Commutation Relations of Baryon "Currents"

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The notion of baryon "currents" is introduced. We make the hypothesis [partially conserved baryon currents (PCBC)] that the divergences of these baryon currents are related to baryon fields in much the same way that the hypothesis of partially conserved axial-vector current (PCAC) relates the divergences of axial-vector meson currents to pseudoscalar meson fields. On the basis of the quark model, we construct the baryon currents as linear combinations of the products of three quarks. Using the canonical anticommutation relations for the quarks, we derive the equal-time commutation relations of the baryon currents with vector and axial-vector meson currents. The right-hand sides of these commutation relations turn out to be linear combinations of the baryon currents themselves. As applications we consider the matrix elements of the commutation relations (1) between vacuum and baryon states and (2) between meson and baryon states. The results consist of algebraic relations between various form factors, which can be checked, in principle, by accurate experimental data when available.

I. INTRODUCTION

THE introductory work of Gell-Mann[1] and of Fubini and Furlan[2] on the commutation relations of meson current components has been pursued by Adler and Weisberger[3] and also by many others[4] in the derivation of various useful sum rules. As further development along this line of investigation one could either find new applications of the same algebra of integrated currents or make refined studies of the techniques involved.

It seems to us that following another line it might be fruitful to investigate commutation relations of new baryon "currents"[5] which bear the same relationship[6] to the baryon fields as do the axial-vector currents to the pseudoscalar-meson fields. In this paper we propose to obtain these commutation relations on the basis of the quark model,[7] which underlies the parallel derivation for the meson currents. The triplet of spin-$\frac{1}{2}$ quarks is assumed to satisfy the canonical anticommutation relations. The meson currents are built out of linear combination of quarks and antiquarks. It is a natural generalization of these ideas to suppose that the baryon "currents" are built out of linear combinations of products of three quarks.

In Sec. II we review the quark model of the meson currents and present a new way of framing the hypothesis of partially conserved axial-vector currents (PCAC). Those who are familiar with the subject may prefer to omit this section, except to note Eqs. (2.10) and (2.11). In Sec. III we suggest the hypothesis of partially conserved baryon currents (PCBC) and then construct the baryon currents as linear combinations of three quarks. Some properties of these currents are examined and discussed. On the basis of the canonical anticommutation relations for the quarks, we obtain in Sec. IV the commutation relations of the baryon current, with the vector and axial-vector meson currents. As application of these commutation relations the matrix elements between vacuum and baryon states on the one hand and between pseudoscalar meson and baryon states on the other hand are studied in Sec. V. Finally the results are summarized and discussed in Sec. VI.

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II. MESON CURRENTS: PCAC

Weak and electromagnetic interactions provide strong support for the existence of, at least, vector and axial-vector currents. The matrix elements of these currents have a well-defined interpretation in terms of leptonic decays and photon couplings.

For later convenience it is useful to introduce the quarks as a triplet of spin-$\frac{1}{2}$ objects $Q^i(x)$ (altogether 12 components) considered as states of the fundamental (nonunitary) representation of the $SU(6,6)$ group.[8] In this quark model, the vector and axial-vector meson currents are obtained quite naturally as a bilinear combination of quarks and antiquarks:

$$J^Z{}_j{}^i=\bar{Q}_j(x)\gamma^Z Q^i(x)\,,\quad (i,j=1,2,3)\,, \tag{2.1}$$

where γ^Z equals γ^μ (vector) or $\gamma^\mu\gamma_5$ (axial vector). By letting γ^Z extend to all sixteen Dirac matrices [i.e., including $\mathbf{1}$ (scalar), γ_5 (pseudoscalar), and $\sigma^{\mu\nu}$ (tensor) in addition to γ^μ and $\gamma^\mu\gamma_5$], one obtains the 144 components of the quark-antiquark system. Equation (2.1) can also be written in the form

$$J_\alpha{}^Z(x)=\bar{Q}(x)\lambda_\alpha\gamma^Z Q(x)\,,\quad (\alpha=0,\cdots,8)\,. \tag{2.2}$$

The correspondence between the two forms is that for $i=j$; λ_α are the Gell-Mann matrices[1] with $\alpha=0$ [$SU(3)$ singlet], $\alpha=3$ (neutral isovector), and $\alpha=8$ (neutral isosinglet); for $i\neq j$ we take λ_α to be $(\lambda_1\pm i\lambda_2)/\sqrt{2}$, $(\lambda_4\pm i\lambda_5)/\sqrt{2}$, or $(\lambda_6\pm i\lambda_7)/\sqrt{2}$ so as to give the currents definite quantum numbers.

Let us analyze this in a language that will become useful later for the construction of baryon currents. The group $SU(6,6)$ possesses the following chain of subgroups

$$SU(6,6)\supset SU(2,2)\otimes SU(3)\supset SL(2,C)\otimes SU(3)\,, \tag{2.3}$$

where the second step involves simply the decomposition of $SU(2,2)$ with respect to $SL(2,C)$, and where $SL(2,C)$ and $SU(3)$ are the usual Lorentz group[9] and the commonly adopted approximate internal symmetry group. In $SU(6,6)$ it is easy to verify the product decomposition.

$$\mathbf{12}\otimes\overline{\mathbf{12}}=\mathbf{1}\oplus\mathbf{143}\,. \tag{2.4}$$

Further decomposition with respect to $SU(2,2)\otimes SU(3)$ goes as

$$\mathbf{1}=[1,1]\,, \tag{2.5a}$$

$$\mathbf{12}=[4,3]\,, \tag{2.5b}$$

$$\overline{\mathbf{12}}=[\bar{4},\bar{3}]\,, \tag{2.5c}$$

$$\mathbf{143}=[1,8]\oplus[15,1]\oplus[15,8]\,. \tag{2.5d}$$

[8] We shall often use the word group in this paper where, in fact, properties of the related algebras only will be relevant to our discussion. The justification of the $SU(6,6)$ attribution becomes apparent in Sec. IV. Almost everywhere, however, it can be thought simply as a convenient way of labeling indices running from 1 to 12 with some symmetry properties. See also Ref. 17.

[9] More precisely the covering of the homogeneous Lorentz group.

The first number [second number] in the square bracket refers to the dimension of the representation of $SU(2,2)$ [$SU(3)$]. The decomposition of the representations of $SU(2,2)$ in terms of the representations of $SL(2,C)$ is given by

$$\mathbf{1}=(1,1)\,, \tag{2.6a}$$

$$\mathbf{4}=(2,1)\oplus(1,2)\,, \tag{2.6b}$$

$$\bar{\mathbf{4}}=(2,1)\oplus(1,2)\,, \tag{2.6c}$$

$$\mathbf{15}=(1,1)\oplus(2,2)\oplus(2,2)\oplus(3,1)\oplus(1,3)\,. \tag{2.6d}$$

In $SL(2,C)$, the representation (1,1) corresponds to a scalar (or a pseudoscalar), (2,2) to a vector (or a pseudovector), $(3,1)\oplus(1,3)$ to an antisymmetric tensor, and $(1,2)\oplus(2,1)$ to a spinor. Other tensors are used in Sec. III. In view of Eqs. (2.4)–(2.6), it is clear that (2.1) and (2.2) exhibit the decomposition of the 144 current components with respect to $SL(2,C)\otimes SU(3)$.

A. PCAC

Aside from their direct interpretation in terms of weak leptonic decays, the axial-vector currents are indirectly related to properties of strong interactions by the generalized partially conserved axial-vector currents hypothesis[10] (PCAC). Denoting the pseudoscalar fields by ϕ_α this hypothesis states that

$$\partial_\mu J_\alpha{}^{\mu 5}(x)=v_\alpha\phi_\alpha(x)\,,\quad (\alpha=0 \text{ excluded}) \tag{2.7}$$

in the notation of (2.2). In the spirit of Cabibbo's model,[11] the constant v_α may be written as

$$v_\alpha=vm_\alpha{}^2 \tag{2.8}$$

where m_α is the mass of the particle associated with the field ϕ_α. Here v is a constant independent of α:

$$v=\frac{i\sqrt{2}M_p}{g_{\pi NN}(0)}g_A\,. \tag{2.9}$$

It depends on the mass of the proton (M_p), the weak axial coupling constant (g_A) in β decay, the pion nucleon form factor evaluated at zero momentum transfer with the normalization $g^2{}_{\pi NN}(m_\pi{}^2)/4\pi=14.7$.

Let us frame the content of (2.7) in the notation of (2.1). Calling $\phi_j{}^i(x)$ $(i,j=1,2,3)$ the nine pseudoscalar meson fields, PCAC should be written as

$$\partial_\mu J^{\mu 5}{}_j{}^i(x)=v_j{}^i{}_l{}^k\phi_k{}^l(x)\,. \tag{2.10}$$

The most general form of $v_j{}^i{}_l{}^k$ satisfying the isospin

[10] M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1960); S. L. Adler, Phys. Rev. **137**, B1022 (1965).

[11] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).

invariance is[12]

$$v_j{}^i{}_l{}^k = a\delta_l{}^i\delta_j{}^k + b\delta_j{}^i\delta_l{}^k + \sqrt{3}c\delta_j{}^i(\lambda_8)_l{}^k + \sqrt{3}d(\lambda_8)_j{}^i\delta_l{}^k + \sqrt{3}e[\delta_l{}^i(\lambda_8)_j{}^k - \tfrac{1}{3}\delta_j{}^i(\lambda_8)_l{}^k - \tfrac{1}{3}(\lambda_8)_j{}^i\delta_l{}^k] + \sqrt{3}f[(\lambda_8)_l{}^i\delta_j{}^k - \tfrac{1}{3}\delta_j{}^i(\lambda_8)_l{}^k - \tfrac{1}{3}(\lambda_8)_j{}^i\delta_l{}^k] + 3g(\lambda_8)_l{}^i(\lambda_8)_j{}^k. \quad (2.11)$$

Let us note that charge conjugation invariance[13] implies

$$v_j{}^i{}_l{}^k = v_i{}^j{}_k{}^l \quad (2.12)$$

or

$$e = f. \quad (2.13)$$

The physical interpretation of the different terms in (2.11) is as follows: a and b are the $SU(3)$-invariant terms (b separates the singlet from the octet whereas a does not); c and d are related to singlet-octet "mixing" in the sense that the divergence of the axial-vector singlet gives rise to octet pseudoscalar field (c) and vice versa (d); e and f induce an $SU(3)$ violation term which transforms as an octet without "mixing"; finally g carries a higher order violation which can be made a pure 27-plet by a suitable linear combination with the other terms.

Equations (2.8) and (2.11) agree if

$$3d = g, \quad (2.14)$$

$$a = \frac{v}{18}(8m_K{}^2 + 7m_\pi{}^2 + 3m_\eta{}^2), \quad (2.15)$$

$$e = f = \frac{v}{18}(5m_\pi{}^2 - 2m_K{}^2 - 3m_\eta{}^2), \quad (2.16)$$

$$g = \frac{v}{18}(3m_\eta{}^2 + m_\pi{}^2 - 4m_K{}^2), \quad (2.17)$$

b and c being arbitrary. It is clear from these formulas that, at the limit of an exact Gell-Mann–Okubo mass formula, g vanishes, hence the contribution from the representation 27 is absent. Also, d equals zero (no mixing); a and $e=f$ are in the ratio

$$\frac{a}{e} = \frac{2m_K{}^2 + m_\pi{}^2}{m_\pi{}^2 - m_K{}^2}. \quad (2.18)$$

B. PCTC

If tensor currents can be related to the nonet of vector mesons $\phi^\nu{}_k{}^l(x)$ in a way analogous to (2.10) and (2.11) then we have

$$\partial_\mu J^{\mu\nu}{}_j{}^i(x) = t_j{}^i{}_l{}^k\phi^\nu{}_k{}^l(x). \quad (2.19)$$

Since the nonet of vector mesons allows for singlet-octet mass mixing we expect in this case that all the terms in the expansion of $t_j{}^i{}_l{}^k$, except perhaps the one analogous to g in (2.11), are different from zero. Aside from their indirect connection to the vector meson fields through PCTC (2.19), no direct (experimental) interpretation of the tensor currents has yet been found.

III. BARYON CURRENTS: PCBC

In this section we turn our attention to the baryons and attempt to construct baryon currents with the property that their divergences are related to the baryon fields. We shall refer to this property as the PCBC (partially conserved baryon current) in the same spirit as PCAC.

A. PCBC

Let us denote the baryon currents by $B_\alpha{}^\mu{}_j{}^i(x)$ describing a nonet ($i, j = 1, 2, 3$) of vector (index $\mu = 0, \cdots, 3$) spinor (index α) currents. PCBC then implies

$$\partial_\mu B_\alpha{}^\mu{}_j{}^i(x) = V_j{}^i{}_l{}^k\psi_{\alpha k}{}^l(x), \quad (3.1)$$

where $\psi_{\alpha k}{}^l(x)$ is a nonet of baryon fields.[14] The most general form of $V_j{}^i{}_l{}^k$ satisfying the isospin invariance is

$$V_j{}^i{}_l{}^k = A\delta_l{}^i\delta_j{}^k + B\delta_j{}^i\delta_l{}^k + \sqrt{3}C\delta_j{}^i(\lambda_8)_l{}^k + \sqrt{3}D(\lambda_8)_j{}^i\delta_l{}^k + \sqrt{3}E[\delta_l{}^i(\lambda_8)_j{}^k - \tfrac{1}{3}\delta_j{}^i(\lambda_8)_l{}^k - \tfrac{1}{3}(\lambda_8)_j{}^i\delta_l{}^k] + \sqrt{3}F[(\lambda_8)_l{}^i\delta_j{}^k - \tfrac{1}{3}\delta_j{}^i(\lambda_8)_l{}^k - \tfrac{1}{3}(\lambda_8)_j{}^i\delta_l{}^k] + 3G(\lambda_8)_l{}^i(\lambda_8)_j{}^k. \quad (3.2)$$

Charge-conjugation invariance does not lead to any constraint on this equation. The physical interpretation of the various terms of (3.2) is exactly the same as given immediately after (2.13) for the case of PCAC. If here again $V_j{}^i{}_l{}^k$ is directly correlated with the mass-splitting operator, then we expect G to be small (no **27** breaking) as well as small C and D (no mixing). The remaining coefficients A, B, E, and F should be related in a simple way to the baryon masses. By analogy with (2.18) we conjecture that $V_j{}^i{}_l{}^k$ is proportional to some power of the mass operator; hence we can derive the forms

$$A = (M_\Sigma{}^n + M_\Lambda{}^n)V, \quad (3.3)$$

$$B = (2M_{Y_0}{}^n - M_\Sigma{}^n - M_\Lambda{}^n)V/6, \quad (3.4)$$

$$E = (M_\Sigma{}^n - M_N{}^n)V/3, \quad (3.5)$$

$$F = (M_\Sigma{}^n + 2M_N{}^n - 3M_\Lambda{}^n)V/6, \quad (3.6)$$

with n being an integer and V some constant of the dimension of mass to the power $(4-n)$.

B. Construction of Baryon Currents

In order to construct the baryon currents and later establish their commutation relations we shall adopt

[12] In view of the identity

$$\delta_l{}^i\delta_j{}^k - \delta_j{}^i\delta_l{}^k + \sqrt{3}\delta_j{}^i(\lambda_8)_l{}^k + \sqrt{3}(\lambda_8)_j{}^i\delta_l{}^k - \sqrt{3}\delta_l{}^i(\lambda_8)_j{}^k - \sqrt{3}(\lambda_8)_l{}^i\delta_j{}^k + 3(\lambda_8)_l{}^i(\lambda_8)_j{}^k - 3(\lambda_8)_j{}^i(\lambda_8)_l{}^k = 0,$$

$(\lambda_8)_j{}^i(\lambda_8)_l{}^k$ can be expressed as a linear combination of the seven terms of Eq. (2.11).

[13] We consider here only the first-class currents.

[14] This does not necessarily imply the existence of a low-lying ninth baryon. By a suitable choice of the values of $V_j{}^i{}_l{}^k$, the singlet and the usual octet can be completely decoupled.

the quark model. The simplest scheme in this model is to build the baryon currents out of linear combinations of products of three anticommuting quarks. We use the language that has been set forth in Sec. II.

In $SU(6,6)$ the direct product of two quarks decomposes as

$$\mathbf{12}\otimes\mathbf{12}=\mathbf{66}\oplus\mathbf{78} \qquad (3.7)$$

where the representation **66** is antisymmetric and **78** symmetric under the interchange of the two quarks. For the product of three quarks one has

$$\mathbf{12}\otimes\mathbf{12}\otimes\mathbf{12} = \mathbf{220}(\mathrm{A})\oplus\mathbf{572}(\mathrm{PS})\oplus\mathbf{572}(\mathrm{PS})\oplus\mathbf{364}(\mathrm{S}) . \qquad (3.8)$$

The symbols in the parenthesis indicate that the corresponding 3-box Young diagrams are totally antisymmetric (A), partially symmetric (PS), or totally symmetric (S).

The decomposition of these $SU(6,6)$ representations with respect to $SU(2,2)\otimes SU(3)$ is

$$\mathbf{66}=[6,6]\oplus[10,\bar{3}], \qquad (3.9a)$$

$$\mathbf{78}=[10,6]\oplus[6,\bar{3}], \qquad (3.9b)$$

$$\mathbf{220}=[\bar{4},10]\oplus[20,8]\oplus[20',1], \qquad (3.9c)$$

$$\mathbf{572}=[\bar{4},8]\oplus[20,1]\oplus[20,8]\oplus[20,10]\oplus[20',8], \qquad (3.9d)$$

$$\mathbf{364}=[4,1]\oplus[20,8]\oplus[20',10], \qquad (3.9e)$$

where we have used for $SU(3)$

$$\mathbf{3}\otimes\mathbf{3}=\bar{\mathbf{3}}\oplus\mathbf{6}, \qquad (3.10)$$

$$\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{3}=\mathbf{1}(\mathrm{A})\oplus\mathbf{8}(\mathrm{PS})\oplus\mathbf{8}(\mathrm{PS})\oplus\mathbf{10}(\mathrm{S}), \qquad (3.11)$$

and for $SU(2,2)$

$$\mathbf{4}\otimes\mathbf{4}=\mathbf{6}\oplus\mathbf{10}, \qquad (3.12)$$

$$\mathbf{4}\otimes\mathbf{4}\otimes\mathbf{4}=\bar{\mathbf{4}}(\mathrm{A})\oplus\mathbf{20}(\mathrm{PS})\oplus\mathbf{20}(\mathrm{PS})\oplus\mathbf{20'}(\mathrm{S}). \qquad (3.13)$$

In turn the representation of $SU(2,2)$ are reduced to representations of $SL(2,C)$ by (2.6) and

$$\mathbf{6}= (1,1)\oplus(1,1)\oplus(2,2), \qquad (3.14a)$$

$$\mathbf{10}= (2,2)\oplus(1,3)\oplus(3,1), \qquad (3.14b)$$

$$\mathbf{20}= (1,2)\oplus(2,1)\oplus(1,2)\oplus(2,1)\oplus(2,3)\oplus(3,2), \qquad (3.14c)$$

$$\mathbf{20'}= (2,3)\oplus(3,2)\oplus(1,4)\oplus(4,1). \qquad (3.14d)$$

The two new types of representations of $SL(2,C)$ which appear in these decompositions are $(2,3)\oplus(3,2)$ and $(1,4)\oplus(4,1)$. The former being a 12-component tensor corresponds to a 16-component tensor $t_\alpha{}^\mu$ constrained by the four conditions[15]

$$_\alpha(\gamma_\mu t^\mu)\equiv\sum_{\beta,\mu} (\gamma_\mu)_{\alpha\beta}t_\beta{}^\mu=0. \qquad (3.15)$$

The latter tensor having eight components corresponds to a 24-component antisymmetric tensor $s_\alpha{}^{\mu\nu}$

$$s_\alpha{}^{\mu\nu}=-s_\alpha{}^{\nu\mu} \qquad (3.16)$$

with the 16 conditions[15]

$$_\alpha(\gamma_\mu s^{\mu\nu})=0. \qquad (3.17)$$

Since the baryon currents are, we propose, to be constructed out of three anticommuting quarks, only the totally antisymmetric combination **220** survives. Since we also want an octet in the traceless part of $B_\alpha{}^\mu{}_j{}^i$ these components are to be found in the [20,8] part of **220** [see (3.9c)]. More precisely we choose to identify our baryon currents with just the $(2,3)\oplus(3,2)$ part of the **20** representation [see (3.14c)]. Consequently, following (3.15) we have

$$_\alpha(\gamma_\mu B^\mu{}_j{}^i)=0. \qquad (3.18)$$

Now we are in a position to proceed with the explicit construction of the baryon currents, uniquely defined up to a factor. We introduce the charge-conjugation matrix C with the usual properties

$$\gamma_\mu{}^t C=-C\gamma_\mu, \quad C^t=-C. \qquad (3.19)$$

Quarks are represented by the spinor $Q_\alpha{}^i(x)$ and we use the notation

$$_\alpha(AQ^i)\equiv A_{\alpha\beta}Q_\beta{}^i, \quad (Q^iA)_\alpha\equiv Q_\beta{}^iA_{\beta\alpha}. \qquad (3.20)$$

First the 66 antisymmetric components of the product of two quarks, as given by (3.9a) and (3.14a,b), are explicitly

$$66\begin{cases}[6,6]\begin{cases}[(1,1),6] & S^{ij}=Q^iCQ^j \quad (3.21a)\\ [(1,1),6] & P^{ij}=Q^iC\gamma_5Q^j \quad (3.21b)\\ [(2,2),6] & A_\mu{}^{ij}=Q^iC\gamma_\mu\gamma_5Q^j \quad (3.21c)\end{cases}\\ [10,\bar{3}]\begin{cases}[(2,2),\bar{3}] & V_{\mu,i}=\epsilon_{ijk}Q^jC\gamma_\mu Q^k \quad (3.21d)\\ [(3,1)\oplus(1,3),\bar{3}] & T_{\mu\nu,i}=\epsilon_{ijk}Q^jC\sigma_{\mu\nu}Q^k \quad (3.21e)\end{cases}\end{cases}$$

where the two quark fields are taken at the same space-time point. It is easily verified, using (3.19) and the anticommutation properties of the quarks, that S, P, and A are symmetrical in i and j, while factors following ϵ_{ijk} in V and T are antisymmetrical in j and k.

In order to obtain the **220** components of the products of three anticommuting quarks at the same space time point, one multiplies the above 66 components of (3.21) by $Q_\alpha{}^l(x)$. These products can be rearranged by linear combinations to yield the decomposition (3.9c), (2.6c),

[15] In fact $t_\alpha{}^\mu=(2,3)\oplus(3,2)\oplus(1,2)\oplus(2,1)$ and the conditions $(\gamma_\mu t^\mu)=0$ eliminate the superfluous spinor $(1,2)\oplus(2,1)$, while $s_\alpha{}^{\mu\nu}=(1,4)\oplus(4,1)\oplus(2,3)\oplus(3,2)\oplus(1,2)\oplus(2,1)$ and the conditions $u_\alpha{}^\nu\equiv{}_\alpha(\gamma_\mu s^{\mu\nu})=0$ eliminate a tensor $u_\alpha{}^\nu$ which evidently has the same content as $t_\alpha{}^\nu$.

(3.14c,d)

$$220\begin{cases}[(2,3)\oplus(3,2),1\oplus 8] & B_\alpha{}^\mu{}_j{}^i = V_{\nu j}{}_\alpha(D^{\mu\nu}Q^i) & (3.22a)\\ [(1,2)\oplus(2,1),8] & B_{\alpha j}{}^i = S^{ik}Q_\alpha{}^l\epsilon_{klj} & (3.22b)\\ [(1,2)\oplus(2,1),8] & B_{\alpha j}{}'^i = P^{ik}Q_\alpha{}^l\epsilon_{klj} & (3.22c)\\ [(1,4)\oplus(4,1),1] & B_\alpha{}^{\mu\nu} = T_l{}^{\rho\sigma}{}_\alpha(E_\rho{}^\mu{}_\sigma{}^\nu Q^l) & (3.22d)\\ [(1,2)\oplus(2,1),10] & B_\alpha{}^{ijk} = S^{ij}Q_\alpha{}^k + S^{jk}Q_\alpha{}^i + S^{ki}Q_\alpha{}^j. & (3.22e)\end{cases}$$

In (3.22a) the singlet and octet parts are given, respectively, by the trace and traceless parts of $B_\alpha{}^\mu{}_j{}^i$. The matrices $D^{\mu\nu}$ and $E_\rho{}^\mu{}_\sigma{}^\nu$ have to be such that $B_\alpha{}^\mu{}_j{}^i$ and $B_\alpha{}^{\mu\nu}$, respectively, obey conditions (3.15) and (3.16), (3.17). It can be shown that these matrices are

$$D^{\mu\nu} = 4g^{\mu\nu} - \gamma^\mu\gamma^\nu, \tag{3.23}$$

$$E_\rho{}^\mu{}_\sigma{}^\nu = 6(\delta_\rho{}^\mu\delta_\sigma{}^\nu - \delta_\rho{}^\nu\delta_\sigma{}^\mu) - 3(\gamma^\mu\gamma_\rho\delta_\sigma{}^\nu - \gamma^\nu\gamma_\rho\delta_\sigma{}^\mu - \gamma^\mu\gamma_\sigma\delta_\rho{}^\nu + \gamma^\nu\gamma_\sigma\delta_\rho{}^\mu) + 2\sigma^{\mu\nu}\sigma_{\rho\sigma}, \tag{3.24}$$

where we have chosen definite normalizations. We remark that the forms obtained above, (3.22), though unique up to a normalization, can be written in many different ways related through identities which follow from the completeness relation of the γ matrices:

$$\sum_{Z=1}^{16} (\gamma^Z)_{\alpha\beta}(\gamma_Z)_{\gamma\delta} = 4\delta_{\alpha\delta}\delta_{\gamma\beta}. \tag{3.25}$$

One such identity, which will be essential later in the derivation of the commutation relations, is proved in appendix A and reads

$$-(Q^iC\gamma_\nu\gamma_5Q^j)\,{}_\alpha(D^{\mu\nu}Q^k) = (Q^iC\gamma_\nu Q^k)\,{}_\alpha(D^{\mu\nu}\gamma_5Q^j) + (Q^jC\gamma_\nu Q^k)\,{}_\alpha(D^{\mu\nu}\gamma_5Q^i). \tag{3.26}$$

In concluding this section we discuss briefly the various components of (3.22). The component (3.22a) satisfies all our *a priori* requirements for the nonet of baryon currents and will be studied further in the next sections. It is not evident to us what obvious use can be made of the other components of **220**. This situation is already met in the case of meson currents where no interesting or fruitful interpretation of the scalar and pseudoscalar currents has yet been proposed. Since the decuplet (N^*, Y^*, Ξ^*, Ω^-) is the best established set of baryon resonances beyond the usual octet, one may ask whether any connection with (3.22e) can be found. The situation does not appear very promising. Indeed an equation of the type

$$\partial_\mu B_\alpha{}^{ijk}(x) = U_l{}^i{}_m{}^j{}_n{}^k\psi_{\mu\alpha}{}^{lmn}(x) \tag{3.27}$$

is unsuitable if $\psi_{\mu\alpha}{}^{lmn}$ is to represent the decuplet of spin-$\frac{3}{2}$ fields since for example the spin would have to be aligned along the momentum (μ index part). A better connection could perhaps be of the kind

$$S^{ij}\partial_\mu Q_\alpha{}^k + S^{jk}\partial_\mu Q_\alpha{}^i + S^{ki}\partial_\mu Q_\alpha{}^j = U_l{}^i{}_m{}^j{}_n{}^k\psi_{\mu\alpha}{}^{lmn} \tag{3.28}$$

which in some way could be interpreted as "orbital excitation" of the constituent quarks. However, we do not feel very secure in proceeding along this line of thought in view of the innumerable underlying ambiguities.

IV. COMMUTATION RELATIONS

As we have already mentioned in the introduction, the commutation relations of the baryon currents are to be derived on the basis of the quark model. As usual we assume that the quarks satisfy the canonical anticommutation relations

$$\{Q_\alpha{}^i(x), Q_\beta{}^j(y)\}_{x_0=y_0} = 0 \tag{4.1}$$

$$\{Q_{\alpha i}{}^*(x), Q_\beta{}^j(y)\}_{x_0=y_0} = \delta_{\alpha\beta}\delta_i{}^j\delta^3(\mathbf{x}-\mathbf{y}). \tag{4.2}$$

From these equations it is then easy to obtain the commutation relations of the meson currents at equal time:

$$[J_\alpha{}^A(x), J_\beta{}^B(y)]_{x_0=y_0} = \delta^3(\mathbf{x}-\mathbf{y})\tfrac{1}{2}Q^*(x)\{[\lambda_\alpha,\lambda_\beta]\{\gamma^0\gamma^A,\gamma^0\gamma^B\} + \{\lambda_\alpha,\lambda_\beta\}[\gamma^0\gamma^A,\gamma^0\gamma^B]\}Q(x) \tag{4.3}$$

or

$$[J^A{}_j{}^i(x), J^B{}_l{}^k(y)]_{x_0=y_0} = \delta^3(\mathbf{x}-\mathbf{y})\{\delta_l{}^i\bar{Q}_j(x)\gamma^A\gamma^0\gamma^BQ^k(x) - \delta_j{}^k\bar{Q}_l(x)\gamma^B\gamma^0\gamma^AQ^i(x)\}. \tag{4.4}$$

These algebraic relations specify the Lie algebra of $SU(6,6)$.

Let us briefly recall the usual interpretation of some of the subalgebra of (4.4). The space integrals of the traces of the tensor components

$$S^{\mu\nu}(x_0) = \int \bar{Q}_i(x)\gamma^0\sigma^{\mu\nu}Q^i(x)d^3x \tag{4.5}$$

are related to the spin part[16] of the generators $M^{\mu\nu}$ of the homogeneous Lorentz group and satisfy the algebra[17] of $SL(2C)$, which is a subalgebra of $SU(6,6)$. On the other hand, the space integrals

$$V_j{}^i(t) = \int \bar{Q}_j(x)\gamma^0Q^i(x)d^3x, \tag{4.6}$$

$$A_j{}^i(t) = \int \bar{Q}_j(x)\gamma^0\gamma_5Q^i(x)d^3x \tag{4.7}$$

[16] See, for example, N. N. Bogoliubov and D. V. Shirkov, *Introduction to the Theory of Quantized Fields* (Interscience Publishers, Inc., New York, 1959), p. 25.

[17] We remark that since we deal with algebra and allow arbitrary combinations of the generators with complex coefficients depending, for example, on the definitions of the 16 γ matrices the distinctions between $SU(6,6)$ and $SU(12)$, between $SU(2)\otimes SU(2)$ and $SL(2,C)$, or between $SU(3)\otimes SU(3)$ and $SL(3,C)$ are not crucial here. We have chosen to refer to (4.3) and (4.4) as the algebra of $SU(6,6)$ since we believe that the interpretation of (4.6) as the elements of the algebra of the spin part of the Lorentz group is quite natural.

generate the familiar chiral algebra[17] $SU(3)\otimes SU(3)$ in the form

$$[V_j{}^i(t),V_l{}^k(t)]=\delta_l{}^iV_j{}^k(t)-\delta_j{}^kV_l{}^i(t)\,, \quad (4.8)$$

$$[V_j{}^i(t),A_l{}^k(t)]=\delta_l{}^iA_j{}^k(t)-\delta_j{}^kA_l{}^i(t)\,, \quad (4.9)$$

$$[A_j{}^i(t),A_l{}^k(t)]=\delta_l{}^iV_j{}^k(t)-\delta_j{}^kV_l{}^i(t)\,. \quad (4.10)$$

We proceed now to the derivation of the commutation relations of the baryon currents. We discuss separately the commutators $[J^0{}_j{}^i,B_\alpha{}^\mu{}_l{}^k]$ and the anticommutators $\{B_\alpha{}^\mu{}_j{}^i,\bar{B}_\beta{}^\nu{}_l{}^k\}$.

A. Commutation Relations between Baryon and Meson Currents

As we have already seen in the preceding section, the proper trilinear combination of three quarks that gives the baryon current is the one in (3.22a):

$$B_\alpha{}^\mu{}_j{}^i(x)=\epsilon_{jkl}Q^k(x)C\gamma_\nu Q^l(x)\;{}_\alpha(D^{\mu\nu}Q^i(x)) \quad (4.11)$$

where $D^{\mu\nu}$ is defined in (3.23). We consider in this subsection only the commutation relations of this current with the vector and axial-vector meson currents. More specifically, our discussion is confined to the time components of these currents only:

$$J^0{}_j{}^i(x)\equiv\bar{Q}_j(x)\gamma^0Q^i(x)\,, \quad (4.12a)$$

$$J_5{}^0{}_j{}^i(x)\equiv\bar{Q}_j(x)\gamma^0\gamma_5Q^i(x)\,. \quad (4.12b)$$

The reason for this restriction is quite clear in view of the recent success in establishing the physical relevance of the chiral algebra.

Consider first the commutation relation between $J_5{}^0{}_j{}^i(x)$ and $B_\alpha{}^\mu{}_l{}^k(y)$ at equal time. Use of (4.1) and (4.2) yields

$$\begin{aligned}[J_5{}^0{}_j{}^i(x),B_\alpha{}^\mu{}_l{}^k(y)]_{x_0=y_0}
&=[Q_j{}^*(x)\gamma_5Q^i(x),Q^a(y)C\gamma_\nu Q^b(y)\;{}_\alpha(D^{\mu\nu}Q^k(y))]_{x_0=y_0}\epsilon_{abl}\\
&=\delta^3(\mathbf{x}-\mathbf{y})[(\delta_j{}^aQ^iC\gamma_\nu\gamma_5Q^b-\delta_j{}^bQ^aC\gamma_\nu\gamma_5Q^i)\;{}_\alpha(D^{\mu\nu}Q^k)\\
&\quad-\delta_j{}^kQ^aC\gamma_\nu Q^b\;{}_\alpha(D^{\mu\nu}\gamma_5Q^i)]\epsilon_{abl}\,. \quad (4.13)\end{aligned}$$

On account of the identity (3.26), the first two terms on the right-hand side of (4.13) may be put in the form of the last term, and we obtain, in terms of the nonet baryon currents defined in (4.11) but with the indices μ and α suppressed for the sake of clarity

$$\begin{aligned}[J_5{}^0{}_j{}^i(x),B_l{}^k(y)]_{x_0=y_0}
&=\delta^3(\mathbf{x}-\mathbf{y})\gamma_5(\delta_j{}^kB_l{}^i+\delta_l{}^iB_j{}^k-\delta_j{}^k\delta_l{}^iB_c{}^c\\
&\quad-\delta_j{}^iB_l{}^k-2\delta_j{}^kB_j{}^i+\delta_j{}^i\delta_l{}^kB_c{}^c)\,. \quad (4.14)\end{aligned}$$

This relation may be simplified if we define

$$\mathcal{A}_j{}^i(x)\equiv J_5{}^0{}_j{}^i(x)-\delta_j{}^iJ_5{}^0{}_c{}^c(x)\,, \quad (4.15)$$

$$\mathcal{B}_\alpha{}^\mu{}_l{}^k(y)\equiv B_\alpha{}^\mu{}_l{}^k(y)-\tfrac{1}{2}\delta_l{}^kB_\alpha{}^\mu{}_c{}^c(y)\,. \quad (4.16)$$

We get

$$\begin{aligned}[\mathcal{A}_j{}^i(x),\mathcal{B}_\alpha{}^\mu{}_l{}^k(y)]_{x_0=y_0}
&=\delta^3(\mathbf{x}-\mathbf{y})(\gamma_5)_{\alpha\beta}[\delta_j{}^k\mathcal{B}_\beta{}^\mu{}_l{}^i(x)+\delta_l{}^i\mathcal{B}_\beta{}^\mu{}_j{}^k(x)]\,. \quad (4.17)\end{aligned}$$

The structure of this commutation relation is not unexpected from group-theoretical considerations. We have seen that $J^0{}_j{}^i$ and $J_5{}^0{}_j{}^i$ are related, through (4.6) and (4.7), to the generators of the subalgebra[17] $SU(3)\otimes SU(3)$. Under commutation with the baryon current which is in the $(2,3)\oplus(3,2)$ representation of $SL(2,C)$, they must transform $(2,3)\oplus(3,2)$ into itself (with the additional factor of γ_5 in the case of $J_5{}^0{}_j{}^i$). Now, the right-hand side of the commutation relation, being a trilinear combination of anticommuting quarks, must be contained in the **220** representation of $SU(6,6)$. The fact that it must also be in the $(2,3)\oplus(3,2)$ representation of the $SL(2,C)$ subalgebra demands, according to (3.22), that it can only be an object that transforms just like our baryon current, i.e., no other components beside the nonet. The particular combination of the baryon currents on the right-hand side of (4.14) is, of course, unexpected. The algebra $SU(3)\otimes SU(3)$ that is generated by $J^0{}_j{}^i$ and $J_5{}^0{}_j{}^i$ then suggests that

$$\begin{aligned}[J^0{}_j{}^i(x),B_\alpha{}^\mu{}_l{}^k(y)]_{x_0=y_0}
&=\delta^3(\mathbf{x}-\mathbf{y})[\delta_l{}^iB_\alpha{}^\mu{}_j{}^k(x)-\delta_j{}^kB_\alpha{}^\mu{}_l{}^i(x)]\,, \quad (4.18)\end{aligned}$$

and this can be verified explicitly.

The normalizations of the singlet components in (4.15) and (4.16) may be altered if we adjust the coefficients a and b in the expansion of $v_j{}^i{}_l{}^k$ in (2.11) for PCAC, and the coefficients A and B in $V_j{}^i{}_l{}^k$, (3.2), for PCBC. We shall therefore not be concerned with the relative magnitudes of the singlet and octet components,[14] and consider just the commutation relations as given in (4.17) and (4.18). Their consequences will be examined in the next section.

B. Anticommutation Relations between Baryon and Antibaryon Currents

We build the antibaryon current out of three antiquarks in exactly the same way that the baryon current is constructed out of three quarks. Thus we have

$$\bar{B}_\beta{}^\nu{}_k{}^l(y)=\epsilon^{lrs}\bar{Q}_r(y)C\gamma_\rho\bar{Q}_s(y)(\bar{Q}_k(y)D^{\rho\nu})_\beta\,. \quad (4.19)$$

The anticommutation relations of this with $B_\alpha{}^\mu{}_j{}^i(x)$ can, in principle, be computed along the same line. We shall not go into an explicit calculation of this here, but present a qualitative discussion.

On account of the canonical commutation relations of the quarks, it is clear that the right-hand side of the anticommutation relation of baryon and antibaryon currents must consist of a linear combination of products of two quarks and two antiquarks, thus of two meson currents. From invariance consideration one obtains the general form

$$\begin{aligned}\{B_\alpha{}^0{}_j{}^i(x),\bar{B}_\beta{}^0{}_l{}^k(y)\}_{x_0=y_0}&=\delta^3(\mathbf{x}-\mathbf{y})\\
&\times\sum_{\zeta\in I}(\Gamma_{YZ}{}^0(\zeta))_{\alpha\beta}X_j{}^i{}_l{}^k{}_r{}^m{}_s{}^n(\zeta)J^Y{}_m{}^r(x)J^Z{}_n{}^s(x)\,. \quad (4.20)\end{aligned}$$

In (4.20) the summation over Y and Z has the same meaning as for (2.1) and (3.25); the superscripts 0 refer to the zeroth component of a Lorentz vector. The tensor $X_j{}^i{}_l{}^k{}_r{}^m{}_s{}^n(\zeta)$ is built out of the Kronecker $\delta_b{}^a$ only; the tensor $\Gamma_{YZ}{}^0(\zeta)$ must be a Lorentz covariant (with indices 0, Y, and Z) 4×4 matrix built out of the γ matrices, $g_{\alpha\beta}$, and $\epsilon_{\alpha\beta\gamma\delta}$. The index ζ runs on the discrete set of all possible tensors X and Γ in any combination.

From group-theoretical considerations in $SU(6,6)$ the left-hand side of (4.20) belongs to

$$220\otimes\overline{220}=1\oplus143\oplus4212\oplus44044 \qquad (4.21)$$

while the right-hand side, being the product of two meson currents belongs to

$$(1\oplus143)\otimes(1\oplus143)=1\oplus143\oplus143\oplus1\oplus143 \oplus143\oplus4212\oplus5005\oplus\overline{5005}\oplus5940. \qquad (4.22)$$

Thus (4.20) can only contain terms transforming as 1, 143, and 4212.

The only structure which has a physical interpretation and seems to be built out of two meson currents taken at the same space-time point is the nonleptonic weak Hamiltonian. A connection could perhaps be found between the baryon-antibaryon anticommutation relations and the weak nonleptonic decays. However we shall not go into more details on this problem now.

V. MATRIX ELEMENTS

In the preceding section we have derived the commutation relations (4.17) between the baryon and the axial-vector currents. As an application we consider now the matrix elements of the commutators of the space integrals of these currents, first between vacuum and baryon states, and then between pseudoscalar-meson and baryon states. After introducing a complete set of intermediate states one obtains a set of linear equations for the matrix elements of the baryon currents. We shall assume that this complete set of intermediate states can be approximated by a few low-lying single-particle states. Using PCAC and PCBC the resulting equations can be expressed as relations between vertex form factors. The linearity in the baryon currents then implies that the overall constant in PCBC [the analogue of V in (3.3)–(3.6)] is not determined by these equations, but that the relative strengths between the various terms of (3.2) are related to the form factors.

A. Between Vacuum and Baryon States

Let us introduce the following notation for the space integrals of the time components of the currents

$$\mathcal{A}_j{}^i(t)=\int \mathcal{A}^0{}_j{}^i(x,t)d^3x, \qquad (5.1)$$

$$\mathcal{B}_{\alpha j}{}^i(t)=\int \mathcal{B}_\alpha{}^0{}_j{}^i(x,t)d^3x. \qquad (5.2)$$

Taking the matrix elements of the integrated form of (4.17) between vacuum $\langle 0|$ and the baryon states $|B_s{}^r\rangle$ one obtains

$$\langle 0|[\mathcal{A}_j{}^i(t),\mathcal{B}_{\alpha l}{}^k(t)]|B_s{}^r\rangle = (\gamma_5)_{\alpha\beta}\langle 0|[\delta_j{}^k\mathcal{B}_{\beta l}{}^i(t)+\delta_l{}^i\mathcal{B}_{\beta j}{}^k(t)]|B_s{}^r\rangle. \qquad (5.3)$$

We shall show that, if we introduce a complete set of intermediate states in the left-hand side of (5.3), the only ones which can contribute in the first term of the commutator are states with $J^P=0^-$:

$$\sum_{n\in(P)} \langle 0|\mathcal{A}_j{}^i(t)|n\rangle\langle n|\mathcal{B}_{\alpha l}{}^k(t)|B_s{}^r\rangle \qquad (5.4)$$

and in the second term states with $J^P=\tfrac{1}{2}^+$:

$$\sum_{n\in(B)} \langle 0|\mathcal{B}_{\alpha l}{}^k(t)|n\rangle\langle n|\mathcal{A}_j{}^i(t)|B_s{}^r\rangle. \qquad (5.5)$$

This restriction follows from PCAC and PCBC and from certain regularity conditions. Indeed we have

$$\langle 0|\mathcal{A}_j{}^i(t)|n\rangle=\langle 0|\int d^3x\ \mathcal{A}^0{}_j{}^i(x)|n\rangle = -\frac{1}{ip_n{}^0}\langle 0|\int d^3x\ \partial_\mu\mathcal{A}^\mu{}_j{}^i(x)|n\rangle = -\frac{1}{ip_n{}^0}v_j{}^i{}_a{}^b\langle 0|\int d^3x\ \phi_b{}^a(x)|n\rangle, \qquad (5.6)$$

where we have assumed that the integrals of the space derivatives (3-dimensional surface terms) vanish and where (2.10) has been used. It is then clear, in the absence of zero-mass hadrons, that the pseudoscalar meson states dominate:

$$\langle 0|\mathcal{A}_j{}^i(t)|P_b{}^a(p)\rangle=i\left(\frac{2\pi}{p^0}\right)^{3/2}\frac{1}{\sqrt{2}}e^{-ip^0t}v_j{}^i{}_a{}^b\delta^3(\mathbf{p}). \qquad (5.7)$$

We have adopted here the convention that $\phi_b{}^a$ annihilates the state $|P_b{}^a\rangle$. A similar argument holds for the baryon currents

$$\langle 0|\mathcal{B}_{\alpha l}{}^k(t)|B_d{}^c(P)\rangle =i(M_d{}^c)^{1/2}\left(\frac{2\pi}{P^0}\right)^{3/2}e^{-iP^0t}V_l{}^k{}_c{}^d u_d{}^c(P)\delta^3(\mathbf{P}). \qquad (5.8)$$

We shall attempt, as far as possible, to use capital letters (lower case) when referring to the baryons (mesons).

The remaining matrix elements that have yet to be evaluated are $\langle P_b{}^a|\mathcal{B}_{\alpha l}{}^k|B_s{}^r\rangle$ and $\langle B_d{}^c|\mathcal{A}_j{}^i|B_s{}^r\rangle$. Proceeding as before, we obtain

$$\langle P_b{}^a(p)|\mathcal{B}_{\alpha l}{}^k(t)|B_s{}^r(P)\rangle$$
$$=\sum_{cd}(2\pi)^3\delta^3(\mathbf{p}-\mathbf{P})\frac{e^{i(p^0-P^0)t}}{i(p^0-P^0)}V_l{}^k{}_c{}^d\frac{1}{M_d{}^c-(\not{p}-\not{P})}\times\langle P_b{}^a(p)|\mathcal{S}_{\alpha d}{}^c(0)|B_s{}^r(P)\rangle,\quad(5.9)$$

$$\langle B_d{}^c(P_n)|\mathcal{A}_j{}^i(t)|B_s{}^r(P)\rangle$$
$$=\sum_{ab}(2\pi)^3\delta^3(\mathbf{P}_n-\mathbf{P})\frac{e^{i(P_n{}^0-P^0)t}}{i(P_n{}^0-P^0)}v_j{}^i{}_a{}^b\frac{1}{m_b{}^{a2}-(P_n-P)^2}\times\langle B_d{}^c(P_n)|\mathcal{J}_b{}^a(0)|B_s{}^r(P)\rangle\quad(5.10)$$

where we have used the notation $\not{p}\equiv\gamma^\mu p_\mu$ and the definitions of the current sources

$$(i\gamma^\mu\partial_\mu+M_d{}^c)\psi_d{}^c(x)=\mathcal{S}_d{}^c(x),\quad(5.11)$$

$$(\partial_\mu\partial^\mu+m_b{}^{a2})\phi_b{}^a(x)=\mathcal{J}_b{}^a(x).\quad(5.12)$$

The most general Lorentz covariant forms of the matrix elements (5.9) and (5.10) of these current sources $\mathcal{S}(x)$ and $\mathcal{J}(x)$, can be found in Appendix B where we have listed all useful matrix elements of that type in terms of their form factors. Collecting all the terms, realizing that the contribution from (5.5) vanishes, and dividing out by common factors, we obtain

$$\sum_{abcd}\Delta_b{}^a{}_d{}^c{}_s{}^r v_j{}^i{}_a{}^b V_l{}^k{}_c{}^d[{}^1G_b{}^a{}_d{}^c{}_r{}^s+(M_s{}^r-m_b{}^a)\,{}^2G_b{}^a{}_d{}^c{}_r{}^s]=i[\delta_j{}^k V_l{}^i{}_r{}^s+\delta_l{}^i V_j{}^k{}_v{}^s],\quad(5.13)$$

where Δ is a known function of masses:

$$(\Delta_b{}^a{}_d{}^c{}_s{}^r)^{-1}=2m_b{}^{a2}(m_b{}^a-M_s{}^r)\times(M_d{}^c+m_b{}^a-M_s{}^r)/M_s{}^r.\quad(5.14)$$

The baryon-meson form factors ${}^1G$ and ${}^2G$ of $\langle P_b{}^a|\mathcal{S}_d{}^c|B_s{}^r\rangle$ are evaluated for the meson $P_b{}^a$ and the baryon $B_s{}^r$ on their mass shell and the momentum transfer squared at $(M_s{}^r-m_b{}^a)^2$. In order to understand the physical content of (5.13) let us study in detail a special case of the "pion-nucleon" commutator:

$$\pi^+:\quad\binom{i}{j}=\binom{1}{2},\quad(5.15\text{a})$$

$$n:\quad\binom{k}{l}=\binom{2}{3},\quad(5.15\text{b})$$

$$p:\quad\binom{r}{s}=\binom{1}{3}.\quad(5.15\text{c})$$

PCAC (2.11) and PCBC (3.2) then imply $a=1, b=2$ and $c=2, d=3$, respectively. Isospin invariance [or (3.2)] requires the equality of the following two relevant coefficients

$$V_3{}^2{}_2{}^3=V_3{}^1{}_1{}^3.\quad(5.16)$$

Using the value of v given by (2.8) and (2.9), (5.13) becomes in this case

$$\frac{M_p{}^2}{m_\pi(m_\pi-M_p)}\frac{g_A}{g_{\pi NN}(0)}[{}^1G_{\pi NN}+(M_p-m_\pi)\,{}^2G_{\pi NN}]=1,\quad(5.17)$$

where as usual we have defined

$${}^iG_{\pi NN}=\frac{1}{\sqrt{2}}\,{}^iG_{\pi^+np}=\frac{1}{\sqrt{2}}\,{}^iG_2{}^1{}_3{}^2{}_1{}^3.\quad(i=1,2).\quad(5.18)$$

Recall that in ${}^iG_{\pi NN}$ one of the nucleons is out of its mass shell and carries a momentum transfer squared equal to $(M_p-m_\pi)^2$.

Equation (5.17), as a special case of (5.13), represents our first result as a constraint on the form factors of the meson baryon vertex. Another relation between g and ${}^iG$ can be obtained directly on the basis that the form factors of (B3) and (B6) are analytic continuations of each other. With all the invariant variables taken at their mass-shell values, one gets

$$g_b{}^a{}_d{}^c{}_e{}^f={}^1G_d{}^c{}_b{}^a{}_e{}^f+{}^2G_d{}^c{}_b{}^a{}_e{}^f M_b{}^a\quad(5.19)$$

where the $SU(3)$ indices (a,b), (c,d) and (e,f) specify the components of the baryons, pseudoscalar mesons and baryons, respectively. For the pion-nucleon case one has then

$$g_{\pi NN}={}^1G_{\pi NN}+{}^2G_{\pi NN}M_p.\quad(5.20)$$

With the help of (5.17) and (5.20), ${}^iG$ can be expressed in terms of g provided we assume that the form factors do not vary significantly between the points where the two equations are evaluated.

We can make a similar analysis for the matrix elements of the integrated form of (4.17) between the vacuum and the spin-$\frac{3}{2}$ decuplet of baryon resonances. In this case it is obvious that the right-hand side vanishes. As before the second term of the commutator does not contribute. Computing the first term we obtain

$$v_j{}^i{}_a{}^b V_l{}^k{}_c{}^d(m_b{}^a)^{-2}\times[{}^1H_b{}^a{}_d{}^c{}_{rst}-{}^2H_b{}^a{}_d{}^c{}_{rst}(m_b{}^a-M^{rst})]=0.\quad(5.21)$$

The form factors ${}^iH$ are evaluated for the pseudoscalar meson and the decuplet on their mass shells and the spin-$\frac{1}{2}$ baryon at $(M^{rst}-m_b{}^a)^2$.

Again, as before, on the mass shells the following relations hold

$$h^*{}_g{}^f{}_j{}^i{}_{abc}={}^1H_j{}^i{}_g{}^f{}_{abc}+M_g{}^f\,{}^2H_j{}^i{}_g{}^f{}_{abc}.\quad(5.22)$$

It is clear that these two equations (5.21) and (5.22) render a complete relationship between h and ${}^iH$ provided, again, that the form factors do not vary too much in the neighborhood of, and in the region between, the points where the equations are valid.

It would be interesting if similar equations could be obtained for other form factors, ${}^iK$ of (B7) for example. However, this requires additional hypotheses such as

PCTC and the commutation relations between the tensor and baryon currents.

B. Between Meson and Baryon States

As a second application of the commutation relations (4.17) we investigate their matrix elements between pseudoscalar meson and baryon states:

$$\langle P_u{}^t|[\mathcal{A}_j{}^i,\mathcal{B}_{\alpha l}{}^k]|B_s{}^r\rangle = \gamma_5\langle P_u{}^t|[\delta_l{}^i\mathcal{B}_{\alpha j}{}^k+\delta_j{}^k\mathcal{B}_{\alpha l}{}^i]|B_s{}^r\rangle. \quad (5.23)$$

As we have mentioned before, we shall approximate the complete set of intermediate states that are to be inserted in the left-hand sides of the equations by a few low-lying single-particle states. Here we limit our considerations to only intermediate pseudoscalar (P) and vector (V) mesons and to spin-$\frac{1}{2}$ (B) and spin-$\frac{3}{2}$ (D) baryons whenever allowed. We shall also find it convenient, following the work of Fubini and Furlan,[2] to consider the limit of infinite momentum for the states. In this limit the equations are considerably simplified since the four-momentum transfers tend to zero.

Let us compute now explicitly the various terms which contribute to the left-hand side of (5.23). Using the relevant matrix elements of $\mathcal{A}$ and $\mathcal{B}$ given in Appendix B and with the help of Appendix C for the sums over intermediate spin states, we have[18] for $\mathbf{p}=\mathbf{P}\to\infty$

$$\langle P_u{}^t(p)|\mathcal{A}_j{}^i|V_f{}^e\rangle\langle V_f{}^e|\mathcal{B}_{\alpha l}{}^k|B_s{}^r(P)\rangle = \delta^3(\mathbf{p}-\mathbf{P})e^{i(p_0-P_0)t}\frac{(2M_s{}^r)^{1/2}}{(m_b{}^a)^2M_d{}^c}v_j{}^i{}_a{}^bV_l{}^k{}_c{}^d \times\frac{1}{4(m_f{}^e)^2[(M_s{}^r)^2-(m_f{}^e)^2]}\{({}^1K+2\,{}^4K)[(m_f{}^e)^2-(M_s{}^r)^2]+2M_s{}^r\,{}^3K\}\theta_u{}^t{}_b{}^a{}_e{}^f u_s{}^r{}_\alpha(P), \quad (5.24)$$

where the $SU(3)$ indices of ${}^iK_f{}^e{}_d{}^c{}_r{}^s$ have been suppressed for brevity; also

$$\langle P_u{}^t(p)|\mathcal{B}_{\alpha l}{}^k|B_f{}^e\rangle\langle B_f{}^e|\mathcal{A}_j{}^i|B_s{}^r(P)\rangle = \delta^3(\mathbf{p}-\mathbf{P})e^{i(p_0-P_0)t}\frac{(2M_s{}^r)^{1/2}}{(m_b{}^a)^2M_d{}^c}v_j{}^i{}_a{}^bV_l{}^k{}_c{}^d \times\frac{1}{[(M_f{}^e)^2-(m_u{}^t)^2](M_f{}^e+M_s{}^r)}\,{}^1G_u{}^t{}_d{}^c{}_e{}^f \times g_f{}^e{}_b{}^a{}_r{}^s u_s{}^r{}_\alpha(P); \quad (5.25)$$

[18] For $\mathbf{p}\to\infty$ we have retained the zeroth-order terms in $|\mathbf{p}|^{-1}$; they are proportional to $u_s{}^r(P)$. The first-order corrections, which we have neglected, have two terms, one proportional to $u_s{}^r(P)$ and the other to $\gamma_0u_s{}^r(P)$. It might prove useful to consider the equations containing γ_0 as further relations on the form factors.

$$\langle P_u{}^t(p)|\mathcal{B}_{\alpha l}{}^k|D^{efg}\rangle\langle D^{efg}|\mathcal{A}_j{}^i|B_s{}^r(P)\rangle = \delta^3(\mathbf{p}-\mathbf{P})e^{i(p_0-P_0)t}\frac{(2M_s{}^r)^{1/2}}{(m_b{}^a)^2M_d{}^c}v_j{}^i{}_a{}^bV_l{}^k{}_c{}^d \times\frac{M^{efg}-M_s{}^r}{6(M^{efg})^2}\,{}^1H_u{}^t{}_d{}^c{}_{efg}h^{efg}{}_b{}^a{}_r{}^s u_s{}^r{}_\alpha(P). \quad (5.26)$$

For the right-hand side we have directly

$$\gamma_5\langle P_u{}^t(p)|[\delta_l{}^i\mathcal{B}_{\alpha j}{}^k+\delta_j{}^k\mathcal{B}_{\alpha l}{}^i]|B_s{}^r(P)\rangle = i\delta^3(\mathbf{p}-\mathbf{P})e^{i(p_0-P_0)t}\frac{(2M_s{}^r)^{1/2}}{M_d{}^c[(M_s{}^r)^2-(m_u{}^t)^2]}\,{}^1G_u{}^t{}_d{}^c{}_r{}^s \times(\delta_l{}^iV_j{}^k{}_c{}^d+\delta_j{}^kV_l{}^i{}_c{}^d)u_s{}^r(P). \quad (5.27)$$

All form factors in the above equations are evaluated for two particles on their mass shell and, since the three-momenta of the states are taken to be infinite, for the momentum transfer square at zero.

The final formula for the matrix elements (5.23) of the commutator relations is obtained by combining linearly the different terms which we have just computed in the following way

$$\text{Eq. (5.24)}-\text{Eq. (5.25)}-\text{Eq. (5.26)}+\cdots=\text{Eq. (5.27)} \quad (5.28)$$

where the dots indicate some possible contributions of higher mass intermediate states.

In order to understand the possible use of this formula let us again consider the special case of the pions and nucleons. In fact, let us investigate the particular components as given by (5.15) supplemented by

$$\binom{t}{u}=\frac{1}{\sqrt{2}}\left[\binom{1}{1}-\binom{2}{2}\right]. \quad (5.29)$$

In each term contained in (5.28) only one state contributes. Collecting them and dividing by the common factors, we have

$$3m_\rho{}^{-2}[{}^1K_{\rho np}+2\,{}^4K_{\rho np}+2M_p(m_\rho{}^2-M_p{}^2)^{-1}\,{}^3K_{\rho np}]\theta_{\pi^0\pi^+\rho} -6M_p{}^{-1}(M_p{}^2-m_\pi{}^2)^{-1}\,{}^1G_{\pi nn}g_{\pi np} -2M_{N^*}{}^{-2}(M_{N^*}-M_p)\,{}^1H_{\pi^0nN^*}h_{\pi^+pN^*}+\cdots =(12i/v)(M_p{}^2-m_\pi{}^2)^{-1}\,{}^1G_{\pi pp}. \quad (5.30)$$

In view of the results of Sec. V.A, all form factors except the ${}^iK$'s can be related to experimentally measurable quantities provided that they vary smoothly with their invariant momentum transfer. Thus this equation may be regarded as a constraint on the ${}^iK$'s, which themselves, in principle, can be measured by a detailed analysis of ρ production processes dominated by nucleon exchange. Further theoretical relations between the ${}^{i}K$'s can be obtained if one assumes, for example, PCTC and suitable commutation relations between the tensor currents and the baryon currents.

One can also investigate the matrix elements of (4.17) between vector mesons and baryon states. The equation is the analogue of (5.23) with $P_u{}^t$ replaced by $V_u{}^t$. The calculation in this case, is completely similar to what we have already considered except that, in addition, the pseudoscalar mesons contribute to the allowed intermediate states. The relevant formulas can easily be obtained by use of Appendices B and C.

VI. SUMMARY AND DISCUSSION

To summarize we have introduced the notion of baryon currents. We have based this notion on the hope that, at least approximately, the baryon field can be related to the divergence of a current. We have given this relation the status of an hypothesis under the name of partially conserved baryon current. In the quark model these currents have been built out of products of three anticommuting quarks. Their commutation relations with axial-vector and vector currents have been derived by use of the canonical commutation relations for the quarks; they are given in (4.17) and (4.18). It is important and interesting that the right-hand side of these commutation relations are linear combinations of the baryon currents themselves.

As applications we have considered two different types of matrix elements: A, between vacuum and baryon states; B, between meson and baryon states. In case A we have obtained relations between different form factors corresponding to different analytic continuations off the mass shell of the same baryon-baryon-meson vertex. These relations are valid for two particles (a,b) on their mass shells ($m_a{}^2$ and $m_b{}^2$) and the third variable evaluated at $(m_a-m_b)^2$.

The applications in case B rely especially on the approximation that only the low-lying single-particle intermediate states contribute significantly to the sum rules. Since the extent to which this approximation is good is not known, we do not regard that it is meaningful to consider these conditions as a set of simultaneous equations to be solved exactly. If detailed experimental information is available for the various form factors involved, the validity of these sum rules can be checked. If new experimental results are obtained for other higher resonances which are allowed to contribute to the intermediate states the approximations can be examined and improved, if necessary.

While exact agreement cannot be expected on account of the approximations, a rough agreement would probably indicate the meaningfulness of the notion of baryon current and the associated hypothesis of PCBC.

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APPENDIX A. THE IDENTITY (3.26)

In order to derive the identity (3.26) let us first fix our convention for the γ matrices

$$\gamma_\mu\gamma_\nu+\gamma_\nu\gamma_\mu=2g_{\mu\nu}\,,$$
$$g_{00}=+1\,,\quad g_{ii}=-1\,,\quad i=1,2,3\,. \tag{A1}$$

To form the complete set of sixteen 4×4 matrices we add

$$\gamma_5=-i\gamma_0\gamma_1\gamma_2\gamma_3\,, \tag{A2}$$

$$\sigma_{\mu\nu}=\tfrac{1}{2}(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)\,, \tag{A3}$$

$$\gamma_{\mu5}=\gamma_\mu\gamma_5\,, \tag{A4}$$

and the unit matrix. We note also

$$\gamma_\mu{}^\dagger=\gamma_0\gamma_\mu\gamma_0\,, \tag{A5}$$

$$\gamma_\mu{}^t=-C\gamma_\mu C^{-1}\,, \tag{A6}$$

$$C^t=-C\,. \tag{A7}$$

The usual completeness relation can then be written as

$$\sum_Z(\gamma^Z)_{\alpha\beta}(\gamma_Z)_{\gamma\delta}=4\delta_{\alpha\delta}\delta_{\gamma\beta} \tag{A8}$$

where

$$Z=S,V,T,\ A,\ \ P\,,$$
$$\gamma^Z=1,\gamma^\mu,\sigma^{\mu\nu},\gamma^\mu\gamma_5,\gamma_5\,, \tag{A9}$$
$$\gamma_Z=1,\gamma_\mu,\sigma_{\nu\mu},\gamma_5\gamma_\mu,\gamma_5\,.$$

We now proceed to the derivation of the identity. We multiply first (A8) by $(Q^aC\gamma_\mu)_{\alpha}$, ${}_\beta(\gamma_5Q^b)$, $Q_\delta{}^c$, and $(4g^{\mu\nu}-\gamma^\nu\gamma^\mu)_\gamma$ and obtain, remembering that two quarks anticommute,

$$(Q^aC\gamma_\mu\gamma^Z\gamma_5Q^b)(4g^{\mu\nu}-\gamma^\nu\gamma^\mu)\gamma_ZQ^c$$
$$=-4(Q^aC\gamma_\mu Q^c)(4g^{\mu\nu}-\gamma^\nu\gamma^\mu)\gamma_5Q^b\,. \tag{A10}$$

Multiplication of (A8) by $(Q^bC\gamma_\mu)_{\alpha}$, ${}_\beta(\gamma_5Q^a)$, $Q_\delta{}^c$, and $(4g^{\mu\nu}-\gamma^\nu\gamma^\mu)_\gamma$ yields

$$(Q^bC\gamma_\mu\gamma^Z\gamma_5Q^a)(4g^{\mu\nu}-\gamma^\nu\gamma^\mu)\gamma_ZQ^c$$
$$=-4(Q^bC\gamma_\mu Q^c)(4g^{\mu\nu}-\gamma^\nu\gamma^\mu)\gamma_5Q^a\,. \tag{A11}$$

We take one fourth of the sum of (A10) and (A11). The left-hand side is therefore symmetrical under the interchange of a and b. It can then be shown, with the help of the antisymmetry properties of the components (3.21d) and (3.21e), that the terms V, P, and A in the sum over Z vanish. The terms S and T give the same contribution, namely,

$$\tfrac{1}{2}(Q^aC\gamma_\mu\gamma_5Q^b)(4g^{\mu\nu}-\gamma^\nu\gamma^\mu)Q^c.$$

Equating the left- and the right-hand sides leads to the identity

$$(Q^aC\gamma_\mu\gamma_5Q^b)(4g^{\mu\nu}-\gamma^\nu\gamma^\mu)Q^c$$
$$=-(Q^aC\gamma_\mu Q^c)(4g^{\mu\nu}-\gamma^\nu\gamma^\mu)\gamma_5Q^b$$
$$-(Q^bC\gamma_\mu Q^c)(4g^{\mu\nu}-\gamma^\nu\gamma^\mu)\gamma_5Q^a\,. \tag{A12}$$

APPENDIX B. COVARIANT FORMS OF MATRIX ELEMENTS

In this appendix we list the matrix elements of the current sources $\mathcal{J}$ and $\mathcal{S}$ which are useful for applications to the commutation relations. All the momentum transfer variables, s_μ or S_μ, are defined as the momentum of the bra state minus the momentum of the ket state

$$\langle P_b{}^a(p)|\,\mathcal{J}_d{}^c(0)\,|V_f{}^e(q)\rangle = n_b{}^a n_f{}^e \theta_b{}^a{}_d{}^c{}_e{}^f s_\mu \epsilon_f{}^{e\mu}\,, \tag{B1}$$

$$\langle V_b{}^a(q)|\,\mathcal{J}_d{}^c(0)\,|V_f{}^e(q')\rangle = n_b{}^a n_f{}^e \omega_b{}^a{}_d{}^c{}_e{}^f \epsilon_b{}^{a\mu} s_\mu s_\nu \epsilon_f{}^{e\nu}\,, \tag{B2}$$

$$\langle B_b{}^a(P)|\,\mathcal{J}_d{}^c(0)\,|B_f{}^e(P')\rangle = N_b{}^a N_s{}^r g_b{}^a{}_d{}^c{}_e{}^f \bar{u}_b{}^a(P)\gamma^5 u_f{}^e(P')\,, \tag{B3}$$

$$\langle D^{abc}(Q)|\,\mathcal{J}_e{}^d(0)\,|B_g{}^f(P)\rangle = N^{abc} N_g{}^f h^{abc}{}_e{}^d{}_f{}^g \bar{u}_\nu{}^{abc}(Q) s^\nu \gamma^5 u_g{}^f(P)\,, \tag{B4}$$

$$\langle D^{abc}(Q)|\,\mathcal{J}_e{}^d(0)\,|D^{fgh}(Q')\rangle = N^{abc} N^{fgh} [{}^1f_f{}^a{}_g{}^b{}_h{}^c{}_e{}^d g^{\mu\nu} + {}^2f_f{}^a{}_g{}^b{}_h{}^c{}_e{}^d s^\mu s^\nu] \bar{u}_\mu{}^{abc}(Q)\gamma^5 u_\nu{}^{fgh}(Q')\,, \tag{B5}$$

$$\langle P_b{}^a(p)|\,\mathcal{S}_d{}^c(0)\,|B_f{}^e(P)\rangle = n_b{}^a N_f{}^e [{}^1G_b{}^a{}_d{}^c{}_e{}^f + {}^2G_b{}^a{}_d{}^c{}_e{}^f \mathbf{S}]\gamma^5 u_f{}^e(P)\,, \tag{B6}$$

$$\langle V_b{}^a(q)|\,\mathcal{S}_d{}^c(0)\,|B_f{}^e(P)\rangle = n_b{}^a N_f{}^e \epsilon_b{}^{a\mu} \{S_\mu[{}^1K_b{}^a{}_d{}^c{}_e{}^f + {}^2K_b{}^a{}_d{}^c{}_e{}^f \mathbf{S}] + \gamma_\mu[{}^3K_b{}^a{}_d{}^c{}_e{}^f + {}^4K_b{}^a{}_d{}^c{}_e{}^f \mathbf{S}]\} u_f{}^e(P)\,, \tag{B7}$$

$$\langle P_b{}^a(p)|\,\mathcal{S}_d{}^c(0)\,|D^{efg}(Q)\rangle = n_b{}^a N^{efg} [{}^1H_b{}^a{}_d{}^c{}_{efg} + {}^2H_b{}^a{}_d{}^c{}_{efg} \mathbf{S}]\gamma^5 S^\mu u_\mu{}^{efg}(Q)\,, \tag{B8}$$

$$\begin{aligned}\langle V_b{}^a(q)|\,\mathcal{S}_d{}^c(0)\,|D^{efg}(Q)\rangle = n_b{}^a N^{efg} \epsilon_b{}^{a\mu} \{&[{}^1L_b{}^a{}_d{}^c{}_{efg} + {}^2L_b{}^a{}_d{}^c{}_{efg} \mathbf{S}] s_\mu s^\nu \\ &+ [{}^3L_b{}^a{}_d{}^c{}_{efg} + {}^4L_b{}^a{}_d{}^c{}_{efg} \mathbf{S}]\gamma_\mu S^\nu + [{}^5L_b{}^a{}_d{}^c{}_{efg} + {}^6L_b{}^a{}_d{}^c{}_{efg} \mathbf{S}]\delta_\mu{}^\nu\} u_\nu{}^{efg}(Q)\,.\end{aligned} \tag{B9}$$

The states labeled by $P(p)$, $V(q)$, $B(P)$, and $D(Q)$ correspond, respectively, to pseudoscalar, vector, spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ (decuplet) particles with momenta p, q, P, and Q. The normalization factors n for the mesons and N for the baryons are

$$n = (2\pi)^{-3/2}(2p^0)^{-1/2}\,, \tag{B10}$$

$$N = (2\pi)^{-3/2}(M/P^0)^{1/2}. \tag{B11}$$

The form factors $[\theta, \omega, f, g, h, G, H, K, L]$ depend on the invariants, namely the masses of the initial and final states and on the square of the momentum transfer (s^2 or S^2). The SU_3 indices on the form factors are arranged with a convention such that if these matrix elements are invariant under SU_3 these form factors may be expanded in tensors built only with $\delta_b{}^a$, ϵ^{abc}, and ϵ_{abc}.

APPENDIX C. SUMS OVER INTERMEDIATE SPIN STATES

In this paper the states are normalized according to

$$\langle k|k'\rangle = \delta^3(\mathbf{k} - \mathbf{k}')\,. \tag{C1}$$

The Dirac equation is given by

$$(\boldsymbol{k} - m)u(k) = 0 \tag{C2}$$

with a normalization

$$\bar{u}(k)u(k) = 1\,. \tag{C3}$$

For spin-1 particles, the polarization vector is chosen to satisfy

$$\epsilon_\mu k^\mu = 0\,, \tag{C4}$$

$$\epsilon_\mu \epsilon^\mu = -1\,. \tag{C5}$$

For spin-$\frac{3}{2}$ particles, we write a wave function $u_\nu(k)$ which obeys

$$(\boldsymbol{k} - m)u_\nu(k) = 0\,, \tag{C6}$$

$$\gamma^\nu u_\nu(k) = 0\,, \tag{C7}$$

$$\bar{u}^\nu(k)u_\nu(k) = 1\,. \tag{C8}$$

These equations imply for the sums over spin states of vector mesons

$$\sum_{\text{spin}} \epsilon_\mu(k)\epsilon_\nu(k) = m^{-2}k_\mu k_\nu - g_{\mu\nu}\,, \tag{C9}$$

spin-$\frac{1}{2}$ spinors

$$\sum_{\text{spin}} u(k)\bar{u}(k) = (\boldsymbol{k} + m)/(2m)\,, \tag{C10}$$

and spin-$\frac{3}{2}$ spinors

$$\begin{aligned}\sum_{\text{spin}} u_\mu(k)\bar{u}_\nu(k) = [&3g_{\mu\nu} - 2m^{-2}k_\mu k_\nu - \gamma_\mu\gamma_\nu \\ &+ m^{-1}\gamma_\mu k_\nu - m^{-1}k_\mu\gamma_\nu](\boldsymbol{k} + m)/(6m)\,.\end{aligned} \tag{C11}$$