present experiment. The mean lifetime of the π^0 as found in the present experiment is in good agreement with the other values quoted in Table VI.

Goldberger and Treiman,[31] using extended dispersion relation techniques, have calculated theoretically a π^0 lifetime value of 0.5×10^{-16} sec. Bose[32] extended the Goldberger-Treiman formula for the charged-pion decay to obtain the rate of neutral-pion decay from a study of the Compton scattering of protons. This calculation yielded a π^0 lifetime value of 1.4×10^{-16} sec. Sternglass,[33] using a semiclassical model of the Bohr-Sommerfeld type, has investigated the relativistic electron-positron–pair system in the limit of high velocities. He showed that a lowest state exists and that the lowest state possesses an energy approximately equal to the π^0 rest energy. The lifetime of the system against annihilation into two gammas is calculated to be 2.06×10^{-16} sec. All of the above theoretical calculations are in reasonable agreement with the value obtained by the present experiment.

[31] M. L. Goldberger and S. B. Treiman, Nuovo Cimento 9, 451 (1958).
[32] S. K. Bose, Nuovo Cimento 23, 408 (1962).
[33] E. J. Sternglass, Phys. Rev. 123, 391 (1961).

ACKNOWLEDGMENTS

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Σ Hypernuclei and Σ⁺-Proton Scattering*

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By analyzing the Σ^+-proton scattering data at low energies, it is found that the triplet Σ-nucleon interaction in the $I=\frac{3}{2}$ state is certainly not strong enough to support the formation of a particle-stable Σ^-n or Σ^+p system. The singlet Σ-nucleon interaction, on the other hand, could be quite strong, but it is probably still not sufficient to create a bound two-body Σ hypernucleus. Also, because of the relative weakness of the triplet interaction, it can be concluded that a bound hypernucleus Σ^-nn very likely does not exist. Using the results obtained in this investigation, speculations are also made concerning the double-Σ hypernuclear systems $\Sigma^-\Sigma^-n$ and $\Sigma^-\Sigma^-nn$.

I. INTRODUCTION

Up to the present time, quite a number of Λ hypernuclei have been observed.[1,2] On the other hand, there has not been a single uniquely identified Σ-hypernucleus event reported. The main reason for this is that when a charged Σ particle becomes bound to a nuclear system containing both neutrons and protons to form a Σ hypernucleus, it would quickly react to produce a Λ particle and hence, such a Σ hypernucleus would not be expected to be observed. Thus, the only types of Σ hypernuclei which could be stable for a time comparable to the lifetime of the Σ hyperon are those which are composed of one or more Σ^- particles with neutrons, and their charge-symmetrical counterparts of Σ^+ particles and protons.

In this investigation, we study the possibility of the formation of particle-stable Σ hypernuclear systems Σ^-n and Σ^-nn. This study is motivated by the fact that scattering results of Σ^+ on protons are beginning to

* Work performed under the auspices of the U. S. Atomic Energy Commission.
[1] R. Levi-Setti in *Proceedings of the International Conference of Hyperfragments, St. Cergue, Switzerland, 1963* (CERN, Geneva, 1964).
[2] C. Mayeur, J. Sacton, P. Vilain, G. Wilquet, D. Stanley, P. Allen, D. H. Davis, E. R. Fletcher, D. A. Garbutt, M. A. Shaukat, J. E. Allen, V. A. Bull, A. P. Conway, and P. V. March, Nuovo Cimento 43, 180 (1966).

appear.[3–6] As we shall see, even though these results are too crude to allow a detailed understanding of the characteristics of the Σ^+-proton interaction, they do allow us to make some definitive statements about the existence of these hypernuclei. In addition, we shall discuss qualitatively the question whether there exist double-Σ hypernuclei, such as $\Sigma^-\Sigma^-n$ and $\Sigma^-\Sigma^-nn$. For this latter discussion, we shall rely upon the experience which we have gained previously from studying the double-Λ hypernuclei $\Lambda\Lambda n$ and $\Lambda\Lambda nn$.[7]

Experimentally, a careful search for Σ^- hyperfragments has been made by Ammar *et al.*, by K^- captures in nuclear emulsions.[8] The results of this search were inconclusive. Although there were several events which could be interpreted as Σ^-n, these authors have remarked that the interpretation was not unequivocal and other explanations for these events might serve just as well.[9]

There is also some information about the strength of the Σ^--neutron interaction in the triplet state. From the results of the experiment $K^-+d\rightarrow\Sigma^-+n+\pi^+$,[10] both Dahl *et al.*,[10] and Chand[11] have concluded that if a hyperfragment exists in the triplet spin state, its binding energy must be less than a few keV.

In Sec. II, we present the results of our analysis of the Σ^+-proton scattering data. From this analysis, we find that Σ^-n quite definitely does not form a bound state in the triplet configuration. In the singlet configuration, the present scattering data could not entirely rule out the formation of a bound state, although the chance of such a formation is rather small. Section III is devoted to a discussion of our calculation on the Σ^-nn system. What we do here is to determine the value of the spin-averaged well-depth parameter which is necessary to form a Σ^-nn system of zero binding. From this value and the results of the Σ^+-proton analysis, we conclude that there is only a very small possibility for the existence of a bound Σ^-nn system. In Sec. IV, we summarize the results of this investigation and discuss qualitatively the question about the possible existence of double-Σ^- hypernuclear systems such as $\Sigma^-\Sigma^-n$ and $\Sigma^-\Sigma^-nn$.

II. ANALYSIS OF Σ^+-PROTON SCATTERING

In this analysis, the result of Dosch *et al.*,[3] will be primarily considered. Using a selection criterion which includes only scattering events with a c.m. scattering angle greater than 60° and assuming an isotropic angular distribution, Dosch *et al.*, have obtained for the Σ^+-proton elastic scattering cross section a value of 185 ± 55 mb, with the Σ^+ momentum in the interval from 135 to 175 MeV/c (c.m. energy range from 3.4 to 5.7 MeV). With this value for the elastic cross section, one obtains 14.7 ± 4.4 mb/sr as the average differential cross section in the angular region from 60° to 180°.[12]

Instead of considering various values of c.m. energy in the energy range of interest, we have calculated mainly with the average value of 4.46 MeV (corresponding to 155 MeV/c). To make some compensation for this simplification, we have increased the uncertainty from 4.4 to 6.4 mb/sr. Hence, the values of the average differential cross section at 4.46 MeV which we will be mostly concerned with are 8.3, 14.7, and 21.1 mb/sr; these will be called σ_1, σ_2, and σ_3, respectively.

Preliminary results of Rubin *et al.*, for Σ^+ momentum in the range from 140 to 175 MeV/c show that the total elastic cross section is equal to 102 ± 26 mb,[13] yielding an average differential cross section of only 8.1 ± 2.1 mb/sr for c.m. angles greater than about 37°, which is somewhat smaller than the value given by Dosch *et al.*[3,6] Hence, in reaching our conclusion from this investigation, we mainly use the results obtained from our calculation with σ_1 and σ_2, rather than these with σ_3.

Upon the completion of this work, we have noticed that Dosch *et al.*,[6] have given in a recent paper Σ^+-proton elastic cross section as a function of momenta in the narrow range of 148 to 178 MeV/c. These results are based on only 19 scattering events and hence, the statistical uncertainties are quite large. We shall not pay too much attention to these results, since they are entirely consistent with the older results (of these authors) which we consider here, and in addition, because of the large uncertainties, very little further information could be extracted from them.

The analysis is done with a model in which the Σ^+-proton interaction is represented by a spin-dependent

[3] H. G. Dosch, R. Engelmann, H. Filthuth, V. Hepp, E. Kluge, and A. Minguzzi-Ranzi, Phys. Letters **14**, 162 (1965).

[4] H. A. Rubin, R. A. Burnstein, T. B. Day, B. Sechi-Zorn, and G. A. Snow, Bull. Am. Phys. Soc. **11**, 37 (1966); H. A. Rubin (private communication).

[5] J. Schultz, W. Chinowsky, R. Kinsey, and N. Rybicki, Bull. Am. Phys. Soc. **10**, 529 (1965).

[6] H. G. Dosch, R. Engelmann, H. Filthuth, V. Hepp, and E. Kluge, Phys. Letters **21**, 236 (1966).

[7] Y. C. Tang and R. C. Herndon, Phys. Rev. Letters **14**, 991 (1965).

[8] R. G. Ammar, N. Crayton, K. P. Jain, R. Levi-Setti, J. E. Mott, P. E. Schlein, O. Skjeggestad, and P. K. Srivastava, Phys. Rev. **120**, 1914 (1960).

[9] Similarly, the event observed by Baldo-Ceolin *et al.* (M. Baldo-Ceolin, W. F. Fry, W. D. B. Greening, H. Huzita, and S. Limentani, Nuovo Cimento **6**, 144 (1957)], which could be interpreted as due to the formation of a Σ^+p hypernucleus, did not have a unique identification. As for the evidence of a Σ^-n bound system reported by Gandolfi *et al.* [E. Gandolfi, J. Heughebaert, and E. Quercigh, Nuovo Cimento **13**, 864 (1959)], see footnote 5 in the paper by Ammar *et al.* (Ref. 8).

[10] O. Dahl, N. Horwitz, D. Miller, and J. Murray, Phys. Rev. Letters **4**, 428 (1960).

[11] R. Chand, Nuovo Cimento **36**, 837 (1965).

[12] In a very recent article (Ref. 6) which came to our attention after the calculation reported here was completed, we have noticed that Dosch *et al.*, have adopted a slightly different selection criterion which includes only events with a c.m. scattering angle between 60° and 115°. This does not affect our conclusion, since in the energy range of interest, the differential cross section is essentially constant after 60°.

[13] We thank Dr. H. Rubin for sending us this value prior to publication.

TABLE I. Parameters of the Σ^+-proton potentials.

Potential type	$r_{\Sigma N}$ (F)	λ (F^{-1})	b (F)	U_0 (MeV)
A	0.35	4.427	1.5	1048
B	0.35	2.724	2.0	397
C	0.35	1.967	2.5	207
D	0.50	3.541	2.0	671

central potential of the form

$$U_{\Sigma N}(r)=[(1+P_{\Sigma N}{}^{\sigma})/2]U_t(r) +[(1-P_{\Sigma N}{}^{\sigma})/2]U_s(r)+U_C(r), \quad (1)$$

where $P_{\Sigma N}{}^{\sigma}$ denotes the spin-exchange operator and the last term represents the Coulomb interaction. The quantities $U_t(r)$ and $U_s(r)$ are the triplet and singlet potentials and are chosen to be of the following exponential type:

$$\begin{aligned} U_t(r)&=\infty\,, && (r<r_{\Sigma N})\\ &=-U_{0t}\exp[-\lambda(r-r_{\Sigma N})], && (r>r_{\Sigma N})\\ U_s(r)&=\infty\,, && (r<r_{\Sigma N})\\ &=-U_{0s}\exp[-\lambda(r-r_{\Sigma N})], && (r>r_{\Sigma N}), \end{aligned} \quad (2)$$

where, for simplicity, we have assumed that both potentials have the same hard-core radius and intrinsic range. A number of different combinations of $r_{\Sigma N}$ and λ will be used. The purpose is to see the sensitivity of the results to these parameters. These different combinations are listed in Table I, where the quantity b denotes the intrinsic range and U_0 denotes the potential depth necessary to yield a Σ^-n system of zero binding.

With a central potential of the form given by Eqs. (1) and (2), the differential cross sections can be easily calculated. For simplicity, we have included only nuclear phase shifts for $l=0$ and 1. This is quite sufficient, since in the energy region of interest, the $l=1$ wave already gives a rather small contribution. In Fig. 1, we plot the differential cross section at 4.46 MeV for the case with potential D and both triplet and singlet well-depth parameters (s_t and s_s) equal to 0.8. As is seen, the differential cross section is almost constant after 60°, which indicates that in computing the average differential cross section, it matters very little whether the angular region is taken as between 60° and 115°,[6] or between 60° and 180°.[3]

The average differential cross section is given by

$$\sigma_{av}=\tfrac{3}{4}\sigma_{av}{}^t+\tfrac{1}{4}\sigma_{av}{}^s\,, \quad (3)$$

with both $\sigma_{av}{}^t$ and $\sigma_{av}{}^s$ defined as

$$\sigma_{av}{}^i=\int_{\theta_1}^{\theta_2}\sigma^i(\theta)\sin\theta d\theta/(\cos\theta_1-\cos\theta_2)\,, \quad (4)$$

where θ_1 and θ_2 are equal to 60° and 180°, respectively. From the inequalities

$$\sigma_{av}{}^t\leq\tfrac{4}{3}\sigma_{av}\,, \quad (5)$$

and

$$\sigma_{av}{}^s\leq 4\sigma_{av}\,, \quad (6)$$

and the experimentally determined value of $\sigma_{av}(\sigma_1, \sigma_2$, or $\sigma_3)$, we can find the upper limits to the well-depth parameters s_t and s_s. For this latter purpose, we have further assumed that both s_t and s_s lie within the range of 0 to 1.2. For $s=0$, it should be noted that all the

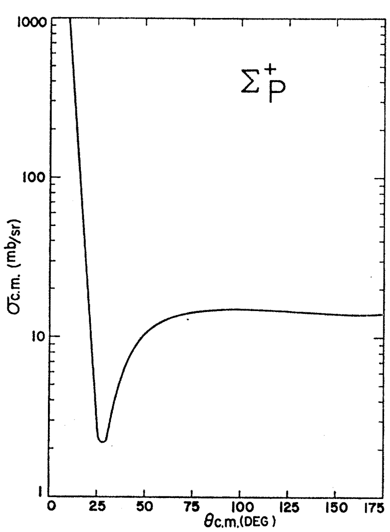

FIG. 1. Differential cross section of Σ^+-proton scattering at c.m. energy of 4.46 MeV with potential D. Both triplet and singlet well-depth parameters are equal to 0.8.

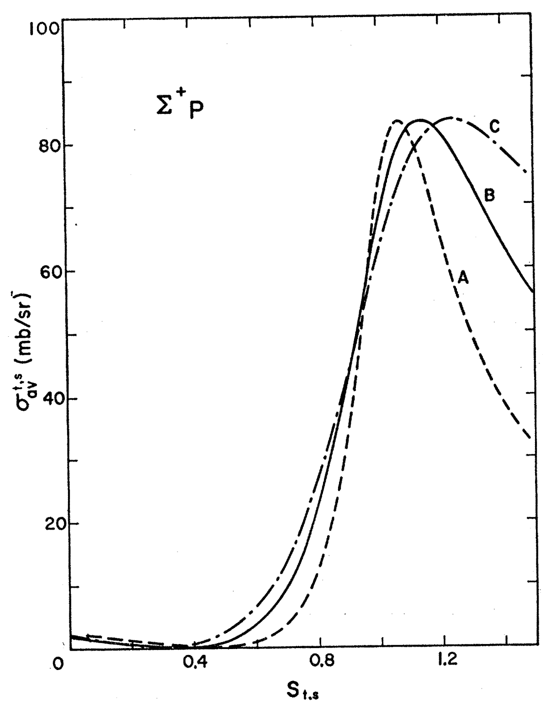

FIG. 2. Average differential cross section $\sigma_{av}{}^t$ or $\sigma_{av}{}^s$ of Σ^+-proton scattering at c.m. energy of 4.46 MeV as a function of s_t or s_s. The curves are for potential A (dashed line), potential B (solid line), and potential C (dot-dashed line).

potentials considered here are repulsive because of the presence of the hard core. For $s=1.2$, Σ^-n would form a bound state, with binding energies equal to 4.2, 1.6, 0.8, and 2.7 MeV for potential A, B, C, and D, respectively. Since these binding energies are rather large, it could be safely assumed that Σ^-n would have been observed in previous searches for this hyperfragment. Hence, the fact that it has not been observed indicates quite strongly that neither s_t nor s_s is as large as 1.2.

The behavior of $\sigma_{av}^i(i=t,s)$ as a function of s_i is shown in Fig. 2 for potential A, B, and C, and in Fig. 3 for potential B and D. From these figures and the experimental value of σ_{av}, we obtain the upper limits to s_t and s_s, which are tabulated in Table II.

From Table II, it is seen that the upper limits to the well-depth parameters are fairly independent of the types of potentials used. This is important since, otherwise, our conclusions will be too model-dependent and have very little value. For the triplet configuration, the values of s_t^{max} are all much smaller than one, even when σ_{av} is equal to σ_3. This indicates that the triplet Σ-nucleon interaction in the $I=\frac{3}{2}$ state could at most be moderately strong and is not enough to support a bound state. For the singlet configuration, no definite statement could be made when σ_{av} is equal to σ_3. But, when σ_{av} is close to either σ_1 or σ_2, then it is possible to rule out the possibility of the existence of a bound state. Since, as has been discussed in a previous paragraph,

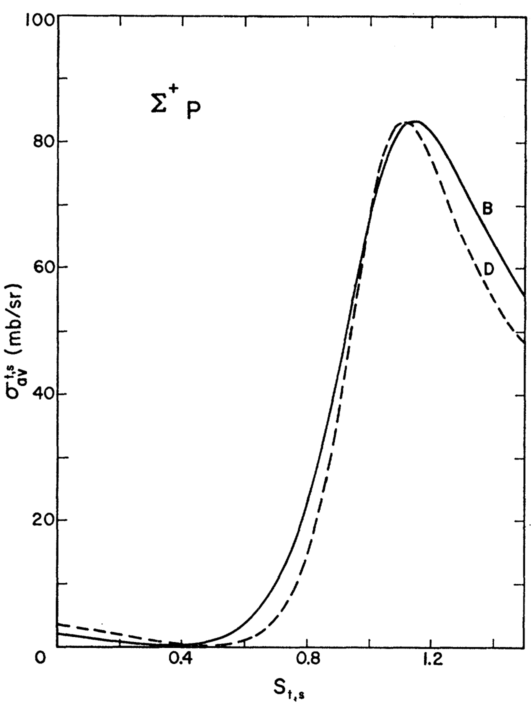

FIG. 3. Average differential cross section σ_{av}^t or σ_{av}^s of Σ^+-proton scattering at c.m. energy of 4.46 MeV as a function of s_t or s_s. The curves are for potential B (solid line) and potential D (dashed line).

TABLE II. Upper limits to s_t and s_s.[a]

σ_{av}	Potential	s_t^{max}	s_s^{max}
σ_1	A	0.792	0.894
	B	0.716	0.857
	C	0.670	0.840
	D	0.778	0.889
σ_2	A	0.843	0.962
	B	0.784	0.962
	C	0.750	0.980
	D	0.830	0.969
σ_3	A	0.878	N.I.
	B	0.831	N.I.
	C	0.810	N.I.
	D	0.868	N.I.

[a] N. I. means that no information could be obtained.

the experimental value of σ_{av} at 4.46 MeV is very likely closer to σ_1 or σ_2 than to σ_3, our analysis of the scattering data does give a strong indication that Σ^-n also does not form a bound system in the singlet configuration.

Under the assumption of complete SU_3 symmetry, the singlet Σ-nucleon interaction in the $I=\frac{3}{2}$ state is identical to the singlet proton-neutron interaction.[14] For the latter system, it is well known that the well-depth parameter is about equal to 0.95, which is close to the values of s_s^{max} given in Table II for the case with σ_{av} equal to σ_2. Thus, even though the result of this investigation does not definitely confirm the prediction of SU_3 symmetry, there is at least no contradiction.

In Fig. 4, we plot σ_{av} as a function of energy in the c.m. system for the case with potential D. In this figure, the curves are obtained with $s_t=s_s=s$, and the averag-

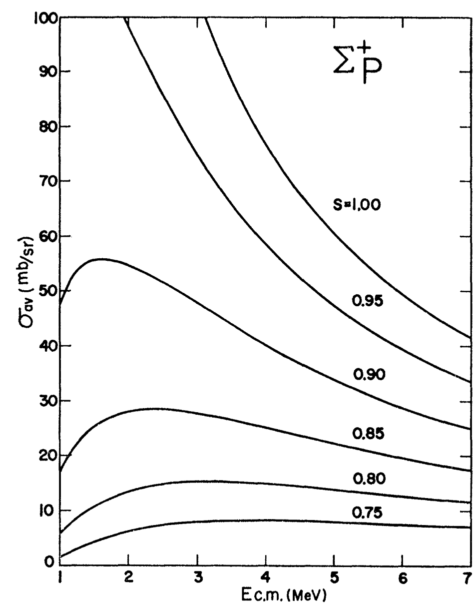

FIG. 4. Average differential cross section of Σ^+-proton scattering as a function of c.m. energies with potential D and various values of the well-depth parameters.

[14] P. D. DeSouza, G. A. Snow, and S. Meshkov, Phys. Rev. **135**, B565 (1964).

ing is done over an angular region of 60° to 115°. Using these curves, one can easily calculate σ_{av} versus $E_{c.m.}$ for any combination of s_t and s_s. In fact, we have used these curves to analyze briefly the recent data of Dosch *et al.*,[6] but because of the large statistical uncertainty associated with these data, we have obtained only slightly more information than is being reported here.

III. $\Sigma^- nn$ HYPERNUCLEUS

The purpose of this calculation is to determine the well depth of the spin-averaged Σ^--neutron potential which would allow the formation of a bound-$\Sigma^- nn$ system with binding energy equal to zero. To achieve this, we shall compute the binding energies of the $\Sigma^- nn$ system with two or three suitably chosen values of the well depth and obtain the desired value by extrapolating to zero binding with an appropriate formula. The question concerning the existence of a bound $\Sigma^- nn$ system can then be answered by deciding whether or not the value of the well depth so determined is compatible with those which are obtained from the experimental value of σ_{av} at 4.46 MeV.

With a trial function which is symmetric with respect to the space exchange of the two neutrons, the spin-averaged Σ^--neutron potential in the $\Sigma^- nn$ system is given by

$$\begin{aligned} U_3(r) &= \infty\,, && (r<r_{\Sigma N}) \\ &= -U_{03}\exp[-\lambda(r-r_{\Sigma N})]\,, && (r>r_{\Sigma N}) \end{aligned} \tag{7}$$

with

$$U_{03}=\tfrac{3}{4}U_{0t}+\tfrac{1}{4}U_{0s}\,. \tag{8}$$

Using the value of U_{03}, the spin-averaged well-depth parameter s_3 is defined as

$$s_3=U_{03}/U_0\,, \tag{9}$$

where the values of U_0 for the various potentials are listed in Table I.

The neutron-neutron potential in the even states is chosen as

$$\begin{aligned} V_{NN}(r) &= \infty && (r<r_{NN}) \\ &= -V_0\exp[-\kappa(r-r_{NN})]\,, && (r>r_{NN}) \end{aligned} \tag{10}$$

with $r_{NN}=0.35$ F, $V_0=216.0$ MeV, and $\kappa=1.97$ F^{-1}. This particular potential is preferred, since with a Coulomb interaction added, the corresponding proton-proton potential fits rather well the effective-range parameters and the $^1S_0$ phase shifts up to about 300 MeV.[15]

The trial wave function adopted for $\Sigma^- nn$ is of a type which has been used in a number of our previous investigations concerning nuclear and hypernuclear few-body problems.[16] It has the form

$$\Psi=\psi\chi\,, \tag{11}$$

with ψ and χ being the spatial and the appropriate spin function, respectively. The function ψ will be chosen as

$$\psi=f(r_{1\Sigma})f(r_{2\Sigma})g(r_{12})\,, \tag{12}$$

with 1 and 2 representing the neutrons. For the function $f(r)$, we use

$$\begin{aligned} f(r) &= u_f(r)/r\,, && (r<d_f) \\ &= A_f r^{-1/2}[\exp(-\alpha_f r) \\ &\qquad +B_f\exp(-\beta_f r)]\,, && (r>d_f) \end{aligned} \tag{13}$$

where $u_f(r)$ is a solution of the equation

$$-\frac{\hbar^2}{2\mu_f}\frac{d^2}{dr^2}u_f(r)+[U_3(r)-e_f]u_f(r)=0\,, \tag{14}$$

with u_f being the reduced mass of the neutron and the Σ^- particle. The constants A_f and B_f are adjusted such that the function $f(r)$ and its first derivative are continuous at the separation distance d_f. There is a total of four variational parameters in this function, namely, α_f, β_f, e_f, and d_f. The function $g(r)$ is defined in an analogous manner, except that μ_f is replaced by μ_g, the reduced mass of two neutrons, and the potential function in Eq. (14) is replaced by the potential $V_{NN}(r)$. The variational parameters in this latter function are α_g, β_g, e_g, and d_g.

The evaluation of the various expectation values is done with a Monte Carlo method which has been discussed in detail previously.[16] The results are shown in Table III, where E_U denotes the upper bound to the eigenvalue.

To obtain the values of U_{03} for the various potentials which would yield a bound $\Sigma^- nn$ system with binding energy equal to zero, we use the interpolation formula

$$U_{03}=a_3+b_3B^{1/2}\,, \tag{15}$$

with two parameters a_3 and b_3. In this formula, the quantity B denotes the binding energy of the $\Sigma^- nn$ system, which will be assumed as equal to the negative of the value of E_U given in Table III. This is, in fact, a quite good approximation. By studying both the upper and lower bounds to the eigenvalue, we have found previously that when a trial wave function of the type described by Eqs. (12)–(14) is used, the upper bound obtained is close to the eigenvalue in all the cases we have studied.[16]

Using Eq. (15) and the sets of values for U_{03} and E_U, we obtain our desired results for the well depths listed in Table IV. In this table, $U_{03}(0)$ denotes the well depth required for binding $\Sigma^- nn$ at zero energy and $s_3(0)$ is the corresponding well-depth parameter. It should be mentioned that the two-parameter interpolation formula seems to be quite adequate for our purpose. For the case with potential A, we have in fact employed a three-parameter interpolation formula of the type used in our previous calculation on Λ hypernuclei[17] and

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[15] E. W. Schmid, Y. C. Tang, and R. C. Herndon, Nucl. Phys. **42**, 95 (1963).

[16] R. C. Herndon and Y. C. Tang, in *Methods of Computational Physics* (Academic Press Inc., New York, 1966), Vol. 6.

[17] R. C. Herndon, Y. C. Tang, and E. W. Schmid, Phys. Rev. **137**, B294 (1965).

TABLE III. Results for $\Sigma^- nn$.[a]

Potential	U_{03} (MeV)	s_3	α_f (F^{-1})	β_f (F^{-1})	e_f (MeV)	d_f (F)	α_g (F^{-1})	β_g (F^{-1})	e_g (MeV)	d_g (F)	E_U (MeV)
A	980	0.935	0.190	7.0	−17.0	1.1	0.140	4.5	−10.0	1.2	-0.96 ± 0.07
	955	0.911	0.160	9.0	−17.0	1.1	0.125	6.0	−12.0	1.2	-0.39 ± 0.07
	940	0.897	0.125	10.0	−19.0	1.1	0.115	6.5	−15.0	1.2	-0.11 ± 0.07
B	352	0.886	0.130	6.0	−22.0	1.1	0.115	5.0	−15.0	1.2	-0.39 ± 0.07
	340	0.856	0.095	7.0	−24.0	1.1	0.115	5.5	−20.0	1.2	-0.03 ± 0.07
C	180	0.870	0.120	6.0	−20.0	1.2	0.115	5.0	−20.0	1.2	-0.24 ± 0.08
	176	0.850	0.105	6.5	−20.0	1.2	0.110	5.5	−20.0	1.2	-0.10 ± 0.08
D	610	0.909	0.130	6.0	−25.0	1.1	0.120	5.0	−15.0	1.2	-0.28 ± 0.06
	598	0.891	0.100	6.0	−22.0	1.1	0.115	5.5	−25.0	1.2	-0.01 ± 0.06

[a] The statistical accuracy in this table is achieved with 50 000 estimates in the Monte Carlo calculation.

found that the resultant value of $U_{03}(0)$ is quite consistent with that given in Table IV.

The crucial problem now is to determine whether the well-depth parameter of the spin-averaged Σ^--neutron potential, defined as

$$s_{av}=\tfrac{3}{4}s_t+\tfrac{1}{4}s_s\,, \tag{16}$$

is larger than the value of $s_3(0)$. If it is not, then this would mean that the strength of the Σ^--neutron interaction is not enough to support the formation of a particle-stable bound state for $\Sigma^- nn$. To resolve this question, we shall try to find the upper limit of s_{av} from the experimental data on Σ^+-proton scattering. That is, we want to determine the maximum value of s_{av} subject to the condition that, at 4.46 MeV, the average differential cross section is equal to that obtained experimentally. Using the method of the Lagrangian multiplier, it can be easily shown that the condition under which s_{av} attains its maximum value is

$$s_t=s_s\,. \tag{17}$$

It should be emphasized that we obtain such a simple condition only because the triplet and singlet Σ^--neutron potentials have been assumed to have the same functional form with the same hard-core radius and intrinsic range. Using Eq. (17) and Figs. 2 and 3, we then obtain the maximum values of s_{av}, denoted as s_{av}^{max}, for the various potentials. These are listed in Table V.

By comparing the values of $s_3(0)$ in Table IV and s_{av}^{max} in Table V, it is seen that regardless of whether σ_{av} is equal to σ_1, σ_2, or σ_3, or whether the Σ^--neutron interaction is represented by potential *A*, *B*, *C*, or *D*, the conclusion remains the same that the probability for the existence of a bound state for the hypernucleus $\Sigma^- nn$ is very small indeed.

TABLE IV. Values of $U_{03}(0)$ and $s_3(0)$ for $B=-E_U$.

Potential	$U_{03}(0)$ (MeV)	$s_3(0)$
A	923.2 ± 7.9	0.881 ± 0.008
B	335.2 ± 6.0	0.844 ± 0.015
C	169.3 ± 4.6	0.818 ± 0.022
D	595.0 ± 8.8	0.887 ± 0.013

In a variational calculation, it is always necessary to discuss the question of how close the upper bound is to the eigenvalue. To investigate this, we use a method which has been employed in our previous calculation on the nuclear few-body problems.[18] What we shall do is to compute the value of $\langle\mathcal{H}\Psi,\mathcal{H}\Psi\rangle$, with $\mathcal{H}$ being the Hamiltonian of the problem, and calculate

$$B+E_U=\Delta B=\frac{\langle\mathcal{H}\Psi,\mathcal{H}\Psi\rangle-E_U{}^2}{\epsilon-E_U}, \tag{18}$$

where ϵ denotes the average excitation energy of the eigenfunctions of the excited states which the trial function mixes into the ground-state eigenfunction. The value of ϵ can be estimated by using the procedure described in detail in our previous publication.[18] For instance, for potential *D* with U_{03} equal to 610 MeV, the value of $\langle\mathcal{H}\Psi,\mathcal{H}\Psi\rangle$ is calculated as 146 MeV^2 and ϵ is estimated to be approximately equal to 600 MeV. Using these numbers, we obtain ΔB as 0.24 MeV, which is rather small compared to the expectation value of the kinetic energy operator (~20 MeV). We realize, of course, that this procedure of estimating the value of *B* and consequently, $s_3(0)$ is an approximate one. However, it is our belief that by doing this, we could at least gain information about how reliable the values of $s_3(0)$ are in Table IV.

TABLE V. Values of s_{av}^{max}.

σ_{av}	Potential	s_{av}^{max}
σ_1	*A*	0.764
	B	0.683
	C	0.632
	D	0.750
σ_2	*A*	0.817
	B	0.749
	C	0.708
	D	0.802
σ_3	*A*	0.850
	B	0.793
	C	0.761
	D	0.839

[18] Y. C. Tang, E. W. Schmid, and R. C. Herndon, Nucl. Phys. **65**, 203 (1965).

TABLE VI. Values of $s_3(0)$ for $B=-E_U+\Delta B$.

Potential	$s_3(0)$
A	0.86 ± 0.01
B	0.80 ± 0.02
C	0.76 ± 0.04
D	0.86 ± 0.02

Using the procedure mentioned above, we have estimated $s_3(0)$ for the various potentials considered. In all these cases, we have tried to be on the safe side by overestimating ΔB according to our experience. The results are shown in Table VI. By comparing the values of $s_3(0)$ in this table with those of s_{av}^{max}, we again find that when σ_{av} is equal to σ_1 or σ_2, there is only a very small probability for the formation of a bound $\Sigma^- nn$ system. On the other hand, when σ_{av} is equal to σ_3, there does appear to be a non-negligible probability for the existence of such a hypernucleus with a small binding energy. However, since there is reason to believe that the experimental value of σ_{av} at 4.46 MeV is apt to be closer to σ_1 or σ_2 than to σ_3, we can still safely conclude from this analysis that the existence of a $\Sigma^- nn$ bound system is unlikely.

IV. CONCLUSION

The results of this investigation show that the triplet Σ-nucleon interaction in the $I=\frac{3}{2}$ state could at most be moderately strong and is certainly not enough to support the formation of a particle-stable $\Sigma^- n$ or $\Sigma^+ p$ system.[19] The singlet Σ-nucleon interaction, on the other hand, could be quite strong; in fact, our analysis cannot rule out the possibility that it has a strength similar to that of the nucleon-nucleon interaction in the $^1S_0$ state. However, from the presently available experimental data on Σ^+-proton scattering, we can still conclude that the existence of a bound $\Sigma^- n$ or $\Sigma^+ p$ hypernucleus in the singlet configuration is rather unlikely.

Also, it seems highly doubtful that there exists a bound $\Sigma^- nn$ system. The main reason for this is that the spin-averaged Σ^--neutron potential effective in the $\Sigma^- nn$ system is heavily weighted by the triplet interaction. As mentioned above, this latter interaction is presumably quite weak and hence, cannot contribute significantly towards the binding of this system.

If the triplet and singlet well-depth parameters of the Σ^--neutron interaction should happen to be close to each other, then we think that there might exist a resonant state for the $\Sigma^- nn$ system. To detect this, it is our opinion that the best way is to stop K^- mesons in a helium bubble chamber through the reaction $He^4(K^-,\pi^+ p)\Sigma^- nn$. The observation of a peak in the summed energy spectra of the pion and the proton would then confirm the existence of such a resonant state. We should mention, however, that unless the resonant energy is very small, say, of the order of a few tenths of a MeV, this resonant state would be quite short-lived, which is due to the fact that the two neutrons themselves do not form a bound state. Thus, even though the final products in this reaction are all charged, the experimental detection of this state might turn out to be not entirely trivial.

Using the results obtained, we can also make some interesting speculations about the existence of double-Σ hypernuclear systems such as $\Sigma^-\Sigma^- n$ and $\Sigma^-\Sigma^- nn$. In the case of $\Sigma^-\Sigma^- n$, we notice that the spin-averaged Σ^--neutron potential is exactly the same as that in $\Sigma^- nn$. Therefore, since the Σ^--Σ^- interaction is probably attractive and comparable in strength to the n-n interaction,[20] our finding that $\Sigma^- nn$ is unlikely to have a bound state implies that $\Sigma^-\Sigma^- n$ will also not form a bound system.

About the existence of a bound $\Sigma^-\Sigma^- nn$ system, the situation is not so clear. In our previous study on light double Λ-hypernuclei,[7] we have found that even though the Λ-Λ interaction is only weakly attractive, it can still contribute 1 MeV or more to the total binding energy. Thus, although $\Sigma^- nn$ will probably not have a bound state, it does not follow that a bound state for $\Sigma^-\Sigma^- nn$ cannot be formed. In fact, if $\Sigma^- nn$ has a resonant state of low energy, then one can already be quite certain that there will be a bound $\Sigma^-\Sigma^- nn$ system. In any case, it is clear that the probability for $\Sigma^-\Sigma^- nn$ to form a bound system is much larger than that for $\Sigma^- nn$. This is interesting, since a detailed study of $\Sigma^-\Sigma^- nn$ may afford the only chance to study low-energy Σ^--Σ^- interaction.

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[19] Similar conclusion about the relative weakness of the triplet Σ-nucleon interaction in the $I=\frac{3}{2}$ state has been reached by DeSouza *et al.* (Ref. 14) and Snow [G. A. Snow, in *Recent Developments in Particle Symmetries*, edited by A. Zichichi (Academic Press Inc., New York, 1966)]. With the assumption of SU_3 invariance and using the p-p scattering data, these authors showed that the triplet contribution to the Σ^+-proton elastic scattering cross section is small.

[20] Except for the Coulomb potential, complete SU_3 symmetry predicts that the Σ^--Σ^- interaction is identical to the n-n interaction.