Modified Analysis of Nucleon Electromagnetic Form-Factor Data*

G. L. KANE
Physics Department, University of Michigan, Ann Arbor, Michigan

AND

R. A. ZDANIS
Physics Department, Johns Hopkins University, Baltimore, Maryland

(Received 15 April 1966; revised manuscript received 1 August 1966)

We suggest that the usual procedure for analyzing nucleon electromagnetic form-factor data be modified by retaining in a suitable manner the dispersion-integral contributions present even after removal of known vector-meson contributions. This allows a more general analysis consistent with all known restrictions on the form factors. The form factors are parametrized and the parameters determined by comparison with experimental data. It is possible to obtain reasonable fits to all of the data with a total of two parameters. The resulting form factors are suitable for the extraction of useful numbers, including the residues at the known vector-meson poles. In addition, analytic continuation to the annihilation region is well-defined and allows meaningful comparison with data there. It is not necessary to make statements about the existence of vector mesons other than ρ, ω, φ. We use our results to discuss the ρ dominance of the isovector form factor and to estimate the vector-meson–nucleon coupling constants.

I. INTRODUCTION

In this paper we would like to suggest a simple modification of the usual procedure for analyzing nucleon electromagnetic form factors in terms of dispersion relations.[1] We assume for our analysis that the form factors satisfy unsubtracted dispersion relations

$$G_{E,M}^{p,n}(t)=G_{E,M}^{S}(t)\pm G_{E,M}^{V}(t)\,, \tag{1}$$

$$G_{E,M}^{S,V}(t)=\frac{1}{\pi}\int_{t_{S,V}}^{\infty}\frac{\bar{g}_{E,M}^{S,V}(t')dt'}{t'-t}\,. \tag{2}$$

Because of the coupling to the photon, the hadrons which contribute to the spectral functions $\bar{g}^{S,V}$ will be isoscalar and isovector mesons with $J^P=1^-$. As is customary, we separate out the contributions of the known vector mesons φ, ω, ρ so that

$$G_{E,M}^{S}(t)=\frac{R_{E,M}^{\omega}}{(m_\omega^2-t)}+\frac{R_{E,M}^{\varphi}}{(m_\varphi^2-t)}+\left(\frac{1}{\pi}\right)\int_{t_S}^{\infty}\frac{g_{E,M}^{S}(t')dt'}{(t'-t)}\,, \tag{3}$$

$$G_{E,M}^{V}(t)=\frac{R_{E,M}^{\rho}}{(m_\rho^2-t)}+\left(\frac{1}{\pi}\right)\int_{t_V}^{\infty}\frac{g_{E,M}^{V}(t')dt'}{(t'-t)}\,. \tag{4}$$

However, we do not neglect the effect of the $J^P=1^-$ background; rather we write

$$\begin{aligned} g_{E,M}^{S,V}(t)&=\sum_{k=0}^{N}c_k(S,V;E,M)g_k^{S,V}(t)\,,\\ &\approx\sum_{k=0}^{N}c_k(S,V;E,M)g_k(t)\,, \end{aligned} \tag{5}$$

where the $g_k^{S,V}(t)$ are functions which we choose to have certain nice properties (see below) and the c_k's are (along with the residues above) to be determined by comparison with the experimental data for the form factors. The g_k's are assumed in what follows to be the same for isoscalar and isovector as in the second part of Eq. (5), an approximation which has negligible effect on the results. We put the vector-meson poles at the physical masses. Initially, we choose $N=2$, so that there are 18 parameters on which the G's depend (12 c's and 6 residues). After we impose conditions such as the known values at $t=0$, requirements as $t\to-\infty$, and equality at $t=4M_N^2$, we have only a few parameters left; the exact counting is done for several situations below.

By following this procedure we achieve several useful results. First, the expressions for the form factors can be analytically continued to the annihilation region, $t>4\mu^2$, (except for t near the square of the mass of one of the vector mesons, in which case we would have to retain the vector mesons with widths in the spectral function). This allows a reliable correlation of scattering and annihilation data, and also permits one to examine the agreement with experiment of theoretical predictions for the annihilation region. Second, we do not have to commit ourselves to the use of conjectured vector mesons to satisfy consistency conditions on form factors,[2] nor must we assume that no other vector

* Research supported in part by the U. S. Atomic Energy Commission, the National Science Foundation, and the U. S. Air Force.

[1] A recent review of the usual procedures is given by F. Pipkin, in *Proceedings of the Oxford International Conference on Elementary Particles, 1965* (Rutherford High Energy Laboratory, Harwell, England, 1966).

[2] V. Barger and R. Carhart, Phys. Rev. **136**, 281 (1964).

mesons exist; rather, the spectral functions $g_{E,M}{}^{S,V}$ which result may or may not show the structure which would accompany additional vector mesons. Third, we can easily satisfy the conditions[2,3]

$$G_E{}^{S,V}(4M^2)=G_M{}^{S,V}(4M^2)$$

for both the real and imaginary parts of the form factors, and we need not predetermine the value of quantities such as $\lim_{t\to-\infty} G_E{}^p(t)/G_M{}^p(t)$, which Sachs has conjectured[4] is unity, but which is very difficult to make unity with 4-pole resonance models for the form factors.[2]

The form factors satisfy $G_E{}^V(0)=\frac{1}{2}$, $G_E{}^S(0)=\frac{1}{2}$, $G_M{}^V(0)=2.353$, $G_M{}^S(0)=0.440$, and we assume[1,2] that

$$\lim_{t\to-\infty}(-t)G_{E,M}{}^{S,V}(t)=C_{E,M}{}^{S,V}. \tag{6}$$

The four C's will be chosen arbitrarily but consistent with existing data.

The functions g_k are chosen as follows:

(i) At threshold ($t_0=t_S, t_V$) they reflect the $J=1$ aspect of the intermediate states by vanishing like $(t-t_0)^{3/2}$.

(ii)
$$\int_{t_0}^{\infty}\frac{g_k(t')dt'}{(t'-t)}\xrightarrow[t\to-\infty]{}\frac{\text{constant}}{t^{k+1}}. \tag{7}$$

Thus the c_k's can be determined because each multiplies a function which decreases with a different power of t for large negative t. The results for the residues are then hardly correlated with those for c_k's.

(iii) They vanish as $t\to-\infty$ sufficiently fast for the dispersion integrals to converge.

II. FITTING PROCEDURE

Explicitly, we proceeded as follows. First define h_n and α_{m+n} by

$$h_n=t^{n+1}(t-t_0)^{3/2}\frac{\exp[-(t/\tau)^{1/2}]}{\tau^{n+5/2}},$$
$$\alpha_{m+n}=\int_{t_0}^{\infty}\frac{t^m h_n(t)dt}{\tau^{m+1}}=\alpha_{n+m}, \tag{8}$$

where τ is a quantity with the dimensions of t. The choice of τ determines the stage at which the large t falloff begins, and τ is used throughout to make various quantities dimensionless. The results depend slightly on its value. Next, it is convenient to define

$$f_0(t)=h_0(t)/\alpha_0,$$
$$f_n(t)=h_n(t)-\sum_{k=0}^{n-1}x_k h_k(t),$$

and choose the x_k's so that

$$\int_{t_0}^{\infty}t^m f_n(t)dt=0 \quad \text{for} \quad 0\leqslant m<n.$$

That is,

$$\int f_0(t)dt\neq 0,\quad \int f_1(t)dt=0;\quad \int tf_1(t)dt\neq 0,\quad \text{etc.}$$

Then

$$\int_{t_0}^{\infty}\frac{f_n(t')dt'}{(t'-t)}\xrightarrow[t\to-\infty]{}\frac{\text{constant}}{t^{n+1}}.$$

Finally, the g_k's are given by

$$g_0(t)=f_0(t),$$
$$g_1(t)=f_1(t)-\left[\frac{\left(\int_{t_0}^{\infty}f_1(t)dt/t\right)}{\left(\int_{t_0}^{\infty}f_3(t)dt/t\right)}\right]f_3(t),$$
$$g_2(t)=f_2(t)-\left[\frac{\left(\int_{t_0}^{\infty}f_2(t)dt/t\right)}{\left(\int_{t_0}^{\infty}f_3(t)dt/t\right)}\right]f_3(t).$$

Thus $\int g_n(t')dt'/(t'-t)$ has the same asymptotic behavior as $\int f_n(t')dt'/(t'-t)$, while $\int g_n(t)dt/t=0$ for $n>0$, so that only g_0 contributes to the threshold values of the form factors. It is clear how to proceed for arbitrary N.

The results are not sensitive to the choice of the falloff factor in Eq. (8), and there is no particular reason for the choices we have made. Drell[5] has emphasized that this is the fastest falloff allowed if the imaginary parts of the form factors are to be bounded everywhere.

We will give results in detail for the case where

$$\tau=200m_\pi{}^2,\quad C_{E,M}{}^{S,V}=0,$$

and

$$G_E{}^{p,n}(4M^2)=G_M{}^{p,n}(4M^2)=0.$$

Some results will be quoted for other cases to show the kind of dependence on the various choices.

First, we note the meaning of some of these choices. Putting $C_{E,M}{}^{S,V}=0$ implies that

$$G_{E,M}{}^{S,V}(t)\xrightarrow[t\to-\infty]{}\text{constant}/t^2,$$

consistent with existing data.[1]

---

[3] S. Bergia and L. Brown, in *Proceedings of the International Conference on Nuclear Structure*, edited by R. Hofstadter and L. I. Schiff (Stanford University Press, Stanford, California, 1963).

[4] R. G. Sachs, Phys. Rev. Letters 12, 231 (1964).

[5] S. Drell, Stanford University Linear Accelerator Center Users Conference, 1965 (unpublished).

We must put $G_E^{p,n}(4M^2)=G_M^{p,n}(4M^2)$ to avoid a spurious singularity in our form factors.[2,3] If we only have e-p scattering data to use to determine our parameters, this gives

$$G_E^{p,n}(4M^2)=G_M^{p,n}(4M^2)=\alpha\pm\beta\,,$$

where α and β are complex constants determined by the fit we obtain, β being some error assigned to α because of the extrapolation of the imperfect fit to the data. Since the point $4M^2=t$ is far from the scattering region, the error β is large, and when we do this we find $\beta\gtrsim\alpha$, so that the results are consistent with $G_{E,M}^{p,n}(4M^2)=0$. Further, preliminary data in the annihilation region indicate an unexpectedly small cross section[6] for $p\bar{p}\rightarrow e^+e^-$, giving a limit on the magnitude of the factors near $t=4M^2$ of about 0.06, and some higher symmetry theories[7] suggest that the form factors will vanish at $t=4M^2$. If we choose to have the form factors have a definite value at $t=4M^2$, with no error, we are essentially adding a datum point in the annihilation region, and consequently we appreciably decrease the errors on all the parameters to be determined. Fortunately, the limit from the annihilation data is small enough so the fits do not change much if $G_{E,M}^{p,n}(4M^2)=0$ is actually specified. We will give our detailed results for $G_{E,M}^{p,n}(4M^2)=0$ and partial results for some other cases.

TABLE I. Results of the case (iv) fit for the c $(S,V;E,M)$.

	S,E	S,M	V,E	V,M
C_0	0.078	0.35	-0.04	$+0.361$
C_1	315	$+415$	-315	-415
C_2	6.35	8.4	-6.35	-8.4

The number of parameters to be determined by the data in each of the above cases is: (i) Equations (3)–(5) plus values at $t=0$ leaves 14 parameters; (ii) case (i) plus specifying the asymptotic behavior leaves 10 parameters; (iii) case (ii) plus requiring $G_E^{p,n}(4M^2)=G_M^{p,n}(4M^2)$ leaves 6 parameters (equality of real and imaginary parts gives four equations); and (iv) case (iii) plus $G_E^{p,n}(4M^2)=G_M^{p,n}(4M^2)=$ (known constant) leaves two parameters. In case (iv) both parameters can be determined by proton form-factor data and the results for the neutron form factors are completely determined. In case (iii) four parameters can be determined by the proton data, leaving two for the neutron data.

III. RESULTS

For case (iv) the results are[8] [choosing $G_E^{p,n}(4M^2)=G_M^{p,n}(4M^2)=0$, $\tau=200$, and $C_{E,M}^{S,V}=0$]

$$\begin{aligned} R_E^{\rho}&=15.0\,m_\pi^2\,, & R_M^{\rho}&=68.7\,m_\pi^2\,,\\ R_E^{\varphi}&=-67.4\,m_\pi^2\,, & R_M^{\varphi}&=-195.9\,m_\pi^2\,,\\ R_E^{\omega}&=52.5\,m_\pi^2\,, & R_M^{\omega}&=120.5\,m_\pi^2\,.\end{aligned}$$

The results for $c_K(S,V;E,M)$ are given in Table I. Figures 1–4 show the resulting form factors in the scattering region. We emphasize that this is a two-parameter fit to all the data; both parameters are determined by the proton data. The curves for the neutron data are completely determined. By inserting these values for $R_{E,M}^{\alpha}$ and the $c(S,V;E,M)$ into Eqs. (3)–(5), one obtains our final expressions for the form factors $G_{E,M}^{S,V}(t)$ for all t.

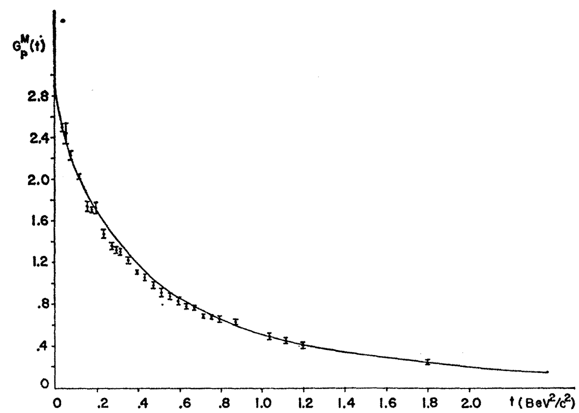

FIG. 1. Proton magnetic form-factor data and our fit [case (iv), a two-parameter fit to all the form-factor data].

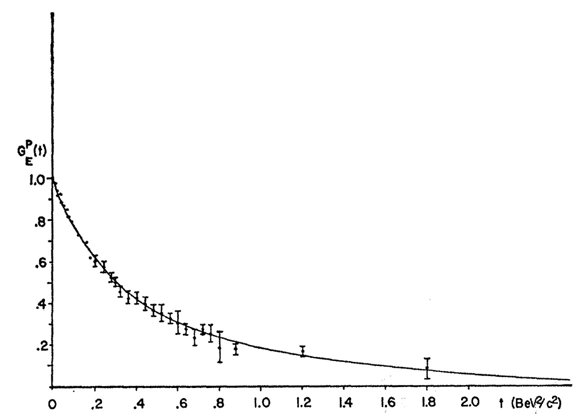

FIG. 2. Proton electric form-factor data and our fit [case (iv), a two-parameter fit to all the form-factor data].

[6] M. Conversi, T. Massam, T. Muller, and A. Zichichi, Nuovo Cimento 40, 690 (1965).

[7] A recent review of predictions of higher symmetries is given by Y. Ne'eman, Lectures given at the Pacific Summer School in Physics (to be published).

[8] We have used data from L. N. Hand, D. G. Miller, and R. Wilson, Rev. Mod. Phys. 35, 335 (1963); T. Janssens, E. B. Hughes, M. R. Yearian, and R. Hofstadter, Phys. Rev. 142, 922 (1966); K. W. Chen, J. R. Dunning, A. A. Cone, N. F. Ramsey, J. K. Walker, and Richard Wilson, *ibid.* 141, 1267 (1966); J. R. Dunning, K. W. Chen, A. A. Cone, G. Hartwig, N. F. Ramsey, J. K. Walker, and Richard Wilson, *ibid.* 141, 1286 (1966).

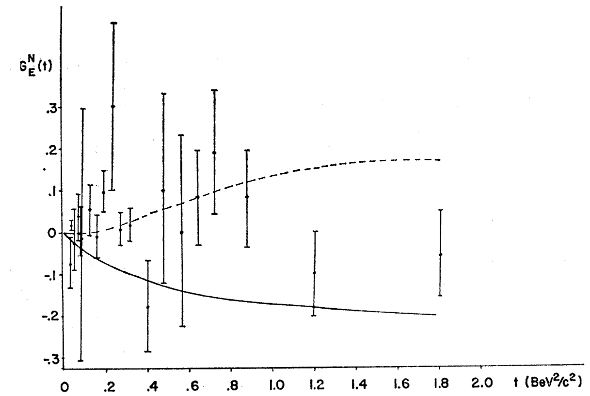

FIG. 3. Neutron electric form-factor data and our fit. The solid line is case (iv) a two-parameter fit to all the form-factor data, with both parameters determined by the proton data, so that the neutron curves are completely determined; the dashed line shows the fit obtained with case (iii), where four parameters are determined by the proton data and two by the neutron data.

The dashed curve in Fig. 4 shows the fit to the neutron data which one can get by going to case (iii) described above, where one has a total of six parameters for all of the data, four parameters being determined by proton data and two by neutron data. ($\chi^2=78$ for 71 proton points and 447 for 21 neutron points for the 6-parameter fit.)

To give some idea of the dependence of the results on the choices of τ, $C_{E,M}{}^{S,V}$, etc., we list in Table II the φ residues obtained for some other cases. In each case, the only changes in the description of case (iv) results are the specified ones. These variations in the φ residues are representative of the effects on all of the parameters of varying the quantities τ, etc., and may be taken as a measure of the errors which should be attached to the residues.

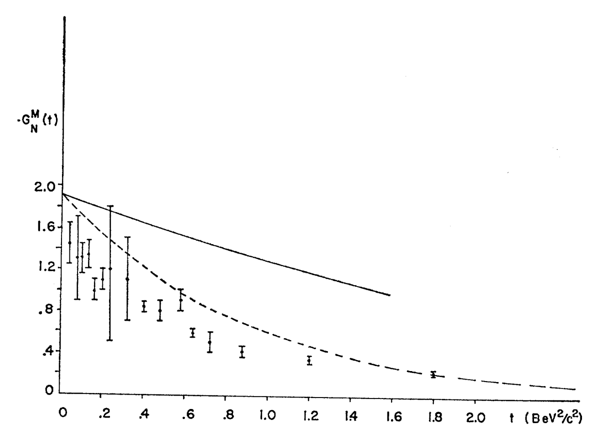

FIG. 4. Neutron magnetic form-factor data and our fit. The solid line is case (iv) a two-parameter fit to all the form-factor data, with both parameters determined by the proton data, so that the neutron curves are completely determined; the dashed line shows the fit obtained with case (iii), where four parameters are determined by the proton data and two by the neutron data.

TABLE II. Effect on the φ residues of changing some of the constants used in the analysis.

Change	Number of free parameters	$R_E{}^\varphi$	$R_M{}^\varphi$
$\tau=400\ M_\pi{}^2$	2	-56.4	-205.5
$C_M{}^p=0.1$	2	$\cdots$	-174.0
$G_E{}^p(4M^2)=0.06+0.06i$	2	-69.9	$\cdots$
$G_M{}^p(4M^2)=0.06+0.06i$	2	$\cdots$	-179.2
$G_E{}^p(4M^2)=G_M{}^p(4M^2)$ (no value specified)	2	-58.7	-248.7
No constraints at $4M^2$	6	-67.4	-195.9

Figure 5 shows $\mathrm{Im}G_M{}^p(t)$ for two case (iv) fits. Curve (a) is the result when $G_E{}^p(4M^2)=G_M{}^p(4M^2)=0.06+0.06i$ is used; curve (b) the result when $G_E{}^p(4M^2)=G_M{}^p(4M^2)=0$ is used. It is clear that we cannot have much confidence in the values obtained for the annihilation cross sections from our analysis. However, all alternatives considered give rising values for both the real and imaginary parts of the form factors for a large distance into the region beyond $4M^2$, so that we would expect the annihilation cross section to increase with energy. When data are obtained in the annihilation region it will be possible to determine the spectral functions more accurately. It should be emphasized that the results for the residues are not sensitive to these effects (see Table II).

IV. DISCUSSION

From the results for the isovector form factor we see that *at* $t=0$ essentially all of the value of $G_E{}^V(t)$ comes from the ρ pole term, none from the "background" integral, so that we have a quantitive measure of the remarkable extent to which the ρ dominates the isovector form factor. Of course, the relative sizes of the two contributions is a fairly rapidly changing function of t. We note that this result is entirely consistent with the rapid falloff of the form factors at large t.

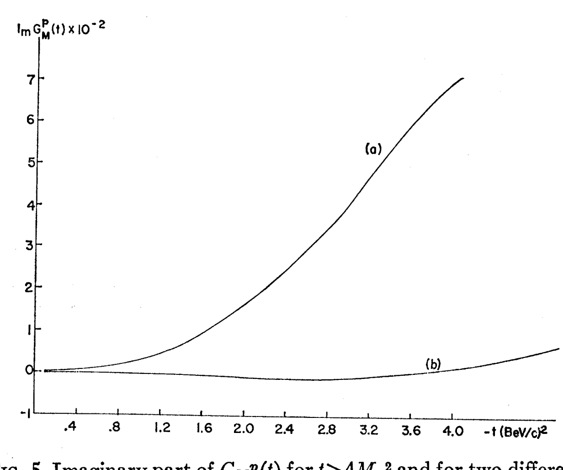

FIG. 5. Imaginary part of $G_M{}^p(t)$ for $t>4M_\pi{}^2$ and for two different choices for $G_M{}^p$ $(4M^2)$ (see discussion in text).

The scattering data and our analysis do not succeed very well in specifying the forms of spectral functions $g_{E,M}{}^{S,V}$ from the background terms. If the spectral functions had simply increased like $(t-t_0)^{3/2}$ near threshold and then fallen smoothly to zero (as we have forced them to), we could have concluded that probably not much interesting structure would manifest itself in the vector-meson spectrum in the future. It is not clear what meaning should be given to the more complicated behavior they exhibit (Fig. 5 shows typical behavior), since the effect of additional thresholds as well as that of other possible vector mesons, should be considered. Also, before taking seriously the structure of $g(t)$ one might require that it be parametrized in a more physically motivated manner (for example, as a suitable product of transition amplitudes which themselves satisfy dispersion relations).

It is perhaps to be emphasized that our procedure is in between the usual fit to a sum of pole terms and an analysis such as that of Peierls, Levinger, and Wang,[9] or Orman.[10] Such analyses, which restrict themselves to a minimum of assumptions about the form which the data should take, will hopefully eventually allow one to draw more definite conclusions about the implications of the data.

As a final use of our results, we note that *the part of the form factors which lead to the vector-meson pole terms* can be interpreted as arising from an effect such as that shown in Fig. 6. Then we can express our residues as

$$R_M{}^\alpha = g_{\gamma\alpha} m_\alpha{}^2 (f_V{}^\alpha + f_T{}^\alpha)\,,$$
$$R_E{}^\alpha = g_{\gamma\alpha} m_\alpha{}^2 (f_V{}^\alpha + m_\alpha{}^2 f_T{}^\alpha / 4m^2)\,,$$

where $g_{\gamma\alpha}$ is an effective photon–vector-meson coupling constant, defined such that at the γ-α vertex one inserts a factor

$$e g_{\gamma\alpha} m_\alpha{}^2 \quad (e^2/4\pi = 1/137)\,,$$

and $f_{V,T}{}^\alpha$ are the vector-meson–nucleon coupling constants defined by the vertex factor

$$f_V{}^\alpha \gamma_\mu + f_T{}^\alpha \sigma_{\mu\nu} q_\nu / 2m\,,$$

where $\alpha=\rho,\ \omega,\ \varphi,\ \sigma_{\mu\nu}=(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)/2i$, m is the nucleon mass, and $q=p_{\text{in}}-p_{\text{out}}$. We take the $g_{\gamma\alpha}$ from the

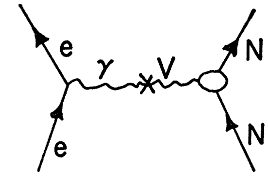

FIG. 6. Interpretation of the form factors in terms of photon–vector-meson and vector-meson–baryon coupling constants.

experiment of Zdanis *et al.*,[11]

$$g_{\gamma\rho}=0.20\,,\quad g_{\gamma\omega}=0.73\,,\quad g_{\gamma\varphi}=0.286\,,$$

and we obtain

$$f_V{}^\rho = 0.76\,,\quad f_T{}^\rho = 10.4\,,$$
$$f_V{}^\omega = 18.0\,,\quad f_T{}^\omega = 38.5\,,$$
$$f_V{}^\varphi = -0.3\,,\quad f_T{}^\varphi = -12.0\,,$$

as estimates for the vector-meson–nucleon coupling constants. Coupling to a universal isospin current would require $(2f_V{}^\rho)^2/4\pi \simeq 2$, whereas we find $(2f_v{}^\rho)^2/4\pi \simeq 1/7$. Our results would approach the former value if $R_M{}^\alpha/R_E{}^\alpha$ were decreased.

We note that our results give a sign for $(dG_E{}^n/dT)$ at $t=0$ opposite to the experimental value.[12] This seems to be inherent in our procedure, in the sense that the parametrization we have used for the form factors, combined with experimental data away from $t=0$, is quite consistent with the slope of incorrect sign. We could, of course, have required the correct result, in which case $G_E{}^n$ would have first increased from zero and then gone negative. However, this procedure changes nothing else in the fits, and it gives us no insight into the physical origin of the zero in the form factor, so we did not do so.

After we had finished most of this work we learned[13] that a similar analysis by Chilton and co-workers was in progress.

ACKNOWLEDGMENTS

We have profited from discussions with G. Feldman, M. Ross, F. Chilton, and V. Barger.

[9] J. S. Levinger and C. P. Wang, Phys. Rev. **138**, B1207 (1965); J. S. Levinger and R. F. Peierls, *ibid.* **134**, B1341 (1964).

[10] B. Orman, Phys. Rev. **138**, B1308 (1965).

[11] R. Zdanis, L. Madansky, R. W. Kraemer, S. Hertzbach, and R. Strand, Phys. Rev. Letters **14**, 721 (1965). The φ production cross section is assumed to be 50 μb and the φ-ω mixing angle is assumed to be 39°.

[12] We would like to thank J. S. Levinger for emphasizing the importance of this experimental result.

[13] F. Chilton (private communcation).