Dynamical Model for Baryon Resonances

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We propose a nonrelativistic three-quark (Q) model of baryons and their resonances with the assumptions of (i) parastatistics which, in constrast to Fermi statistics, allows totally symmetric wave functions, and (ii) operation of Q-Q forces of the factorable type in s and p waves which facilitate an exact solution of the three-body problem. It is found that the s-wave force gives rise to a strong attraction in a spatially symmetric (S) state of $L=0$, but repulsion in a mixed-symmetric (M) state. This provides a natural dynamical realization of the **56** representation of $SU(6)$ for the familiar octet and decuplet of baryons. The same s-wave force also predicts a set of negative-parity resonances but these are of much too high energy to be of any physical consequence. The p-wave force leads to a set of negative-parity $L=1$ resonances via strong attraction in M-type spatial states of $L=1$, thus making up the representation (70,3) of $SU(6)\times O(3)$. A p-wave spin-orbit Q-Q force splits these states in a manner which fits in rather well with Dalitz's recent analysis of experimental data for the negative-parity baryonic resonances. Finally, the p-wave force generates a strong attraction in a spatially antisymemtric state (A) of even parity and $L=1$, giving rise to the representation (20,3) of $SU(6)\times O(3)$, of which the lowest states are an octet and a singlet of $J^P=\frac{1}{2}^+$, the central mass of each lying lower than the corresponding lowest mass multiplet of negative parity. This provides a natural explanation of the so-called "Roper" resonance (at 1450 MeV), and in addition, strongly predicts an even-parity singlet of a mass *lower* than the $Y_0^*(1405)$. The distinction between Fermi statistics and paraststistics is discussed in the context of the above results, and it is argued that while Fermi statistics could in principle generate negative-parity $L=1$ resonances (via M functions), it could not possibly account for the **56** of baryons, since with Q-Q forces alone,the A function of $L=0$ that should go with it has a strongly repulsive kernel.

WE present here a dynamical model of baryon resonances as three-quark states, which leads to the striking prediction of a new unitary singlet at low mass. The idea that the structure of baryons and their resonances can be understood in terms of nonrelativistic quarks (Q), is both physically interesting and mathematically tractable.[1–4] Even without any detailed dynamical assumptions about the Q-Q forces, a quark model already predicts several interesting $SU(6)$ results like the F/D ratio, the ratio of the nucleon magnetic moments, the $M1$ transition amplitudes between $N_{3/2}^*$ and p, and so on.[1–4] One however needs more detailed assumptions about the Q-Q interactions to predict, in a dynamical fashion, the group structures of the various baryon resonances, their central masses, and their modes of splitting due to spin-orbit, $SU(3)$-violating, etc., forces. While speculations of a general nature can be made on these group structures and multiplicities on the basis of certain spatial symmetries (S, A, or M) of the $3Q$ wave functions with spin and unitary–spin-independent forces, it is still an open question how such symmetries can be explicitly brought about through conventional types of Q-Q forces. For example in a Fermi model of three Gellmann-Zweig[5] quarks, we have been unable to construct an antisymmetric state (A) of $L=0$ which must go with the **56** through p-wave Q-Q forces because such a state usually turns out to be repulsive.[6] In a dynamical model, wave functions of definite symmetry should, in principle, be obtained automatically through the (input) Q-Q forces. We report here the results of a simple dynamical model which generates certain bound states of $3Q$, exhibiting characteristic group structures of $SU(6)\otimes O(3)$, thus providing a dynamical realization of several representations of this group.[7] In the limit of full symmetry of the interaction, the following states of $[SU(6)\otimes O(3)]^P$ appear in ascending order of energy levels:

$$(\mathbf{56},1)^+,\quad (\mathbf{20},3)^+,\quad (\mathbf{70},3)^-,\quad (\mathbf{70},1)^+,\quad (\mathbf{70},5)^+. \quad (1)$$

While the **56** is of course realized as the state of lowest energy, there appear some positive-parity states before the usual negative-parity ones get their turn. The immediate implication of this result is that even after spin-orbit forces have split the $(\mathbf{20},3)^+$ states, there remain the $(\mathbf{8},P_{1/2}^+)$ and the $(\mathbf{1},P_{1/2}^+)$ somewhat *below* the levels of the corresponding negative parity states. The main assumptions required for the above derivation are (1) dominance of s- and p-wave Q-Q forces and (2) parastatistics which allows symmetric $3Q$ functions.[8]

The spin and unitary–spin-dependent Q-Q forces in the s and p states, consistent with parastatistics are of the form

$$V=(P_\sigma^+P_u^+ + P_\sigma^-P_u^-)V_s + (P_\sigma^-P_u^+ + P_\sigma^+P_u^-)V_p, \quad (2)$$

[1] G. Morpurgo, Physics **2**, 65 (1965).
[2] Y. Nambu, in *Symmetry Principles at High Energy*, edited by B. Kursunoglu, A. Perlmutter, and I. Sakmer (W. H. Freeman and Company, San Francisco, 1965).
[3] A. N. Tavkhelidze, in the International Seminar on High Energy Physics and Elementary Particles, Trieste, 1965 (unpublished).
[4] R. H. Dalitz, in *Proceedings of the Oxford International Conference on Elementary Particles, 1965* (Rutherford High Energy Laboratory, Harwell, England, 1966).
[5] M. Gell-Mann, Phys. Letters **8**, 214 (1963); G. Zweig, CERN Report No. 8182/Th 401, 1964 (unpublished).
[6] A. N. Mitra, Phys. Rev. **142**, 1119 (1966); referred to as A.
[7] K. T. Mahanthappa and E. C. G. Sudarshan, Phys. Rev. Letters **14**, 163 (1965).
[8] O. W. Greenberg, Phys. Rev. Letters **13**, 598 (1964).

where $P_\sigma^\pm$ are the triplet and singlet spin-projection operators, and $P_u^\pm$ are the corresponding operators in unitary-spin space for the $2Q$ states of 6 and 3*, respectively.[9] With such a force, one obtains broadly the following structures for the $3Q$ states. The s-wave force V_s can yield S or M functions which satisfy Schrödinger equations of two distinct symmetry types. These equations, which are especially easy to reduce with the assumption of factorable shapes[6] for V_s and V_p, can be shown to be attractive for S, repulsive for M.[10] This immediately yields the result that only the states $(\mathbf{10},S_{3/2}{}^+)$ and $(\mathbf{8},S_{1/2}{}^+)$ are attractive, making up the desired **56**, while the remaining **70** states associated with M functions are repulsive. The expression for the central mass of the **56** is the same as the one give in Sec. 5 of A, with s-wave forces in the "long-range" approximation. The splitting of the **8** and **10** can be brought about through an $SU(3)$ invariant term of the form

$$(P_\sigma{}^+P_u{}^+ - P_\sigma{}^-P_u{}^-)V_s{}', \tag{3}$$

and further splittings within each multiplet, by the usual $SU(3)$-violating terms. However, except for the demonstration of the existence of the **56** as the ground level in our scheme, these other details are not of immediate interest to this discussion. The same force also yields a set of negative parity states in $(\mathbf{56},3)^-$, but the attraction in these states is appreciably weaker than in $(\mathbf{56},1)^+$, so that these are again not of much physical interest.

The more interesting results are due to the p-wave interaction, which gives states of both positive and negative parity. The spatial functions are now of the types A or M, but not S. For V_p in (2) we assume the factorable form given in A. For the negative-parity states, the M functions only can be shown to have strongly attractive kernels.[10] These lead to the following multiplicity of states:

$$(\mathbf{10},P_{1/2}{}^-),\quad (\mathbf{10},P_{3/2}{}^-),\quad (\mathbf{8},P_{1/2}{}^-)^2,\quad (\mathbf{8},P_{3/2}{}^-)^2,$$
$$(\mathbf{8},P_{5/2}{}^-),\quad (\mathbf{1},P_{1/2}{}^-),\quad (\mathbf{1},P_{3/2}{}^-), \tag{4}$$

giving rise to the representation $(\mathbf{70},3)$ of $SU(6)\otimes O(3)$. For the positive parity states of $L=1$, only A functions have strongly attractive kernels,[10] and yield the following states

$$(\mathbf{1},{}^4P_{1/2}{}^+),\quad (\mathbf{1},{}^4P_{3/2}{}^+),\quad (\mathbf{1},{}^4P_{5/2}{}^+),$$
$$(\mathbf{8},{}^2P_{1/2}{}^+),\quad (\mathbf{8},{}^2P_{3/2}{}^+), \tag{5}$$

making up a $(\mathbf{20},3)^+$. There is a second set of positive parity states of $L=0$, which have attractive kernel only for M functions,[10] *but not A*,[11] leading to the following states of $(\mathbf{70},1)^+$

$$(\mathbf{10},S_{1/2}{}^+),\quad (\mathbf{8},S_{1/2}{}^+),\quad (\mathbf{8},S_{3/2}{}^+),\quad (\mathbf{1},S_{1/2}{}^+). \tag{6}$$

Finally the p-wave interaction generates a set of positive parity states of $L=2$, for which only the M functions have attractive kernels[10] giving rise to the multiplicity $(\mathbf{70},5)^+$ of $SU(6)\otimes O(3)$. These states are appreciably higher in energy than the others listed in (1), and are likely to be strongly affected by d-wave Q-Q forces. As a result it is not possible to speak more quantitatively about them.

We have considered the splitting of these various states due to a p-wave spin-orbit force of the (factorable) form

$$M_Q\langle\mathbf{p}|V_{LS}|\mathbf{p}'\rangle = -3\lambda_{LS}P_\sigma{}^+P_u{}^-i(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{p}\times\mathbf{p}'e^{-\frac{1}{2}(p^2+p'^2)/\beta^2}, \tag{7}$$

taking for convenience the same (Gaussian) shape factor as for the central interaction.[6] The appearance of $P_\sigma{}^+P_u{}^-$ in (7) indicates that this force shifts only the **8** and **1** states, but not the **10**. It also mixes up the various L states with the same J^P. If, however, the coupling to the repulsive states, as well as those between different L states is ignored, an extension of the calculational techniques of A yields the following approximate formula for the energy levels splitting of the $(\mathbf{70},3)^-$ states into $SU(3)$ multiplets[10]

$$M_Q\Delta E^{(-)} = -\tfrac{3}{2}\beta^2\xi\sigma_{LS}[1+(16/25)\zeta^{-1/2}] - (24/25)\beta^2\sigma[(1+\xi\sigma_{LS}/\sigma)^{1/2}-1]. \tag{8}$$

The central mass which coincides with that of the states $(\mathbf{10},\frac{1}{2}^-)$ and $(\mathbf{10},\frac{3}{2}^-)$, is

$$E_0{}^{(-)}(\mathbf{70},3) = -\tfrac{3}{2}\beta^2\sigma M_Q{}^{-1}[(41/25)+(16/25)\zeta^{-1/2}] + 3M_Q. \tag{9}$$

Here σ and σ_{LS} are dimensionless parameters measuring the respective strengths of the central and spin-orbit forces, and the number ξ has the values given by Table I.

For the $(\mathbf{20},3)^+$ states, the formula for the (split) energy levels of the $SU(3)$ multiplets is

$$E^{(+)}(\mathbf{20},3) = -\tfrac{3}{2}\beta^2(\sigma+\eta\sigma_{LS})M_Q{}^{-1}[1+(64/25)\zeta^{-1/2}] + 3M_Q. \tag{10}$$

TABLE I. Spin-orbit parameters for various SU(3) states.

State[a]	ξ	State[a]	η
$(\mathbf{1},\frac{1}{2}^-)$	$\frac{8}{3}$	$(\mathbf{1},\frac{1}{2}^+)$	$\frac{5}{3}$
$(\mathbf{1},\frac{3}{2}^-)$	$-\frac{4}{3}$		
$(8,\frac{1}{2}^-)_{\mathrm{I}}$	4	$(\mathbf{1},\frac{3}{2}^+)$	$\frac{2}{3}$
$(8,\frac{1}{2}^+)_{\mathrm{II}}$	2	$(\mathbf{1},\frac{5}{2}^+)$	$-\frac{4}{3}$
$(8,\frac{3}{2}^-)_{\mathrm{I}}$	-2	$(8,\frac{1}{2}^+)$	$\frac{4}{3}$
$(8,\frac{3}{2}^-)_{\mathrm{II}}$	2	$(8,\frac{3}{2}^+)$	$-\frac{2}{3}$
$(8,\frac{5}{2}^-)$	-2		

[a] For simplicity the letter P for $L=1$ states has been omitted.

[9] With Fermi statistics, the brackets multiplying V_s and V_p would be interchanged.

[10] A. N. Mitra (to be published).

[11] This fact is the main reason why, with the assumption of Fermi statistics, it is dynamically hard to construct an A function to go with the **56** with reasonable Q-Q forces which must be mainly in p waves.

The values of the quantity η for the different $SU(3)$ states are also given in Table I.

As for the $(\mathbf{70},1)^+$ states, these are not split by the spin orbit force in the above approximation, and their central mass is given by

$$E_0^{(+)}(\mathbf{70},1)=-\tfrac{3}{2}\beta^2\sigma M_Q^{-1}[1+(56/25)\zeta^{-1/2}]+3M_Q. \quad (11)$$

It is clear that a p-wave force along with spin-orbit coupling is capable of generating a large number of states. The results seem to be quite reasonable for the odd parity multiplets most of which, according to Dalitz's analysis[12] of experimental data, appear to have been discovered, except for $(\mathbf{10},{}^2P_{1/2}^-)$ and $(\mathbf{8},P_{3/2}^-)$. While it would be unreasonable to make a detailed comparison with the experimental data without the inclusion of (1) coupling between the states of $(\mathbf{56},3)^-$ (obtained via the s-wave interaction), and those of $(\mathbf{70},3)^-$, (2) coupling to repulsive states and (3) coupling between different L states, it appears that the general trend of the $(\mathbf{70},3)^-$ levels as predicted by Eq. (8) and Table I, is not in qualitative disagreement with experiment.[4] For example, the proximity of the $(\frac{3}{2}^-)$ $N^*(1518)$ and the $(\frac{1}{2}^-)$ $N^*(1510)$ has a "place" in Table I. The $(\frac{5}{2}^-)$ of **8** is reasonably high up according to Table I, as is indeed indicated by the "experimental position" of $(\frac{5}{2}^-)$ $N_{1/2}^*$ at 1688.[13] The relative positions of the two singlets $(\frac{1}{2}^-)$ and $(\frac{3}{2}^-)$ are maintained, thus making it possible to identify them with the (1405) Y_0^* and (1520) Y_0^*, respectively. An important shortcoming of our formula is that it does not predict the appreciably lower masses of the singlets compared with the octets, though the latter maintain their own relative positions. However, the effect of coupling to the repulsive states could substantially remedy this defect, since there are several repulsive states in **8**, and fewer in **1**. On the whole, the negative-parity states are qualitatively accounted for in our model.

There are, however, some positive-parity states in the lower mass region. While the $(\mathbf{70},1)^+$ need not pose a serious problem, since its central mass, according to (11), is roughly in the same region as that of $(\mathbf{70},3)^-$ so that its various $SU(3)$ states could stand a good deal of correction, one has to be more concerned about the $(\mathbf{20},3)^+$, whose central mass is definitely lower. According to Table I, the two lowest lying states of this set are $(\mathbf{1},{}^4P_{1/2}^+)$ and $(\mathbf{8},2P_{1/2}^+)$ which are strongly depressed by the spin-orbit force. The octet seems to fit rather well with the state $N_{1/2}^*$ (1450) (which Dalitz's analysis[4] describes as a "puzzle"). The physical reason for a natural appearance of this state in our scheme is that an A function of the axial-vector type, as already discussed in Ref. (6), has a strongly attractive kernel. This result is a very general feature of the p-wave interaction, and the separable approximation to its radial part does not affect its validity.

The big experimental challenge is clearly posed by the singlet $\frac{1}{2}^+$ whose existence is rather strongly predicted by the model. On the basis of Eqs. (8)–(10) and Table I, the mass of this particle is estimated at 1280–1350, taking the 1405 and 1520 as a calibration points and $\sigma/\sigma_{LS}\sim 5$–8, the higher value corresponding to the lower mass. It may appear surprising at first sight how such a low-mass state could exist at all without being detected. A possible answer to this paradox could be that because it is an even-parity singlet, it is extremely hard to produce through conventional types of reaction. Even its decay into a baryon and a meson is greatly inhibited by the *doubly strong* recoil effect (due to its axial-vector structure), as compared with the decay of an odd-parity baryon whose polar-vector structure causes only an ordinary recoil effect. It may be recalled in this connection that the history of detection of the $(\frac{1}{2}^+)$ $N^*(1450)$ is much more recent than that of the odd-parity resonances. However, one mechanism of detection of $N^*(1450)$, viz., a fairly strong production in a high-energy p-p reaction in the forward direction,[14,15] gives a possible clue to a similar experiment to detect this singlet, viz., to shoot a high-energy beam of Λ particles against a target nucleus, and look for such an "object" through its decay. While such experiments would no doubt be harder by several orders of magnitude, it should be of great interest to await their outcome when they become feasible, as these would have a direct bearing on the very validity of the quark model of baryon resonances through p-wave forces. Such forces which appear essential for an understanding of the structure of the odd parity resonances, predict, *without extra assumptions*, strongly bound axial-vector states.[16]

Finally, we make a few comments on the role of parastatistics in this investigation. As has been already remarked an antisymmetric state of $L=0$ is dynamically difficult to realize with conventional Q-Q forces. Parastatistics, on the other hand, has been shown here to yield the **56** through s-wave forces in a very natural manner.[17] It has also been found[18] that baryon form factors with antisymmetric functions show nodal structures at rather small (~ 20 F^{-2}) values of the square of the momentum transfer, in total disagreement with experiment. With symmetric functions, on the other

[12] It may be noted that according to our calculations, doublet and quartet states get strongly mixed by spin-orbit forces so it is not possible to specify the spin multiplicity of the octet states. This conclusion differs from that of Ref. 4.

[13] P. Bareyre *et al.*, Phys. Letters **18**, 342 (1965).

[14] G. Bellitini *et al.*, Phys. Letters **14**, 164 (1965).

[15] E. W. Anderson *et al.*, Phys. Rev. Letters **16**, 855 (1966).

[16] A similar mechanism with p-wave forces between neutrons was recently shown to produce n^3 states of the type $(LSJ)^P=(1\,\frac{3}{2}\,\frac{1}{2})^+$ as those of strongest attraction [A. N. Mitra and V. S. Bhasin, Phys. Rev. Letters **16**, 523 (1966)].

[17] One could perhaps use Fermi statistics and yet obtain symmetric wave functions in **56**, but the price for this is an extension of the 3-quark model. See, e.g., M. Y. Han and Y. Nambu, Phys. Rev. **139**, B1006 (1965).

[18] A. N. Mitra and R. Majumdar, Phys. Rev. **150**, 1194 (1966).

hand, this difficulty is completely avoided. In short, within a simple 3-quark model, we need parastatistics not only to get the **56** dynamically, but to predict a qualitatively correct shape of the form factors as well. As for the $3Q$ states with p-wave interaction, these could also have been brought about through Fermi statistics, with the interchange of **10** and **1** states. One consequence of this would have been that instead of the even parity singlet of the above discussion one would have been faced with the existence of a decuplet of states at that energy. This contrast makes parastatistics *prima facie* more attractive than Fermi statistics. It is of course recognized that this much defense of parastatistics is clearly inadequate and that in order to be accepted at all, many more tests will be necessary.

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Physical Reductions in Higher Symmetries*

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It is shown that only reductions with respect to particular subgroups occur in a certain class of physical theories of higher symmetry. It is proven that each irreducible representation (except the trivial one) of a simple Lie group of rank three contains representations of different triality of any SU_3 subgroup thus embedded. This paper is a sequel to a work, by the same author, which has used these results for the discussion of the electromagnetism of hadrons and higher symmetries.

I. INTRODUCTION

In theories of higher symmetry, the reduction of each of the irreducible representations of a compact semisimple symmetry group with respect to those of a particular semisimple subgroup are often considered. We show that only reductions with respect to particular subgroups are pertinent to a certain class of theories and hereafter call them "physical reductions."

We prove that if any of the simple Lie groups of rank three is subjected to a physical reduction with respect to its A_2 (SU_3) content, each irreducible representation (except the trivial one) contains mixed trialites. This paper is a sequel to a work which has used these results for the discussion of the electromagnetism of hadrons and higher symmetries.[1]

In Sec. II physical reductions are defined and their role in physical theories is demonstrated. Section III contains a discussion of physical reductions of rank-three simple Lie groups with respect to SU_3 content. An example of a nonphysical reduction may be found in Appendix A, which also contains a discussion of a familiar theory which does not belong to the class of theories being considered. In Appendix B we prove that only the trivial representation of a simple Lie group of rank l has a weight diagram which is completely contained in an $(l-1)$-dimensional hyperplane. Thus, an element of a Cartan subalgebra of the algebra of a simple Lie group may be represented by the unit matrix only in the trivial representation.

II. PHYSICAL REDUCTIONS

In this section we consider the reduction of any irreducible representation of a compact semisimple invariance group G with respect to those of a semisimple subgroup $G' \subset G$ contained in that representation.

The class of physical theories we consider are those such that G (rank l) was constructed from G' (rank l') by adjoining to G' $(l-l')$ additively conserved quantities which are G' singlets. Let us call these "canonical theories." Note that the theory discussed in Ref. 1 is included ($l=3$ or 4, $l'=2$), as are the usual theories combining hypercharge with isospin, whereas Wigner's SU_4 (supermultiplet) theory[2] and its generalization (to elementary particles) SU_6 *are not* of this type. In the latter however, the chain[3] whereby the reduction is to $SU_4 \times SU_2 \times U_1 \subset SU_6$ obeys the theorem of this section, although the next step in that chain does not.

We show that only those subgroups which are embedded in a particular way, to be called "physically embedded," are involved in reductions which arise in

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[1] William B. Rolnick, Phys. Rev. Letters **17**, 416 (1966).

[2] E. Wigner, Phys. Rev. **51**, 106 (1937). We shall discuss this theory in Appendix A.

[3] As employed by F. Gursey, A. Pais, and L. A. Radicati, Phys. Rev. Letters **13**, 299 (1964).