Two-Channel Calculation for the ρ Meson by the Effective-Potential Method

S. M. VAIDYA
Tata Institute of Fundamental Research, Bombay, India
(Received 17 December 1965; revised manuscript received 3 May 1966)

The width and the mass of the ρ meson are calculated in the coupled $\pi\pi$-$K\bar{K}$ problem, using an effective potential in a Schrödinger equation to unitarize the input. It is found that the coupling to the closed $K\bar{K}$ channel has a considerable narrowing effect on the ρ resonance. The calculation is free of any arbitrary parameter such as cutoffs or the subtraction points. With about 10% self-consistency we obtain $m_\rho \approx 5.5$ pion masses and the reduced width ≈ 0.36. We also include the effect of f^0 to obtain the reduced width ≈ 0.215 and mass ≈ 4.6. The width of the ρ comes out considerably smaller than in the usual claculations.

I. INTRODUCTION

MANY bootstrap calculations of the parameters of the ρ meson have been reported. Most of these calculations[1] have used the N/D method to unitarize the input. In all these calculations, the reduced width (or what amounts to the same thing, the square of the $\rho\pi\pi$ coupling constant) always comes out several times larger than the experimental value. A hope is often expressed that the inclusion of the nearby inelastic channels would greatly reduce this disagreement. Recently Balázs[2] proposed an effective-potential approach for calculations with strong interactions and this is just another device to unitarize an input by the use of a Schrödinger equation. By using this method a calculation was made to bootstrap[3] simultaneously ρ and f^0 in π-π scattering. The width of the ρ came out considerably narrower than in the usual calculations which ignore the f^0. In the lowest approximation, the method is quite simple and does not depend on any arbitrary parameters such as cutoffs or subtraction points, and would on this account seem to be more reliable. In the present paper, we shall apply the effective-potential approach to the coupled $\pi\pi$-$K\bar{K}$ problem and see the effect of the closed inelastic $K\bar{K}$ channel on the width of the ρ.

II. THE $\pi\pi$-$K\bar{K}$ SYSTEM

In the coupled $\pi\pi$-$K\bar{K}$ channels the input forces arise due to the single-particle exchanges shown in Fig. 1. In our calculation masses of the π, K, K^*, and ϕ mesons are all assumed to be different and taken from the experiment.[4] The ρ mass and $\rho\pi\pi$ coupling are determined self-consistently while all other couplings are supposed to be given in terms of $\rho\pi\pi$ coupling constant by the $SU(3)$ unitary symmetry scheme.[5] For $l=1$ coupled channel problem we have the coupled Schrödinger equations[6]

$$d^2u/dr^2+[q_1^2-V_1(r,q_1^2)-2/r^2]u=V_{12}v\,, \quad (1)$$

$$d^2v/dr^2+[q_2^2-V_2(r,q_2^2)-2/r^2]v=V_{12}u\,. \quad (2)$$

In the above equations $s^{1/2}=$ total c.m. energy, $m=m_K$, $q_1=[(s/4)-1]^{1/2}=$ momentum of pion, $q_2=[(s/4)-m^2]^{1/2}=$ momentum of kaon, $u=$ wave function for channel $1(\pi$-$\pi)$, and $v=$ the wave function for channel 2 $(K\bar{K})$.

By straightforward generalization of the procedure followed in B, we obtain for the isotopic spin $I=1$ case

$$V_1=-12\frac{\Gamma_1}{\sqrt{s}}\left(s+\frac{m_\rho^2-4}{2}\right)\frac{e^{-m_\rho r}}{r}, \quad (3)$$

$$V_2=+3\frac{\Gamma_1}{\sqrt{s}}\left(s+\frac{m_\rho^2-4m^2}{2}\right)\frac{e^{-m_\rho r}}{r} -9\Gamma_1\frac{1}{\sqrt{s}}\left(s+\frac{m_\phi^2-4m^2}{2}\right)\frac{e^{-m_\phi r}}{r}, \quad (4)$$

$$V_{12}=-3\sqrt{2}\frac{\Gamma_1}{\sqrt{s}}\{m_{K^*}^2-(m+1)^2\}\{m_{K^*}^2-(m-1)^2\} \times\left(1+\frac{2sm_{K^*}^2}{\{m_{K^*}^2-(m+1)^2\}\{m_{K^*}^2-(m-1)^2\}}\right)\frac{e^{-m_{K^*}r}}{r}. \quad (5)$$

Here Γ_1 is the reduced width of ρ (i.e., $2(q_R^3\Gamma_1/m_\rho)$ is the half-width in the q^2 variable), $q_R^2=\frac{1}{4}m_\rho^2-1$, and in deriving the above expressions for the potentials we have made a zero-width resonance approximation for the exchanged particles.

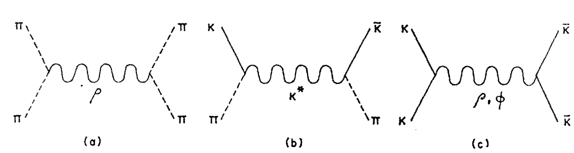

FIG. 1. Exchange diagrams in the $\pi\pi$-$K\bar{K}$ problem.

[1] For a review of bootstrap methods, see F. Zachariasen, in *Strong Interactions and High Energy Physics*, edited by R. G. Moorehouse (Plenum Press, New York, 1964); and also B. M. Udgaonkar, in *Proceedings of the Seminar on High Energy Physics and Elementary Particles, 1965* (International Atomic Energy Agency, Vienna, 1965). The latter contains an exhaustive list of references.

[2] L. A. P. Balázs, Phys. Rev. **137**, B1510 (1965). Hereafter this paper will be referred to as B.

[3] L. A. P. Balázs and S. M. Vaidya, Phys. Rev. **140**, B1025 (1965).

[4] A. H. Rosenfeld, A. Barbaro-Galtieri, Walter H. Barkas, P. L. Bastien, J. Kirz, and M. Roos, Rev. Mod. Phys. **36**, 977 (1964).

[5] R. H. Capps, Phys. Rev. Letters **10**, 312 (1963).

[6] R. G. Newton, Ann. Phys. (N. Y.) **4**, 29 (1958); R. G. Newton, J. Math. Phys. **1**, 319 (1960); J. J. de Swart and C. D. Dullemond, Ann. Phys. (N. Y.) **16**, 263 (1961).

The coupled equations (1) and (2) are solved numerically for the $l=1$ partial waves.[7] The calculation of the $\pi\pi$ scattering phase shift below the threshold for the reaction

$$\pi+\pi\rightarrow K+\bar{K}$$

is quite straightforward. One has only to remember that q_2^2 is negative in this case and that the proper behavior at infinity is that the wave function corresponding to $K\bar{K}$ decreases exponentially.[6] Since out of two coupled channels, only one channel is open, the S matrix is one dimensional. We have, however, two linearly independent regular solutions for the Schrödinger equations. By taking an appropriate linear combination to satisfy the boundary condition at infinity for the closed $K\bar{K}$ channel, we are left with only one scattering solution[6]; the phase shift is real. Having obtained the phase shift, we plot

$$\Phi(q_1^2)=2s^{-1/2}q_1^3\cot\delta\,, \tag{6}$$

where $\delta=$ the phase shift for $l=1$, and $I=1$. If we have a resonance at $q_1^2=\nu_R$, then $\Phi(\nu_R)=0$ and the reduced width Γ_1 is

$$\Gamma_1=-1/\Phi'(\nu_R)\,. \tag{7}$$

The mass m_ρ and the reduced width Γ_1 for the resonance is determined by the self-consistency requirement. The mass and the reduced width as calculated from (6) and (7) must be the same as assumed in the expressions for V_1, V_2, and V_{12} in Eqs. (3) and (5). If we take as input $m_\rho=5.5$ and $\Gamma_1=0.36$, we obtain the outputs $m_\rho=5.8$, $\Gamma_1=0.326$. These values are to be compared with the values $m_\rho=4.2$, $\Gamma_1=0.47$ obtained in the single-channel calculation.[3] The corresponding experimental values are $m_\rho=5.5$ and $\Gamma_1=0.18$. (We are taking mass $=765$ MeV and width $=110$ MeV for the ρ. See Ref. 4.)

We see that the inclusion of the closed $K\bar{K}$ inelastic channel results in a narrowing of the width of the ρ, and the mass of ρ also increases, both changes being in the right direction. The mass of ρ is surprisingly enough in excellent agreement with the experiment. Further, if we plot the $I=1$, $l=1$ partial-wave cross section $\sigma_1^1(q^2)=12\pi\sin^2\delta/q^2$, we see from Fig. 2 that it does fall off fairly rapidly above the position of the resonance. The cross section is more or less symmetric about its maximum and gives a width which is about the same as given by Eq. (7) above. It has been noted that the shape of the ρ resonance is not correctly given by the N/D method. For example, Fulco, Shaw, and Wong[8] find that the cross section is rather asymmetrical about the maximum of the resonance and does not fall off as rapidly above the position of the resonance as would seem to be suggested by experiment. Our results differ from those of Fulco, Shaw, and Wong in another respect in that they find that the $K\bar{K}$ channel does not affect the resonance appreciably.

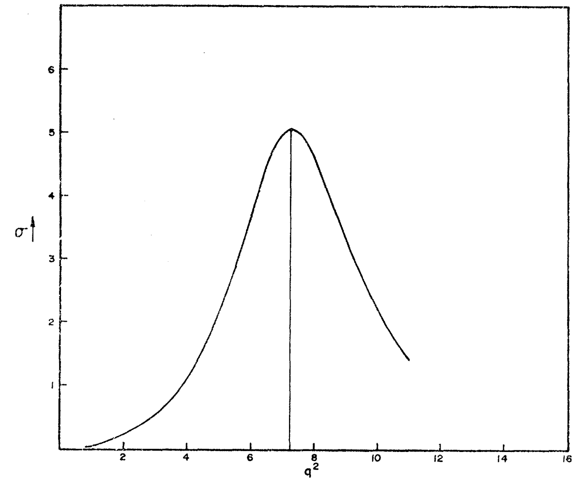

FIG. 2. The $I=1$, $l=1$ partial-wave cross section in the coupled $\pi\pi$-$K\bar{K}$ problem for $m_\rho^{\text{in}}=5.5$, $\Gamma_1^{\text{in}}=0.36$. We are using pion-mass units.

We also include the exchange of f^0 with the ρ and do a coupled $\pi\pi$-$K\bar{K}$ channel calculation for ρ, but only a one-channel ($\pi\pi$) calculation for f^0. The results are

$$m_\rho=4.6,\; m_{f^0}=8.2,\; \Gamma_\rho=0.215,\; \Gamma_{f^0}=0.0065 \text{ for the input,}$$

and

$$m_\rho=4.8,\; m_{f^0}=8.3,\; \Gamma_\rho=0.177,\; \Gamma_{f^0}=0.0055 \text{ for the output.}$$

The width of the ρ is reduced further though the mass of the ρ decreases somewhat. We have done only a single-channel calculation for f^0 and if we had done a two-channel calculation for f^0 instead it would have given extra attraction in the f^0 state, thus making possible an increase in the mass of ρ, the decrease in attraction due to a higher value of ρ mass being compensated by the extra attraction due to the inclusion of $K\bar{K}$ channel for f^0 also.

The present calculation does not depend on any arbitrary parameters, such as cutoffs or subtraction points and seems to be capable of reproducing the main features of the ρ resonance; the agreement with the reduced width is better.

ACKNOWLEDGMENTS

Sincere thanks are due to Professor B. M. Udgaonkar and Professor Virendra Singh for their encouragement. The author would like to thank Professor L. A. P. Balázs for discussions which suggested this investigation and Dr. G. Rajasekaran for a discussion about the solution of the differential equations. The numerical calculations were carried out on the CDC-3600 computer of the Institute.

[7] For the numerical solution of the coupled differential equations we use a generalization of the Cowell's method. See, e.g., P. Henrici, *Discrete Variable Methods in Ordinary Differential Equations* (John Wiley & Sons, Inc., New York, 1962).

[8] J. R. Fulco, G. L. Shaw, and D. Y. Wong, Phys. Rev. **137**, B1242 (1965).