Mean Life of the π^0 Meson*

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(Received 6 June 1966)

An estimate of the π^0 mean lifetime has been obtained from a sample of 232 $K_{\pi 2}^+$ ($K^+ \to \pi^+ + \pi^0$) decays at rest. The π^0's were detected by the Dalitz mode ($\pi^0 \to e^+ + e^- + \gamma$), and the decay distances from the K^+ ending to the π^0 decay points were measured. These events were found in an area scanning of an emulsion stack of 600-μm Ilford *G*5 emulsion pellicles exposed to a 400 MeV/*c* separated K^+ beam. The observed mean π^0 lifetime is $(1.0 \pm 0.5) \times 10^{-16}$ sec.

I. INTRODUCTION

THE first determinations of the mean lifetime of the π° meson set a series of decreasing upper limits as experimental techniques improved.[1,2] However, in recent years, several experiments[3–6] using nuclear-emulsion techniques have measured the mean π° lifetime with both upper and lower limits. These recent emulsion experiments, with one exception,[5] used the method of Harris, Orear, and Taylor.[2] This method utilizes the $K_{\pi 2}^+ (K^+ \to \pi^+ + \pi^\circ)$ decay mode of the K^+ meson at rest. In this decay a neutral pion is emitted with a unique velocity[7] $\beta = 0.835$ in the direction opposite to the observed π^+ track, followed by the "Dalitz-pair"[8] decay of the π° ($\pi^\circ \to \gamma + e^+ + e^-$). The lifetime of the π° is obtained from the direct measurement of the flight distances of the neutral pions. The major difficulty in this method is that one is dealing with an average effect as small as 0.05 μm which is masked by a large error distribution. (A typical emulsion-grain radius is 0.3 μm).

Recently Shwe *et al.*[5] at Berkeley attempted to overcome the above difficulty by using a new method that utilizes the relativistic flight-path dilation produced by high pion velocity. This method uses the Dalitz-pair decay mode of the π° produced by the interaction stars of 3.5 BeV/*c* π^- in Ilford *K*5 nuclear emulsion, yielding a mean π° decay distance of the order of 0.5 μm. The longer decay distance is a distinct advantage over the $K_{\pi 2}^+$ method, but the knowledge of the unique π° momentum vector has been sacrificed.

Von Dardel *et al.*[9] performed a counter experiment that measured the yield of positrons from the decay of high-energy π°'s that were produced in platinum foils of 3–60 μm thickness exposed to the internal proton beam of the CERN proton synchroton. The neutral pions decay into two photons and the photons convert into electron-positron pairs in the target itself. Neutral pions are produced uniformly throughout the foil, but due to the finite π° decay length the positron yield is not expected to depend linearly on the target thickness. A deviation of 1.5% was observed and the π° decay length was derived from this effect.

Another method, first suggested by Primakoff,[10] was used by Bellettini *et al.*[11] to determine the mean lifetime of the π^0. The photoproduction of the π^0 in the Coulomb field of a heavy nucleus is measured, and the cross section for this process is inversely proportional to the π^0 lifetime. However, this approach also has difficulties. The Primakoff cross section is rather small and the angular distribution of the mesons is strongly collimated about the direction of the incident photon. At these small angles, there are in addition to the lifetime effect, other possible effects such as inelastic production via the usual nuclear photoproduction process, and multiple π° production giving rise to correlated γ-γ events. To interpret the experimental results, one has to deal with a theory involving several unknown parameters.

Presented here are the results of an experiment[12] to measure the lifetime of the π° by the method of Harris, Orear, and Taylor.[2] However, the number of events used is two to three times greater than the number used in previous experiments using this method.

* Research supported in part by grants from the National Science Foundation and by an equipment loan contract with the U. S. Office of Naval Research. This paper is based on a thesis submitted (by P. S.) in partial fulfillment of the requirements for the Ph.D. degree in Physics at Stevens Institute of Technology.
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‡ Supported by a National Aeronautics and Space Administration traineeship.
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II. METHODS FOR ESTIMATING LIFETIME

A. Maximum Likelihood

In the determination of the mean lifetime of the neutral pion by the method of Harris, Orear, and Taylor,[2] one observes the distance between the K^+ ending, for $K_{\pi 2}{}^+$ decay at rest, and the origin of the Dalitz pair along the extrapolated π^+ line of flight. A correct statistical procedure is to calculate the relative *a priori* probability that the n observed decay distances and their n errors turn out the way they did as a function of the lifetime. This relative probability is known as the likelihood function. Since the distribution of true flight distances is exponential, and the distribution of measured gaps is assumed to be Gaussian distributed about each true distance, the likelihood function for n events is[2]

$$Q(\tau) \propto \prod_{i=1}^{n} \frac{1}{(2\pi)^{1/2}\sigma_i} \frac{1}{\beta\gamma c\tau} \times \int_0^\infty \exp\left\{-\left[\frac{l_i}{\beta\gamma c\tau}+\frac{(g_i-l_i)^2}{2\sigma_i^2}\right]\right\} dl_i, \quad (1)$$

where l_i is the true unknown flight distance for the ith event, g_i is the measured flight distance, σ_i is the standard deviation of g_i about l_i, and τ is the mean π° lifetime; β and γ are given by: $\beta=v/c$, $\gamma=(1-\beta^2)^{-1/2}$, where c is the velocity of light in vacuum, and v is the velocity of the π°. The measured flight distances g_i are given by

$$g_i = x_i \sec\alpha_i, \quad (2)$$

where α_i is the dip angle (the angle between the π° momentum and the plane of observation) for the ith event and x_i is the distance between the K^+ decay point and the intersection of the projections of the π^+ and pair momenta on the plane of observation. The K^+ decay point is determined by the method first used by the Oxford group,[13] in which the foot of the perpendicular from the center of the last K^+ grain to the projection of the π^+ momentum is taken as the K^+ decay point. The errors σ_i are assumed to be of the form[14]

$$\sigma_i^2 = [A^2 + B^2(\cot^2\theta)_i]\sec^2\alpha_i, \quad (3)$$

where $(\cot^2\theta)_i$ is the square of the cotangent of the angle between the intersection of the projections of the π^+ and pair momenta on the plane of observation and A and B are constants to be determined. Note that σ_i is a minimum for $\theta=90°$, and becomes infinite at $\theta=0°$. Once A and B are known, the method of maximum likelihood consists of choosing, as an estimate of the unknown true value of τ, the particular value that renders Q as great as possible. Since $\ln Q$ attains its maximum for the same value of τ as Q we thus have to solve the likelihood equation

$$\partial \ln Q/\partial\tau = 0. \quad (4)$$

The value of τ that maximizes Q will be denoted by τ_{ml}. It can be shown[15] that the standard deviation of τ_{ml} is given approximately by

$$\sigma_{ml} = 1\bigg/\left(-\frac{\partial^2 \ln Q}{\partial\tau^2}\right)^{1/2}_{\tau_{ml}}. \quad (5)$$

The constants A and B are determined by measurements on τ^+ decays ($K^+ \rightarrow \pi^- + \pi^+ + \pi^+$). Each τ^+ decay is considered as a "model" of a $K_{\pi 2}{}^+$ plus associated Dalitz-pair event, using the τ^+ secondaries of least ionization and next to least ionization as "models" of an electron and π^+ respectively, of a true event. Since any measured gap between the τ^+ ending and the intersection of the projections of the "model" π^+ and pair momenta on the plane of observation must be due to error alone, the likelihood function for n such events is

$$Q_\tau(A,B) \propto \prod_{i=1}^{n} \frac{1}{(2\pi)^{1/2}\sigma_i} \exp[-g_i^2/2\sigma_i^2], \quad (6)$$

where all quantities refer to the "model" τ^+ events. The only unknowns in this function are the constants A and B in σ_i, which are assumed to be the same as the A and B for a true $K_{\pi 2}{}^+$ event. Hence the maximum likelihood values of A and B may be found by solving simultaneously the equations

$$\partial Q_\tau/\partial A = 0, \quad \partial Q_\tau/\partial B = 0. \quad (7)$$

B. Weighted Mean

Another correct statistical procedure for estimating the lifetime is to calculate the weighted mean. The standard deviation of the ith event in the over-all gap distribution is

$$S_i(\tau) = [\sigma_i^2 + (\beta\gamma c\tau)^2]^{1/2}. \quad (8)$$

The weighting factors are taken as the reciprocals of the S_i^2; hence the weighted mean estimate of τ is

$$\tau_{wm} = \frac{1}{\beta\gamma c} \sum_{i=1}^{n} g_i/S_i^2(\tau_{wm}) \bigg/ \sum_{i=1}^{n} 1/S_i^2(\tau_{wm}). \quad (9)$$

Since both the right and left side of Eq. (9) depend on the lifetime, the solution is obtained by iteration. The error of τ_{wm} is given by

$$\sigma_{wm1} = \frac{1}{\beta\gamma c}\left[\frac{1}{n-1}\sum_{i=1}^{n} (g_i - \beta\gamma c\tau_{wm})^2 / S_i^2(\tau_{wm}) \bigg/ \sum_{i=1}^{n} 1/S_i^2(\tau_{wm})\right]^{1/2}. \quad (10)$$

[13] R. F. Blackie, A. Engler, and J. H. Mulvey, Phys. Rev. Letters **5**, 384 (1960).

[14] This form is exactly equivalent to the form $\sigma_i^2 = (a^2 + b^2 \csc^2\theta_i)\sec^2\alpha_i$ used in Ref. 13.

[15] H. Cramér, *Mathematical Methods of Statistics* (Princeton University Press, Princeton, New Jersey, 1946).

Another direct estimate of the error of τ_{wm} is obtained from

$$\sigma_{wm2}=\frac{1}{\beta\gamma c}\left(\sum_{i=1}^{n} 1/S_i^2(\tau_{wm})\right)^{-1/2}. \quad (11)$$

Equations (1) through (11) will be used in Secs. V and VI below to estimate the lifetime of the π° and its associated error.

III. EXPOSURE AND SCANNING

An 84-pellicle stack of 6 in.×8 in.×600 μm Ilford G5 emulsion was exposed to a 400 Mev/c separated K^+ beam at the Bevatron of the Lawrence Radiation Laboratory of the University of California.[16] The beam kaons came to rest near the center of each pellicle, in an area ~1.5 cm×4 cm. It was desired that the density of stopped kaons be relatively high in order that the scanning time required to find a kaon decay event be fairly short. Therefore the stack was inserted in the beam after only one stage of separation, and a background of approximately 10 beam pions for each stopping K^+ was present. These pions were of minimum ionization, and traversed the entire stack. The density of kaon endings in the stopping region of the exposed stack was $\sim 2\times 10^4$ K^+/cm^3. The individual pellicles were alligned for scanning and track following by the method outlined in Ref. 17.

The stack was systematically area-scanned for K^+ meson endings with multiple secondaries and this scanning sample was separated into two groups. One group consisted of three secondary events, each with two or three of the secondaries having greater than 1.5 times minimum ionization, and the results have been previously reported.[18–21] The other group consisted of 269 events with two or more minimum ionizing secondaries. All were assumed to be events with an associated Dalitz pair or a nearby electron pair from an associated γ.

Two of these events were unmeasurable due to the emulsion peeling off the plates and were eliminated from further consideration. Whenever possible, obvious high-grain-density secondaries were followed to their endings and identified by their decay characteristics as either π^+ from τ'^+ $(K^+\rightarrow\pi^++2\pi^\circ)$ decay or μ^+ from $K_{\mu3}^+(K^+\rightarrow\mu^++\nu_\mu+\pi^\circ)$ decay. The charged secondary from the K^+ decay for each of the remaining events was grain-counted.[22] Secondary tracks determined to be greater than 1.56 times minimum ionization were followed to their endings and identified whenever possible. All other charged secondaries were determined to be less than 1.56 times minimum ionization by at least 2.5 standard deviations. The group of 269 events consisted of 33 τ'^+ decays, 6 $K_{\mu3}^+$ decays, one K_{e3}^+ $(K^+\rightarrow e^++\nu_e+\pi^\circ)$ decay, 2 unmeasureable events, one $K_{\mu3}^+$ or K_{e3}^+ decay with an associated double Dalitz pair, 4 unidentified K^+ decays involving a secondary track of greater than 1.56 times minimum ionization, and 222 K^+ decays with a secondary track of less than 1.56 times minimum ionization. This grain counting criterion eliminated all τ'^+ events and approximately one-quarter of the $K_{\mu3}^+$ events[23] from the sample of 222 events, which was assumed to be $K_{\pi2}^+$ decays at rest with an associated Dalitz pair or external pair, plus a "background" of $K_{\mu3}^+$ and K_{e3}^+ events of approximately 26%. These 222 events, plus 14 previously reported Columbia events[2] (herein re-analyzed) were considered as candidates to be used in the determination of the neutral pion lifetime.

Note that a scanning loss of $K_{\pi2}^+$ decays with an associated Dalitz pair introduces no bias to the gap distribution. This is true because the π° flight distance is much less than the mean gap length between two grains for minimum ionizing tracks. Hence, the probability for overlooking such an event cannot be a function of the π° decay distance.

IV. MEASUREMENT TECHNIQUE

Following the Columbia method,[2] *camera lucida* drawings of each event to be used in the determination of the mean π^0 lifetime were made at a drawing board magnification of about 6000×.[24] A typical *camera lucida* drawing is shown in Fig. 1. The *camera lucida* had been first

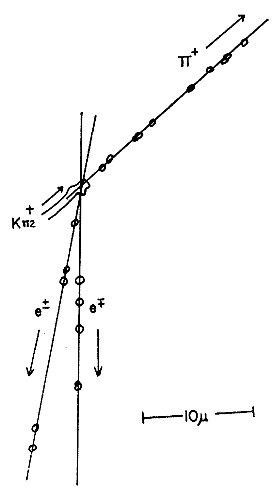

FIG. 1. *Camera lucida* drawing of a typical event.

[16] G. Goldhaber *et al.*, Lawrence Radiation Laboratory Report No. BEV-483, 1960 (unpublished).

[17] S. Taylor, G. Harris, J. Orear, P. Baumel, and J. Lee, Rev. Sci. Instr. **30**, 244 (1959).

[18] T. Huetter, S. Taylor, E. L. Koller, P. Stamer, and J. Grauman, Phys. Rev. **140**, B655 (1965).

[19] E. L. Koller, S. Taylor, T. Huetter, and P. Stamer, Phys. Rev. Letters **9**, 328 (1962).

[20] P. Stamer, T. Huetter, E. L. Koller, S. Taylor, and J. Grauman, Phys. Rev. **138**, B440 (1965); E. L. Koller, S. Taylor, T. Huetter, and P. Stamer, *ibid.* **129**, 1381 (1963).

[21] S. Taylor, E. L. Koller, T. Huetter, P. Stamer, and J. Grauman, Phys. Rev. Letters **14**, 745 (1965).

[22] For grain count versus energy data, see W. Barkas and D. Young, University of California Radiation Laboratory Report No. UCRL-2579 Rev. (unpublished).

[23] Estimated from the $K_{\mu3}^+$ energy spectrum; see V. Bisi, G. Borreani, R. Cester, A. Debenedetti, M. I. Ferrero, C. M. Garelli, A. Marzari-Chiesa, B. Quassiati, G. Rinaudo, M. Vigone, and A. E. Werbrouck, Phys. Rev. Letters **12**, 490 (1964).

[24] Four events were drawn with a reduced magnification of about 3300× and one event with about 4800× in order to have a sufficient number of resolved grains for each track.

checked to assure that there was a constant magnification and no appreciable angular distortion over the central part of the field of view. Two drawings, by the same observer, were made for each event. These two drawings were checked for consistency and if found inconsistent, they were discarded and two new drawings were made. Any pertinent comments relevant to the identification of the last grain of the K^+ ending, the identification of grains for each track, and the resoluion of the pair near the K^+ ending, were put on the drawing. A minimum of three resolved grains was required for each track and an average of about seven grains per track was realized. The completed drawings, including comments, were then checked on the microscope by an independent observer, and any disagreements were resolved.

The determination as to which track was the π^+ and which two tracks constituted the Dalitz pair was first made qualitatively at the microscope by looking for the multiple scattering of the electrons. This procedure was no doubt also influenced by the angles between the tracks.

Verifax copies[25] were made of each drawing. To obtain the required x_i and $(\cot^2\theta)_i$ for Eqs. (2) and (3), respectively, two independent observers carried out the necessary geometric constructions on the copies of the original *camera lucida* drawings. The sign of the flight distance was taken to be positive if the decay point of the π^0, as defined above, lay on the opposite side of the K^+ decay point from the physically existing π^+ track. The idealized geometry of an event is shown in Fig. 2. For the sake of clarity, only one electron of the Dalitz pair is shown.

"Best-fit" lines were drawn by eye through the π^+ grains and through the pair grains (or through the grains of each electron if they were resolved near the K^+ ending). The drawing of these lines was standardized so that the π^+ line always was drawn first and then the pair line (or lines) was drawn. In the drawing of all lines, the observers took into account any comments written on the drawing. The grains closest to the K^+ ending were given the most weight, particularly in case of curvature of a track, either due to emulsion distortion or scattering. The observers also made a deliberate attempt not to be influenced by the position of the K^+ ending. At the completion of the drawing of a set of lines for an event, the center of the last K^+ grain was located and a perpendicular was dropped to the π^+ line. All geometric constructions were checked by an independent observer and any disagreements were resolved.

Measurements of the gaps $x_{ijk}(j, k=1, 2)$, as defined above, and the angles θ_{ijk} between the π^+ and pair lines, were carried out. The index i denotes the ith event, j denotes *camera lucida* drawing 1 or 2, and k denotes either the first or second set of geometric constructions for a fixed value of the index j. The dip angle α_i for the π^+ was measured as described below.

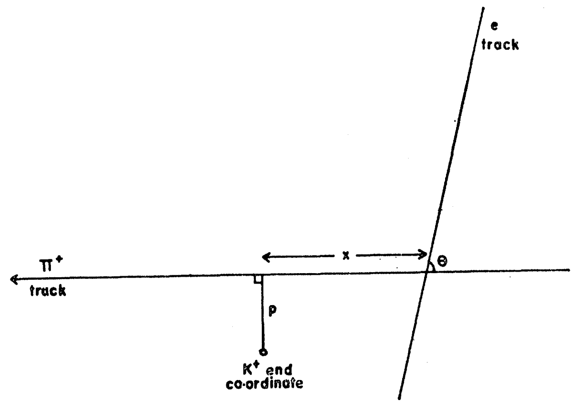

FIG. 2. Reconstructed event showing geometry and measured quantities, x and θ.

The identification of the π^+ and pair was checked by measurements which yielded the laboratory-system space angles between the tracks. The tangents of the dip angles between the plane of the emulsion and the secondaries were measured by using the fine z motion of the microscope and a calibrated eyepiece grid. Using these measured dip angles and the plane angles θ_{ijk}, described above, each event was analyzed to determine the laboratory-system space angle between each pair of tracks. It has been shown that the distribution of the opening angle of the Dalitz pair is strongly peaked for small angles.[8] Therefore, unless the "qualitative appearance" of the tracks of an event in the microscope strongly indicated otherwise, the Dalitz pair was assumed to be the pair of tracks having the smallest space angle between them. These measurements resulted in the reidentification of the π^+ and pair for 7 events. Except for one event, the final choice of the Dalitz pair was the pair of tracks with the smallest space angle between them. As a check, the experimental distribution of these opening angles in the center-of-mass system was calculated and compared with the theoretical distribution.[26] The χ^2 value is 19.3 for eleven degrees of freedom, corresponding to *a* χ^2 probability of $\sim 6\%$, which is not unreasonable.

The data for Eq. (6) were obtained from 200 τ^+ decays. The τ^+ sample was chosen such that the distribution of the plane angle between the two pion secondaries used in this analysis was the same as the experimental distribution of the plane angle between the π^+ and each electron of a true event. Two *camera lucida* drawings, by the same observer, were made for each event in the τ^+ sample. The same geometric construction and measurement process that was followed in the true $K_{\pi2}^+$ events was applied to the τ^+ drawings, yielding four sets of data, x_{ijk} and θ_{ijk}.

[25] Any error due to distortion introduced by this reproduction process was completely negligible.

[26] D. W. Joseph (private communication).

V. TREATMENT OF EXPERIMENTAL DATA

A. Determination of A and B

For each τ^+ event the measurements from the two sets of geometric constructions for each *camera lucida* drawing were averaged, and then the data from the two *camera lucida* drawings were averaged, to give a single set of data. The weighting functions for this averaging procedure were assumed to be of the same form as $1/\sigma_i^2$. Hence, preliminary values of the constants A and B were needed at each stage of the averaging. The preliminary A and B used in the first stage were obtained by inserting each set of measured τ^+ gaps, x_{ijk}, and τ^+ angles, θ_{ijk}, for a given j and k into Eq. (7) and solving for the respective set of constants, A_{jk} and B_{jk}. Then A_{jk} and B_{jk} were used to define the weighting functions

$$w_{ijk}=1\bigg/\left[\left(\frac{A_{j1}+A_{j2}}{2}\right)^2+\left(\frac{B_{j1}+B_{j2}}{2}\right)^2\cot^2\theta_{ijk}\right],$$

and the weighted averages of the two sets of geometric constructions for a given *camera lucida* drawing were determined:

$$x_{ij}=\sum_{k=1}^{2} w_{ijk}x_{ijk}\Big/\sum_{k=1}^{2} w_{ijk},$$

$$(\cot^2\theta)_{ij}=\sum_{k=1}^{2} w_{ijk}\cot^2\theta_{ijk}\Big/\sum_{k=1}^{2} w_{ijk}.$$

The second stage consisted of carrying out a weighted average of the data for the two *camera lucida* drawings. Again, preliminary values of the constants A and B were needed. These were generated by inserting each set of x_{ij} and $(\cot^2\theta)_{ij}$, for a given j, into Eq. (7) and solving for the corresponding maximum likelihood values A_j and B_j. New weighting functions were defined,

$$w_{ij}=1\bigg/\left[\left(\frac{A_1+A_2}{2}\right)^2+\left(\frac{B_1+B_2}{2}\right)^2(\cot^2\theta)_{ij}\right],$$

and the final weighted averages, x_i and $(\cot^2\theta)_i$, were calculated. These x_i and $(\cot^2\theta)_i$ were then inserted into Eq. (7) and the final maximum likelihood values of A and B were determined, yielding $A=0.18$ μm and $B=0.14$ μm. These values of A and B were used in the determination of the σ_i for the lifetime events. Note that A and B are comparable in magnitude to the radius of a grain (in the present experiment, $\sim$0.33 μm) which is just what one might expect for the inherent accuracy of this procedure. The values of A_{jk}, B_{jk}, A_j, B_j, A, and B obtained by the above procedure are presented in Table I. Note that the value of $(A^2+B^2)^{1/2}$ is approximately constant at each stage of finding A and B, and also that the value of $(A^2+B^2)^{1/2}$ gets smaller at successive stages, as it should.

B. Determination of g_i and σ_i

To obtain the g_i and σ_i for the $K_{\pi2}^+$ events, the four sets of data for any one electron (or unresolved pair) were averaged in the same manner as the data for a τ^+ event, using the same preliminary A and B at each stage as in the corresponding stage for the τ^+ data, and finally obtaining the averages x_i and $(\cot^2\theta)_i$. At this point the averaging procedure was complete for one electron and unresolved pair[27] events, and g_i and σ_i were calculated from Eqs. (2) and (3), respectively. However, for events involving two resolved electrons, a weighted average of the two gaps was obtained from

$$x_i=\sum_{l=1}^{2} w_{il}x_{il}\Big/\sum_{l=1}^{2} w_{il},$$

where

$$w_{il}=\frac{1}{A^2+B^2(\cot^2\theta)_{il}},$$

and the index l denotes either electron 1 or 2 for a fixed i. The flight distance g_i was calculated from Eq. (2). Since the two gaps involve independent intersections of the π^+ and electron lines, but the same K^+ decay point, the error is

$$\sigma_i^2=A^2/2+1\bigg/\left[\frac{1}{A^2/2+B^2(\cot^2\theta)_{i1}}+\frac{1}{A^2/2+B^2(\cot^2\theta)_{i2}}\right],$$

where, for any one measurement, the constant part of the error is assumed to arise equally from the K^+ decay point and from the intersection of the π^+ and electron

TABLE I. Computed values of A_{jk}, B_{jk}, A_j, B_j, A, and B (in units of μm) for the sample of 200 "model" τ^+ events.

j	1		2	
k	1	2	1	2
$A_{jk}, B_{jk}, [(A_{jk}^2+B_{jk}^2)^{1/2}]$	0.22, 0.14, [0.26]	0.24, 0.16, [0.29]	0.16, 0.19, [0.25]	0.19, 0.20, [0.27]
$A_j, B_j, [(A_j^2+B_j^2)^{1/2}]$	0.22, 0.14, [0.26]		0.16, 0.19, [0.25]	
$A, B, [(A^2+B^2)^{1/2}]$	0.18, 0.14, [0.23]			

[27] In five events, only one electron of the Dalitz pair was found. In 37 events the opening angle of the Dalitz pair was so small that it was impossible to ascertain the track assignment of the first few grains.

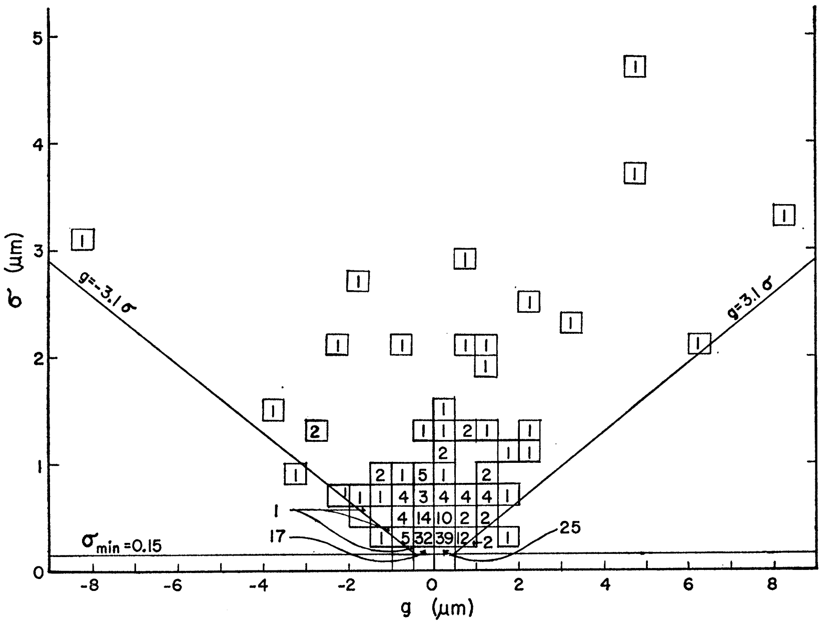

FIG. 3. Histogram of the 232 events used in the calculation of the π^0 lifetime. The numbers in the boxes are the number of events observed in the indicated increments of g and σ, where g and σ are defined in the text. The lines $g=\pm 3.1\,\sigma$ correspond very closely to the acceptance criterion given by Eq. (12).

lines. Thus, at the completion of the averaging procedure for the $K_{\pi 2}^+$ plus Dalitz pair events, there was a measured flight distance g_i and error σ_i for each event.

A correction was applied to eliminate external pairs from the sample. Let $P(d)$ be the probability of having a distance d from the origin of an electron pair (of small opening angle) to the first developed pair grain. Let $R(d)$ be the probability of either one of a pair of real γ's (having small opening angle and appropriate energy) converting to an electron pair at a distance d from their origin. The distance d was found such that $P(d)=R(d)$. This calculated distance was 12 μm. Any event where the distance from the last K^+ grain to the first pair grain exceeded 12 μm was discarded as a $K_{\pi 2}^+$ decay plus external pair. This procedure is appropriate since the mean gap length between grains and the mean free path for pair production in nuclear emulsion are large compared with the mean π^0 lifetime gap. Four of the Stevens events were discarded on the basis of this criterion. It is estimated that about 13 $K_{\pi 2}^+$ plus external pair events are within the 12-μm cutoff and remain in the sample. Some of these events are probably eliminated by the statistical process described in the next section.

The 218 remaining Stevens events plus 14 Columbia events[28] were used in the estimation of the π^0 lifetime. Figure 3 shows a histogram of these 232 events distributed according to flight distance g, and σ. Table II tabulates the different types of events.

TABLE II. Tabulation of $K_{\pi 2}^+$ plus electron-pair events.

	Unusable		Usable			
	External pair	Unmeasurable	One electron	Unresolved pair	Resolved pair	Total
Columbia[a]	0	0	0	8	6	14
Stevens	4	2	5	29	184	218

[a] Reference 2.

VI. RESULTS

The first maximum-likelihood estimate of the lifetime τ_{ml} was obtained by inserting the 232 average measured gaps g_i and errors σ_i into Eq. (1) and solving Eq. (4). The first weighted mean estimate τ_{wm} and the corresponding errors σ_{wm1} and σ_{wm2} were obtained from Eqs. (9), (10), and (11), thus obtaining the preliminary results listed as zeroth iteration in Table III.

Effects, such as the misidentification of the last K^+ grain, the scattering or curving of an electron or π^+ near the K^+ ending, the wrong assignment of grains near the K^+ ending, or the existence of remaining external pairs could give rise to events with spurious π^0 flight distances. An argument first used by Evans[6] indicates that such events, having large spurious values of g_i/σ_i (of either sign), have different effects upon the

TABLE III. Calculated values of τ's and σ's obtained at successive stages of applying the acceptance criterion of Eq. (12). All lifetimes and errors are in units of 10^{-16} sec.

Iteration	No. of events	τ_{ml}	σ_{ml}	τ_{wm}	σ_{wm1}	σ_{wm2}
0th	232	2.50		0.99	0.60	0.41
1st	229	1.50		0.88	0.53	0.41
2nd	227	1.05	0.47	0.74	0.52	0.42

[28] The emulsion pellicles for eight of the Columbia events were unavailable for remeasurement and the g_i and σ_i were obtained from original *camera lucida* drawings, using appropriate A and B values.

estimate of τ depending upon the method of calculation. In the weighted mean estimate, Eq. (9), both positive and negative values of g_i/σ_i are admitted with equal weight. However, the likelihood function, Eq. (1), involves the exponential decay distribution and thus large negative gap-to-error ratios, g_i/σ_i, contribute very little to the shape of the likelihood curve, whereas large positive gap-to-error ratios strongly affect the position of the maximum. Hence, τ_{ml} is expected to be greater than the true estimate if such events are present.

A criterion was introduced which was used to reject events that were inconsistent with the main set. If the measured flight distances of the 232 events used in the first lifetime estimate are assumed to have a Gaussian distribution about $\beta\gamma c\tau_{ml}$, less than one-half of an event would be expected outside of the range given by

$$|(g_i-\beta\gamma c\tau_{ml})/S_i(\tau_{ml})|\leq 3.1. \qquad (12)$$

Actually the measured flight distances are not Gaussian-distributed, but since $\beta\gamma c\tau_{ml}\ll\sigma_i$, Eq. (12) gives a reasonable acceptance criterion. All events were first tested against this acceptance criterion, and then an attempt was made to reinterpret those events that failed. By examining them at the microscope, it was found that for some of these events the track assignment of certain grains near the K^+ ending, or the location of the last K^+ grain could be plausibly reinterpreted. These reinterpreted events were then tested against the acceptance criterion. All events still failing this test were rejected. New estimates of τ_{ml}, τ_{wm}, σ_{wm1}, and σ_{wm2} were obtained from the revised set of events. Again, all events were tested using the new value of τ_{ml} and the above analysis was repeated. After two such iterations, all 227 events in the final revised set, including six of those that were reinterpreted, passed the acceptance criterion. Figure 4 shows the final likelihood function $Q(\tau)$ (normalized to unity at the maximum). The standard deviation of τ_{ml}, σ_{ml}, was obtained by solving Eq. (5) graphically. The results of the above calculations are tabulated in Table III.

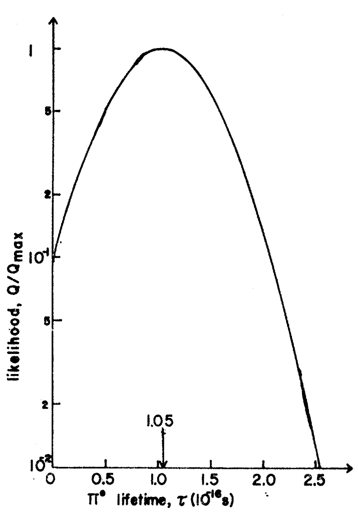

FIG. 4. The likelihood function Q normalized to unity at the maximum, and plotted versus the mean π^0 lifetime τ. The maximum likelihood value is $\tau_{ml}=1.05\times10^{-16}$ sec (before correction for "background").

The final maximum likelihood estimate of the lifetime τ_{ml} is in reasonable agreement with the final weighted mean estimate τ_{wm}. The three errors are also in reasonable agreement indicating that the data are internally consistent with the assumed form of σ_i and the values of A and B. Since the maximum-likelihood estimate uses the data most efficiently, but the weighted-mean estimate is less subject to systematic effects, the "best estimate" of the π^0 lifetime is taken to be the average of τ_{ml} and τ_{wm}, with an error given by the average of the errors. Hence, the result (still uncorrected for $K_{\mu3}^+$ and K_{e3}^+ "background") is $\tau_{\text{expt}}=(0.89\pm0.47)\times10^{-16}$ sec.

A correction to the final experimental value of the lifetime τ_{expt} is required by the presence of "background" events involving a π^0 from one of the three-body decay modes of K^+, namely, $K_{\mu3}^+$ and K_{e3}^+. In these three-body decay modes the direction and momentum of the π^0 is not unique. However, these "background" events are analyzed as though they were $K_{\pi2}^+$ decays, and hence contribute false flight distances to the experimental gap distribution. Thus, the mean experimental gap is given by

$$G_{\text{expt}}=B_{\pi2}G_{\pi2}+B_{e3}G_{e3}+B_{\mu3}G_{\mu3},$$

where $G_{\pi2}$, G_{e3}, and $G_{\mu3}$ are the unknown mean experimental gaps for π^0 from $K_{\mu2}^+$, K_{e3}^+, and $K_{\mu3}^+$ decay and $B_{\pi2}$, B_{e3}, and $B_{\mu3}$ are the respective relative proportions of the various modes in the experimental sample. The values of the B's are[29]

$$B_{\pi2}=0.74,\ B_{e3}=0.17,\ \text{and}\ B_{\mu3}=0.09.$$

The unknown gap $G_{\pi2}$ is given by $G_{\pi2}=\beta\gamma c\tau_{\pi^0}$, where τ_{π^0} is the true π^0 lifetime, whereas τ_{expt} is, in effect, defined to be given by

$$G_{\text{expt}}=\beta\gamma c\tau_{\text{expt}},$$

where β and γ are those for the π^0 from $K_{\pi2}^+$ decay. The mean gap for a "background" mode is

$$G_{\mu3,e3}=\langle(\beta\gamma c\cos\phi)_{\mu3,e3}\rangle_{\text{av}}\tau_{\pi^0},$$

where ϕ is the angle between the backward extrapolation of the charged lepton momentum and the neutral-pion momentum. The averages $\langle(\beta\gamma c\cos\phi)_{\mu3,e3}\rangle_{\text{av}}$ are taken over the energy and angular distribution of the π^0 and over the Dalitz-pair electron distribution (since it is the electrons which are observed experimentally).

[29] The branching ratios used to calculate $B_{\pi2}$, B_{e3}, and $B_{\mu3}$ were taken from Ref. 7, but the $K_{\mu3}^+$ branching ratio was multiplied by 0.75, since approximately one-quarter of this decay mode was eliminated by grain counting.

The corrected π^0 lifetime is given by

$$\tau_{\pi^0}=\tau_{\text{expt}}\left[(1-B_{e3}-B_{\mu3})\left(1+\frac{B_{e3}\langle(\beta\gamma c\cos\phi)_{e3}\rangle_{\text{av}}}{B_{\pi2}\beta\gamma c}+\frac{B_{\mu3}\langle(\beta\gamma c\cos\phi)_{\mu3}\rangle_{\text{av}}}{B_{\pi2}\beta\gamma c}\right)\right]^{-1}.$$

Note τ_{π^0} is approximately of the form $\tau_{\text{expt}}(1+\epsilon)$, where ϵ is small. The averages were estimated from available information on K_{e3} and $K_{\mu3}$ decay,[30] and the error in ϵ was conservatively estimated at 50%. The result is $\tau_{\pi^0}=\tau_{\text{expt}}(1.11\pm0.05)$. Hence, the final corrected value of the mean lifetime of the neutral pion is

$$\tau_{\pi^0}=(1.0\pm0.5)\times10^{-16}\text{ sec}.$$

VII. CHECKS ON THE METHOD

A comparison of the Columbia[2] and Oxford[13] methods for the determination of the K^+ decay point was carried out. In the Columbia method, the K^+ decay point is taken as the intersection of a line drawn through the last few K^+ grains with a line drawn through the π^+ grains. The Oxford method is described in Sec. II above.

TABLE IV. Computed values of A, B, and $(A^2+B^2)^{1/2}$ (in units of μm) as the number of "model" τ^+ events n was increased.

n	A	B	$(A^2+B^2)^{1/2}$
65	0.203	0.143	0.248
100	0.181	0.143	0.231
150	0.181	0.136	0.226
200	0.175	0.144	0.227

The methods were compared by using 71 of the "model" τ^+ events. The "best" determination of the K^+ decay point was taken as the intersection of the angle bisectors of the triangle formed by fitting lines through each of the three pion secondaries. The distances between this "best" decay point and the decay points given by the Columbia and Oxford methods were then compared for various values of the angle between the K^+ ending and the "model" π^+ line. It was found that the two methods were in good agreement for values of this angle near 90°, but that the Oxford method gave significantly better K^+ endings for small values of the angle.

In order to check the sensitivity of the lifetime and error results to the assumed form of the σ_i, the estimates τ_{ml}, τ_{wm}, σ_{wm1}, and σ_{wm2} have been completely recalculated using

$$\sigma_i^2=[A'^2+B'^2(\cot^2\theta/2)_i]\sec^2\alpha_i. \quad (13)$$

TABLE V. Computed values of the estimates τ_{ml}, τ_{wm}, σ_{wm1}, and σ_{wm2} for an unbiased sample of 55 events, using sets of A and B values that corresponded approximately to the ranges of A and B observed as the number of τ^+ events was increased. All values of τ's and σ's are in units of 10^{-16} sec and A and B are in μm.

A	B	$(A^2+B^2)^{1/2}$	τ_{ml}	τ_{wm}	σ_{wm1}	σ_{wm2}
0.181	0.101	0.207	2.50	1.62	1.19	0.77
0.203	0.101	0.227	2.25	1.60	1.23	0.84
0.181	0.143	0.231	2.25	1.62	1.09	0.84
0.203	0.143	0.248	2.00	1.63	1.12	0.92

The application of the criterion of Eq. (12) rejected the same events as before. The final results for 227 events are $\tau_{ml}=1.05\times10^{-16}$ sec, $\tau_{wm}=0.72\times10^{-16}$ sec, $\sigma_{wm1}=0.51\times10^{-16}$ sec, and $\sigma_{wm2}=0.40\times10^{-16}$ sec which are consistent with those obtained by using σ_i of the form of Eq. (3). However, for the numerical values of A and B found in the present experiment, Eq. (3) can be expanded in the form of Eq. (13) plus fairly small terms; thus the two error functions are not completely independent.

The "stability" of the constants A and B was checked by calculating their numerical values at successive stages as the number of τ^+ decays was increased. See Table IV. Note that after the first 100 τ^+'s the values of A and B are reasonably stable.

The sensitivity of the lifetime to the values of A and B was checked by calculating τ_{ml}, τ_{wm}, σ_{wm1}, and σ_{wm2} for an unbiased sample of 55 events, using sets of A and B values that correspond approximately to the ranges of the values of A and B observed in the "stability" test above. See Table V. The range of the lifetime estimates is seen to be small compared to the errors. Note that in τ_{ml} and σ_{wm2}, $(A^2+B^2)^{1/2}$ is the significant quantity, while τ_{wm} is insensitive to changes in A and B over the range tested.

VIII. COMPARISON WITH OTHER EXPERIMENTS AND THEORY

Table VI summarizes the results of recent measurements of the π^0 lifetime, using the various methods described in the Introduction. The methods used in the first three measurements are very similar to that of the

TABLE VI. Summary of recent measurements of the π^0 lifetime, where τ is the mean lifetime and n (only given for emulsion experiments) is the number of events used in the calculation.

τ (10^{-16} sec)	n	Experiment
1.9 ± 0.5	76	Glasser *et al.*[a]
$2.3_{-1.0}^{+1.1}$	45	Tietge *et al.*[b]
$1.6_{-0.5}^{+0.6}$	67	Evans[c]
1.7 ± 0.5	103	Shwe *et al.*[d]
1.05 ± 0.18		von Dardel *et al.*[e]
0.73 ± 0.105		Bellettini *et al.*[f]
1.0 ± 0.5	232	Present experiment

[a] Reference 3.
[b] Reference 4.
[c] Reference 6.
[d] Reference 5.
[e] Reference 8.
[f] Reference 10.

[30] J. L. Brown, J. A. Kadyk, G. H. Trilling, R. F. Van deWalle, B. P. Roe, and D. Sinclair, Phys. Rev. Letters **7**, 423 (1961); G. L. Jensen, F. S. Shaklee, B. P. Roe, and D. Sinclair, Phys. Rev. **136**, B1431 (1964); S. Furuichi, Nuovo Cimento **7**, 269 (1958).

present experiment. The mean lifetime of the π^0 as found in the present experiment is in good agreement with the other values quoted in Table VI.

Goldberger and Treiman,[31] using extended dispersion relation techniques, have calculated theoretically a π^0 lifetime value of 0.5×10^{-16} sec. Bose[32] extended the Goldberger-Treiman formula for the charged-pion decay to obtain the rate of neutral-pion decay from a study of the Compton scattering of protons. This calculation yielded a π^0 lifetime value of 1.4×10^{-16} sec. Sternglass,[33] using a semiclassical model of the Bohr-Sommerfeld type, has investigated the relativistic electron-positron–pair system in the limit of high velocities. He showed that a lowest state exists and that the lowest state possesses an energy approximately equal to the π^0 rest energy. The lifetime of the system against annihilation into two gammas is calculated to be 2.06×10^{-16} sec. All of the above theoretical calculations are in reasonable agreement with the value obtained by the present experiment.

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ACKNOWLEDGMENTS

We wish to thank Dr. E. Lofgren, Dr. W. Wenzel, and the staff of the Lawrence Radiation Laboratory, University of California, for making the exposure possible. We also thank Dr. J. Mulvey for his extensive collaboration during the exposure and for arranging to have certain of our early lifetime calculations carried out on the Oxford Ferranti "Mercury" computer. Most of the calculations in this experiment were carried out on the Stevens IBM 1620 computer and we acknowledge the cooperation of the staff at the Stevens Computer Center. We gratefully acknowledge the invaluable help of D. Moran, G. Taplin, and O. Wayne, who carried out the great bulk of the scanning. We also thank E. Dulberg, L. Frank, and A. Radkowski for help with scanning and computing.

PHYSICAL REVIEW VOLUME 151, NUMBER 4 25 NOVEMBER 1966

Σ Hypernuclei and Σ^+-Proton Scattering*

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(Received 28 June 1966)

By analyzing the Σ^+-proton scattering data at low energies, it is found that the triplet Σ-nucleon interaction in the $I=\frac{3}{2}$ state is certainly not strong enough to support the formation of a particle-stable Σ^-n or Σ^+p system. The singlet Σ-nucleon interaction, on the other hand, could be quite strong, but it is probably still not sufficient to create a bound two-body Σ hypernucleus. Also, because of the relative weakness of the triplet interaction, it can be concluded that a bound hypernucleus Σ^-nn very likely does not exist. Using the results obtained in this investigation, speculations are also made concerning the double-Σ hypernuclear systems $\Sigma^-\Sigma^-n$ and $\Sigma^-\Sigma^-nn$.

I. INTRODUCTION

Up to the present time, quite a number of Λ hypernuclei have been observed.[1,2] On the other hand, there has not been a single uniquely identified Σ-hypernucleus event reported. The main reason for this is that when a charged Σ particle becomes bound to a nuclear system containing both neutrons and protons to form a Σ hypernucleus, it would quickly react to produce a Λ particle and hence, such a Σ hypernucleus would not be expected to be observed. Thus, the only types of Σ hypernuclei which could be stable for a time comparable to the lifetime of the Σ hyperon are those which are composed of one or more Σ^- particles with neutrons, and their charge-symmetrical counterparts of Σ^+ particles and protons.

In this investigation, we study the possibility of the formation of particle-stable Σ hypernuclear systems Σ^-n and Σ^-nn. This study is motivated by the fact that scattering results of Σ^+ on protons are beginning to

* Work performed under the auspices of the U. S. Atomic Energy Commission.
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