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Sodium-Electron Spin-Exchange Collisions

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A spin-exchange optical pumping experiment to study collisions between sodium atoms and free electrons is reported. In this experiment, electrons in a weak magnetic field were polarized by spin-exchange collisions with optically pumped sodium atoms. The sodium-electron collisions were the principal source of the electron resonance linewidth, and they also produced a shift in the electron resonance frequency. The magnitudes of the linewidth and the frequency shift depend upon the scattering amplitude for sodium-electron collisions, the sodium polarization, and the sodium atom density. The ratio of the frequency shift to the linewidth was 0.03. The sodium polarization was approximately 30%. The electron resonance frequency was lower when the sodium polarization was positive. The measurements of the electron linewidth and the frequency shift agree with theoretical values of the linewidth and frequency shift obtained from the sodium-electron scattering phase shifts which have been recently calculated by Garrett.

INTRODUCTION

In recent years, several authors have investigated theoretically the low-energy elastic scattering of electrons from alkali atoms.[1–4] It would be desirable to experimentally check the calculated scattering phase shifts and cross sections in the lower energy ranges. In general, however, it is quite difficult to measure electron-alkali scattering cross sections at very low energies.

Spin-exchange optical pumping experiments provide a limited but convenient means for studying electron–alkali atom collisions at temperatures between 300 and 450°K. In such experiments, an alkali vapor in a magnetic field is polarized by the absorption of circularly polarized optical resonance radiation incident along the direction of the magnetic field. The polarization is monitored by observing the transmission of the resonance radiation through the flask containing the alkali vapor. Free electrons produced in the flask by a radio-frequency discharge are polarized by spin-exchange collisions with the alkali atoms. If a radio-frequency field is adjusted so as to depolarize the electrons, the alkali atoms will be partially depolarized through spin-exchange collisions with the electrons, and the intensity of the light transmitted by the flask will decrease accordingly. Because the electron–alkali atom spin-flip cross section is quite large, the alkali atom–electron collisions are generally the dominant electron spin relaxation mechanism. When this is the case, the alkali atom–electron collisions are the principal source of the width of the electron resonance signal. They also give rise to a shift in the electron resonance frequency. Both the frequency shift and the linewidth are functions of the electron–alkali atom scattering phase shifts.

Measurements of the spin-flip cross section and frequency shift in a rubidium-electron system and in a cesium-electron system have already been reported.[5,6] In both experiments, the frequency shifts were small and the spin-flip cross sections were large, indicating that the scattering was nearly all s-wave and that the difference between the triplet and singlet s-wave phase shifts was approximately $\pi/2$. These conclusions were reached under the assumption that the phase shifts did not vary appreciably with energy. The results of the cesium-electron measurements were compared with the phase-shift calculations of Stone and Reitz.[4] They predicted a much larger frequency shift and a much smaller spin-flip cross section.

This paper reports a spin-exchange optical pumping experiment performed to study sodium-electron colli-

* This research was conducted while pursuing a Postdoctoral Resident Research Associateship.

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[2] A. Salmona and M. J. Seaton, Proc. Phys. Soc. (London) **77**, 617 (1961).
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[5] L. C. Balling, R. J. Hanson, and F. M. Pipkin, Phys. Rev. **133**, A607 (1964).
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sions at temperatures near 420°K. The theoretical numerical values for the phase shifts are quite sensitive to the form of the polarization potential used in the calculations. Since sodium has fewer electrons than either cesium or rubidium, it seems likely that the calculations of the sodium-electron scattering phase shifts would be more reliable than the calculations for cesium.

The first part of this paper summarizes the theory of spin-exchange optical pumping experiments. The theory predicts the dependence of the frequency shift and linewidth of the electron resonance upon the sodium-electron scattering phase shifts; the second part briefly describes the apparatus; the third part summarizes the measurements; the concluding part compares the results of the measurements with available theoretical calculations and with other measurements.

THEORY

A complete theoretical treatment of spin-exchange optical pumping experiments has been given in a previous paper.[5] The purpose of this section is to summarize the theoretical expressions for the electron linewidth and frequency shift which were derived in that paper. In order to calculate the form of the electron resonance signal, one must understand the effect upon the sodium and electron spin-systems of the pumping light and the radio-frequency field and how the two systems are coupled through spin exchange. A suitable way to characterize the electron and sodium spin system quantum mechanically is by the sodium and electron spin-space density matrices. The form of the electron resonance signal is arrived at by calculating the time rate of change of these density matrices.

If one assumes that there is no spin-orbit coupling during an electron-sodium collision, the scattering can be described in terms of singlet and triplet scattering phase shifts. One can then show that the time dependence of the electron density matrix due to electron-sodium collisions is given by the expression[5]

$$\frac{d\rho(e)}{dt}=\begin{bmatrix} \dfrac{P(\mathrm{Na})-P(e)}{2T_{ee}} & -\dfrac{1-i\kappa P(\mathrm{Na})}{T_{ee}}\rho_{12}(e) \\ -\dfrac{1+i\kappa P(\mathrm{Na})}{T_{ee}}\rho_{21}(e) & \dfrac{P(e)-P(\mathrm{Na})}{2T_{ee}} \end{bmatrix}, \quad (1)$$

where $P(e)$ and $P(\mathrm{Na})$ are the electron polarization and the sodium electronic polarization given by the equations

$$P(e)=\rho_{11}(e)-\rho_{22}(e) \quad (2)$$

and

$$P(\mathrm{Na})=\mathrm{Tr}\sigma_{ez}\rho(\mathrm{Na}), \quad (3)$$

where σ_{ez} is the Pauli spin matrix which operates on the electron coordinate. The spin-exchange relaxation time is given by the equation

$$1/T_{ee}=V_{e\mathrm{Na}}N_{\mathrm{Na}}\sigma_{\mathrm{SF}}. \quad (4)$$

Here, $V_{e\mathrm{Na}}$ is the velocity of the electrons relative to the sodium atoms. N_{Na} is the density of sodium atoms in the optical pumping cell, and σ_{SF} is the sodium-electron spin-flip cross section.

In terms of the phase shifts, the spin-flip cross section σ_{SF} and the frequency shift parameter κ are given by the equations

$$\sigma_{\mathrm{SF}}=\left(\frac{\pi}{k^2}\right)\sum_{l=0}^{\infty}(2l+1)\sin^2(\delta_l{}^3-\delta_l{}^1), \quad (5)$$

$$\kappa=\frac{1}{\sigma_{\mathrm{SF}}}\frac{\pi}{2k^2}\sum_{l=0}^{\infty}(2l+1)\sin 2(\delta_l{}^3-\delta_l{}^1), \quad (6)$$

where $\delta_l{}^3$ and $\delta_l{}^1$ are the triplet and singlet phase shifts for each partial wave; k is the absolute value of the momentum of the electrons relative to the sodium atoms divided by $\hbar$.

An equation analogous to Eq. (1) describes the time behavior of the sodium density matrix.[5]

If one includes the effects of relaxation mechanisms other than spin-exchange collisions and calculates the time rate of change of the sodium and electron density matrices due to the pumping light and radio-frequency field, one arrives at an expression for the change in light transmitted by the absorption flask when there is a radio-frequency field present which can depolarize the electrons. Assuming that there is a static magnetic field H_0 along the direction of the light beam, and a radio-frequency field $2H_1\cos\omega t$ perpendicular to the light beam, it is shown in Ref. 5 that the change in transmitted light δI is

$$\delta I=(\mathrm{const})\frac{\omega_1{}^2\tau_1\tau_2}{1+\omega_1{}^2\tau_1\tau_2+(\omega_0-\delta\omega_0-\omega)^2(\tau_2)^2}; \quad (7)$$

ω_1 is the resonance frequency of the electrons in the field H_1; ω_0 is the resonance frequency of the electrons in the field H_0; τ_1 and τ_2 are algebraic functions of all the spin relaxation times which describe the physical situation in the optical pumping cell. Explicit expressions for τ_1 and τ_2 are given in Ref. 5. Under most experimental conditions

$$\tau_1\cong\tau_2\cong T_{ee}. \quad (8)$$

Equation (7) indicates that the electron line will have a Lorentzian shape and a line center which is shifted by the amount $\delta\omega_0$ due to spin-exchange collisions with the sodium atoms. The direction and magnitude of this frequency shift depend upon the scattering phase shifts and the sodium polarization, and is given by

$$\delta\omega_0=P(\mathrm{Na})\kappa/T_{ee}. \quad (9)$$

The theory can be generalized to allow for the possibility of spin-orbit coupling in an electron–alkali atom collision.[6] The scattering can then no longer be de-

scribed by singlet and triplet phase shifts only, and the expressions for the linewidth and frequency shift are more complicated. The assumption of no spin-orbit coupling is made in electron–alkali atom scattering calculations and it will be made here.

APPARATUS

The apparatus used in these experiments was similar in most respects to the apparatus described in two previous papers.[5,6] A block diagram of the experimental setup is shown in Fig. 1. The magnetic field of 50 mG was produced by a solenoid 12 in. in diameter and 36 in. long. The solenoid was enclosed in three concentric cylindrical magnetic shields to reduce the perturbing effects of external magnetic fields. The full width at half-maximum of the sodium resonance (700 cps/mG) due to magnetic field inhomogeneity was less than 100 cps.

The light source was a commercial sodium arc lamp which was operated in an oven. No D-1 filter was used. The circular polarizer could be rapidly changed to give either left or right circularly polarized light.

The resonance signal was measured by amplitude modulating the radio-frequency field with a coaxial relay and observing the demodulated absorption signal with a lock-in detector.

The absorption flasks were Pyrex cylinders 8 cm in diameter and 8 cm long. After sodium was distilled into them, they were filled with a helium or neon buffer gas. A continuous radio-frequency discharge between two glass-covered electrodes was the source of free electrons. The electrodes were located in a turret on top of the absorption cylinder. The electrons produced by the discharge diffused down into the main absorption flask.

The flasks were located inside a cylindrical oven. The heating element was wound on a $\frac{1}{8}$-in.-thick copper tube running the length of the oven and closely surrounding the absorption flask. This tube held temperature gradients across the flask to a minimum. The gradient was less than 2°C. The temperature of the flask was measured with three copper Constantan thermocouples attached to its surface. The radio-frequency field was produced by a signal generator operating into a single coil surrounding the flask inside the copper tube.

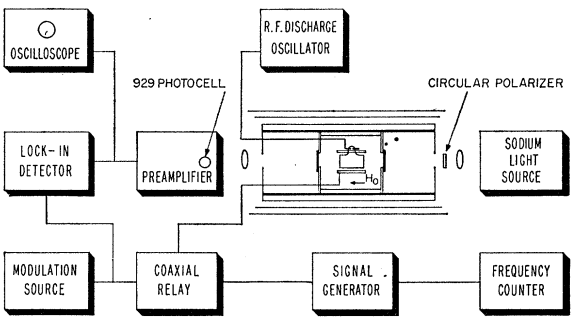

FIG. 1. Block diagram of the optical pumping apparatus.

MEASUREMENTS

The first measurements were directed toward a determination of τ_1 and τ_2 as a function of temperature. Equation (7), which describes the amplitude of the electron signal, indicates a method for determining τ_1 and τ_2. A plot of the square of the full width at half-maximum of the electron signal versus $(\omega_1/2\pi)^2$ should yield a straight line with a slope of $4(\tau_1/\tau_2)$ and with a zero radio-frequency intercept $(1/\pi\tau_2)^2$. Thus, measurements of the electron linewidth as a function of radio-frequency field strength at various temperatures yields τ_1 and τ_2 as a function of temperature. The radio-frequency field strength ω_1 was obtained by keeping the radio-frequency signal generator set at the electron resonance frequency and increasing the magnetic field H_0 until the sodium resonance was visible on the oscilloscope. This signal showed the characteristic modulation of the pumping light ("wiggles") due to the nutation of the sodium moment. Since the magnetic moment of the electron is approximately four times the atomic moment of the sodium atom, ω_1 was obtained from the relationship

$$\omega_1 = 8\pi\nu(\mathrm{Na}). \tag{10}$$

Here ν(Na) is the frequency of the sodium "wiggles."

Figure 2 shows typical plots of (linewidth)^2 versus $(\omega_1/2\pi)^2$ for various temperatures. These measurements were made without the surrounding copper tube in order to observe the "wiggles" over a wider range of radio-frequency field strengths. An absorption flask with all the sodium in one small spot at the bottom of the flask was used. It contained 43 mm Hg of helium. A thermocouple was attached to the flask at the location of the

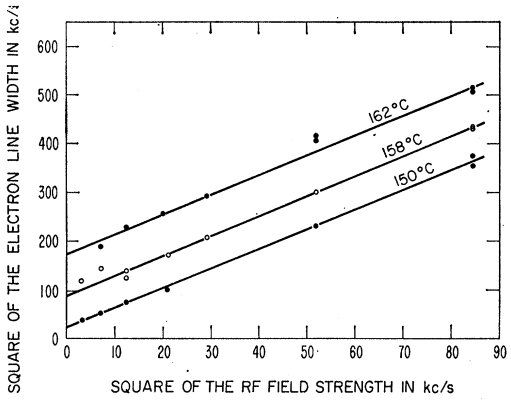

FIG. 2. The square of the full width at half-maximum of the electron line in kc/sec plotted versus the square of the rf field strength in kc/sec for three different temperatures. The slope of these lines is equal to $4\tau_1/\tau_2$; the intercept of the lines is $(1/\pi\tau_2)^2$. These measurements were made in a flask containing sodium condensed in one spot on the wall of the flask.

sodium. The slopes of the resulting straight lines yielded a value for τ_1/τ_2 of 1.0±0.05.

In Ref. 5 it is shown that $\tau_1=\tau_2\cong T_{ee}$ if spin-exchange collisions are the dominant electron-spin relaxation mechanism. Referring to Eqs. (4) and (5), one sees that

$$\frac{1}{T_{ee}}\propto N_{\mathrm{Na}}V_{e\mathrm{Na}}\frac{1}{k^2}\propto\frac{p(\mathrm{Na})}{T^{3/2}}, \tag{11}$$

where p(Na) is the vapor pressure of sodium, and T is the temperature in °K. If T_{ee} is indeed the principal source of the electron linewidth, then the zero radio-frequency intercepts ($1/\pi\tau_2$) of the straight-line plots should have the temperature dependence of $1/T_{ee}$ as given by Eq. (11).

The measurements of $1/\pi\tau_2$ as a function of temperature were made with an absorption flask with 60% of its wall surface completely covered with sodium. The end windows were also partially coated with a thin film of sodium. This was to prevent the walls from absorbing the sodium and lowering the sodium density in the flask. The buffer gas was neon at a pressure of 45 mm Hg. In order to be certain that wall absorption did not lower the density of sodium in the experimental bulb, the sodium density in the experimental bulb was compared with the sodium density in a bulb with 95% of its surface coated with sodium. This comparison was made by measuring the amplitude of the sodium signal as a function of temperature in both bulbs under identical light source and buffer-gas conditions. There was no observable difference between the sodium densities in the two bulbs. A plot of $1/\pi\tau_2$ as a function of temperature is shown in curve A in Fig. 3. The copper tube was employed in these measurements to minimize temperature gradients. The dashed curve shows the temperature dependence of $p(\mathrm{Na})/T^{3/2}$. Curve B shows the values of $1/\pi\tau_2$ for the flask containing sodium in one spot. The large discrepancy between the results demonstrates the necessity of having the walls well coated. Table I shows in another fashion the temperature dependence of the zero radio-frequency linewidth $\Delta\nu$ of the electron signal. It is believed that departures of the measured values of τ_2 from the proper temperature dependence reflect errors in the measurement of the average temperature of the absorption flask. We conclude that, within the temperature range investigated, $\tau_1=\tau_2\cong T_{ee}$.

In order to measure the frequency shift $\delta\nu_0$ of the electron resonance, the electron signal was observed first with left circularly polarized light and then with right circularly polarized light. This operation reversed the sign of the sodium polarization and hence the sign of the frequency shift. That is,

$$2\delta\nu_0=\nu_{\mathrm{right}}-\nu_{\mathrm{left}}. \tag{12}$$

It was found that the electron resonance frequency was lower when the sodium polarization was positive. The temperature dependence of $2\delta\nu_0$ is shown in Fig. 4. The dashed curve shows how the zero radio-frequency linewidth varied with temperature. According to Eq. (9), the frequency shift should have the same temperature dependence as $1/T_{ee}$. In fact,

$$2\delta\nu_0/\Delta\nu=2\pi T_{ee}\delta\nu_0=P(\mathrm{Na})\kappa. \tag{13}$$

Table II lists the measured values of $2\delta\nu_0$. The polarization P(Na) of the sodium atoms during these measurements was (30±10)%. This was determined by a measurement of the relative amplitudes of the six Zeeman transitions of sodium. These could be fitted to the population scheme where the population of the Zeeman sublevel of magnetic quantum number m is proportional to $e^{m\beta}$. The magnitude of β reflects the degree of polarization.

TABLE I. The values of $\Delta\nu$ and τ_2 obtained at various temperatures. $\Delta\nu$ is the electron linewidth at zero rf. The values of τ_2 and $\Delta\nu$ referred to 140°C (413°K) were obtained from the equation

$$\tau_2(413^\circ\mathrm{K})/\tau_2(T)=N_{\mathrm{Na}}(T)/N_{\mathrm{Na}}(413^\circ\mathrm{K})\left[\frac{413}{T}\right]^{1/2}=\frac{\Delta\nu(T)}{\Delta\nu(413^\circ\mathrm{K})}.$$

$N_{\mathrm{Na}}(T)$ is the number of sodium atoms per cm³ in the flask at the temperature T. (T is in °K.)

Temperature (°C)	$\Delta\nu$ (kc/sec)	τ_2 (10^{-5} sec)	$\Delta\nu$(140°C) (kc/sec)	τ_2(140°C) (10^{-5} sec)
132	8.17	3.89	14.5	2.19
136	10.0	3.18	13.3	2.39
140	11.35	2.80	11.35	2.80
143	13.7	2.32	11.1	2.86
146	16.85	1.89	11.1	2.86
152	23.65	1.34	10.4	3.06
156	29.3	1.08	9.96	3.19

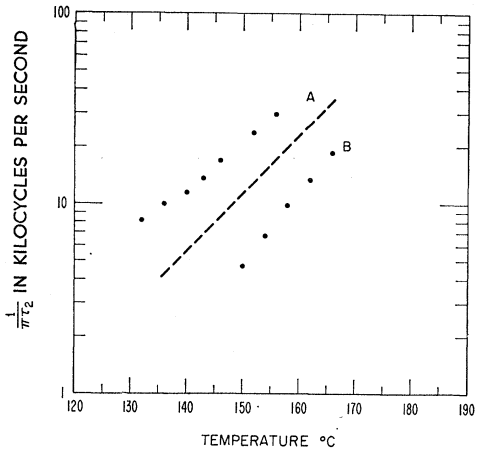

FIG. 3. A plot of $1/\pi\tau_2$ as a function of temperature. The values of $1/\pi\tau_2$ were obtained at each temperature by measuring the zero rf field intercepts of straight-line plots such as those shown in Fig. 2. Curve A is a plot of measurements made in a flask with its walls saturated with sodium; curve B is a plot of measurements in a flask with the sodium condensed in one spot; the dashed curve shows the temperature dependence of the function $p(\mathrm{Na})T^{-3/2}$. This is the temperature dependence one expects for $1/\pi\tau_2$ if electron-sodium collisions are the principal source of linewidth.

INTERPRETATION

The average measured electron linewidth referred to 140°C is

$$\Delta\nu(140)=(11.7\pm2)\text{ kc/sec},\qquad(14)$$

where the quoted error represents the spread in the data. This linewidth was obtained with the absorption bulb, the walls of which were coated with sodium.

The measured ratio of twice the frequency shift to the electron linewidth is

$$0.04\leq 2\delta\nu_0/\Delta\nu\leq 0.09.\qquad(15)$$

At a fixed electron velocity one has from Eqs. (5), (6), and (13)

$$\kappa=-\frac{1\sum_{l=0}^{\infty}(2l+1)\sin2(\delta_l^3-\delta_l^1)}{2\sum_{l=0}^{\infty}(2l+1)\sin^2(\delta_l^3-\delta_l^1)}=\frac{1}{P(\mathrm{Na})}\left[\frac{2\delta\nu_0}{\Delta\nu}\right].\qquad(16)$$

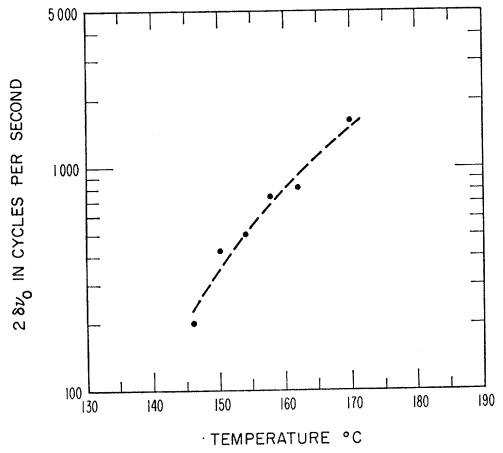

FIG. 4. A plot of twice the frequency shift $2\delta\nu_0$ as a function of temperature as measured in a single run. The dashed line represents the observed temperature dependence of $1/\pi\tau_2$. One expects the frequency shift to have this same temperature dependence.

From Eqs. (15) and (16) and the measured values of the sodium polarization $P(\mathrm{Na})$ we have

$$0.1\leq\kappa=-\frac{1\sum_{l=0}^{\infty}(2l+1)\sin2(\delta_l^3-\delta_l^1)}{2\sum_{l=0}^{\infty}(2l+1)\sin^2(\delta_l^3-\delta_l^1)}\leq 0.4.\qquad(17)$$

Also,

$$\Delta\nu=\frac{1}{\pi T_{ee}}=N_{\mathrm{Na}}V_{e\mathrm{Na}}\frac{1}{k^2}\sum_{l=0}^{\infty}(2l+1)\sin^2(\delta_l^3-\delta_l^1).\qquad(18)$$

In the studies of the rubidium-electron and cesium-electron spin-exchange collisions, one interpreted the frequency shift and linewidth by assuming that the scattering phase shifts did not vary appreciably over the energy range spanned by thermal electrons.

TABLE II. The measured values of twice the frequency shift $\delta\nu_0$ at various temperatures. $P(\mathrm{Na})\kappa$ is obtained from the equation $P(\mathrm{Na})\kappa=2\delta\nu_0/\Delta\nu$. $P(\mathrm{Na})$ is the electronic polarization of the sodium atoms which was estimated to be $(30\pm10)\%$ for these measurements. $\Delta\nu$ is the electron linewidth at zero rf. The absorption flask used for these measurements contained 43 mm Hg of helium. The sodium was located in one spot on the wall of the flask.

Temperature (°C)	$2\delta\nu_0$ (cps)	$\Delta\nu$ (kc/sec)	$P(\mathrm{Na})\kappa$
146	200	3.17	0.063
150	414	4.70	0.088
154	487	6.70	0.073
158	740	10.0	0.074
158	454	10.0	0.045
162	800	12.65	0.063
162	430	11.40	0.038
166	870	16.5	0.053
166	822	17.0	0.047
170	1650	20.0	0.083

A list of the singlet and triplet phase shifts for $l=0$, 1 at low energies, as calculated by Garrett,[1] is given in Table III. Contributions from higher values of l are negligible. Calculated values of κ and $\sin^2(\delta_0^3-\delta_0^1)+3\sin^2(\delta_1^3-\delta_1^1)$ are also shown. It is clear that one can not justify ignoring the energy dependence of the phase shifts. In order to compare the calculations of Garrett with the present experiment, one must average the theoretical values of κ and $[\sin^2(\delta_0^3-\delta_0^1)+3\sin^2(\delta_1^3-\delta_1^1)]/V_{e\mathrm{Na}}$ over a Maxwell velocity distribution characterized by $T=140$°C. In order to obtain theoretical expressions for the frequency shift and linewidth, one must know the density of sodium atoms in the bulb. Two commonly used vapor-pressure curves[7,8] are plotted in Fig. 5. Using the vapor-pressure data of

TABLE III. Singlet and triplet phase shifts for $l=0$, 1 as calculated by Garrett (Ref. 1) and the corresponding values of κ and of $\sin^2(\delta_0^3-\delta_0^1)+3\sin^2(\delta_1^3-\delta_1^1)$.

E (Ry)	δ_0^1	δ_0^3	δ_1^1	δ_1^3	κ	$\sin^2(\delta_0^3-\delta_0^1)+3\sin^2(\delta_1^3-\delta_1^1)$
0.00050	12.3625	12.4587	6.3460	9.4017	−5.13	0.03
0.0010	12.2609	12.3912	6.4740	9.3528	−2.84	0.22
0.0015	12.183	12.336	6.680	9.307	−1.51	0.75
0.0020	12.116	12.288	6.960	9.262	−0.78	1.69
0.0025	12.060	12.240	7.283	9.219	−0.31	2.64
0.0030	12.006	12.206	7.573	9.177	0.03	3.04
0.0040	11.913	12.133	7.765	9.100	0.31	2.89
0.0050	11.832	12.069	8.113	9.031	0.86	1.95
0.0060	11.759	12.010	8.193	8.968	1.14	1.53
0.0100	11.526	11.814	8.025	8.762	1.23	1.43
0.0200	11.126	11.966	7.845	8.415	1.70	0.99

[7] R. W. Ditchburn and J. C. Gilmour, Rev. Mod. Phys. 13, 310 (1941).

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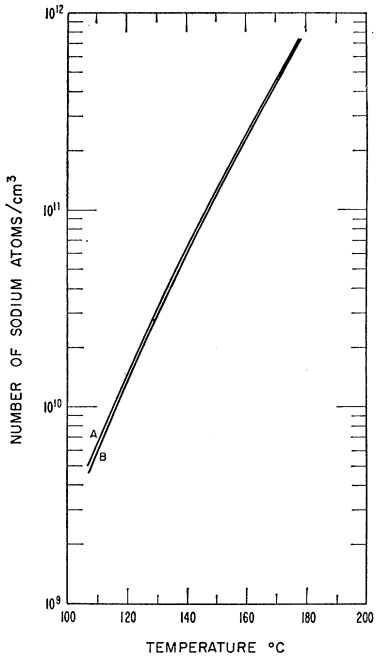

FIG. 5. Plots of the sodium density as a function of temperature. Curve A is derived from the equation (Ref. 7)

$$\log_{10}p(\mathrm{Na}) = -5567/T + 9.235 - 0.5\log_{10}T,$$

where p(Na) is the vapor pressure of sodium in mm Hg and T is the temperature in °K. Curve B is derived from the equation (Ref. 8)

$$\log_{10}p(\mathrm{Na}) = -5775/T + 11.744 - 1.274\log_{10}T.$$

Ditchburn and Gilmour, one finds that

$$\Delta\nu(140) = 11.6 \text{ kc/sec (theoretical)} \tag{19}$$

and

$$\kappa = \frac{1}{P(\mathrm{Na})}\frac{2\delta\nu_0}{\Delta\nu} = 0.2 \text{ (theoretical)}. \tag{20}$$

These average values do not vary appreciably over the range of temperatures investigated experimentally. The agreement between the experimental results and the calculated shift and linewidth is quite good. The use of the vapor-pressure data is justified by the fact that the density of sodium atoms in the bulb was not lowered by wall absorption.

Garrett's calculations for sodium are in disagreement with those made by Salmona and Seaton. Garrett finds that the triplet scattering length A^- is equal to 1.634 and the singlet scattering length A^+ is equal to 6.511. These differ from the sodium-electron scattering lengths calculated by Salmona and Seaton.[2] Their results were $A^-=9$ and $A^+=-12$.

In order to compare the results of this experiment with the previous experiments with rubidium and cesium, one must make the unjustified assumption that the phase shifts do not vary appreciably with energy. If one makes this assumption and allows only s- and p-wave contributions to the scattering, the measured value of the shift parameter κ demands that either

$$0.9 < \sin^2(\delta_0^3-\delta_0^1) + 3\sin^2(\delta_1^3-\delta_1^1) < 1.1, \tag{21}$$

or

$$2.4 < \sin^2(\delta_0^3-\delta_0^1) + 3\sin^2(\delta_1^3-\delta_1^1) < 4.0. \tag{22}$$

Using the vapor-pressure data of Ditchburn and Gilmour and the average measured value of the linewidth given in Eq. (14), one finds that

$$\sum_{l=0}^{\infty}\sin^2(\delta_l^3-\delta_l^1) = 1.2\pm0.2. \tag{23}$$

The quoted error does not include possible error in the use of the vapor-pressure data.

On the basis of Eqs. (21), (22), and (23), one would conclude that

$$\sin^2(\delta_0^3-\delta_0^1) + 3\sin^2(\delta_1^3-\delta_1^1) \cong 1.0, \tag{24}$$

$$(0.8)\pi/2 \pm n\pi \leq (\delta_0^3-\delta_0^1) \leq (1.2)\pi/2 \pm n\pi, \tag{25}$$

and

$$0 \pm n\pi \leq (\delta_1^3-\delta_1^1) \leq (0.2)\pi/2 \pm n\pi. \tag{26}$$

Similar conclusions were reached in the work with rubidium and with cesium.

Equation (24) would yield a value for the spin-flip cross section σ_{SF} of

$$\sigma_{SF} = (2.2\pm0.2)\times10^{-14} \text{ cm}^2. \tag{27}$$

Dehmelt,[9] in his report of the first spin-exchange optical pumping experiment, concluded that at a temperature of 400°K,

$$\sigma_{SF} > 2.3\times10^{-14} \text{ cm}^2. \tag{28}$$

He obtained this result by analyzing the experimental width and amplitude of the electron resonance signal.

CONCLUSIONS

It is somewhat surprising that the results of the spin-exchange studies of electron–alkali atom scattering should be so similar for cesium, rubidium, and sodium, since one expects their polarization potentials to be significantly different in each case.

The calculations of Garrett are supported by the results obtained in this experiment. His calculations indicate that the simple interpretation of spin-exchange experiments, in which the energy dependence of the phase shifts was ignored, is of doubtful value. Because of the energy dependence of the theoretical phase shifts and the resulting necessity of averaging over a wide range of values for the parameters which determine the frequency shift and linewidth, the spin-exchange experiment is not a particularly sensitive test of the theory.

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[9] H. G. Dehmelt, Phys. Rev. **109**, 381 (1958).