To get a feeling for their relative magnitudes, we give below the numerical values of these parameters for the case II D of Table III: $F_S=-2.95$, $G_S=-5.9$, $F_P=4.9$, $G_P=-2.9$, $F_V\approx 0$, $G_V=5.9$, (all in units of $10^{-5}/M_N^2$); and $\lambda_S=0.3$, $\lambda_P=0.73$, $\eta_S=0.02$, $\eta_P=-0.26$, $\lambda_V=\eta_V=0$. The inclusion of the mesonic poles of the VV, VS, and SS types will not modify the general form of this Hamiltonian. In particular, $\eta_V=\lambda_V=0$ will hold even then.

Summarizing, then, we have considered a simple dynamical model for the weak interaction $\Lambda p\rightarrow np$, restricting ourselves to an octet-type weak Hamiltonian satisfying the $\Delta I=\frac{1}{2}$ rule and using one-boson-exchange forces for the strong-interaction part. The model appears to be quite successful in reproducing the phenomenological deductions of BD.

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Current Algebra and ω-ϕ Mixing Parameter

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Assuming the current-generated algebra and the partially conserved axial-vector current condition for the strangeness-changing axial-vector current, an attempt is made to relate the F/D ratio parameter f with the ω-ϕ mixing angle. We conclude that the Adler-Weisberger type of sum rule obtained here gives results consistent with previous work.

1. INTRODUCTION

THE purpose of this note is to calculate the ω-ϕ mixing parameter within the framework of current-generated algebra.[1] Starting with the assumptions of equal-time commutation relation and partially-conserved-axial-vector-current (PCAC) hypothesis for the strangeness-changing axial-vector current, the use of the Fubini-Furlan-Adler-Weisberger[2] technique leads us to a simple relation between the F/D ratio parameter f and the ω-ϕ mixing angle θ. Our values of these parameters are in excellent agreement with those obtained by previous authors.[3]

2. THE SUM RULE BETWEEN f AND θ

In the quark model,[1,4] we denote the vector-current and pseudovector-current octets by $(V_b{}^a)_\mu$ and $(P_b{}^a)_\mu$, respectively.[5] The corresponding charges are defined by the integrals of the fourth component of the respective currents over the spatial coordinates, namely,

$$A_b{}^a(t)=i\int d^3x(V_b{}^a)_4\,,\qquad B_b{}^a(t)=i\int d^3x(P_b{}^a)_4\,. \tag{1}$$

Now we can write the equal-time commutation relations. Since we shall be concerned with the strangeness-changing pseudovector current, we assume the following commutation relation:

$$[B_3{}^1,B_1{}^3]=A_3{}^3-A_1{}^1=Y+Q\,. \tag{2}$$

Secondly, we shall assume the PCAC hypothesis. Following Gell-Mann and Lévy,[6] we express it in the form

$$\partial_\mu(P_1{}^3)_\mu=C\phi_{K^+}\,, \tag{3}$$

where C is a constant of proportionality and ϕ_{K^+} is the renormalized kaon field that destroys a K^+ meson. Following the method of Adler,[7] one can evaluate the

[1] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); Physics **1**, 63 (1964).
[2] S. Fubini and G. Furlan, Physics **1**, 229 (1965); S. Adler, Phys. Rev. Letters **14**, 1051 (1965); W. I. Weisberger, *ibid.* **14**, 1047 (1965).
[3] There is a lot of literature on the determination of both f and θ. Here we mention only three papers: A. W. Martin and K. C. Wali, Phys. Rev. **130**, 2455 (1963); S. Okubo, Phys. Letters **5**, 165 (1963); R. F. Dashen and D. H. Sharp, Phys. Rev. **133**, B1585 (1964). The widely accepted values are $f=0.25$ and $\theta=38°$.
[4] M. Gell-Mann, Phys. Letters **8**, 214 (1964).
[5] Our notation is similar to that used by L. K. Pandit and J. Schechter, Phys. Letters **19**, 56 (1965).
[6] M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1960).
[7] S. Adler, Phys. Rev. **137**, B1022 (1965).

constant C by taking the matrix element of Eq. (3) between single p and Λ states. A simple calculation gives the result

$$C=g_A{}^{\Lambda}(M_p+M_\Lambda)M_K{}^2/g^{Kp\Lambda}K^{Kp\Lambda}(0). \quad (4)$$

Here $K^{Kp\Lambda}(0)$ is the form factor at the $Kp\Lambda$ vertex to be evaluated at zero momentum transfer.

Hereafter, we make use of the Fubini-Furlan-Adler-Weisberger[2] method. We take the matrix element of Eq. (2) between single K^+ states and introduce intermediate states on the left-hand side,

$$\sum_n\{\langle K^+|B_3{}^1|n\rangle\langle n|B_1{}^3|K^+\rangle-(B_3{}^1\leftrightarrow B_1{}^3)\}=\langle K^+|Y+Q|K^+\rangle=2. \quad (5)$$

Here we have suppressed the momentum factors. Now, the left-hand side of Eq. (5) can be written in two parts: (i) the contribution from single-particle intermediate states and (ii) the contribution from many-particle intermediate states. In order to evaluate (i), we introduce here the concept of ω-ϕ mixing.[8] In the standard fashion, we can write

$$\omega_{\text{ob}}=\omega_1\cos\theta+\omega_8\sin\theta, \quad (6)$$
$$\phi_{\text{ob}}=-\omega_1\sin\theta+\omega_8\cos\theta,$$

where θ is the ω-ϕ mixing angle and ω_1, ω_8 belong, respectively, to the singlet and the vector-meson octet. Now, since the ω_8 part of both ω_{ob} and ϕ_{ob} can decay into $K\bar{K}$, we see that the assumption of ω-ϕ mixing, conservation of parity, and angular momentum, and G-conjugation leads to the conclusion that the single-particle intermediate states allowed are ρ^0, ϕ, and ω mesons. For $\rho K\bar{K}$ and $\omega_8 K\bar{K}$ interactions we consider the following Lagrangians:

$$L_{\rho KK}=ig_{\rho KK}\boldsymbol{\rho}_\mu\cdot\bar{K}\boldsymbol{\tau}\partial_\mu K,$$
$$L_{\omega_8 KK}=ig_{\omega_8 KK}(\omega_8)_\mu(\bar{K}\partial_\mu K-\partial_\mu\bar{K}K). \quad (7)$$

The above considerations coupled with the PCAC condition (3) give the following single-particles contribution[9] to Eq. (5):

$$\frac{(g_A{}^\Lambda)^2(M_p+M_\Lambda)^2}{(g^{Kp\Lambda})^2}\left[\frac{1}{m_\rho{}^2}g_{\rho KK}{}^2+\frac{1}{m_{\phi_{\text{ob}}}{}^2}g_{\omega_8 KK}{}^2\cos^2\theta+\frac{1}{m_{\omega_{\text{ob}}}{}^2}g_{\omega_8 KK}{}^2\sin^2\theta\right]. \quad (8)$$

In the above equation we have made a simplifying approximation, i.e.

$$\frac{[K^{\rho KK}(0)]^2}{[K^{Kp\Lambda}(0)]^2}\approx\frac{[K^{\omega_8 KK}(0)]^2}{[K^{Kp\Lambda}(0)]^2}\approx1.$$

The many-particle contribution is evaluated in exactly the same manner as by Adler and Weisberger, using the Fubini-Furlan technique.[2] Thus, finally Eq. (5) becomes

$$\frac{(g_A{}^\Lambda)^2(M_p+M_\Lambda)^2}{(g^{Kp\Lambda})^2}\left[\frac{1}{m_\rho{}^2}g_{\rho KK}{}^2+\frac{1}{m_{\phi_{\text{ob}}}{}^2}g_{\omega_8 KK}{}^2\cos^2\theta+\frac{1}{m_{\omega_{\text{ob}}}{}^2}g_{\omega_8 KK}{}^2\sin^2\theta+\mathcal{I}\right]=2, \quad (9)$$

where $\mathcal{I}$ is the integral

$$\mathcal{I}=\frac{1}{[K^{Kp\Lambda}(0)]^2}\frac{2}{\pi}\int_{2m_K}^{\infty}\frac{WdW}{(W^2-m_K{}^2)}\times\{\sigma_{K^+K^-}(W,0)-\sigma_{K^+K^+}(W,0)\}+\mathcal{I}'. \quad (10)$$

Here $\mathcal{I}'$ denotes the contribution from the unphysical region lying between the lowest possible threshold $2m_\pi$ and the physical threshold $2m_K$. Since except for the ρ meson, there are apparently no reliable indications of any large $\pi\pi$-$K\bar{K}$ interaction in this region, we have no alternative but to drop the term $\mathcal{I}'$ in Eq. (10). In order to introduce the physical scattering cross sections in Eq. (10), we make another simplifying assumption[10] similar to that made by Pandit and Schechter[5] and by Mathur and Pandit,[11] and write

$$\sigma_K{}^{l,\mathcal{I}}(W)\approx\frac{1}{[K^{Kp\Lambda}(0)]^2}\sigma_K{}^{l,\mathcal{I}}(W,0). \quad (11)$$

Firstly, we try to fix the various coupling constants. Using $SU(3)$[12] and also bootstrap results,[13] we see that $g_{\rho KK}$ is simply related to $g_{\rho\pi\pi}$,

$$g_{\rho KK}{}^2/4\pi=0.5g_{\rho\pi\pi}{}^2/4\pi. \quad (12)$$

Here $g_{\rho\pi\pi}{}^2/4\pi\sim2.2$. Again, using the Lagrangian of Eq. (8) and the experimental width of the ϕ_{obs} resonance, one gets

$$g_{\omega_8 KK}{}^2/4\pi\approx1.6/\cos^2\theta. \quad (13)$$

Secondly, introducing the F/D ratio parameter f and using the Martin-Wali[3] and Pandit-Schechter[5] values,

[8] See for example, J. J. Sakurai, Phys. Rev. Letters **9**, 472 (1962).

[9] The one-particle (e.g., ρ^0) term is of the form

$$\sum_{\text{spin}}\int\frac{d^3q}{(2\pi)^3}\langle K^+(p)|B_1{}^3(t)|\rho^0(q)\rangle\langle\rho^0(q)|B_3{}^1(t)|K^+(p')\rangle.$$

Using the proper normalizations and summing over the spins, we get the term $c^2g_{\rho KK}{}^2/m_K{}^4m_\rho{}^2$. Of course, in this process, one has to take the limit $p_0\rightarrow\infty$.

[10] Recently some work has appeared regarding the mass-shell corrections, see, e.g., B. Renner, Phys. Letters **20**, 72 (1966) and the references quoted therein. As shown in this paper, in the case of strangeness-changing transitions (involving a zero-mass kaon), the mass-shell corrections are expected not to produce much change in the results.

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[12] M. Gell-Mann, California Institute of Technology Report No. CTSL-20, 1961 (unpublished).

[13] R. H. Capps, Phys. Rev. Letters **10**, 312 (1962).

TABLE I. Corresponding values of parameters f and θ.

f	Mixing angle θ (deg) Without f'	With f'
0.1	40.7	37.5
0.2	39.6	36.1
0.25	39.5	35.9
0.3	39.8	36.3
0.4	41.3	38.1

we have

$$g^{KN\Lambda}=-\sqrt{3}^{-1}(1+2f)g^{\pi NN}$$

$$(g_A{}^{\Lambda})^2=\frac{2(1+2f)^2}{(1+2f)^2+2.79(1-2f)^2+6.18}. \quad (14)$$

Here, $(g^{\pi NN})^2/4\pi\sim 14.6$. Feeding these values into Eq. (9), we get the following expression

$$(1+2f)^2+2.79(1-2f)^2+0.13 = 4.53\tan^2\theta+7.1\mathcal{I}m_K{}^2/4\pi. \quad (15)$$

In order to make an estimate, one can see that even if $\mathcal{I}$ on the whole contributes negligibly, the above sum rule is consistent with the present accepted values[3] of f and θ.

Lastly we try to make a plausible estimate of $\mathcal{I}$ in the face of very limited experimental data. As has been mentioned by Mathur and Pandit[14] and as is physically obvious, the maximum contribution to the integral comes from low-lying resonances in the particular channel under consideration. So far, except for the ϕ meson and f' (1500 MeV) resonance,[15] there are no clearly established resonances in the $K\bar{K}$ system. However, recently Armenteros *et al.*[16] conclude some evidence for the existence of a peak in the $K\bar{K}$ charged system at about 1025 MeV, i.e. close to the $K\bar{K}$ threshold. The J^{PG} quantum numbers of this $(I=1)$ likely resonance are still to be established. Again, for K^+K^+ scattering we have no direct experimental data. Ferro-Luzzi *et al.*[17] have concluded the existence of a peak in this system at about 1280 MeV, they also obtained strong evidence of yet another distinct peak[18] in the system at about 1050 MeV. However, the last three resonances are still to be established and, consequently, we do not take these into account in our further calculations. We calculate the integral $\mathcal{I}$ by taking into account the f' resonance only. For this purpose, we use the full Breit-Wigner one-level formula and do the integration numerically. Our parametrizations of the reduced widths, etc., are similar to those used by Adler.[19] The contribution to the integral $\mathcal{I}$ from the f' resonance comes out to be ~ 5.0 (BeV)$^{-2}$. On putting in this value, Eq. (15) becomes

$$(1+2f)^2+2.79(1-2f)^2-0.56=4.53\tan^2\theta. \quad (16)$$

Thus, finally, after the aforementioned assumptions and simplifications, we get a sum rule connecting the f parameter and the mixing angle θ. In Table I, we sum up the various fits to the above expression. Here we make it clear that Eq. (16) enables us to calculate $|\tan\theta|$ only, but it is plausible to assume that θ lies in the first quadrant. Our results show that the best fit is obtained with $f=0.25$ only. These values $f=0.25$ and $\theta=36°$ are in very good agreement with those obtained by previous authors.[3] Here, it is of interest to mention that $f=0.25$ corresponds to $(g^{KN\Lambda})^2/4\pi=10.85$ and $|g_A{}^{\Lambda}|=0.7$.

3. CONCLUSION

In conclusion, we would like to remark that notwithstanding the various approximations made while calculating the various matrix elements, it is gratifying to note that the simple assumptions of equal-time commutation relation [Eq. (2)] and the PCAC hypothesis [Eq. (3)] have enabled us to arrive at a relation between the F/D ratio parameter f and the ω-ϕ mixing parameter θ. The sum rule shows that a variation in f does not produce a large variation in θ. In other words, apparently θ does not depend on f very critically. However, it is clear that the best value for θ is obtained only with $f=0.25$. So, we conclude that the Adler-Weisberger type of sum rule generated with the help of the current-generated algebra is consistent with previous work[3] regarding the ω-ϕ mixing angle.

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