Levels in ^{239}U†

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Thirty-seven levels have been observed in ^{239}U up to an excitation energy of 1692 keV with 12-MeV deuterons and the reaction ^{238}U$(d,p)^{239}$U. The ground-state Q value was determined as 2579±7 keV. A germanium detector, operated as a pair spectrometer for high-energy capture gamma rays and as a total-energy anticoincidence spectrometer for low-energy gamma rays, has been used to measure 27 high-energy transitions between 3150 and 4850 keV and 27 low-energy transitions between 490 and 900 keV. Analysis of the combined data suggests fairly definite assignments for 13 states below 373 keV. The presence of the 152-neutron energy gap is clearly indicated by the absence of states between 373 and 692 keV. From 692 keV upward, nine additional tentative assignments are made. The spectroscopic assignments are [band-head energy (keV) in parenthesis, followed by spin, parity, and (in brackets) asymptotic quantum numbers]: (ground state), $\frac{5}{2}+$[622] with superimposed rotational band to 11/2+; (133), $\frac{1}{2}+$[631] with rotational band to $\frac{9}{2}+$; (~173), $\frac{7}{2}+$[624] with rotational band to 13/2+; (692), $\frac{1}{2}+$[620] with rotational band to $\frac{5}{2}+$, and (742), $\frac{1}{2}-$[761] with anomalous rotational spacings to 11/2−. A table of $C_{j,l}$ values for deformation β of 0.20, 0.25, and 0.30 is presented in the Appendix for all Nilsson orbitals of interest in the actinide region. The relative intensities or signatures of the bands are found for the most part to be in good agreement with theory. Absolute magnitudes are a factor of 2 too high. This discrepancy is ascribed to Coulomb stripping effects.

I. INTRODUCTION

THIS study has three important aims. The first is the study of the levels in ^{239}U. There are no known parent radioactivities which populate ^{239}U, so decay-scheme studies are not possible. There are two complementary methods, both involving nuclear-reaction spectroscopy, which can be applied to the study of the levels in ^{239}U. The first is the study of neutron-capture gamma rays. High-energy and low-energy neutron-capture gamma-ray work have previously been done.[1–3] The second utilizes the (d,p) reaction.[4,5] The appropriate reaction is ^{238}U$(d,p)^{239}$U. Both neutron-capture gamma-ray spectroscopy and the (d,p) reaction are used in the present study. It is to be expected that the (d,p) reaction should populate most of the one quasiparticle states, whereas thermal-neutron capture in ^{238}U should populate a restricted set of states in ^{239}U with low spin. A compound state in ^{239}U with spin parity $\frac{1}{2}+$ is produced by thermal-neutron capture. Dipole depopulation results in excitation of states with spin parity $\frac{1}{2}\pm$ and $\frac{3}{2}\pm$. The complementarity of the two methods should make spin-parity assignments easier.

The second goal of the present research is to look for evidence of Coulomb stripping in these heavy nuclei. With 12-MeV deuterons and ~20-MeV Coulomb barrier the distance of closest approach of the deuteron to the nucleus is ~1.8 ^{239}U radii. Under these conditions it is essentially only the tail of the wave functions of the Nilsson orbitals which are involved in nuclear reactions. Evidence for this effect might be found in abnormal cross sections.

The third goal of the present research is to test the validity of the theoretical methods in calculating cross sections for the (d,p) reactions in the deformed region of the actinide elements. It has already been shown that in the rare-earth region it is possible to get good agreement between the computed theoretical differential cross section and the experimentally observed cross sections. Since the collective properties of the actinide nuclei appear to be as well described by the unified model as the rare-earth nuclei, it seems probable that calculations of the differential cross section involving the distorted-wave Born approximation (DWBA) and the Nilsson wave functions would compare equally well in this region of nuclei. However, if the (d,p) reaction results essentially from Coulomb stripping, additional difficulties may result.

During the course of this work a paper[5] on the levels in ^{239}U obtained by the reaction ^{238}U$(d,p)^{239}$U appeared in the literature. The (d,p) reaction spectroscopy results presented here agree for the most part with the measurements presented in this recent paper but differ considerably in their interpretation.

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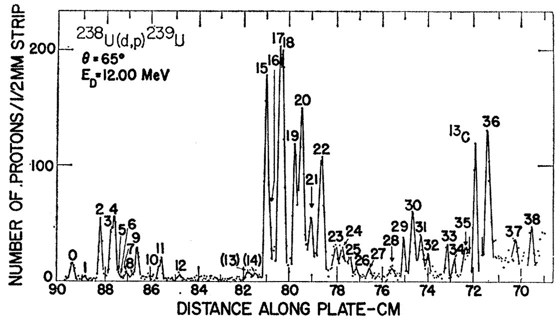

FIG. 1. Proton spectrum from the reaction $^{238}U(d,p)^{239}U$ observed with 12-MeV deuterons at a laboratory angle of 65°.

II. EXPERIMENTAL METHODS AND RESULTS

A. $^{238}U(d,p)^{239}U$ Reaction Spectroscopy

For this experiment a deuteron beam of 12.0 MeV was supplied by the Florida State University Tandem Van de Graaff accelerator.[6] The beam was focused through a series of slits which collimated it to a $\frac{1}{4}$-mm $\times$3-mm spot on the target, which then served as a line source for the 60-cm broad-range magnetic spectrograph. The beam current was approximately 0.7 μA of deuterons. The acceptance solid angle of the spectrograph was set at 1.78×10^{-4} sr for the normally deflected ray. The analyzed protons were recorded on an 8-mm $\times$100-cm strip along a series of 50-μ nuclear track plates which were spring loaded against hyperbolic rails to form the correct focal surface for the spectrograph. After development, the plates were scanned with a microscope in $\frac{1}{2}$-mm$\times$8-mm strips along the exposed region. The spectrograph was calibrated against ^{210}Po alpha particles and the calibration was checked by recording multiply charged oxygen atoms scattered from a ^{165}Ho target.

Target preparation was accomplished as follows: A layer of spectroscopically pure carbon was evaporated on glass slides over a substrate of Teepol. The carbon layer was 40–60 μg/cm^2 in thickness. A layer of ^{238}U as UO_2 was evaporated on the slides to a thickness of approximately 100 μg/cm^2. The ^{238}U was of essentially 100% purity. At Florida State University, the carbon-backed targets were floated off the glass in ion-exchanged water and picked up on aluminum target frames having an area of 0.8 cm^2. Loss of resolution because of carbon buildup on the beam spot of the target during exposure was reduced by pumping the target chamber with a mercury diffusion pump through a liquid-nitrogen trap. In addition, the beam spot was changed at regular intervals during the run.

The protons resulting from bombardment of the ^{238}U target with 12.0-MeV deuterons were analyzed at 65° with respect to the incident beam. The spectrum for 65° is shown in Fig. 1. The data are presented as proton tracks per $\frac{1}{2}$-mm strip of plate as a function of distance along the plate array. Energy resolution is approximately 12 keV full width at half-maximum, (FWHM). Computer least-squares fits of skewed Gaussian functions were made to lines in the entire spectrum. The energies of the groups are then determined from the positions of the centroids of the fitted Gaussians. Such a procedure is more accurate than one which selects, for example, the $\frac{1}{3}$ height of the leading edge of a group, since the former method utilizes all the data composing the group. In addition to improving the energy determination of single groups, this method allows the energies and areas of close-lying overlapping groups to be determined. This analysis was essential for treatment of the higher energy groups in Fig. 1.

The results of the above analysis are listed in Table I, where the Q values and excitation energies of ^{239}U levels are given. The ground-state Q value was measured with respect to the ground-state Q value for the reaction

TABLE I. Energy levels and Q values from the reaction $^{238}U(d,p)^{239}U$ at a laboratory angle of 65°. (g.s. = ground state.)

Level number	Q value (keV)	Excitation energy E(keV)	ΔE(keV)
0	2579	0(g.s.)	Ref.
1	2536	43	3
2	2481.7	97	2
3	2445.6	133.1	2
4	2438.6	145.2	3
5	2414.0	165	5
6	2406.0	173	5
7	2390.3	188.5	4
8	2359.0	220	5
9	2349.7	229.1	3
10	2279.0	300	4
11	2271.5	307.2	3
12	2205.7	373	4
13[a]	1956.0	623	5
14[a]	1928.3	651	5
15[b]	1885.7	693.2	2
16	1861.0	718	4
17	1836.6	742.3	2
18	1824.8	754	4
19	1780.4	798.6	2
20[b]	1756	823	3
21	1725	854	4
22	1685	894	5
23	1636	943	5
24	1612	967	5
25	1580	999	6
26	1554	1025	6
27	1512	1067	6
28[b]	1424	1155	6
29	1374	1205	6
30	1341	1238	5
31	1315	1264	5
32	1294	1285	5
33	1210	1369	5
34	1182	1397	7
35[c]	1138	1441	5
36[b]	1059	1520	3
37	956	1623	6
38	887	1692	5

[a] Possible impurity.
[b] Possible complex structure.
[c] The region corresponding to excitations from 1440 to 1510 keV is obscured by a strong proton group from carbon.

[6] The operation of the Florida State University Tandem Van de Graaff has been facilitated through support of the Office of Air Research of the U. S. Air Force and the Nuclear Program of the State of Florida.

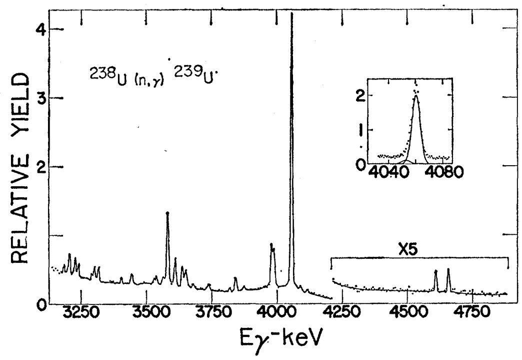

FIG. 2. Spectrum of $^{238}U(n,\gamma)^{239}U$ observed with a 3-mm-deep Li-drifted germanium detector placed inside a large NaI annulus. In this energy region the system is used as a 2-quantum escape spectrometer by gating the Ge detector on with a 1022±100-keV pulse from the annulus.

$^{12}C(d,p)^{13}C$ which is known to be 2721 keV. The Q value was determined to be 2579±7 keV. Peak 17 in the (d,p) spectrum has a measured Q value of 1836.6 keV. This corresponds within 4 keV to the strong neutron-capture line of 4059 keV described in the next section. The Q value also agrees well with the value of 2588±20 keV which was determined by Macefield and Middleton.

Because of our higher resolution, we have resolved doublets which were not resolved in the work of Macefield and Middleton, corresponding to several new levels. In addition, we report here levels beyond the limit of approximately 1500-keV excitation of the previous work.

B. $^{238}U(n,\gamma)^{239}U$

High-energy gamma rays following thermal-neutron capture by ^{238}U have been observed by the Chalk River Group[1] with a pair spectrometer operated to give a linewidth of 40 keV. Lines were seen at 3576, 3662, and 4062 keV, with a suggestion of significant unresolved radiation in the tail of the 4062-keV line. This group also reported seven resolved lines in the region between 540 and 640 keV. In a later measurement of the low-energy portion of the spectrum by Maier[2] with the bent-crystal spectrometer at Risø, some 38 certain and an additional 23 uncertain lines were reported up to an energy of 780 keV.

In the present work the portions of the spectrum between 490 and 900 keV and between 3150 and 4850 keV were investigated with a 3-mm-deep lithium-drifted germanium detector placed inside a large NaI annulus. The system was operated in the total energy (anti-coincidence) mode at low energies and in the 2-quantum escape (coincidence) mode at high energies. The addition of the NaI escape shield results in a substantial improvement in the ratio of line height to background in both modes.

In these experiments, capturing targets were contained in a beryllium holder and placed inside the thermal column of the Omega West Reactor at a position 6 m from the detector. Careful collimation of the gamma-ray beam permitted it to be intercepted by the germanium detector but not by the NaI annulus. The low-energy spectrum was obtained with a 29-g target of uranium enriched to 6000:1 in ^{238}U. At higher energies, where the detector efficiency is much reduced, the target was increased to 56 g and the gamma-ray beam was hardened with a $\frac{1}{2}$-in. Pb absorber. The resulting spectrum is shown in Fig. 2.

The high-energy portion of the spectrum was later repeated with a target of ordinary uranium; this part

TABLE II. Gamma rays from $^{238}U(n,\gamma)^{239}U$.

Energy E_γ(keV)	ΔE_γ(keV)	Intensity (per 10^3n) I_γ	ΔI_γ
498	2	7	3
522	1.5	28	4
537	3	43	8
542	3	19	7
552	2	105	15
561	3	13	3
580	2	24	5
588	3	14	8
592.4	1	48	11
601	5	20	5
605	5	12	5
612	2	80	15
629	4	23	5
638	3	18	4
658	4	6	2
683	4	11	2
690	3	24	5
698	4	9	2
713	4	7	2
722	4	7	3
750	3	7	2
770	3	6	2
788	3	8	2
797	3	7	2
834	3	18	4
847	3	16	4
856	3	13	3
3186	2	2.4	0.7
3201	3	2.2	0.7
3208	2	6.5	1.5
3229	2	6.3	1.5
3242	2	3.8	0.8
3293	3	2.0	0.7
3304	2	4.8	1.2
3318	2	4.5	1.2
3406	3	2.0	0.7
3446	3	4.1	1.2
3532	4	1.4	0.6
3540	3	2.3	0.7
3567	3	2.2	0.7
3584	2	25	4
3612	2	8.4	1.5
3639	2	6.8	1.3
3653	2	6.3	1.5
3739	3	2.3	0.7
3818	3	1.2	0.5
3843	2	5.6	1.5
3873	3	1.3	0.5
3982	2	15	2
3991	2	15	2
4052	4	5	2
4059.4	2	110	12
4090	5	2.2	0.5
4117	6	0.8	0.4
4610	2	2.1	0.5
4659	2	2.2	0.5

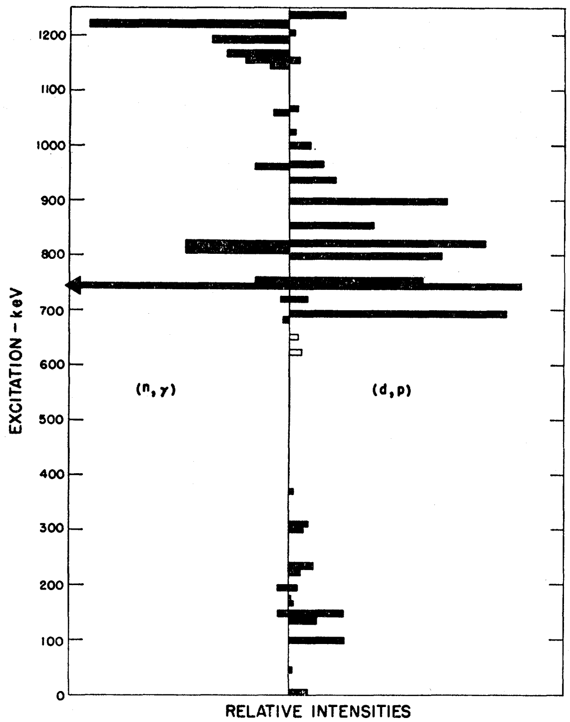

FIG. 3. Comparison of the energies and intensities of the (d,p) and (n,γ) spectra. The intensity scales are arbitrary.

of the experiment was necessary to confirm that none of the observed lines was in fact from high-energy fission product activity. At low energies the spectrum was repeated with a 1.7-g target of uranium enriched to 59 000:1 in ^{238}U. Although the counting rate was quite low, it was possible to make identification with the lines seen from the larger target and thus to make sure that fission-product activity was making a negligible contribution to the spectrum.

Computer least-squares fits of Gaussian line distributions were made to the individual peaks in the entire spectrum. Table II lists the line energies and intensities found in both the low- and high-energy scans. Note that the low-energy portion extends from 490 to 900 keV, and that the high-energy portion extends from 3150 to 4850 keV, corresponding to excitations up to 1650 keV. The intensities in gamma rays per 1000 neutrons captured were obtained by comparison with lines in the spectrum from $^{14}N(n,\gamma)^{15}N$, where the partial cross sections are known to $\pm 10\%$. The nitrogen spectrum was also used to establish the energy scale for the high-energy part of the ^{238}U capture spectrum; thus the high-energy line energies are limited to an absolute accuracy of ± 2 keV, but the accuracy of energy differences is largely limited by the statistical accuracy of the least-squares fitting, and involves the individual line intensities.

With the least-squares fitting code it is possible to determine linewidth, as well as centroid position and intensity, to a high degree of accuracy. In the initial treatment of the data, the strong line at 4059 keV showed a slightly greater width than that expected from a single line. In order to investigate whether or not it was complex, the energy region from 3600 to 4200 keV was examined in detail, using simultaneously targets of ^{238}U and Mg. This made it possible to compare the line in question with the strong single 3920-keV

TABLE III. Levels in ^{239}U from the (d,p) and (n,γ)[a] reactions.

(d,p) E (keV)	ΔE (keV)	I_{rel}	ΔI_{rel}	(n,γ) E (keV)	ΔE[b] (keV)	I_γ[c]	ΔI_γ
0	2	70	10				
43	3	5	3				
97	2	202	20				
133.1	2	99	15				
145.2	3	205	30	142.8	2	2.2	0.5
165	5	14	7				
173	5	7	5				
189	4	28	8	191.5	2	2.1	0.5
220	5	34	15				
229.1	3	92	15				
300	4	40	20				
307.2	3	80	20				
373	4	16	6				
623[d]	5	41	12				
651[d]	5	30	10				
693.2[e]	2	796	80	685	5	0.8	0.3
718	4	60	30	713	4	2.2	0.5
742.3	2	840	150	742.3		110	12
754.1	4	480	100	750	3	5	2
798.6	2	550	50				
				811.0	0.5	15	2
				819.9	0.5	15	2
823[e]	3	710	70				
854	4	300	40				
894	5	570	60				
				928.4	2	1.3	0.5
943	5	160	30				
967	5	120	20				
				983.6	2	1.2	0.5
999	6	75	25				
1025	6	17	8				
1067	6	30	10				
1155[e]	6	35	10	1149.1	1	6.3	1.5
				1162.4	1	6.8	1.3
				1189.7	1	8.4	1.5
1205	6	53	6				
				1217.9	1	25	4
1238	5	200	20	1234.5	2	2.2	0.7
1264	5	100	15	1269.4	3	1.4	0.6
1285	5	80	25				
				1355.5	2	4.1	1.2
1397	7	30	8	1395.9	2	2.0	0.7
1441	5	85	25				
				1483.3	1.5	4.5	1.2
				1498.1	1.5	4.8	1.2
				1508.4	2	2.0	0.7
1520[e]	3	510	50				
				1560.1	1.5	3.8	0.8
				1572.3	1.5	6.3	1.5
				1594.1	1.5	6.5	1.5
				1600.7	2	2.2	0.7
				1614.9	1.5	2.4	0.7
1623	6	45	15				
1692	5	100	15				

[a] Excitation energies corresponding to the high-energy (n,γ) transitions are based on a neutron binding energy for ^{239}U of 4801.7 keV.
[b] Errors on the level energies determined in the (n,γ) experiment are relative to the 742.3-keV level.
[c] Intensities in gamma rays per 10^3 neutrons captured.
[d] Possible impurity.
[e] Possible complex structure.

line from capture in ^{24}Mg; the energy difference is small enough so that the normal increase in linewidth with energy could be neglected. The width of the Mg line was determined to be 7.35±0.11 keV, and an attempt was made to fit the uranium line as a doublet using this characteristic width for both members. This process resulted to two lines with energies of 4059.4±2 keV and 4051.5±4 keV, and respective intensities in the ratio 20:1. (See insert of Fig. 2.)

C. Comparison of High-Energy (n,γ) and (d,p) Spectra

Capture of a thermal neutron by a ^{238}U nucleus results in a unique spin-parity assignment of $\frac{1}{2}+$ for the capturing state. Lines in the high-energy capture spectrum would thus correspond to excitation of low-lying $\frac{1}{2}-$ and $\frac{3}{2}-$ states by $E1$ transitions, $\frac{1}{2}+$ and $\frac{3}{2}+$ states by $M1$ transitions, and $\frac{5}{2}+$ states by $E2$ transitions. A prominent feature of the high-energy (n,γ) spectrum is the absence of $E1$ transitions up to an excitation of 742 keV. Only two states are excited below the 152-neutron energy gap, which extends from 400 to 600 keV, while the (d,p) reaction excites some eleven states below the gap. The absence of states within the gap is easily seen in both the (n,γ) and (d,p) spectra. The $E2$ transition to the $\frac{5}{2}+$ ground state of ^{239}U is not observed in the (n,γ) spectrum.

In contrast, the (d,p) reaction can excite low-lying positive-parity states up to high-spin values. One would therefore expect the (d,p) reaction to be the more powerful approach in observing the low-lying states in ^{239}U. The lack of dual excitation of states up to 650 keV is evident from the histogram of Fig. 3, which shows the states excited in each reaction. Table III lists the levels in ^{239}U excited in the (d,p) reaction and by high-energy gamma rays.

The sum of the (d,p) ground-state Q value, 2579±7 keV, and the deuteron binding energy, 2225 keV, gives a value for the neutron binding energy in ^{239}U of 4804±7 keV. If it is assumed that the 4059.4-keV line in the (n,γ) spectrum excites the strong 742.3-keV state seen in the (d,p) spectrum, then the binding energy can be determined as 4801.7±3 keV.

III. DISCUSSION

A. General

The spectra of ^{239}U can best be understood in terms of the unified model of Bohr and Mottelson.[7] Indeed, the specific spectroscopic assignments have been shown to follow directly from the Nilsson model in a large variety of different data.

The Nilsson levels are shown in Fig. 4 for deformation $(\beta)=0.25$. A large gap of several hundred keV is immediately obvious in the figure between the $\frac{7}{2}+[624]$ and the $\frac{1}{2}-[761]$ orbitals. This gap corresponds to the stability of the 152nd neutron.

The observations of Holm, Burwell, and Miller[4] of the two large peaks in the reaction ^{238}U$(d,p)^{239}$U may be understood in terms of the grouping of many unresolved proton groups in the first 300 keV of excitation and a second grouping of much more intense proton groups in the 700–900-keV region. The large dip in cross section between these two complex groups shows the presence of the gap immediately after neutron number 152. With more resolution these same features are shown in Fig. 1.

The success of the Nilsson model in expanding the differential cross sections in (d,p) reactions on rare-earth deformed nuclei is becoming increasingly documented.[8–16] It is obvious that this is a very powerful spectroscopic tool. The theory developed by Satchler[17] is particularly simple in the case of stripping on an even-even target nucleus, leaving the residual nucleus in a state of angular momentum I with Nilsson quantum numbers Ω and N. The differential cross section is

$$d\sigma_{I,l,\Omega,N}(\theta)/d\omega=2C_{I,l}^2(\Omega,N)\phi_l(\theta)U^2\,, \qquad (1)$$

where l is the orbital angular momentum of the captured neutron, the $C_{j,l}(\Omega,N)$ ($j=1$ for an even-even target neucleus) are normalized coefficients of the expansion of the Nilsson wave function $\chi(\Omega,N)$ in a series of wave functions of definite j which are related to the Nilsson coefficients $a_l(\Omega,N)$ by the Clebsch-Gordan transformation:

$$C_{j,l}(\Omega,N)=\sum\nolimits_\lambda a_{l,\lambda}(\Omega,N)\langle l,\tfrac{1}{2},\lambda,\Sigma\,|\,I,\Omega\rangle. \qquad (2)$$

The $\phi_l(\theta)$ are intrinsic single-particle cross sections which contain all the angular dependence of the cross section. The distorted-wave Born approximation (DWBA) calculation normally used to define $\phi_l(\theta)$ has been published by Macefield and Middleton[5] for ^{239}U.

The U^2 in Eq. (1) is the probability that an orbital in the target nucleus was available for occupation at the time of the reaction. Without pairing correlations it would be zero for all hole states and 1.0 for particle states. With pairing correlations, states close to the ground state have a U^2 of $\sim$0.5. In view of the fact that most of the states of interest here are relatively near the

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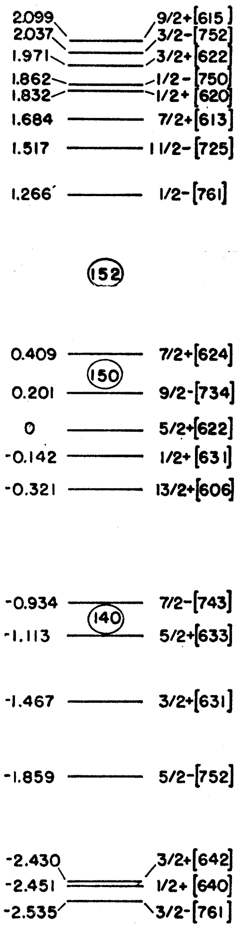

FIG. 4. Nilsson single-particle levels for odd-n nuclei for a deformation $\delta=0.25$. $\mu=0.45$ and $k=0.05$ for the $N=6$ and $N=7$ shells.

ground state and because of the uncertainties resulting from Coulomb stripping the U^2 are assumed to be 0.5 below 320 keV and 1.0 above 650 keV.

The table of $C_{j,l}$ values for rare-earth deformed nuclei[8] has proven particularly valuable in assigning Nilsson orbitals from (d,p) cross sections. It therefore seems desirable to have a similar table for nuclei with $Z>82$ and $N>126$. Such a table for deformation (β) of 0.20, 0.25, and 0.30 is presented in the Appendix for all Nilsson orbitals of interest in the actinide region. Since they are already contained in the previous table,[8] those orbitals which are expected to be common to the rare-earth region and the actinide region, i.e., $\frac{3}{2}-[501]$, $\frac{5}{2}-[503]$, and $\frac{1}{2}-[501]$ orbitals, are not repeated in the Appendix. The spectroscopic assignments of Nilsson levels in the next section lean heavily on this Appendix. It should be noted that this is not the first use of this spectroscopic tool in the actinide region. Both Macefield and Middleton[5] and especially Erskine[18] and collaborators have shown its usefulness in the interpretation of actinide spectra.

B. Spectroscopic Assignments in ^{239}U

1. The 5/2+[622] Band

The measurement of angular distributions of various proton groups for the ^{238}U$(d,p)^{239}$U reaction has recently been performed by Macefield and Middleton,[5] who also determined the l values for the capture of neutrons into excited states up to about 900 keV by comparing the measured angular distributions with DWBA calculations. Combining the results with the predictions of the Nilsson model, they have also identified the ground and 92 ± 5-keV levels with the $\frac{5}{2}+$ and $\frac{9}{2}+$ members of the $\frac{5}{2}+[622]$ rotational band. Our measurements are in complete agreement with this assignment, with the ground state the $\frac{5}{2}+$ member, and the 97-keV state the $\frac{9}{2}+$ member of the band. We also see evidence for the $\frac{7}{2}+$ member of this band in an extremely weak proton group at 43 keV. This state has an extremely low spectroscopic factor and is seen in (d,p) spectroscopy because of the Coriolis coupling with the $\frac{7}{2}+[624]$ band head. The 11/2+ member of this band should be observed at ~165 keV. Some proton tracks are observed above background at this energy. However, this assignment is at best highly tentative.

2. The $\frac{7}{2}+[624]$ Band

The suggestion of Macefield and Middleton[5] that the $\frac{7}{2}+[624]$ band occurs at ~160 keV (band head unobserved), at 222 keV ($\frac{9}{2}+$ state), and at 301 keV (11/2+ state) has at least two serious difficulties. The

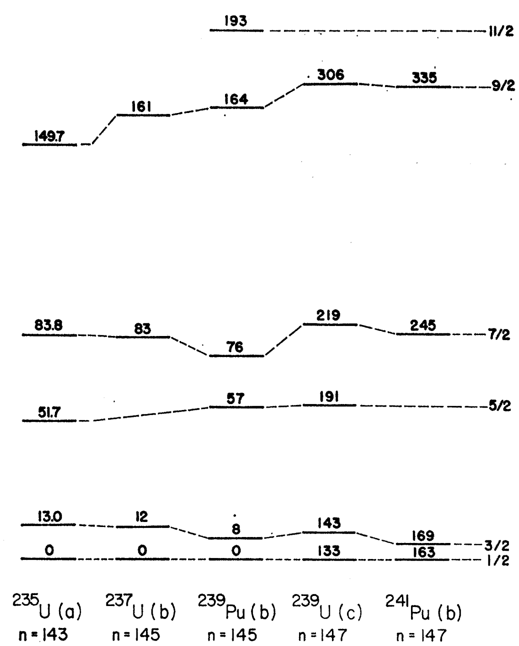

FIG. 5. The energy systematics of the $\frac{1}{2}+[631]$ band. Energy of the levels is in keV. (a) Landolt-Bornstein [*Numerical Data and Functional Relationships in Science and Technology* (Springer-Verlag, Berlin, 1961)]. (b) T. H. Braid *et al.* (Ref. 18). (c) This work.

[18] T. H. Braid, R. R. Chasman, J. R. Erskine, and A. M. Friedman, Phys. Letters 18, 149 (1965). Also, J. R. Erskine (private communication).

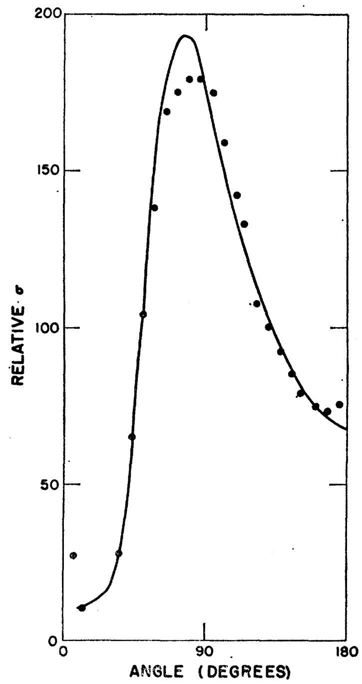

FIG. 6. Comparison of the theoretical angular distribution of Macefield and Middleton (Ref. 5).

calculated cross sections indicate that the 160-keV line should definitely be observed and the ratio of the intensities of the $\frac{9}{2}$ and 11/2 members are calculated to be 0.3 but experimentally measured (Ref. 5) to be 1.38. We believe there is evidence for the $\frac{7}{2}+$ band head at ~173 keV. Because of the probable presence of the 11/2+ member of the $\frac{5}{2}+[622]$ band at ~165 keV, however, this suggestion must be considered tentative. The higher resolution available in our experiment suggests that the states at ~222 and 301 keV are in fact doublets with energies 220 and 229 and 300 and 307 keV. The states at 229 and ~300 keV are then the $\frac{9}{2}+$ and 11/2+ members of the $\frac{7}{2}+[624]$ band. Thus the angular distribution should show $l=4$ and $l=4+6$ for the two doublets. The observed $l=4$ and $l=6$ distributions could not easily be distinguished from that which is expected. This splitting up of the doublets reduces the intensity anomaly but does not solve it. The anomalously large intensity of the $\frac{9}{2}+$ state at 229 keV in the (d,p) reaction is almost certainly due to the Coriolis coupling between it and the $\frac{9}{2}+$ state at 97 keV from which it attains considerable intensity.

3. *The 1/2+[631] Band*

In addition to the $\frac{5}{2}+[622]$ and $\frac{7}{2}+[624]$ rotational bands, $\frac{1}{2}+[631]$, 13/2+[606], and $\frac{9}{2}-[734]$ rotational bands are to be expected in the low-energy region of excitation (see Fig. 4). The latter two bands, however, will not be appreciably excited by either (d,p) or (n,γ) reactions because of their high K values as well as the nature of their single-particle wave functions. Thus, the 133- and 145-keV levels observed in this (d,p) work and the one or possibly two levels observed in these (n,γ) experiments can probably be identified with members of the $\frac{1}{2}+[631]$ rotational band. The rotational band, based on the Nilsson level $\frac{1}{2}+[631]$, is actually observed as the ground-state rotational band in ^{239}Pu. Using these experimental results and identifying the 133-keV level in our experiment with the lowest $\frac{1}{2}+$ member of this band, we can predict the positions of other members of this band. Figure 5 includes the predicted level sequence for the $\frac{1}{2}+[631]$ band in ^{239}U. The correspondence between the predicted levels and the observed ones is very good; thus we can identify the states at 133, 142.8, 191, 219, and 306 keV with the $\frac{1}{2}+$, $\frac{3}{2}+$, $\frac{5}{2}+$, $\frac{7}{2}+$, and $\frac{9}{2}+$ members of this band.

There would appear to be one serious difficulty with this assignment. Macefield and Middleton showed that the l value of the mixed angular distribution corresponding to the excitation to 133- and 143-keV states is $l=1$. According to our assignment, however, this must be a mixture of $l=0$ and $l=2$, which does not contradict the angular-distribution data of Macefield and Middleton.[5] Based on our assignment, we found that the predicted mixed angular distribution of $l=0$ and $l=2$ also agrees with their experimental curve, as shown in Fig. 6. In obtaining the theoretical curve in this figure, we used the results of the DWBA calculations of Macefield and Middleton and the mixing ratio predicted from the Nilsson model.

A normalizing factor of 0.26 is necessary to get absolute instead of relative agreement with experiment. This factor can be identified with U^2 [Eq. (1)] or with the effect of Coluomb stripping (see Sec. III D), or both. Since the $\frac{1}{2}+[631]$ orbital is a hole orbital the value of U^2 should be somewhat less than 0.5; 0.26 is not unreasonable. Thus we can conclude that the angular distribution observed by Macefield and Middleton can be explained assuming an $l=0$ and $l=2$ mixture.

Finally, in the high-resolution low-energy (n,γ) data of Maier[2] a gamma ray of 133.80±0.08 keV was observed to have an intensity (uncorrected for internal conversion) of 23 per hundred captures. This transition should probably be associated with the 133-keV state of this work and is the type of intense transition expected to depopulate a low-spin band head in the low region of excitation.

An unusual aspect of the $\frac{1}{2}+[631]$ band is the fact that the high-energy (n,γ) experiment fails to populate the $\frac{1}{2}+$ state at 133 keV but populates both the $\frac{3}{2}+$ and $\frac{5}{2}+$ states at 144 and 193 keV, respectively. This may indicate that the $M1$ transition (the only multipole possible to the $\frac{1}{2}+$ 133-keV state) is considerably slower than the $E2$ transition to the $\frac{3}{2}+$ and $\frac{5}{2}+$ states.

This appears to be additional evidence for the individuality of the capture state and suggests a direct-reaction mechanism rather than a compound-nucleus mechanism for the high-energy neutron-capture process in ^{238}U.

4. *Higher Excited States*

No states are observed in the energy range from 400 to 600 keV in either (d,p) or (n,γ) reactions. This is presumably evidence for the "semi-magic" character of neutron number 152. Above 700 keV, many levels are strongly populated in both reactions. The large cross section for these levels suggests that the K values of these excited states should be small because the strong peaks of the (d,p) reaction come from the capture of neutrons of low l values ($l \leq 2$), while the neutron-capture γ ray can only feed states of low angular momenta.

As is seen from Fig. 4, there are many such orbitals, for example, $\frac{1}{2}-[761]$, $\frac{1}{2}+[620]$, $\frac{1}{2}-[750]$, $\frac{3}{2}+[631]$, $\frac{3}{2}+[622]$, $\frac{3}{2}-[752]$, and $\frac{3}{2}-[761]$ which are expected to appear above 700 keV. It is, therefore, difficult to identify uniquely the Nilsson orbital with these observed levels. The extremely strong transition observed in the (n,γ) reaction to a state at about 742 keV, however, suggests an $E1$ transition and thus a negative-parity state at this energy. The lowest negative-parity state expected to appear in this energy region is the $\frac{1}{2}-[761]$ orbital. Using the value for the decoupling parameter predicted from the Nilsson model and the empirical value of the moment of inertia and also assuming that the lowest member of this band lies at 742 keV, we calculated the energy of other members of the band. The results are presented in Fig. 7, together with the observed levels in this energy region. The correspondence between the predicted energies and the observed ones is good; the relative intensity of the $\frac{7}{2}-$ members of this band, however, is too low.

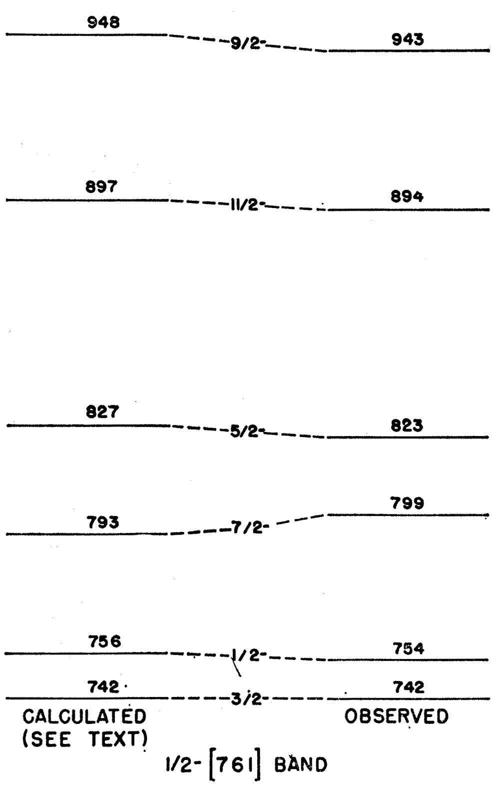

FIG. 7. Calculated and experimental energies of the $\frac{1}{2}-[761]$ band. In this calculation, the theoretical decoupling parameter was used. It should be noted that with a decoupling parameter of -1.0, the six band members would collapse to three doubly degenerate sets of levels without seriously affecting the agreement with experiment.

The gamma-ray of 4052±4 keV ($E_{ex}=750\pm3$ keV) observed in the (n,γ) spectrum corresponds in energy to a transition to the $\frac{1}{2}-$ member of the $\frac{1}{2}-[761]$ band predicted at 754 keV. Its relative weakness (~5% of the transition to 742 keV) may result from the individuality of the capture state, which is not a unique occurrence.

To emphasize the tentativeness of this assignment, it should be noted that an alternative explanation is possible. If the $\frac{1}{2}-[761]$ band should have a decoupling parameter of -1.0 (slightly less negative than the theoretical prediction), then one would expect degenerate $\frac{1}{2}-$ and $\frac{3}{2}-$ states at 742 keV, degenerate $\frac{5}{2}-$ and $\frac{7}{2}-$ states at 799 or 823 keV, and degenerate $\frac{9}{2}-$ and $11/2-$ states at 894 or 943 keV. Such an explanation leaves additional states in the (d,p) spectrum unaccounted for, but does not affect the agreement between the predicted and observed (d,p) intensities. In this paper we have chosen to accept the first of these interpretations, as shown in Fig. 8.

The existence of states at 692 and ~715 keV may correspond to the $\frac{1}{2}+$ and $\frac{3}{2}+$ members of the $\frac{1}{2}+[620]$ band. This is suggested from the cross sections of the (d,p) reaction to these states. Then, using the theoretical value of the decoupling parameter ($a=0.30$) and also the value of the moment of inertia extracted from the observed energy difference, the $\frac{5}{2}+$ member of this band is predicted to appear at 736 keV. This is nearly the same energy as that of the band head of the $\frac{1}{2}-[761]$ Nilsson state. Of course, the transition of the neutron-capture γ ray to the $\frac{5}{2}+$ state will be weak. However, the cross section of the (d,p) reaction is expected to be relatively strong. The observed strong intensity of the 742-keV peak in the (d,p) reaction can thus be reasonably explained (see Fig. 9).

A $\frac{1}{2}+$, $K-2$ gamma vibrational band built on the $\frac{5}{2}+[622]$ ground state can confidently be predicted to be in the energy region from 700 to 900 keV in ^{239}U. Such a vibrational band would be expected to mix with the $\frac{1}{2}+[620]$ Nilsson state. Thus the band beginning with the 692-keV state may have some $K-2$ gamma vibrational admixture from the $\frac{5}{2}+[622]$ Nilsson state. In view of the relatively strong (d,p) cross section for populating the 692-keV band, however, it would appear that the major component is the $\frac{1}{2}+[620]$ intrinsic orbital.

It should be clearly understood, however, that these two band assignments are considerably less certain than the assignments below the gap. The level density and resolution available make it probable that some states

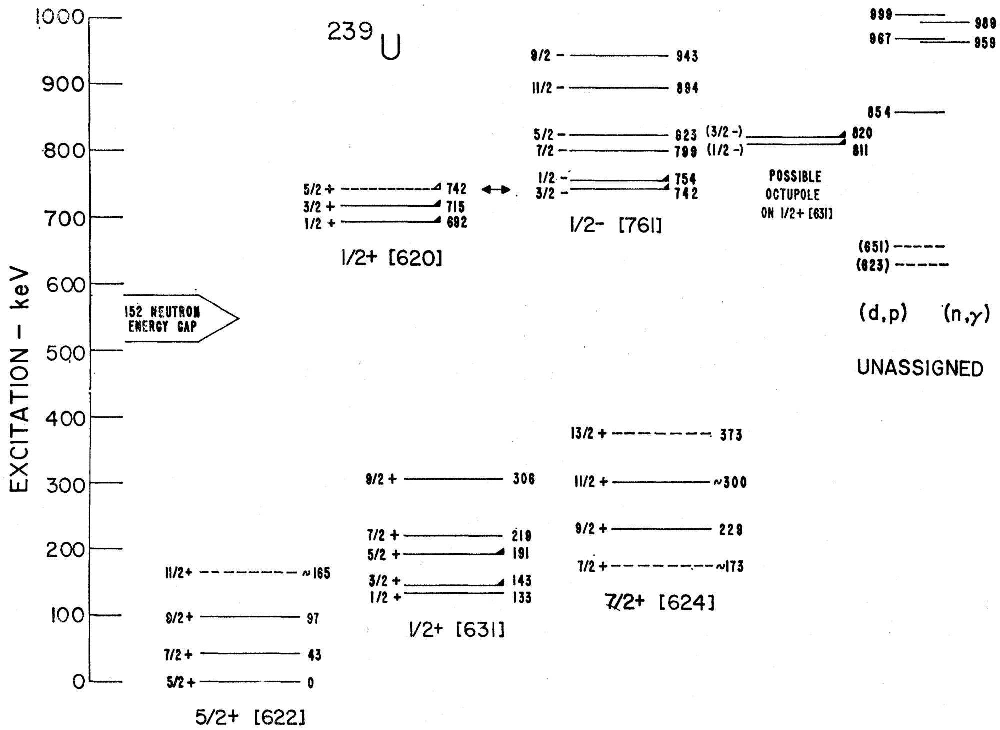

FIG 8. Energy levels in ^{239}U. See text (Sec. III C) for explanation of symbols and conventions used in the illustration.

necessary for a complete description of the spectroscopy are being missed.

Finally, there is the moderately strong doublet at 811 and 820 keV observed in the (n,γ) reaction. If the interpretation of the 823-keV state observed in the (d,p) experiment as the $\frac{5}{2}-$ member of the [761] band is correct, the upper member of the doublet observed in the (n,γ) spectrum cannot correspond to the same state. There is no correspondence between (n,γ) and (d,p) for the 811-keV state. However, the failure to observe a $\frac{1}{2}-$ or $\frac{3}{2}-$ state in the (d,p) reaction which has been populated in the (n,γ) reaction is not surprising. The $\frac{3}{2}-$[761], $\frac{3}{2}-$[752], and $\frac{1}{2}-$[750] Nilsson orbitals all provide $\frac{1}{2}-$ or $\frac{3}{2}-$ states which would not be appreciably populated by the (d,p) reaction. We have also considered the possibility of low-spin states in ^{239}U being related to Nilsson assignments from the $N=5$ shell. Although the positions of the $\frac{1}{2}-$[501] and $\frac{3}{2}-$[501] states are reasonably well known for small deformations, the positions for high deformation where they merge into the states from the $N=6$ shell as applicable for ^{239}U are not well determined.[19] The $\frac{1}{2}-$[501] state would be a deep-hole state in ^{239}U and, therefore, not be excited in the (d,p) reaction, but it would be expected that a strong transition to the $\frac{1}{2}-$ and $\frac{3}{2}-$ states might occur in the (n,γ) reaction. We have rejected this assignment for the 742-keV state, but it is possible that the $\frac{1}{2}-$[501] state is related to the 811- and 820-keV states. It should be pointed out that Erskine[18] has suggested the assignment of the $\frac{1}{2}-$[501] Nilsson stats in ^{233}U and ^{235}U.

Perhaps the most plausable interpretation of this doublet would explain it as the octupole band built on

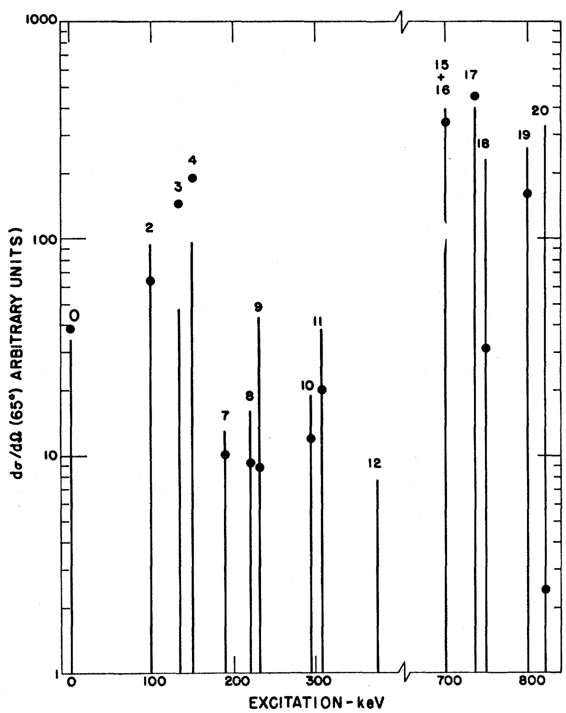

FIG. 9. Comparison of experimental and theoretical relative cross sections for the more intense proton groups in the reaction ^{238}U$(d,p)^{239}$U.

[19] J. J. Griffin (private communication); also, J. W. Starner (private communication).

TABLE IV. Transitions between low-lying states in ^{239}U. Energies are in keV. Transition energies in the body of the table are from the Ge(Li) low-energy (n,γ) spectrum.

	1/2+ 692±2	3/2+ 715.5±2	5/2+ (742.3±2)	3/2− 742.3±2	1/2− 754±2	7/2− 799±2	5/2− 823±3	11/2− 894±5	9/2− 943±5
1/2+ 133.80±0.08[a]	561±3	580±2		612±2					
3/2+ 142.8±2	552±2		601±5	601±5	612±2		683±4		
5/2+ 191±2		522±2	552±2	552±2		605±5	629±4		
7/2+ 219±2			522±2			580±2	605±5		722±4
9/2+ 306±2						498±2		588±3	638±3

[a] Energy value from Ref. 2.

the $\frac{1}{2}+[631]$ orbital. The fact that an octupole band is observed in ^{238}Pu at 605 keV, which compares in this case with an energy difference of 676 keV, seems to support this interpretation. Comparative (d,p) intensities have not been calculated for octupole bands.

These suggestions must be considered highly tentative. However, there can be little doubt about the presence of Nilsson orbitals arising from the neutron shell above 184 neutrons in the region of ~700 keV in ^{239}U. The very large (d,p) cross section and the simultaneous observation of $\frac{1}{2}$ or $\frac{3}{2}-$ states from the (n,γ) reaction seems to demand this explanation. This is of considerable interest because neutron orbitals from the shell above 184 neutrons have not been previously identified and because of the expected mixing of $N=7$ and $N=5$ states which appear on the basis of this study to be close to each other.

C. The Level Scheme

Figure 8 summarizes the experimental data described in the previous sections. A solid line indicates that the energy of a level has been deduced accurately; the dashed lines at ~165 and ~173 keV indicate the presence of a broad, weak proton group at ~169 keV in the (d,p) spectrum assumed to be a doublet. In spite of the certainty in its energy, the state at 373 keV (corresponding to peak 12) is shown as a dashed line because its intensity is somewhat large for its interpretation as the 13/2+ member of the $\frac{7}{2}+[624]$ Nilsson band. A triangle at the right-hand edge of a level indicates that it is directly populated by the (n,γ) as well as the (d,p) reaction. Note that the gamma ray exciting the 742-keV $\frac{5}{2}+$ member of the $\frac{1}{2}+[620]$ band is degenerate with the gamma ray exciting the $\frac{3}{2}-$ member of the $\frac{1}{2}-[761]$ band, and consequently the triangle is shown hollow. The Nilsson assignments have previously been discussed, but it should be kept in mind that assignments above the 152 neutron energy gap are much less certain than those below the gap.

Thirteen of the 27 low-energy gamma rays given in Table II have been fitted into the level scheme of Fig. 8. Within the energy accuracy of these gamma rays and of the levels, it is possible to account for them as 20 energy differences in the level scheme. This is shown in Table IV. Since the low-energy (n,γ) work reported here was aimed primarily at finding transitions that traversed the 152-neutron energy gap, interband transitions were not observed. During the treatment of the data it was apparent that many of the lines in the low-energy spectrum were actually composed of more than one component.

The level energies given in Table IV and in Fig. 8 represent in some cases a combination of the (d,p) and (n,γ) data to produce a more reliable value than either experiment could yield by itself.

Unassigned levels up to 1 MeV are shown at the right-hand side of Fig. 8. The dashed levels at 623 and 651 keV are very weak lines observed in the (d,p) spectrum which may be from contamination.

It is particularly interesting to note that all of the gamma-ray depopulation of the low-lying levels in ^{239}U reported here proceed through the $\frac{1}{2}+[631]$ band. This is consistent with the extremely strong 133.8 ±0.08-keV gamma ray reported by Maier[2] between the $\frac{1}{2}+[631]$ band head and the $\frac{5}{2}+[622]$ ground state.

A disturbing aspect of the decay scheme is the failure of the 754-keV $\frac{1}{2}-$ level to depopulate except by a gamma ray to the 133-keV $\frac{1}{2}+$ level. This gamma ray is degenerate with another transition, as shown in Table IV. The necessity of additional extreme-resolution low-energy (n,γ) spectroscopy is obvious. The degeneracies assumed here could thereby be removed and the very low-energy interband transitions, which are missing in this level scheme, might be found.

D. Coulomb Stripping

The comparison of the relative intensities of the observed (d,p) reaction with the theory is made in Fig. 9. Although over-all agreement is fairly good, some discrepancies can be clearly seen. In the low-excitation region, the disagreement is most serious for the 229-keV state (peak 9). As already mentioned in Sec. III B, however, this difficulty should be removed by considering the Coriolis force coupling between $\frac{5}{2}+[622]$ and $\frac{7}{2}+[624]$ Nilsson states. In the higher excitation region, larger discrepancies are seen in the 754- and 823-keV peaks (peaks 18 and 20). Perhaps there are accidental degeneracies involving levels having rather large (d,p) cross sections; there is, in fact, a slight indication that the 823-keV peak is indeed a doublet.

The absolute intensities of the (d,p) cross section were measured by Macefield and Middleton.[5] Using their data, we can also compare the absolute experimental cross sections with the calculated ones. Although the agreement between experiment and theory for relative intensities is fairly good, the absolute experimental cross sections are systematically two times larger than the calculated ones.

The minimum distance of approach for deuterons at 12 MeV is ~1.8 uranium radii for the ^{238}U target nucleus. Thus it is only the tail of the wave function of the captured neutron which is effective in the (d,p) reaction; this phenomenon is known as Coulomb stripping. It has been argued by several authors that in such a Coulomb-stripping process, the finite range of the neutron-proton interaction and the polarization effect caused by the asymmetry of the distorting potential of the target nucleus on the incident deuteron due to the Coulomb potential increase the differential cross section. According to Kerman and Gibson,[20] these effects increase the differential cross section for the ^{209}Bi$(d,p)^{201}$Bi reaction by a variable amount as large as 16%. The variation in the cross sections depends on the Q and l values of the captured neutron. Dar *et al.*,[21] however, suggested that the enhancement amounts to a factor ~4.5 for $Q=0$ and increases slowly with the Q of the reaction.

When the cross sections for the reaction ^{238}U$(d,p)^{239}$U are calculated, using the zero-range approximation and neglecting polarization effects on the deuteron, they are systematically smaller than the experimental cross section by a factor of approximately 2. This factor is between the two estimated values in Refs. 18 and 19. This suggests that the discrepancy may be removed by taking into account Coulomb-stripping effects. It is to be hoped that these experiments will serve as a stimulus for the appropriate calculations.

ACKNOWLEDGMENTS

It is a pleasure to acknowledge the suggestion of Professor Arthur Kerman regarding Coulomb stripping, and calculations of Nilsson levels and band systematics by J. W. Starner. We are indebted to John Povalites of the Los Alamos Scientific Laboratory for the UO_2 evaporation for target preparation in the (d,p) experiment.

APPENDIX: VALUES OF $C_{j,l}$ FOR DEFORMATION (β) OF 0.20, 0.25, AND 0.30 FOR NILSSON LEVELS OF INTEREST IN THE ACTINIDE DEFORMED ODD-A NUCLEI

Nilsson level	j,l \ δ	0.20	0.25	0.30
3/2−[761]	3/2,1	0.01316	0.02254	0.03390
	5/2,3	0.01027	0.01886	0.03022
	7/2,3	0.08662	0.12566	0.16654
	9/2,5	0.02367	0.03787	0.05509
	11/2,5	0.34306	0.40879	0.46428
	13/2,7	0.03987	0.05283	0.06675
	15/2,7	0.93401	0.90112	0.86438
1/2+[640]	1/2,0	−0.07646	−0.13380	−0.18166
	3/2,2	0.10132	0.07613	0.04826
	5/2,2	−0.29951	−0.39869	−0.45683
	7/2,4	0.37588	0.35956	0.34441
	9/2,4	−0.43588	−0.44721	−0.41045
	11/2,6	0.72286	0.63628	0.58475
	13/2,6	0.20096	0.28847	0.35666
3/2+[642]	3/2,2	0.12485	0.15138	0.17115
	5/2,2	0.30114	0.29271	0.27879
	7/2,4	0.38566	0.44737	0.48963
	9/2,4	0.50730	0.38876	0.29573
	11/2,6	0.64339	0.67043	0.67700
	13/2,6	−0.27147	−0.30109	−0.32782
5/2−[752]	5/2,3	0.00602	0.01015	0.01514
	7/2,3	0.05467	0.07695	0.09960
	9/2,5	0.02658	0.04007	0.05521
	11/2,5	0.29825	0.35338	0.40068
	13/2,7	0.05868	0.07541	0.09231
	15/2,7	0.95072	0.92834	0.90429
3/2+[631]	3/2,2	−0.01087	−0.03127	−0.05056
	5/2,2	−0.30869	−0.37374	−0.42042
	7/2,2	0.17301	0.14638	0.12344
	9/2,4	−0.64155	−0.64773	−0.62988
	11/2,6	0.64288	0.57827	0.53857
	13/2,6	0.22312	0.28975	0.34445

Nilsson level	j,l \ δ	0.20	0.25	0.30
5/2+[633]	5/2,2	0.13911	0.13707	0.13383
	7/2,4	0.31384	0.35136	0.37894
	9/2,4	0.44150	0.36186	0.30121
	11/2,6	0.80135	0.81829	0.82349
	13/2,6	−0.21230	−0.23921	−0.26387
7/2−[743]	7/2,3	0.02528	0.03466	0.04399
	9/2,5	0.02176	0.03143	0.04179
	11/2,5	0.23992	0.28241	0.31920
	13/2,7	0.07009	0.08802	0.10556
	15/2,7	0.96768	0.95410	0.93984
13/2+[606]	13/2,6	1.00000	1.00000	1.00000
1/2+[631]	1/2,0	0.37183	0.34914	0.32399
	3/2,2	0.54602	0.54872	0.54513
	5/2,2	0.23369	0.12691	0.04308
	7/2,4	0.36745	0.30404	0.25581
	9/2,4	−0.42294	−0.45160	−0.45976
	11/2,6	−0.42239	−0.48258	−0.51940
	13/2,6	0.12914	0.17789	0.22228
5/2+[622]	5/2,2	−0.20937	−0.24302	−0.26907
	7/2,4	0.02336	−0.0273	−0.02482
	9/2,4	−0.80052	−0.80817	−0.80313
	11/2,6	0.52244	0.47316	0.44281
	13/2,6	0.20457	0.25281	0.29305
9/2−[734]	9/2,5	0.01273	0.01779	0.02304
	11/2,5	0.17295	0.20213	0.22747
	13/2,7	0.07380	0.09100	0.10745
	15/2,7	0.98208	0.97496	0.96757
7/2+[624]	7/2,4	0.18841	0.20683	0.22102
	9/2,4	0.34766	0.29840	0.26186
	11/2,6	0.90504	0.91423	0.91764
	13/2,6	−0.15666	−0.17989	−0.20131

[20] A. K. Kerman and F. P. Gibson, Argonne National Laboratory Report No. ANL-6848, 1964 (unpublished), p. 48.
[21] A. Dar, A. de-Shalit, and A. S. Reiner, Phys. Rev. **131**, 1732 (1963).

APPENDIX (*continued*).

Nilsson level	$j,l \diagdown \delta$	0.20	0.25	0.30
1/2−[761]	1/2,1	0.09349	0.14947	0.19640
	3/2,1	0.23682	0.29327	0.32122
	5/2,3	0.14530	0.24205	0.32309
	7/2,3	0.50920	0.49581	0.44440
	9/2,5	0.21156	0.32694	0.40651
	11/2,5	0.64707	0.45763	0.29102
	13/2,7	0.28170	0.37557	0.41022
	15/2,7	−0.33462	−0.36037	−0.37168
11/2−[725]	11/2,5	0.10037	0.11641	0.13034
	13/2,7	0.06927	0.08408	0.09800
	15/2,7	0.99254	0.98964	0.98661
7/2+[613]	7/2,4	0.04896	0.06724	0.08286
	9/2,4	0.90731	0.91434	0.91544
	11/2,6	−0.38622	−0.35108	−0.32876
	13/2,6	−0.15883	−0.19029	−0.21686
1/2+[620]	1/2,0	−0.34335	−0.37291	−0.39374
	3/2,2	0.19100	0.17585	0.16216
	5/2,2	−0.54944	−0.50257	−0.45875
	7/2,4	0.53644	0.50643	0.48500
	9/2,4	0.38999	0.43871	0.47231
	11/2,6	−0.31007	−0.33776	−0.35691
	13/2,6	−0.08801	−0.12007	−0.15013

Nilsson level	$j,l \diagdown \delta$	0.20	0.25	0.30
1/2−[750]	1/2,1	0.03188	0.01872	−0.00640
	3/2,1	−0.05620	−0.13090	−0.20566
	5/2,3	0.16084	0.17001	0.15763
	7/2,3	−0.19283	−0.32103	−0.40872
	9/2,5	0.42726	0.42987	0.41323
	11/2,5	−0.29476	−0.36606	−0.36096
	13/2,7	0.80505	0.69442	0.60823
	15/2,7	0.12344	0.22260	0.30784
3/2+[622]	3/2,2	0.38790	0.40328	0.41254
	5/2,2	0.35937	0.30187	0.25375
	7/2,4	0.68861	0.66263	0.63971
	9/2,4	−0.32872	−0.35915	−0.37971
	11/2,6	−0.36260	−0.40777	−0.44022
	13/2,6	0.08167	0.10909	0.13497
3/2−[752]	3/2,1	0.09727	0.11306	0.12269
	5/2,3	0.13838	0.18757	0.22736
	7/2,3	0.37493	0.36445	0.34277
	9/2,5	0.32494	0.41033	0.46981
	11/2,5	0.60253	0.45323	0.33408
	13/2,7	0.52750	0.58843	0.60915
	15/2,7	−0.28972	−0.31492	−0.33522
9/2+[615]	9/2,4	0.23191	0.20589	0.18704
	11/2,6	0.96668	0.97031	0.97176
	13/2,6	−0.10842	−0.12690	−0.14388

Erratum

Cluster-Model Calculation for Phase Shifts of α-α Scattering, SUEJI OKAI AND SHIM C. PARK [Phys. Rev. **145**, 787 (1966)]. We report here that a part of the result obtained by us regarding the energy independence of the effective α-α interaction and the discussion regarding Fig. 3 was similarly obtained by R. Tamagaki and H. Tanaka, Progr. Theoret. Physics (Kyoto) **34**, 191 (1965). The two investigations were done completely independently of each other.