vertical cylindrical tube with open ends, could be carried out if warranted by an experimental situation. It should also be noted that there is nothing in the present calculation that precludes its applicability to superconducting objects.[4b]

One aspect of the role of the supporting constraints was alluded to early in Sec. II, but not discussed further. This is the fact that there is a gravitation-induced electric field associated with them as well as with the object that they support. If the object is a closed metallic shell and the constraints are on the outside, it is apparent that they have no effect on the field inside. In general, if the constraints are themselves metallic, they may be considered as extensions of the object, and sufficiently remote parts contribute a negligible amount to the field. However, the field around the sphere of Sec. IV will depend to some extent on the size, shape, and orientation of the constraints.

While the general theory developed in this paper is applicable to nonconducting objects and constraints as well, the calculation would differ considerably in detail. Since it seems likely that stray charges would greatly complicate any measurement of the gravitation-induced electric field near a dielectric, no attempt has been made to obtain a theoretical result. There is, however, one situation that can be approached in a straightforward manner: that in which various metallic parts of the object and constraints are separated by thin nonconducting wafers. Their direct contribution to the field can be made very small by making the wafers very thin, and they then serve merely to permit the metallic parts to be at different potentials under the influence of the test charge, and to prevent the flow of induced charge from one part to another.

It is natural to think of enhancing the gravitation-induced electric field by substituting for gravitational force the much larger centrifugal force that can be obtained by rapid rotation. However, we have been unable to obtain a treatment as simple and general as that presented here, in the case of a rotating solid. This does not perhaps seem surprising when it is remembered that a rotating superconductor has qualitatively different properties from a rotating normal metal,[5] while as we have seen they have the same behavior with respect to gravitation when at rest.

[4b] *Note added in proof.* For a discussion of a related phenomenon, see B. S. DeWitt, Phys. Rev. Letters **16**, 1092 (1966).

[5] F. London, *Superfluids* (Dover Publications, Inc., New York, 1961), Vol. 1, p. 78.

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Primary Cosmic-Ray Spectrum at High Energies and Spectra of γ Rays and Muons in the Atmosphere*

YASH PAL AND S. N. TANDON
Tata Institute of Fundamental Research, Bombay, India
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The shape of the primary cosmic-ray spectrum at very high energies and its connection with the spectra of high-energy γ-rays and muons in the atmosphere have been discussed. Using a simple formulation for calculating secondary spectra arising from power-law primary spectra cutoff at arbitrary points, it is shown that a specific two-component model for the primary spectrum explains the present observations on the high-energy γ-ray spectrum at all altitudes and the muon spectrum at sea level, without invoking any change in the character of the interactions at high energies. Such a spectrum is also consistent with air-shower observations.

1. INTRODUCTION

ABOVE 100 GeV the methods which have been so far used for measuring the energy of primary cosmic-ray particles are essentially calorimetric in character. For energies greater than $\sim 10^{14}$ eV air showers are used, while below this the current information comes from the study of the energy going into the soft component in interactions produced by a primary particle in heavy-metal emulsion assemblies and arrangements using absorbers and ionization chambers. Transforming a measured particle energy spectrum into an energy-per-nucleon spectrum requires additional information on the chemical composition of the primary nuclei as a function of energy, which is usually not available. However, the following statements may be made regarding the chemical composition:

(i) The relative number of high-energy interactions produced by primary particles of different charges in large emulsion assemblies exposed at the top of the atmosphere suggests that, at least up to 10^{13} eV/nucleon, the primary composition is not very different from that found at low energies.

* A preliminary version of this paper was presented at the International Cosmic Ray Conference in London in 1965.

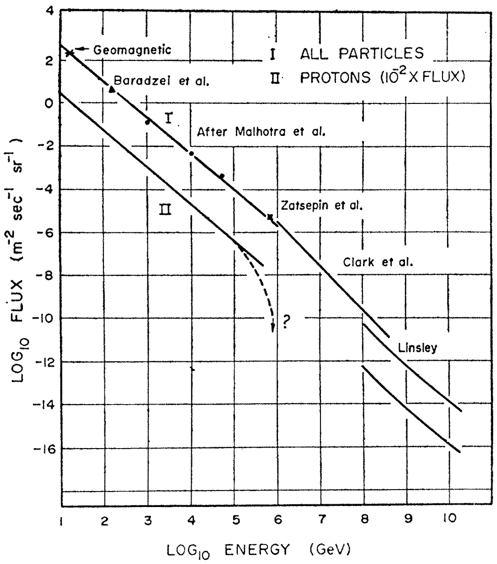

FIG. 1. Integral particle energy spectrum (curve I) from 10 to 10^{10} GeV. The points below 10^5 GeV are obtained by multiplying the measured or estimated proton flux by a factor of 2 (see text). Curve II gives the integral energy spectrum of protons on the assumptions that up to 10^5 GeV the chemical composition is same as around 10 GeV and that beyond 10^8 GeV only protons are present. The dotted curve corresponds to a rigidity cutoff. (a) L. T. Baradzei, V. I. Rubtsov, Y. A. Smorodin, M. V. Solovyov, and B. V. Tolkachev, J. Phys. Soc. Japan, Suppl. A–III **17**, 433 (1962). (b) G. T. Zatsepin, S. I. Nikolsky, and G. B. Khristiansev, in *Proceedings of the 1963 IUPAP Cosmic Ray Conference, Jaipur, India* (Commercial Printing Press, Ltd., Bombay, India, 1963), Vol. 4, p. 100. (c) G. Clark, H. Bradt, M. LaPointe, V. Domingo, I. Escobar, K. Murakami, K. Suga, Y. Toyoda, and J. Hersil, *ibid.* p. 65. (d) J. Linsley, *ibid.* p. 77. (e) Reference 7.

(ii) Changes in the core structure and other characteristics of air showers beyond 10^{15} eV suggest that there may be a dominance of heavy nuclei at these energies.[1–3]

(iii) The smallness of fluctuations in the number of muons in showers of fixed size suggest that the primary beam is relatively pure in its composition beyond 10^{17} eV and presumably consists only of protons.[4,5]

It can be shown that, for a chemical composition like that observed for the latitude-sensitive part of the cosmic-ray spectrum, the total particle flux at any energy would be almost exactly twice the proton flux at that energy. (If α is the ratio of nuclei of charge Z to protons above a certain rigidity, then the ratio above the same total energy will be αZ^γ, where γ is the power-law exponent of the integral energy spectrum.)

The particle energy spectrum given by curve I in Fig. 1 is based on geomagnetic measurements and the results of other relevant experiments listed therein, and in this figure the points below 10^{14} eV are obtained by multiplying the measured or estimated proton fluxes by a factor of 2. Curve II gives the proton energy spectrum based on the assumption that the chemical composition up to 10^{14} eV is the same as at low energies and that beyond 10^{17} eV only protons are present.

Though there is some discrepancy in the absolute value of the flux in the air-shower region, the following features are fairly clear from this collection of data:

(a) From 10^{10} eV to $\sim 10^{15}$ eV the particle energy spectrum may be represented fairly well by the power-law expression:

$$N(>E) = (2.08\times10^4)[E/(1\text{ GeV})]^{-1.67}\text{ m}^{-2}\text{ sr}^{-1}\text{ sec}^{-1}.$$

The flux of protons is about one-half, and the flux of all nucleons (including those forming parts of heavy nuclei) is about two-thirds (more nearly 0.675), of the flux of nuclei above the same energy. The differential flux of all nucleons from 10^{10} eV to a few times 10^{14} eV is then

$$N(E) = (2.35\times10^4)[E/(1\text{ GeV})]^{-2.67}\text{ m}^{-2}\text{ sr}^{-1}\text{ sec}^{-1}\text{ GeV}^{-1}.$$

(b) Beyond 5×10^{17} eV the slope of the particle energy spectrum is ~ 1.7.

(c) From 6×10^{15} eV to 5×10^{17} eV the slope of the particle energy spectrum is ~ 2.2.

The observed discontinuities in the particle energy spectrum, and possibly also in the chemical composition, have led to the hypothesis that the primary spectrum may contain at least two distinct components which may arise from different sources and, generally, may have different chemical compositions and spectrum slopes.[1,2] If the low-energy spectrum is subject to a rigidity cutoff at 10^{14}–10^{15} eV, this would be accompanied by a change in chemical composition of the incident beam, because nuclei of increasing charge will be cut off at progressively increasing total energy. As already mentioned, evidence for this effect has been given by the Sydney group[3] who find an increase in the fraction of multicored showers at a size of $\sim 10^6$. Several other properties of air showers in this size range also suggest that the showers are superpositions of a large number of low-energy showers produced by individual nucleons of a heavy nucleus.[1,2]

If there is a steepening in the energy-per-nucleon spectrum (implied by a rigidity cutoff), this should be consistently reflected in the spectra of secondary cosmic-ray particles in the atmosphere. Indeed, discontinuities in the γ-ray spectra in the atmosphere have been observed.[6] The position in energy of this discontinuity seems to depend on the altitude of observation.

[1] B. Peters, Nuovo Cimento **22**, 800 (1961).
[2] B. K. Chatterjee, Ph.D. thesis, Bombay University, 1964 (unpublished).
[3] A. D. Bray, D. F. Crawford, D. L. Jauncey, C. B. A. McCusker, P. C. Poole, M. H. Rathgeber, J. Ulrichs, R. H. Wand, M. M. Winn, and A. Ueda, Nuovo Cimento **32**, 827 (1964).
[4] J. Linsley and L. Scarsi, Phys. Rev. Letters **9**, 123 (1962).
[5] Y. Toyoda, K. Suga, K. Murakami, H. Hasegava, S. Shibata, V. Domingo, I. Escobar, K. Kamata, H. Bradt, G. Clark, and M. Lapointe, in *Proceedings of the International Conference on Cosmic Rays, London, 1965* (The Institute of Physics and the Physical Society, London, 1966), Vol. 2, p. 708.
[6] In this paper "γ-ray flux" incudes the flux of γ rays and electrons.

Thus the γ-ray spectrum steepens from an integral slope of $\sim$1.7 to a slope of more than 2 at $\sim 3\times10^3$ GeV near the top of the atmosphere,[7] while the steepening occurs at $\sim 1.5\times10^3$ GeV at airplane altitude[8]; on the other hand, at mountain altitude ($\sim$550 g/cm^2) the spectrum seems to have a slope of $\sim$2.2 right from $\lesssim 10^3$ GeV to $\sim 10^4$ GeV. It is of interest to investigate whether this apparently peculiar behavior of the γ spectrum with atmospheric depth can be understood in terms of changes in the primary spectrum and if so, to determine the parameters which characterize these changes. (It is clear that if the steepening of the γ-ray spectrum were due to a change in the character of the interaction of nucleons at an energy where they can produce γ rays of a few thousand GeV, then the steepening of the γ spectra would occur at about the same energy at all depths in the atmosphere, if this change did not affect the elasticity of nucleon interaction.) Therefore in this paper we calculate the high-energy γ-ray and muon spectra for a two-component primary spectrum; the first component (A) with a slope of 1.67 and breaking at a rigidity E_c, to be fixed by comparison with experiment, and beyond that a second component (B) whose intensity and slope have to be assigned so as to agree with the γ-ray data and the available information on the high-energy end of the air shower spectrum. It is shown that a consistent solution is possible, without invoking any change in the character of the interaction.

2. CALCULATION OF HIGH-ENERGY γ-RAY AND MUON SPECTRA

We have made a simple general formulation by which various secondary spectra for a primary power-law spectrum with a cutoff either on the low-energy side or on the high-energy side can be calculated. The secondary spectra in the atmosphere are easily calculated for a power-law primary spectrum without any break (see for example Pal and Peters[9]). If a power-law spectrum with a break can be fitted, in the neighborhood of the break, to a sum of many continuous power laws, then the secondary fluxes will be given by the sum of individual contributions from these spectra for energy regions corresponding to the neighborhood of the break; while for energy regions where primary energies contributing to the flux are far away from the break, the fluxes are again the same as obtained by using the asymptotic slope on the relevant side of the break.

With this in view a least-squares fit to a step function

$$\begin{aligned} f(x)&=1.0, \quad \text{for} \quad x<1; \\ &=0.0, \quad \text{for} \quad x>1, \end{aligned} \tag{2.1}$$

was made in the following form:

$$f(x)=\sum_{j=1}^{11} c_j x^{(j-6)/10}, \tag{2.2}$$

for $10^{-3}<x<10^3$. Now a power-law spectrum of integral slope γ with a break at $E=1$ on the high-energy side can be approximated by

$$N(E)dE=\frac{S_0}{E^{\gamma+1}}dE\sum_{j=1}^{11} c_j E^{(j-6)/10}, \tag{2.3}$$

and one with a break on the low-energy side by

$$N(E)dE=\frac{S_0}{E^{\gamma+1}}dE\Big(1-\sum_{j=1}^{11} c_j E^{(j-6)/10}\Big), \tag{2.4}$$

for $10^{-3}<E<10^3$.

With this procedure, the basic calculation required is that for a continuous power-law primary spectrum of an arbitrary slope. Pal and Peter's model[9] for nucleon propagation and meson production by cosmic rays has been used to derive formulas for secondary spectra for a power-law primary (with no cutoff); these formulas are presented below:

(a) The differential muon flux at very high energies at any depth x is given by

$$F_\mu(E,x)=\frac{S_0\langle B(\gamma)\rangle}{E^{\gamma+2}}\frac{E_\pi}{r^{\gamma+1}\lambda}1\Big/\left(\frac{1}{\lambda_\pi}-\frac{1}{\Lambda(\gamma)}\right) \times\sum_{i=0}^{\infty}\frac{(-1)^i}{(i+1)(i+1)!}\left[\left(\frac{x}{\lambda_\pi}\right)^{i+1}-\left(\frac{x}{\Lambda(\gamma)}\right)^{i+1}\right]. \tag{2.5}$$

(b) Using the stationary solutions, given by Rossi and Greisen[10] for the propagation of a power-law spectrum of γ rays, the flux of γ rays at any depth x can be written as

$$\begin{aligned} F_V(E,x)=&\frac{S_0\langle B(\gamma)\rangle}{E^{\gamma+1}}\frac{1}{\lambda(\gamma+1)(K_1-K_2)} \\ &\times\Bigg[\Bigg(K_2\Big/\left(\frac{1}{\Lambda(\gamma)}+\lambda_2(\gamma)\right) \\ &-K_1\Big/\left(\frac{1}{\Lambda(\gamma)}+\lambda_1(\gamma)\right)\Bigg)e^{-x/\Lambda(\gamma)} \\ &+K_1\Big/\left(\frac{1}{\Lambda(\gamma)}+\lambda_1(\gamma)\right)e^{x\lambda_1(x)} \\ &-K_2\Big/\left(\frac{1}{\Lambda(\gamma)}+\lambda_2(\gamma)\right)e^{x\lambda_2(\gamma)}\Bigg], \end{aligned} \tag{2.6}$$

where

$$K_1=1+\frac{C(\gamma)}{\mu_0+\lambda_1(\gamma)} \quad \text{and} \quad K_2=1+\frac{C(\gamma)}{\mu_0+\lambda_2(\gamma)}.$$

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[7] P. K. Malhotra, P. G. Shukla, S. A. Stephens, B. Vijayalakshmi, J. Boult, M. G. Bowler, P. H. Fowler, H. L. Hackforth, J. Keereetaveep, V. M. Mayes, and S. N. Tovey, Nuovo Cimento **40**, 385 (1965).

[8] M. G. Bowler, P. H. Fowler, and D. H. Perkins, Nuovo Cimento **26**, 1182 (1962).

[9] Yash Pal and B. Peters, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **33**, No. 15 (1964).

[10] B. Rossi and K. Greisen, Rev. Mod. Phys. **13**, 240 (1941).

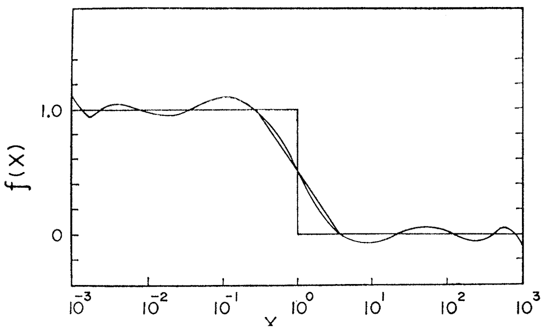

FIG. 2. The fit [Eq. (2.2)] to the step function [Eq. (2.1)] which has been used to simulate cutoffs in the primary cosmic-ray energy spectrum.

Here $S_0/E^{\gamma+1}$ represents the primary differential energy spectrum, $\langle B(\gamma)\rangle$ gives the ratio of rate of charged-pion production (per nucleon interaction length λ) to the flux of nucleons of the same energy at any point; $E_\pi = m_\pi h_0/\tau_\pi c$ is the energy at which the pion-decay mean free path equals the scale height of the atmosphere, h_0, and has a value 128 GeV for $h_0=7$ km; r is the effective fractional energy a muon receives in pion decay; λ_π is the interaction mean free path of pions; and $\Lambda(\gamma)$ is the attenuation length of nucleons;

$$\lambda_{1,2}(\gamma)=-\frac{[A(\gamma)+\mu_0]}{2}\pm\{[A(\gamma)-\mu_0]^2+4B_e(\gamma)C(\gamma)\}^{1/2};$$

$$A(\gamma)=(\tfrac{4}{3}+2b)\left[\frac{d}{d\gamma}\ln(\gamma!)+R-1+\frac{1}{\gamma+1}\right]+\tfrac{1}{2}-\frac{1}{(\gamma+1)(\gamma+2)};$$

$$B_e(\gamma)=2\left[\frac{1}{\gamma+1}-(\tfrac{4}{3}+2b)\left(\frac{1}{\gamma+2}-\frac{1}{\gamma+3}\right)\right];$$

$$C(\gamma)=(\tfrac{4}{3}+2b)\left(\frac{1}{\gamma}-\frac{1}{\gamma+1}\right)+\frac{1}{\gamma+2};$$

$$\mu_0=7/9-b/3;\quad b=[18\ln(183Z^{-1/3})]^{-1};$$

and R is Euler's constant. λ and λ_π have been taken to be 75 and 120 g/cm², respectively. To calculate $\langle B(\gamma)\rangle$ and $\Lambda(\gamma)$ it has been assumed that the fractional energy k given to a pion and the fractional energy η retained by a nucleon in an interaction have square distributions. Then we get

$$\langle B(\gamma)\rangle=N_\pi(k_{\max}{}^{\gamma+1}-k_{\min}{}^{\gamma+1})/(\gamma+1)(k_{\max}-k_{\min}), \quad (2.7)$$

and

$$\Lambda(\gamma)=\lambda/\{1-(\eta_{\max}{}^{\gamma+1}-\eta_{\min}{}^{\gamma+1})/(\gamma+1)(\eta_{\max}-\eta_{\min})\}. \quad (2.8)$$

Values of $k_{\max}$, $k_{\min}$, $\eta_{\max}$, and $\eta_{\min}$ have been taken as 0.14, 0.03, 0.75, and 0.34, respectively. Such a choice of these parameters is consistent with observations on low-energy cosmic rays (corresponding to a primary energy ~1000 GeV per nucleon). *However, the calculated fluxes mainly depend on the value of the parameter* $\langle B(\gamma)\rangle$ *and not on the specific choice of distributions of* k *and* η.[9]

The calculations were done with the help of a CDC 3600 computer.

3. RESULTS AND DISCUSSION

Figure 2 shows the fit, Eq. (2.2), to the step function, Eq. (2.1). This function, $f(x)$, when multiplied with a power law in x, does not give an abrupt cutoff at $x=1$; in fact the actual form of the steepening so obtained, though arbitrary, is more realistic than an abrupt cutoff.

The first object is to determine the position of the cutoff. For this the γ-ray flux at balloon and airplane altitudes has been calculated, using a slope of 1.67 for the primary integral spectrum and a high-energy (per nucleon) cutoff at an arbitrary value E_c. It is found that the calculation represents quite well the experimental data at balloon and airplane altitudes if the value of E_c is taken as 2×10^{14} eV. (Curves and points marked I and II in Fig. 3.) *Thus the component A of the primary spectrum is tentatively specified as having a chemical composition more or less the same as at low energy, an exponent for the integral spectrum of* ~1.67, *and a rigidity cutoff at about* 2×10^{14} eV *for protons.* It is interesting to note that from an analysis of the flux of high-energy nucleons in the atmosphere, Hayakawa *et al.*[11] concluded that the "bending" of the primary energy-per-nucleon spectrum should occur at 3×10^{14} eV.

Currently available γ-ray data at balloon and airplane altitudes extend only up to a few thousand GeV of γ-ray energy and can be explained without invoking any contribution from component B of the primary spectrum. However, the component B does exist and its effect would be felt in γ-ray spectra up to a few thousand GeV at lower altitudes. Guided by the air-shower data, we assume this component (i) to have the same slope as the component A (~1.67), (ii) to consist only of protons, and (iii) to have an intensity 1/20 the nucleon intensity or ~1/15 the proton intensity of the component A (see Fig. 1). With the above specifications for components A and B of the primary spectrum, the γ-ray spectrum at mountain altitude has been calculated and shown alongside the experimental points by curve III of Fig. 3. It is seen that the fit to the experimental data is quite good. The dotted curve near III gives the expected spectrum if the primary spectrum is a continuous power law without any cutoff.

Once the distributions of the parameters k and η are specified, the absolute values of γ-ray fluxes depend on the value of the parameter N_π, which determines

[11] S. Hayakawa, J. Nishimura, and Y. Yamamota, Progr. Theoret. Phys. (Kyoto) Suppl. 32, 104 (1964).

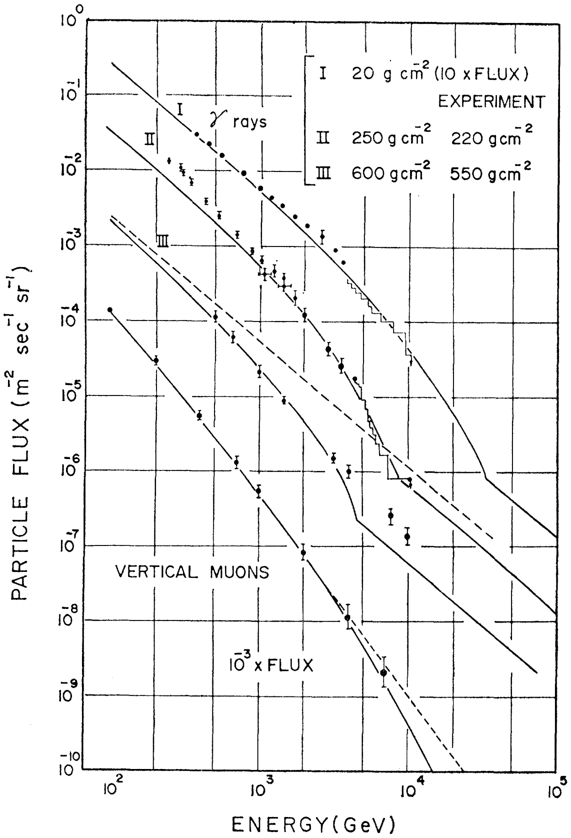

FIG. 3. Calculated integral energy spectra of γ rays with experimental points for vertical atmospheric depths of 20 g/cm^2 (I), 220 g/cm^2 (II), and 550 g/cm^2 (III) and of vertical muons at sea level. For calculations of γ-ray flux corresponding to measurements at vertical depths of 220 and 550 g/cm^2, effective depths of 250 and 600 g/cm^2, respectively, have been assumed. Experimental points are: I: Malhotra *et al.* (Ref. 7). II: Bowler *et al.* (Ref. 8). III: M. Akashi *et al.*, in *Proceedings of the 1963 IUPAP Cosmic Ray Conference, Jaipur, India* (Commercial Printing Press, Ltd., Bombay, India, 1963), Vol. 5, p. 326. Muons: after Menon and Ramana Murthy (Ref. 13). The dashed curves near III and the muon curve give the fluxes expected if the primary spectrum had no cutoff.

(through 2.7) the quantity $\langle B(\gamma)\rangle$ occurring in Eq. (2.5); N_π is determined by a simultaneous calculation of the high-energy muon spectrum, normalizing it to the 100-GeV point[12] of Menon and Ramana Murthy.[13] It is assumed in these calculations that pions are the only parents of muons and γ rays, and that charge independence is valid. The calculated and experimental vertical muon spectra at sea level are also shown in Fig. 3. As already shown by Duthie *et al.*,[14] the simultaneous fitting of γ-ray and muon intensity supports the conclusion that pion production remains dominant over kaon and hyperon production for primaries of energy up to $\sim 10^{14}$ eV/nucleon.

TABLE I. Calculated γ-ray flux at three depths for a few energies. Corresponding to each energy the effective attenuation length between 250 and 600 g/cm^2 is given in last column. The change in attenuation length with energy is due to a cutoff in component A of the primary spectrum (see text).

Energy (GeV)	Integral γ-ray flux per (m^2 sec sr) 20 g/cm^2	250 g/cm^2	600 g/cm^2	Attenuation length between 250 and 600 g/cm^2 (g/cm^2)
200	8.25×10^{-3}	1.19×10^{-2}	$7.6\ \times10^{-4}$	$\sim$127
500	1.79×10^{-3}	2.32×10^{-3}	1.22×10^{-4}	$\sim$119
1000	$5.2\ \times10^{-4}$	6.21×10^{-4}	$2.5\ \times10^{-5}$	$\sim$109
2000	1.52×10^{-4}	1.38×10^{-4}	4.54×10^{-6}	$\sim$102
4000	$3.8\ \times10^{-5}$	2.02×10^{-5}	$5.8\ \times10^{-7}$	$\sim$99

Thus we conclude that a primary spectrum with two components—the first having a rigidity cutoff around 2×10^{14} eV for protons, beyond which the second component (purely protons) with a slope the same as and intensity $\sim 1/20$ of the nucleon intensity of the first one dominates—explains the present data on altitude variation of γ-ray spectrum and muon spectrum at sea level without invoking any change in interaction characteristics at high energies. Though this picture of the primary spectrum is in general agreement with air-shower observations in the high-energy region, some discrepancies do remain in regard to the cutoff energy for the component A. McCusker *et al.*[3] find that multicored showers begin to dominate at a shower size of $\sim 10^6$, which would correspond to a primary energy of more than 10^{15} eV, while Chatterjee[2] finds that all the heavy nuclei seem to disappear at size 10^6, which would place the cutoff for protons at an energy of $\sim 3\times10^{14}$ eV, quite close to the position of the cutoff in component A arrived at by the present analysis. Past experience leads one to be wary of the results on γ-ray flux measurements; however, in order to increase the cutoff energy from 2×10^{14} to 10^{15} eV, the relative integral flux of 5000-GeV γ rays at airplane altitude will have to be increased by a factor of five.

Finally it is to be noted that with such a primary spectrum the measured attenuation length of high-energy γ rays will be a strong function of energy, as shown in Table I.

[12] The flux of muons of energy greater than 100 GeV given by Menon and Ramana Murthy is about 47% higher than the magnetic-spectrograph measurements of P. J. Hayman and A. W. Wolfendale [Proc. Phys. Soc. (London) 80, 710 (1962)] to which the calculations of Y. Pal and B. Peters (Ref. 9) were normalized.

[13] M. G. K. Menon and P. V. Ramana Murthy, in *Progress in Cosmic Ray and Elementary Particle Physics* (North-Holland Publishing Company, Amsterdam, to be published), Vol. 9.

[14] J. Duthie, P. H. Fowler, A. Kaddoura, D. H. Perkins, and K. Pinkau, Nuovo Cimento 24, 122 (1962).