupling constant G_V can be of form

$$G_A/G_V=-1+O(m/M)=-1.15\pm0.02 \tag{10}$$

which places M in the range 7 to 10 BeV. The form (10) is suggested by the zero-nucleon-mass limit, where we are back to a "neutrino-type nucleon" with $G_A/G_V=-1$; $O(m/M)$ represents the effect of strong-interaction renormalization when the nucleon mass is switched on. Note that the same consideration does not necessarily apply to $O(m_\pi/M)$, where in the limit of zero-pion mass, G_A retains a possibly small renormalization effect and $G_A\neq -G_V$.[48]

In contrast to the above explicit estimate of quark mass, Kokkedee and Van Hove[49] have shown that the scattering of low-mass hadrons at high energy can lead to interesting consequences if the pion is regarded as composed of two units $(q\bar{q})$ and the nucleon of three units (qqq). These quasifree quarks are expected to have *rather low mass* ($\sim\frac{1}{3}$ to $\frac{1}{4}$ of the nucleon mass) on the basis of their magnetic moments, assuming that these are not abnormally large for quarks. The situation is reminiscent of the effective mass m^* of electrons in crystals compared with the free-electron mass m. For the case of high diamagnetic susceptibility such as in bismuth and gamma-brass, $m^*/m\ll 1$.

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[43] G. Altarelli, R. Gatto, L. Maiani, and G. Preparata, Phys. Rev. Letters **16**, 918 (1966).

[44] Y. Dothan, M. Gell-Mann, and Y. Ne'eman, Phys. Letters **17**, 148 (1965).

[45] The leptonic unitary group is $U_2\times U_2$; see T. D. Lee, Nuovo Cimento **35**, 945 (1965).

[46] In terms of the quark model (Ref. 30), it is unclear whether the pseudoscalar-meson mass formula should be quadratic or linear. The same question has been raised on a more empirical level by A. J. Macfarlane and R. H. Socolow, Phys. Rev. **144**, 1194 (1966). The meson spectrum has been analyzed by O. Sinanoğlu, *ibid.* **145**, 1205 (1966).

[47] F. Gürsey, T. D. Lee, and M. Nauenberg, Phys. Rev. **135**, B467 (1964). We thank Professor Lee for a helpful discussion.

[48] We thank Professor S. B. Treiman for this comment.

[49] J. J. J. Kokkedee and L. Van Hove, Nuovo Cimento **42A**, 711 (1966).