Level Spectra of Odd-Even $1f_{7/2}$-Shell Nuclei in the Coriolis Coupling Model*

F. B. MALIK AND W. SCHOLZ
Yale University, New Haven, Connecticut
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The negative parity levels of odd-even nuclei with protons and neutrons in the $1f_{7/2}$ shell are calculated using the strong-coupling symmetric-rotator model including the Coriolis coupling between bands. The single-particle energy levels and wave functions in the deformed well are computed for a spin-orbit strength $C=-0.26\hbar\omega_0$ which is consistent with the observed splitting in Ca^{41}. The well-flattening parameter D is taken to be $-0.06\hbar\omega_0$ in the middle of the $1f_{7/2}$ shell and $-0.035\hbar\omega_0$ otherwise. The band-head energies are calculated from the appropriate summation over the occupied single-particle energy levels using $\hbar\omega_0=41/A^{1/3}$ MeV for the energy of the oscillator quantum. The moment of inertia is taken from the excitation energy of the first excited 2^+ state of neighboring even-even nuclei assuming a rotational character for this state. The same value is used for all bands in any given nucleus. The matrix elements of the Coriolis coupling are computed from the single-particle wave functions. The final excitation spectra are obtained by diagonalizing the Coriolis coupling term with the rotational wave function based on the ten available particle or core excited states in the $1f$-$2p$ shell. Energy levels and wave functions are calculated as a function of the deformation parameter β; the level spectrum for each individual nucleus is given for a tentative choice of β. The effect of a quenching of the Coriolis interaction on the calculated level spectrum is investigated. The computed level spectra for the nuclei (i) Sc^{43}, Sc^{45}, Sc^{47}, (ii) Ca^{43}, Ti^{45}, V^{47}, V^{49}, V^{51}, (iii) Ca^{45}, Ti^{47}, Cr^{49}, Mn^{51}, Mn^{53}, and (iv) Ti^{49}, Cr^{51} compare favorably with experiment. The model predicts the correct ground-state spin for all nuclei, including the anomalous cases of Ti^{47}, Cr^{49}, and Mn^{51} which have a $\frac{5}{2}^-$ ground state. Thus the observed $\frac{7}{2}^-$ (and occasionally $\frac{5}{2}^-$) ground-state configuration cannot be adduced as evidence supporting the validity of the spherical-shell model in the $1f_{7/2}$ shell. The positions of negative parity states with known spin—especially the low-lying $\frac{3}{2}^-$ states—are well accounted for in this model, in contrast with difficulties encountered in the shell-model treatment. In particular, the model reproduces the observed ground-state triplets in V^{47} and in V^{49} and the low-lying $\frac{3}{2}^-$ state in Cr^{51}. Further, this model predicts the correct number of levels below about 2.5-MeV excitation energy for nuclei in the upper half of the $1f_{7/2}$ shell. For nuclei in the lower half of the $1f_{7/2}$ shell, more levels are observed than predicted, which suggests the presence of positive parity states arising from core excitation of the $2s$-$1d$ shell. The natural classification of the $1f_{7/2}$ nuclei in this model according to their odd-nucleon count is borne out by the similarity of the observed spectra. The natural shell-model classification scheme in terms of the conjugate and equivalent pairs, on the other hand, does not seem to be supported by the experimental data.

I. INTRODUCTION

EXTENSIVE calculations of the low-lying levels of nuclei in the $1f_{7/2}$ region of the periodic table are available in terms of the spherical-shell model including residual interaction between nucleons outside the doubly closed shell of Ca^{40}.[1–10] These investigations are based on two different approaches. In one, the residual interaction between the nucleons outside the closed shell is assumed as a perturbation; the two-body matrix elements are then evaluated and the level spectrum is obtained by diagonalization using the strengths of the two-body potentials as free parameters.[1–3,8,9] In the other, all the matrix elements of the residual interaction are treated as parameters and are obtained by fitting the observed spectra of the $1f_{7/2}$-shell nuclei[6]—particularly of the Ca isotopes.[10] Apart from a few exceptions,[8–10] in either case the nucleons outside the closed shell are restricted usually to states of the $(1f_{7/2})^n$ configuration. (Henceforth denoted as the shell model with the $1f_{7/2}$ configuration mixture.)

The experimental information on the properties of the odd nuclei in this region of the nuclear chart is, on the other hand, relatively sparse. In general, with a few exceptions such as V^{51} the ground-state spin and the approximate level scheme are the only available data. Despite this, the predictions of the extensive computations within the framework of the above-mentioned spherical-shell model show in a number of cases significant discrepancies with the observed level spectra, as noted below.

The above-mentioned shell model predicts that the cross-conjugate pairs, i.e., the pair ($Z=20+a$, $N=20+b$) and ($Z=28-b$, $N=28-a$), and the equivalent pairs (i.e., the nuclei having the same number of particles or holes) should possess roughly the same excitation spectra. On this basis the group Ca^{43}, Ca^{45}, V^{51}, Mn^{53} should exhibit about the same level pattern, as they do only to a first approximation. Other cross-conjugate pairs in the $1f_{7/2}$ shell are (Cr^{51}, Sc^{45}), (Ti^{47}, V^{49}), (Ti^{45}, Mn^{51}), (Sc^{47}, Ti^{49}), and (V^{47}, Cr^{49}). Although experimental information is by no means complete there are differences between the spectra of the cross-conjugate and equivalent pairs; e.g., the

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excitation energy of the first $\frac{3}{2}^-$ state in Ti^{49} and V^{51} is almost twice as high as in Sc^{47} and Ca^{43}, respectively. Other discrepancies are found in the spectra of the pairs (Ti^{47}, V^{49}) and (V^{47}, Cr^{49}), where one of the members shows a ground-state doublet as opposed to a triplet in the other member.

Many of the spectra of these pairs are not in accord with the theoretical shell-model calculations, e.g., (i) Sc^{45} and Sc^{43} exhibit at least ten and five states, respectively, below the first computed level. It is improbable that all these extra levels have positive parity which eludes a $1f_{7/2}$-shell-model-configuration treatment. (ii) The first excited state in Cr^{51} at 0.78-MeV excitation energy is observed to have the assignment $\frac{3}{2}^-$, whereas the first excited state predicted by the shell model is an $11/2^-$ state at 1.5 MeV. (iii) Near their ground state V^{47} and V^{49} both exhibit a characteristic triplet which remains unexplained within the framework of the shell-model computation with the $1f_{7/2}$ configuration.[6]

Extension of the shell model to include the effect of $2p_{3/2}$ (and $1g_{9/2}$ for the positive parity states) configurations on the level scheme of the Ca isotopes[8–10] still does not entirely solve the problem—the computed level density of Ca^{45} between 1 and 2 MeV excitation energy is twice the observed one.[9] Although it remains possible that a careful mixture of different configurations may explain any one or more of these gross discrepancies, it is equally legitimate to search for an alternative approach to this problem—particularly in view of the extensive computational techniques and the large numbers of parameters involved in mixing such configurations within the framework of the spherical-shell model. Such a search becomes particularly imperative in view of the consistent enhancement by an order of magnitude of the electric quadrupole transition rates in the entire $1f_{7/2}$ shell.[11–14] Historically, for heavy nuclei this suggested[15,16] a collective participation of a deformed core in determining the level spectrum and transition rates; however, for nuclei in the $1f_{7/2}$ shell the typical band structure or band spacing is not observed. The usual approach to attribute the ground-state spin to the lowest state of a band based on the Nilsson level occupied by the last odd nucleon at a first glance predicts ground-state spins which are in contradiction with the observed $\frac{7}{2}$ or $\frac{5}{2}$ spin of the ground states of the odd nuclei in this region of the nuclear chart.

In general, however, the Coriolis coupling would be expected to mix different bands in a deformed-core model for odd-even nuclei.[17] The coupling strength of this perturbation is inversely proportional to the moment of inertia of the system, which is in turn roughly proportional to $A^{5/3}$. Thus the coupling strength of this perturbation, which is of the order of 10 keV for the heavy elements, e.g., the uranium isotopes, is expected to increase by at least a factor of 10 in the light nuclei. This large increase in the Coriolis coupling introduces a very strong mixing of bands which may in some cases destroy the original band structure entirely since the spacing of the single-particle Nilsson levels becomes of the same order of magnitude as the rotational energies. Moreover, the state of the lowest predicted energy need no longer correspond to the orbit occupied by the last odd nucleon.

With this view in mind, the strong-coupling model of Bohr and Mottelson[16] was applied to V^{51} (Ref. 18). The large strength of the indicated Coriolis coupling warranted a proper diagonalization of the Coriolis interaction between ten available bands in the $1f$-$2p$ major shell. The theory predicted (i) the correct ground-state spin, (ii) a reasonable level spectrum, and (iii) the experimental transition rates and static moments without requiring the introduction of effective charges.

In this paper we apply the Coriolis coupling model to all odd-even nuclei in the $1f_{7/2}$ shell. We restrict considerations to the level spectra and investigate the success of the model in predicting the following qualitative features of the spectra of these nuclei:

(a) The ground-state spin. The elementary single-particle shell model[19,20] of Mayer, Jensen, Suess, Feenberg, Hammack, Haxel, and Nordheim predicts a $\frac{7}{2}$ spin for the ground state, as it is observed for all odd nuclei except Ti^{47}, Cr^{49}, and Mn^{51}, which have a $\frac{5}{2}$ ground-state spin. This $\frac{5}{2}$ ground-state spin can be reproduced under certain conditions in the shell model involving a careful configuration mixture with an appropriate residual interaction.[6] The Nilsson model, without the Coriolis coupling, cannot account for the ground-state spin.

(b) The observed triplet near the ground state of V^{49} and V^{47}. This has not yet been successfully understood in terms of the shell model.

(c) The presence of the low-lying $\frac{3}{2}^-$ states, e.g., in V^{49} and Cr^{51}. In the shell-model computation of McCullen, Bayman, and Zamick[6] the observed low-lying $\frac{3}{2}^-$ state is not always well reproduced.

(d) The gap of 1.4 MeV between the ground and the first excited state in Ti^{49}. This can be understood in terms of the $(1f_{7/2})^n$-shell model.

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The present incomplete experimental information on the level structure of odd nuclei does not yet warrant a detailed analysis as has been done for V^{51}. The pıimary aim of the present paper is to examine to what extent the Coriolis coupling model shows promise in reproducing the qualitative features of the level structure observed so far and make predictions concerning the properties of the odd nuclei in the $1f_{7/2}$ shell in this model.

II. THE MODEL

The basis underlying the present treatment is the strong-coupling model as suggested by Bohr and Mottelson. In the model all but the last odd nucleon are incorporated in the deformed core. This deformed core has angular momentum **R** and the total Hamiltonian is given by

$$H=\frac{\hbar^2}{2\mathcal{J}_x}R_x^2+\frac{\hbar^2}{2\mathcal{J}_y}R_y^2+\frac{\hbar^2}{2\mathcal{J}_z}R_z^2+H_p,$$

where the first three terms describe the core rotation and H_p represent the single-particle motion with respect to the core in the body-fixed coordinates. $\mathcal{J}_x$, $\mathcal{J}_y$, and $\mathcal{J}_z$ and R_x, R_y, and R_z are, respectively, the Cartesian components of the moment of inertia, and the angular momentum **R** in this coordinate system. For a deformation symmetric relative to the z axis $\mathcal{J}_x=\mathcal{J}_y=\mathcal{J}$ and the Hamiltonian may be expressed in terms of the total angular momentum **I**

$$H=(\hbar^2/2\mathcal{J})[I^2+j^2-2(\mathbf{I}\cdot\mathbf{j})]+H_p, \quad (1)$$

where

$$\mathbf{I}=\mathbf{R}+\mathbf{j} \quad (2)$$

and $\mathbf{j}$ is the intrinsic nucleon angular-momentum operator.

The Coriolis coupling term $(\mathbf{I}\cdot\mathbf{j})$ couples the intrinsic motion of the last odd particle with core rotation. The first-order estimate of the mixing of two rotational bands due to the Coriolis interaction was studied by Kerman[17] in W^{183}.

The intrinsic motion is described by the Nilsson[21]-model Hamiltonian

$$H_p=-(\hbar^2/2\mu)\Delta+\tfrac{1}{2}\mu(\omega_p^2x^2+\omega_p^2y^2+\omega_z^2z^2) +C\mathbf{l}\cdot\mathbf{s}+D\mathbf{l}\cdot\mathbf{l}, \quad (3)$$

where μ is the reduced mass of the last nucleon; C is the strength of the average spin-orbit splitting and D serves to depress the higher angular-momentum states. The anisotropy is assumed to have rotational symmetry about the z axis; hence the x and y oscillator frequencies equal ω_p; ω_z is the oscillator frequency in the z direction. Because of this anisotropy, the projection Ω of the angular momentum j on the axis of the symmetry—and not j itself—is a good quantum number.

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The eigenvectors of H_p may be expanded in terms of the orthonormal basis of the isotropic harmonic-oscillator wave functions $|j\Omega\rangle$,

$$\chi_{\Omega,\nu}=\textstyle\sum_j c_{j,\Omega,\nu}|j\Omega\rangle \quad (4)$$

and for the negative projection correspondingly,

$$\chi_{-\Omega,\nu}=\textstyle\sum_j (-1)^{j-1/2}c_{j,\Omega,\nu}|j-\Omega\rangle. \quad (5)$$

The extra index ν has been introduced to distinguish between different Nilsson levels in the same oscillator shell with the same projection quantum number Ω.

The expansion coefficients $c_{j,\Omega,\nu}$ and the eigenvalues $E_{\Omega,\nu}$ are obtained by diagonalizing the single-particle Hamiltonian.

Assuming that the Nilsson single-particle energies are derived from the basic two-nucleon potential, the total energy of the system is not the sum of the single-particle energies but rather is given by the following expression:

$$E=\textstyle\sum_\nu \tfrac{1}{2}(1+\mu/2M)E_{\Omega,\nu}-(\mu/4M)\langle C\mathbf{l}\cdot\mathbf{s}+D\mathbf{l}\cdot\mathbf{l}\rangle, \quad (6)$$

where $E_{\Omega,\nu}$ is the Nilsson energy of the individual nucleons and M is the nucleonic mass; the summation runs over all occupied levels.

Single-particle states have been constructed by placing the last unpaired nucleon in any of the unoccupied levels; similarly hole states or core-excited states have been generated by lifting a core particle and pairing it with the odd nucleon. Under the assumption that the pairing energy within the same oscillator shell does not depend on the occupied Nilsson level, the energy of the hole states and their eigenfunctions are also obtained from (6) and (4), respectively. (Other types of hole states obtained by lifting a core particle to any state other than that occupied by the odd nucleon or by lifting more than one core particle are not considered herein.)

The eigenfunction of the operator $(\hbar^2/2\mathcal{J})I^2$ are the normalized D functions. In the absence of the Coriolis coupling $\mathbf{I}\cdot\mathbf{j}$, the total wave function Φ is a properly symmetrized product of the D function, the single-particle function $\chi_{\Omega,\nu}$ and the wave function of the core ϕ_c

$$\Phi(IMK)=\left(\frac{2I+1}{16\pi^2}\right)^{1/2}[D_{MK}{}^I(\theta)\chi_{\Omega,\nu} +(-1)^{I-1/2}D_{M-K}{}^I(\theta)\chi_{-\Omega,\nu}]\phi_c, \quad (7)$$

where K is equal to Ω for a symmetric rotator.

Φ is not an adequate wave function when the Coriolis interaction is included in the model Hamiltonian; however, we may use Φ as a suitable basis and expand the total wave function Ψ in terms of Φ.

$$\Psi(I,M)=\left(\frac{2I+1}{16\pi^2}\right)^{1/2}\sum_{K=\Omega}\sum_\nu C_{K,\nu}[D_{MK}{}^I(\theta)\chi_{\Omega,\nu} +(-1)^{I-1/2}D_{M-K}{}^I(\theta)\chi_{-\Omega,\nu}]\phi_c, \quad (8)$$

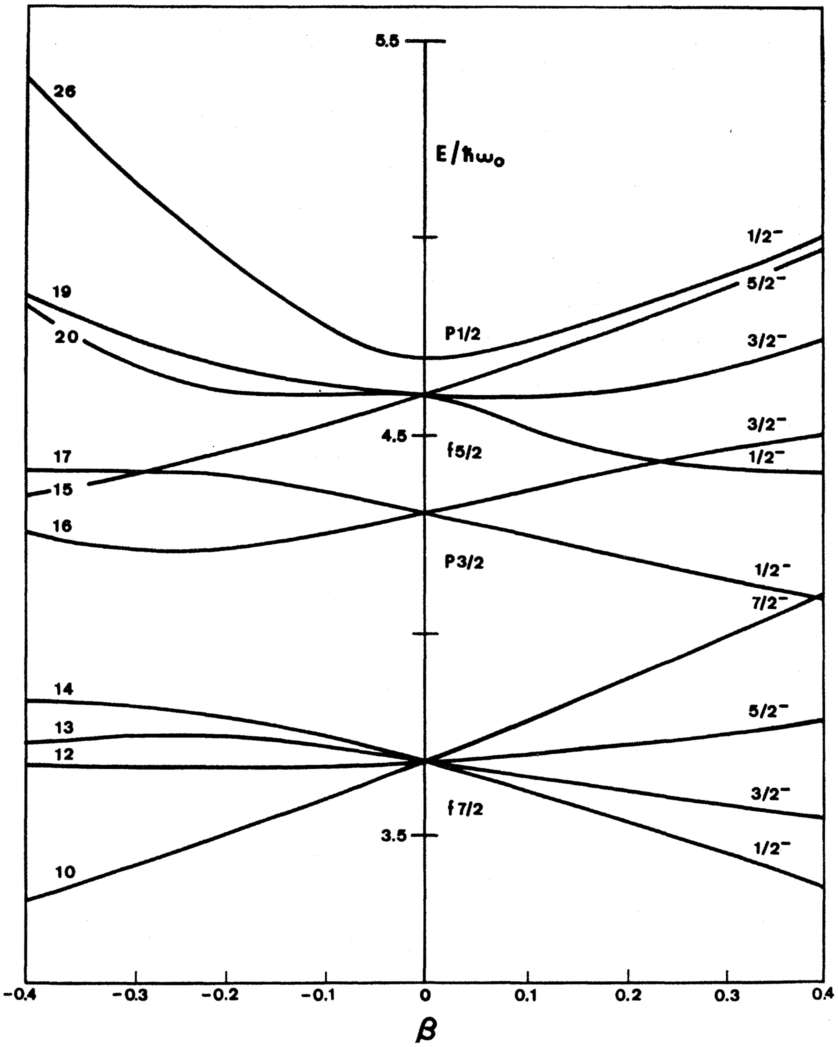

FIG. 1. Single-particle energy eigenvalues in the $1f$-$2p$ shell as a function of the deformation parameter β in the unit of $\hbar\omega_0$ for a spin-orbit strength $C=-0.26\hbar\omega_0$ and a well-flattening parameter $D=-0.035\hbar\omega_0$. The number adjacent to the levels on the left denotes the Nilsson particle levels in the deformed potential. These levels are further identified in the limit of zero deformation by the spherical-shell-model notation and at the nonzero deformation by the projection Ω of the angular momentum on the axis of symmetry and their parity.

where the summation extends in our convention from the allowed positive to the negative values of K in steps of two. The coefficients $C_{K,\nu}$ are determined by the Jacobi diagonalization procedure. In this representation the Coriolis coupling connects states $|\Delta K|=\pm1$ and $\Delta K=0$ for $K=\frac{1}{2}$ bands. The diagonal and off-diagonal matrix elements are given by the following expressions:

$$\langle IMK\nu|H(\mathrm{D})|IMK\nu\rangle = (\hbar^2/2\mathcal{J})[I(I+1)-K^2-\Omega^2], \tag{9}$$

$$\langle IMK\nu_1|H(\mathrm{OD})|IM-(K\pm1)\nu_2\rangle = -(\hbar^2/2\mathcal{J})\{[(I\mp K)(I\pm K+1)]^{1/2} \times\sum_j c_{j,\Omega,\nu_1}c_{j,-(\Omega\pm1),\nu_2}(-1)^{I-j}[(j\mp\Omega)(j\pm\Omega+1)]^{1/2} +\sum_j c_{j,\Omega,\nu_1}c_{j,\Omega,\nu_2}j(j+1)\}, \tag{10}$$

where $H(\mathrm{D})$ and $H(\mathrm{OD})$ are, respectively, the diagonal and the off-diagonal part of the model Hamiltonian (1) with respect to the chosen basis (8).

The matrix elements (9) and (10) are written under the specific assumptions that the moment of inertia is a scalar and the core wave function does not depend on the quantum state of the odd nucleon. This is only reasonable for a rigid core. In practice the moment of inertia differs appreciably from that of a rigid rotator; thus the core wave function is not expected to be completely independent of the total and the intrinsic spins. This dependence of the core wave function on the particle state $\chi_{\Omega,\nu}$ of the odd nucleon stems from the residual interaction between the core and the particle outside the core. The net effect of this implicit dependence of the core wave function is that the core overlap integral $[\phi_c(I,\Omega,\nu), \phi_c(I,\Omega',\nu')]$ may deviate from its assumed value of unity for the off-diagonal matrix elements. To the first order such an effect can be accounted for by multiplying the off-diagonal matrix elements (10) with the core overlap integral which is one for the ideal case of a rigid rotator but may be less than one in reality.

If the Coriolis coupling is strong in comparison with the single-particle energy spacing, the ground-state spin is not known *a priori*. However, the ground-state spin by definition is that associated with the lowest eigenvalue obtained after the diagonalization. The complete level spectrum and the associated spins are given by the magnitude and the spin of the successive eigenvalues.

III. DETAILS OF THE CALCULATION

In this calculation the usual value of $\hbar\omega_0=41/A^{1/3}$ MeV has been chosen for the energy of the oscillator quantum and $R=1.2A^{1/3}$. F has been taken for the nuclear radius. The observed spin-orbit splitting between the $1f_{7/2}$ and $1f_{5/2}$ levels in Ca^{41} yields $C=-0.26\ \hbar\omega_0$ as the strength of the spin-orbit interaction. The well flattening parameter D can not be determined empirically for most nuclei due to the lack of sufficient experimental information. For nuclei with five neutrons or protons in the $1f_{7/2}$ shell this parameter is chosen to be $-0.06\hbar\omega_0$ from the position of the $l=1$ states in Ti^{47}. For other nuclei we have taken the smallest absolute value of $D=-0.035\hbar\omega_0$ which preserves the shell-model-level ordering at zero deformation. The present experimental data do not warrant a proper search for the actual value of D. The computed single-particle energy levels for the latter choice of D are plotted in Fig. 1 as a function of the deformation β.

The rotational constant $A=\hbar^2/2\mathcal{J}$ is usually taken from the mean of the observed energies of the first 2^+ state in the adjacent even-even nuclei assuming a rotational character for these levels. In some cases the constant A is obtained by extrapolation using Fig. 2. *For each nucleus only one rotational constant is used for all bands and for the strength of the Coriolis coupling in that nucleus.* It may be mentioned that the rotational constants given in Fig. 2 correspond to moments of inertia between 20% and 50% of the rigid-body value.

The lowest single-particle intrinsic state is constructed by filling the lowest available Nilsson levels. The excited particle states are obtained by lifting the last odd nucleon to all unfilled levels in the $1f$-$2p$ shell; the core excited states are generated by lifting a nucleon from a single-particle state in the $1f$-$2p$ shell below the occupied level of the last odd nucleon and then pairing it off with this last nucleon, e.g., in Ti^{49}, and for positive deformation, three core-excited states may be constructed by lifting a nucleon from one of the Nilsson levels 12, 13, or 14 and pairing it off with the nucleon in level 10 (Fig. 1). The single-particle energies and wave functions depend implicitly on the deformation and are specified by the deformation parameter β, which is essentially the only free parameter entering into the present model calculation. The band-head energies are computed from expression (6). *In this computation no adjustment of the band-head energies has been allowed.*

In the band-mixing problem for each nucleus all bands based on the ten available single-particle and core-excited levels in the $1f$-$2p$ shell have been included. These are the only bands in the $1f$-$2p$ shell that are coupled to the band based on the lowest possible intrinsic state by the Coriolis coupling. *All matrix elements of the Coriolis coupling are computed from expressions (9) and (10) using our single-particle wave functions* and are determined completely by the choice of the deformation.

$_{26}$Fe							1.41
$_{25}$Mn					*0.150*		*0.237*
$_{24}$Cr				*0.132*	0.79	*0.185*	1.434
$_{23}$V			*0.150*		*0.150*		*0.247*
$_{22}$Ti		*0.156*	0.887	*0.156*	0.99	*0.213*	1.56
$_{21}$Sc	*0.215*		*0.150*		*0.185*		
$_{20}$Ca	1.52	*0.223*	1.159	*0.190*			
N =	22	23	24	25	26	27	28

FIG. 2. Observed excitation energies in MeV of the first 2^+ states of the even-even nuclei in the $1f_{7/2}$ shell (normal-type numerals) and the rotational constant used in the computation of the level spectrum (italic numerals) for nuclei with the mass number between 42 and 54. N indicates the neutron number.

If the residual interaction is neglected completely, there is no overlap between the bands based on the particle-excited and on the core-excited states and the core overlap integral is unity otherwise. However, application of this model to the odd nuclei in the $2s$-$1d$ shell, where rotational structure is evident in the observed spectra of many odd nuclei, indicates that the core overlap integral is indeed less than one for many nuclei (Ref. 22). In the $2s$-$1d$ shell this deviation of the core overlap integral from one is generally more pronounced for nuclei in the mass region half-way between O^{16} and Si^{28}. We have therefore made a preliminary study of this effect for the nuclei with five protons or neutrons in the $1f_{7/2}$ shell and taken the core-overlap integral to be 0.75. In addition a small overlap of 0.25 between bands based on the particle-excited states and bands based on the core-excited states has been introduced; this however has very little influence on the spectra because the Coriolis interaction does not connect bands with $|\Delta K|=\pm2$. This yields a considerably improved agreement between the computed and the observed spectra for these nuclei in the $1f_{7/2}$ shell. For other nuclei in the $1f_{7/2}$ shell the variation introduced in the excitation spectra by choosing a core overlap integral different from one can also be obtained equivalently by changing the deformation and keeping the core overlap integral to be unity; consequently for all these nuclei the core overlap has been taken to be one.

The final spectra have been obtained by diagonalizing the Coriolis interaction with the rotational wave functions based on all ten available particle- and core-excited states. The magnitudes and the spins of the successive eigenvalues determined the spectrum.

[22] W. Scholz and F. B. Malik (to be published).

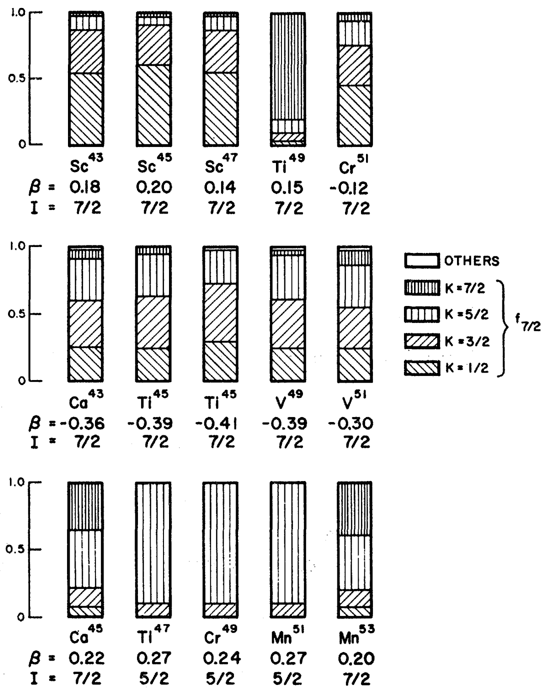

FIG. 3. The squared amplitudes $|C_{K,\nu}|^2$ of the components of different bands into the ground states of the odd nuclei in the $1f_{7/2}$ shell. The ground-state wave function is normalized to unity. The ground-state spin and the values of the deformation β used for each nucleus are shown below each column. The band marked others includes the sum of the squared amplitudes of six bands based on the Nilsson levels originating from the $2p_{3/2}$, the $1f_{5/2}$, and the $2p_{1/2}$ subshells.

IV. RESULTS AND DISCUSSION

We have computed the low-lying spectra of Ca^{43}, Ca^{45}, Sc^{43}, Sc^{45}, Sc^{47}, Ti^{45}, Ti^{47}, Ti^{49}, V^{47}, V^{49}, V^{51}, Cr^{49}, Cr^{51}, Mn^{51}, and Mn^{53}. In a broad sense these nuclei may be classified in four groups of equivalent nuclei on the basis of the odd nucleon count above the $2s$-$1d$ shell. This classification is motivated by the fact that all members of a particular group would have exhibited exactly the same spectrum in this model, had they possessed the same moment of inertia and the same deformation. This expected similarity among members of the same group is clearly evident in most of the observed spectra, although in fact the moment of inertia and the deformation are not the same for every member of the same group. As a result we observe some differences. In this classification Sc^{43}, Sc^{45}, and Sc^{47} form Group I with only one proton outside the proton closed shell. Group II consists of Ca^{43}, Ti^{45}, V^{47}, V^{49}, and V^{51}. Group III and Group IV contain, respectively, Ca^{45}, Ti^{47}, Cr^{49}, Mn^{51}, Mn^{53} and Ti^{49}, Cr^{51}. In the Nilsson model without the band mixing one expects, for prolate deformation, ground-state spins of $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, and $\frac{7}{2}$ for nuclei in Groups I, II, III, and IV, respectively. For oblate deformation the spin sequence is reversed. However, the strong Coriolis coupling mixes bands considerably, as can be seen from the squared-band amplitudes $|C_{K,\nu}|^2$ of the different bands in the ground-state wave functions (Fig. 3), and in consequence of this mixing the model predicts the experimentally observed ground-state spin in all cases. The occurrence of a low-lying $\frac{7}{2}^-$ state in the decoupled $K=\frac{1}{2}$ band based on the Nilsson level 14 together with the strong Coriolis coupling is responsible for the predicted $\frac{7}{2}$ ground-state spin for most nuclei in this shell. However, for a sufficiently large deformation a $\frac{5}{2}$ ground-state spin can be obtained for nuclei in Groups II and III which have a low-lying $K=\frac{5}{2}$ band.

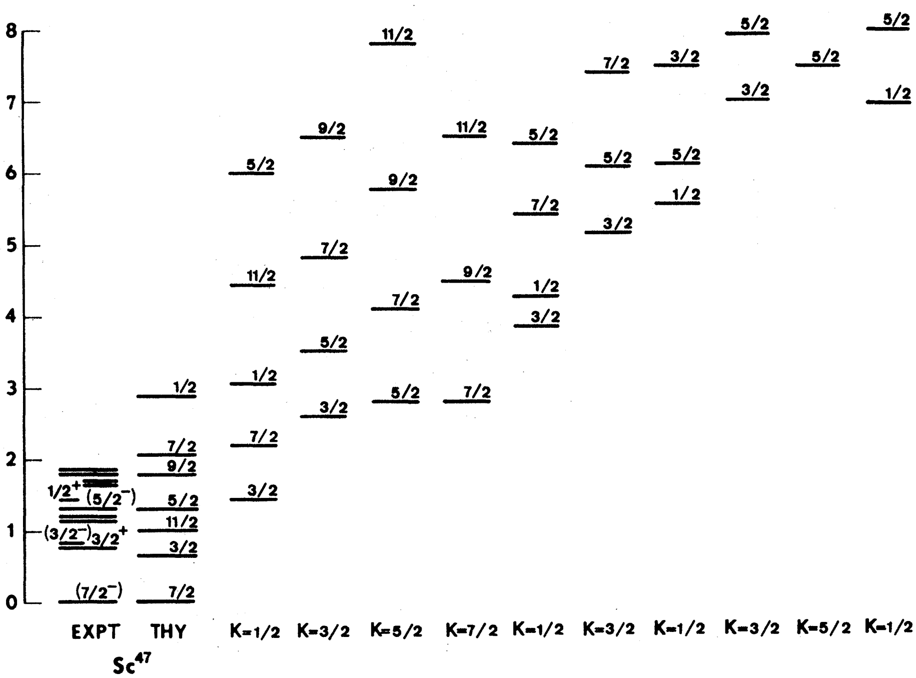

FIG. 4. The effect of the Coriolis coupling on the spectrum of Sc^{47}. The ten columns to the right show the unperturbed bands based on the ten available single-particle states in the $1f$-$2p$ shell. The band head energies are computed from Eq. (6). The column marked "THY" is the final spectrum obtained after diagonalization of the Coriolis coupling term. To the left is the experimental spectrum. The scale is in MeV.

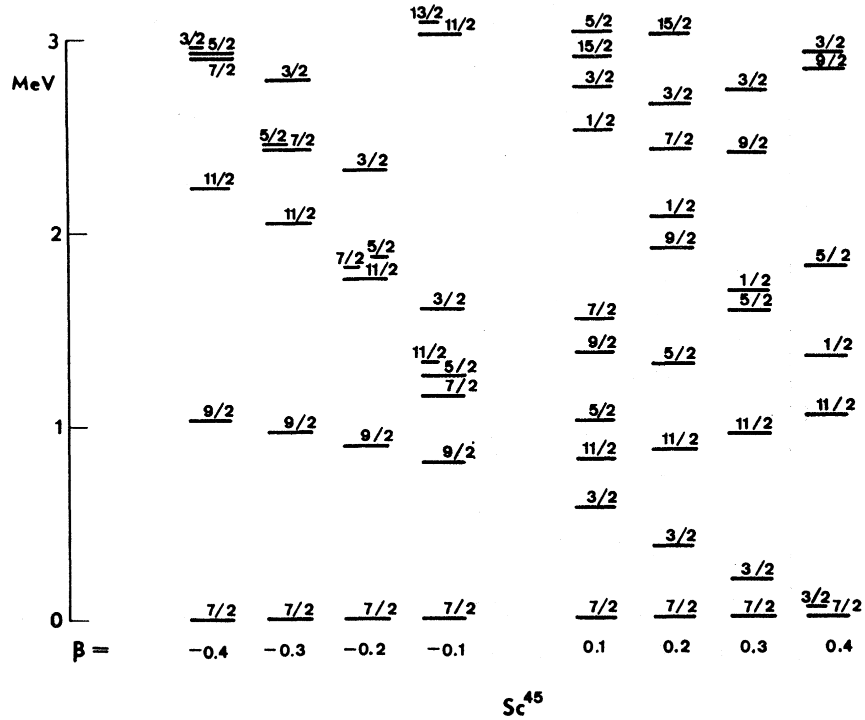

FIG. 5. The dependence of the computed spectrum of Sc^{45} on the deformation β. Similar dependence holds also for Sc^{43} and Sc^{47} which have nearly the same rotational constant. $\frac{7}{2}$ remains the ground-state spin for a wide range of the deformation.

An example of the band structure before and after the diagonalization is carried out in Fig. 4 for the case of Sc^{47}. Band-head energies, diagonal and off-diagonal matrix elements are computed for $\beta=0.14$ using expressions (6), (9), and (10). The two leftmost columns represent the experimental and the computed spectrum after the band mixing. The strong Coriolis coupling completely obscures any residual band structure in the final spectrum and prevents any empirical adjustment of band-head energies as is usually done in the level fits with the Nilsson model to nuclei in the $2s$-$1d$ shell and to heavy nuclei. On the other hand, this strong Coriolis coupling makes the final spectrum relatively insensitive to slight changes ($\sim$200 keV) of the band-head energies.

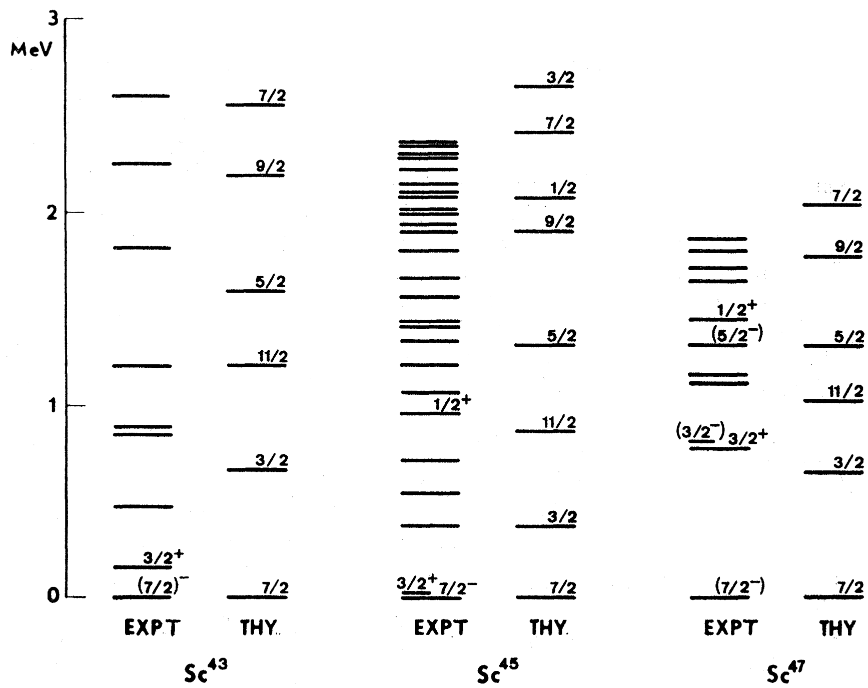

FIG. 6. The experimental (EXPT) and the theoretical (THY) spectra of nuclei in Group I. The deformation β is 0.18, 0.20, and 0.14 for Sc^{43}, Sc^{45}, and Sc^{47}, respectively. Since the spin of none of the excited negative-parity states in Sc^{43} and Sc^{45} is known, their theoretical spectra must be read in conjunction with Fig. 5 which provides the approximate dependence of their respective spectrum on the deformation.

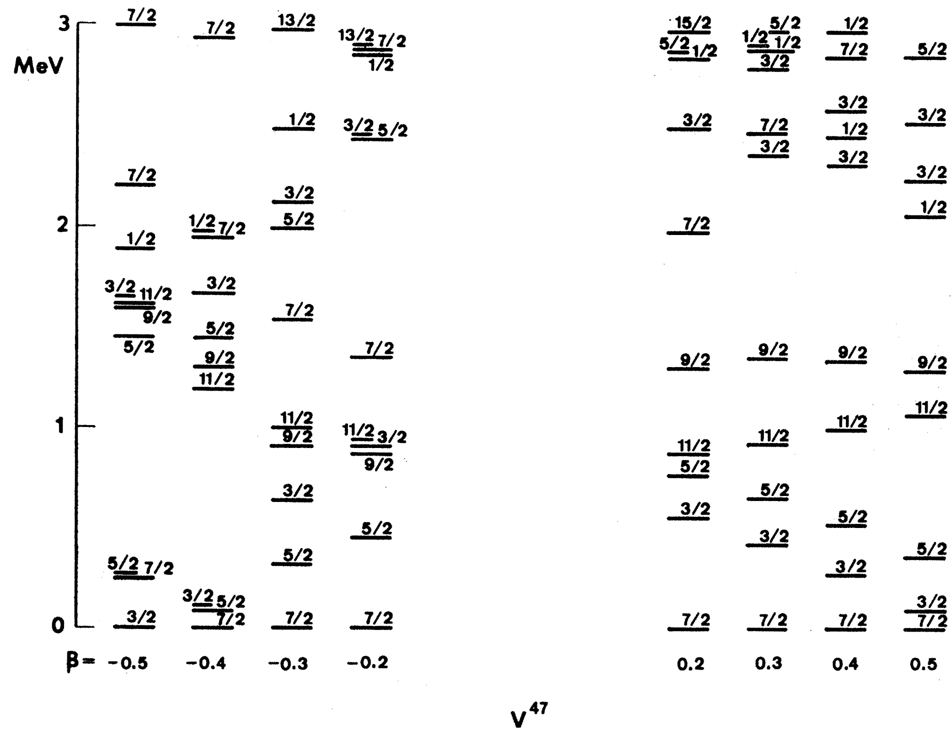

FIG. 7. The dependence of the final spectrum of V^{47} on the deformation. The same dependence holds also for V^{49} which has the same rotational constant. The ground-state spin remains $\frac{7}{2}$ for a wide range of deformation.

The present experimental data do not permit a conclusive statement concerning the sign of the deformation for many of the nuclei in the $1f_{7/2}$ shell because in some cases the observed level scheme is reproduced equally well for both negative and positive deformations.

In the following section the level spectra below about 2.5-MeV excitation energy will be considered for nuclei in each group separately. At higher excitation energies states arising from multiparticle excitations are likely to occur. Such states are beyond the scope of the present treatment which is basically an extreme single-particle model.

A. Nuclei of Group I: Sc^{43}, Sc^{45}, Sc^{47}

This group is characterized by a single odd proton above the closed $2s$-$1d$ shell. The change in the level spectrum for this group with deformation is illustrated in Fig. 5 using Sc^{45} as an example. The levels of Sc^{43} and Sc^{47} move in very much the same way with deformation, the only difference being in the rotational constants which are 215, 150, and 185 keV for Sc^{43}, Sc^{45}, and Sc^{47}, respectively. For a wide range of deformation a $\frac{7}{2}$ ground-state spin is predicted. Only in the case of Sc^{47} are enough experimental data known[23–29] to fix the sign and the value of the deformation. Figure 6 shows the computed level schemes for Sc^{43}, Sc^{45}, and Sc^{47} together with the experimental information.[23–30] The theoretical spectra for Sc^{43} and Sc^{45} should be considered as tentative and a final choice of β must await the spin and parity assignment to some of the observed levels. In particular, the possibility that many of the low-lying states may be of positive parity, and therefore outside the scope of this model, presents a major difficulty in choosing the appropriate deformation. In fact, the first excited state for the three scandium isotopes is $\frac{3}{2}^+$ (Refs. 25, 26, 29); the presence of low-lying positive parity states, originating from hole states in the $2s$-$1d$ shell, is not surprising for our choice of the spin-orbit splitting which results in a very small energy gap between the $2s$-$1d$ and the $1f_{7/2}$ shell.

The Coriolis coupling model reproduces the locations of $\frac{7}{2}^-$, $\frac{3}{2}^-$, and $\frac{5}{2}^-$ states in Sc^{47} rather well, and five of the eleven states observed below 2 MeV in excitation energy can be accounted for. The situation in Sc^{45} and Sc^{43} is similar. It is worth mentioning, that four negative-parity states are predicted in this model for all Sc isotopes below the first excited state given by the shell model at about 1.5 MeV.[6]

B. Nuclei of Group II: Ca^{43}, Ti^{45}, V^{47}, V^{49}, V^{51}

This group is characterized by having three particles of the odd species above the closed $2s$-$1d$ shell. A

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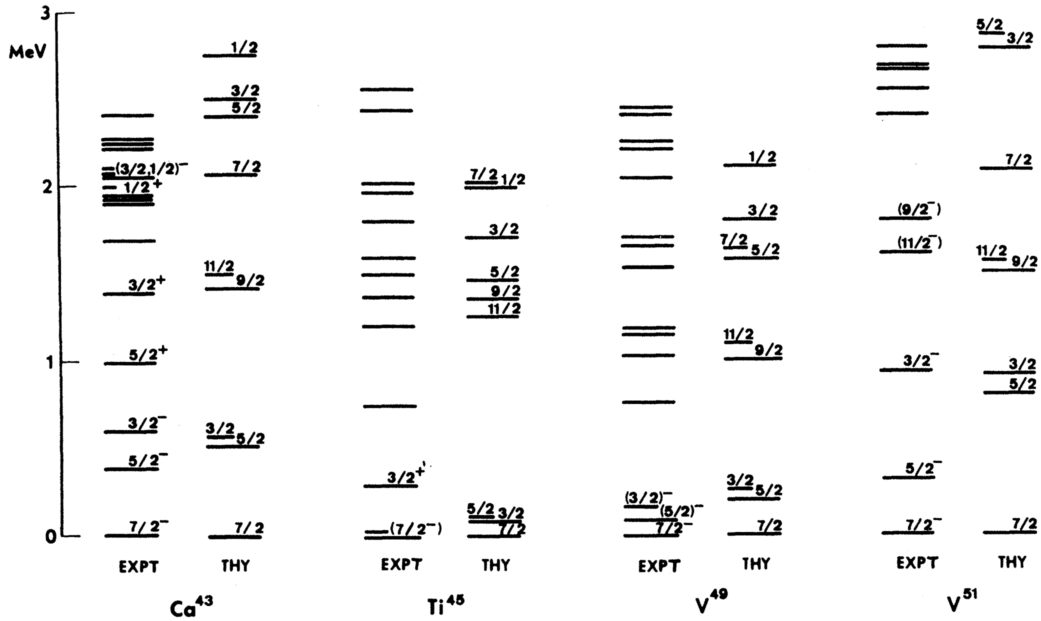

FIG. 8. The experimental (EXPT) and the theoretical (THY) excitation spectra of odd nuclei in Group II. The spectrum of V^{47} is not shown because it is similar to that of V^{49}. The deformation is chosen as −0.36, −0.41, −0.39, and −0.30 for Ca^{43}, Ti^{45}, V^{49}, and V^{51}, respectively.

previous application of the Coriolis coupling model to V^{51} (Ref. 18) is in remarkable agreement with the observed data on the level spectrum, static moments, and transition rates and favors an oblate deformation for V^{51}.

This group shows rather clearly the increase of the moment of inertia in moving towards the middle of the $1f_{7/2}$ shell. This is indicated by the values of the rotational constant (derived from the first excited 2^+ state in neighboring even-even nuclei) which are 223, 156, 150, and 247 keV for Ca^{43}, Ti^{45}, (V^{47}, V^{49}), and V^{51}, respectively. It is noteworthy that the rotational constants obtained in this way are rather large for nuclei with a closed proton or neutron shell like Ca^{43} or V^{51}.

Figure 7 shows the computed level positions as a function of the deformation for V^{47}. Exactly the same behavior of the levels holds also for V^{49} which has the same rotational constant. For a considerable range of the deformation the ground-state spin remains $\frac{7}{2}$.

Around $\beta=-0.4$ an interesting crossover takes place for the single-particle energy levels obtained from expression (6) corresponding to Nilsson levels 12 and 13 (Fig. 1). After the crossover the $K=\frac{5}{2}$ band is based on an excited single-particle state rather than being the lowest particle state and therefore in our model is no longer coupled by the Coriolis interaction to the $K=\frac{7}{2}$ core-excited band based on Nilsson level 10. The effect on the final spectrum associated with this crossover can be seen from the spectra given in Fig. 8 for V^{49} (before the crossover) and Ti^{45} (after the crossover). In practice the residual interaction between the nucleons may well introduce some overlap between the two bands mentioned above, particularly near the crossing point. In this case the actual spectrum is expected to lie between the two extremes as shown in Fig. 8.

For both positive and negative deformations the model reproduces the characteristic $\frac{7}{2}^-$, $\frac{5}{2}^-$, $\frac{3}{2}^-$ triplet near the ground state, followed by a gap in the level spectrum as observed in V^{49} (Refs. 23, 31–33). This ambiguity in the sign of the deformation results from the relative insensitivity of the final spectrum to slight changes in band-head energies, mentioned above, and applies to all nuclei in this group. The shell-model calculation with the $1f_{7/2}$ configuration can not account for this triplet.

The observed level spectrum of V^{47} (Refs. 23, 31, 32) is similar to that of V^{49} except that two states in the energy region around 1 MeV in V^{49} appear to have moved down in excitation. It is interesting to note that the model predicts two states less than observed in that energy region for V^{49}. In view of the fact that Ca^{43}, with three neutrons outside the closed $2s$-$1d$ shell, has two positive parity states below 1.5-MeV excitation energy, it would be of particular interest to determine the parity of the low-lying states in V^{47} and V^{49} which have three protons outside the closed shell. There is some evidence[23] from the β decay of V^{47} to Ti^{47} that the ground state of V^{47} is the $\frac{5}{2}$ member of the predicted triplet. The shell

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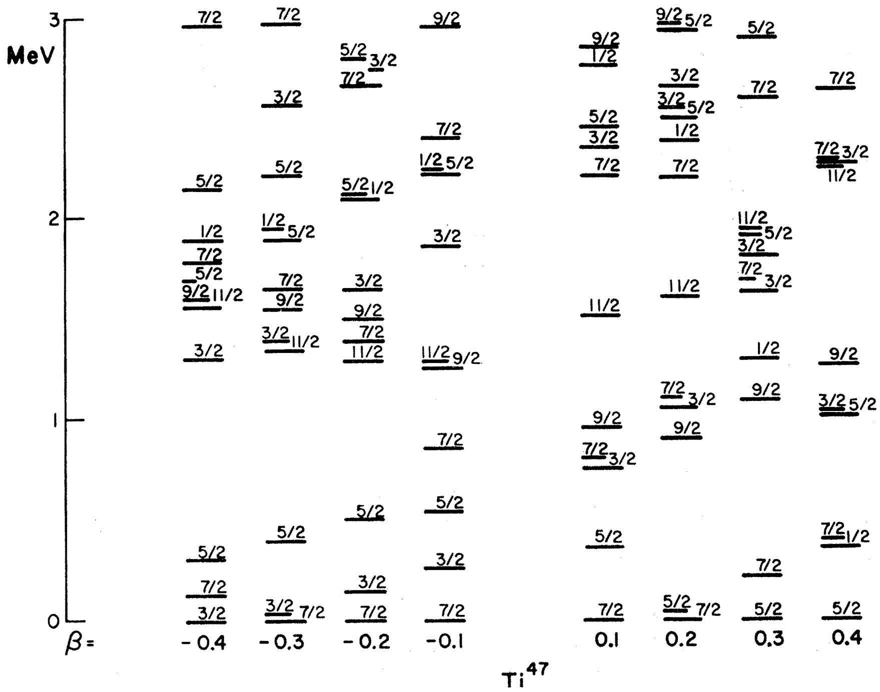

FIG. 9. The dependence of the final spectrum of Ti^{47} on the deformation. For reasonable prolate shape the ground-state spin changes from the $\frac{7}{2}^-$ to the observed $\frac{5}{2}^-$ value. A similar dependence holds also for Cr^{49} and Mn^{51} which have about the same rotational constant.

model with $1f_{7/2}$ configuration mixture[7] predicts only one excited level below 1-MeV excitation energy in V^{47}.

Experimental information on the levels of Ca^{43} is available from the β decay of K^{43} and of Sc^{43} (Ref. 23), and from the $Ca^{42}(d,p)Ca^{43}$ and the $Ca^{43}(dd')Ca^{43}$ reactions.[34,35] The Coriolis coupling model reproduces the ground and the known negative parity states at 0.37 and 0.59 MeV. The 2.05 MeV state is known to be $\frac{3}{2}^-$ or $\frac{1}{2}^-$ from an analysis of the deuteron stripping reaction on Ca^{42} (Ref. 34) and may correspond to the $\frac{3}{2}^-$ state predicted at 2.5 MeV. Negative parity has been tentatively assigned to five more states between 1.5 and 2.5 MeV on the basis of inelastic deuteron scattering,[35] while four more negative parity states are predicted in this region (Fig. 8). The states at 0.99, 1.39, and 1.96 MeV have positive parity and are beyond the scope of this model.

Shell-model calculations with the pure $1f_{7/2}$ configuration[6] predict the second-excited-$\frac{3}{2}^-$-state excitation to be 700 keV too high. Inclusion of the $2p_{3/2}$ configuration for all three nucleons and treatment of the strength of the different terms in the two-body potential as a parameter[9] results in the correct position for the second excited $\frac{3}{2}^-$ state but predicts additional low-lying $\frac{5}{2}^-$ and $\frac{9}{2}^-$ states which have not been observed. These two states, however, do not occur if the $p_{3/2}$ configuration admixture is restricted to one nucleon only.[10] More experimental information concerning these nuclei would be helpful.

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The observed spectrum for Ti^{45} shows a doublet near the ground state.[23,33,36] The next excited state at 0.33 MeV has been assigned to be $\frac{3}{2}^+$ from the angular distribution of the deuterons in the $Ti^{46}(p,d)Ti^{45}$ reaction[36]; however, the assignment is somewhat ambiguous (see Ref. 6). The two states at 0.74 and 1.23 MeV are seen in the $Sc^{45}(p,n)Ti^{45}$ reaction (Ref. 33) but not in the $Ti^{46}(p,d)Ti^{47}$ reaction.[36] For each one of the next six levels a predicted counterpart can be found (Fig. 8) while the shell model with $1f_{7/2}$ configuration[6] predicts only three states in this region.

C. Nuclei of Group III: Ca^{45}, Ti^{47}, Cr^{49}, Mn^{51}, Mn^{53}

Nuclei of this group have either five protons or five neutrons outside the closed $2s$-$1d$ shell. The observed spectra of all five nuclei exhibit a characteristic doublet near the ground state followed by a gap of about 1 MeV. This similarity in the level spectrum is expected in this model for all five nuclei if the deformation and the rotational constant were the same for each nucleus. However, the rotational constants of 190 and 237 keV for Ca^{45} and Mn^{53}, respectively, are considerably higher than those for Ti^{47}, Cr^{49}, and Mn^{51}, which are 156, 132, and 150 keV, respectively. Here again we observe the drop of the rotational constant as we move away from the closed-shell configuration for the nucleons of the even type. The presence of the $K=\frac{5}{2}$ band, based on Nilsson level 12, at a low excitation energy generates a low-lying $\frac{5}{2}$ state in all these nuclei and the difference in the rotational constants is responsible for the anomalous $\frac{5}{2}$ ground-state spin in Ti^{47} and possibly in Cr^{49} and Mn^{51}.

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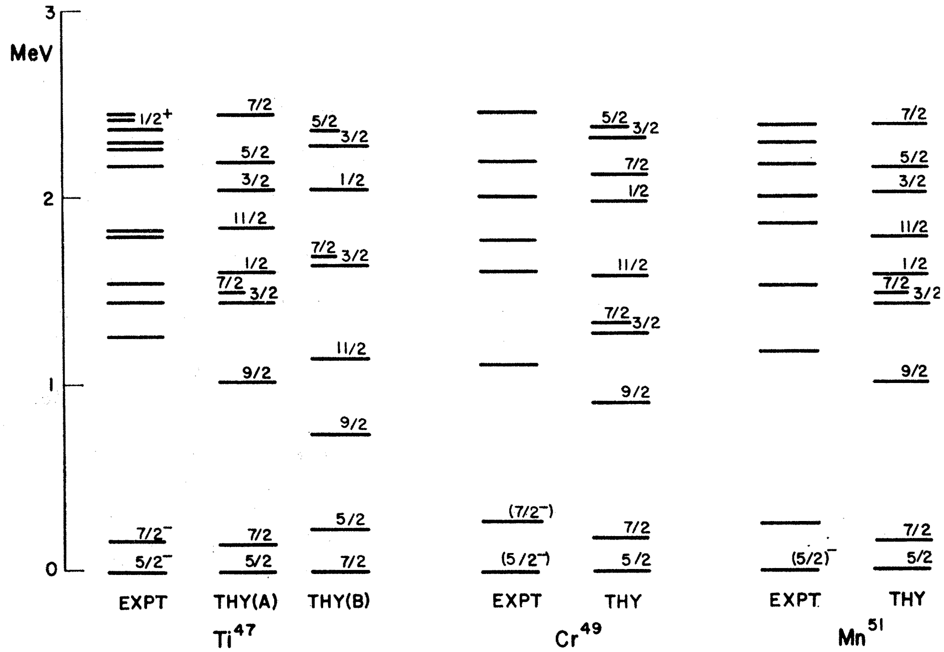

FIG. 10. The experimental and theoretical level spectrum of three members Ti^{47}, Cr^{49}, and Mn^{51} of Group III. All of them have about the same moment of inertia. The deformation is taken to be 0.27, 0.24, and 0.27 for Ti^{47}, Cr^{49}, and Mn^{51}, respectively.

The dependence of the level spectrum on the deformation is shown for Ti^{47} in Fig. 9 using a well-flattening parameter $D=-0.06\hbar\omega_0$ and 0.75 for the core overlap integral. For reasonable prolate deformation we get the observed ground-state spin of $\frac{5}{2}$. With the core overlap integral of the order of 1, it is difficult to get the correct ground-state spin of $\frac{5}{2}$ and at the same time a reasonable spectrum for the positive deformation (as can be seen from Fig. 10); however, for an oblate deformation one can get the $\frac{5}{2}$ ground-state spin. But in that case the model predicts a triplet near the ground-state which has not been observed in any one of these nuclei. Thus the observed doublet together with the ground-state spin of $\frac{5}{2}$ favors a prolate deformation and a core overlap integral less than 1. This same value for the core overlap integral has been used for all nuclei in this group because the present experimental data do not merit treating this quantity as a free parameter. This parameter affects primarily the position of the high-spin states.

The ground state of Ti^{47} is known to be $\frac{5}{2}^-$ (Ref. 37). A large amount of information on the location of the excited states has been obtained from the $Ti^{48}(p,d)Ti^{47}$ (Ref. 36), $Ti^{46}(d,p)Ti^{47}$ (Refs. 38–44), and $Ti^{47}(p,p')Ti^{47}$ (Ref. 45) reactions. Some information on the levels in Ti^{47} is also available from the observation of gamma rays from the $Ti^{46}(n,\gamma)Ti^{47}$ reaction.[46,47] The first excited state at 160 keV has also been observed in Coulomb excitation[48,49] studies. A summary of the levels below 2.5 MeV in excitation energy observed in these experiments is presented, together with the model predictions in Fig. 10. The states at 0.157, 1.545, and 1.788 MeV are strongly excited in the $Ti^{46}(d,p)Ti^{47}$ reaction and orbital angular-momentum transfer $l=3$, 1, and 1, respectively, have been assigned to them. These $l=1$ states may correspond to either of the predicted $\frac{1}{2}$ and $\frac{3}{2}$ states in their vicinity; however, the $\frac{3}{2}$ state predicted around 1.5 MeV might not be excited very strongly in the $Ti^{46}(d,p)Ti^{47}$ or in the $Ti^{47}(p,p')Ti^{47}$ reaction because it is mainly based on $K=\frac{3}{2}$ and $K=\frac{1}{2}$ bands arising from the $1f_{7/2}$ configuration and therefore has small overlap with the ground-state wave function which is based primarily on the $K=\frac{5}{2}$ band (Fig. 3). The 1.25-MeV level is excited strongly in the $Ti^{47}(p,p')Ti^{47}$ reaction but only weakly in the $Ti^{46}(d,p)Ti^{47}$ reaction. This is often characteristic of a high-spin state. The theory predicts the second excited state to be $\frac{9}{2}$.

Thus the experimental information on the levels of Ti^{47} available to date is in general agreement with the predictions of this model. The shell model with the $1f_{7/2}$ configuration mixture predicts the first $\frac{3}{2}^-$ state

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[49] D. A. Bromley, J. A. Kuehner, and E. Almqvist, Phys. Rev. **115**, 586 (1959).

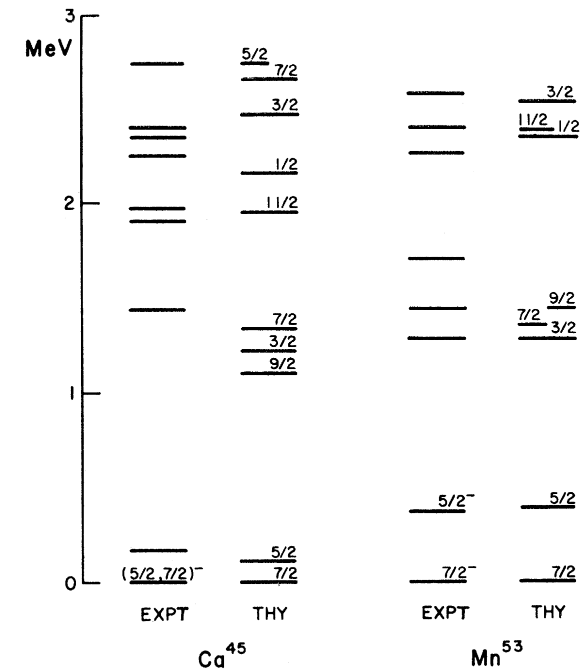

FIG. 11. The theoretical and the experimental spectra of the two members with relatively high rotational constant of Group III. The deformation for Ca^{45} and Mn^{53} is 0.22 and 0.20, respectively. Experimentally there is some evidence for three more states in Ca^{45} around 1.5 MeV in excitation energy (Ref. 55).

around 0.8 MeV where no state has been reported in any but one[41] experiment. Moreover, according to the above-mentioned shell model Ti^{47} should have about the same spectrum as its cross conjugate V^{49} which does not appear to be the case on the basis of the experimental information available so far.

Experimental data on Cr^{49} are available from the $Cr^{50}(p,d)Cr^{49}$ (Ref. 50) and the $Cr^{50}(He^3,\alpha)Cr^{49}$ (Ref. 51) reactions. A summary of the observed levels is given in Fig. 10. The angular distributions of the alpha particles from the $Cr^{50}(He^3,\alpha)Cr^{49}$ reaction to the ground and the first excited states of Cr^{49} indicate an $l=3$ pattern. The β decay from Cr^{49} to the triplet near the ground state of V^{49} argues for a $\frac{5}{2}^-$ spin assignment to the ground state of Cr^{49}. These facts are consistent with the theoretical predictions of the model. Six levels observed between 1 and 2.5 MeV in excitation energy compare favorably with eight levels predicted in this region. The over-all agreement between the theoretical and experimental level pattern is reasonably good. According to the shell model with the $1f_{7/2}$ configuration mixture, Cr^{49} and V^{47} should show a similar level spectrum. This is not supported by the experimental evidence.

The levels of Mn^{51} have been observed in the $Fe^{54}(p,\alpha)Mn^{51}$ reaction (Refs. 52, 53). The level scheme is very similar to the Cr^{49} or Ti^{47} spectrum. The β-decay probability to the states of Cr^{51} suggest a possible $\frac{5}{2}$ ground-state spin for Mn^{51} (Ref. 23). The Coriolis coupling model again predicts a doublet near the ground state and eight levels between 1 and 2.5 MeV in excitation energy (Fig. 10). Experimentally the ground-state doublet and seven levels between 1 and 2.5 MeV have been observed.

Levels of Ca^{45} have been observed in the $Ca^{44}(d,p)Ca^{45}$ reaction.[54,55] The similarity of the proton angular distributions to the ground and the first excited state of Ca^{45} to those observed from the corresponding states in Ca^{43} and Ti^{47} argues for spin assignments of $\frac{7}{2}$ and $\frac{5}{2}$, respectively, to these states.[56] This spin assignment is also in agreement with the model prediction (Fig. 11). The high rotational constant for Ca^{45} is primarily responsible for the $\frac{7}{2}$ ground-state spin. The angular distributions of the protons groups populating the 1.43-, 1.90-, and 2.25-MeV states show an $l=1$ pattern. This is in accord with the prediction of one $\frac{1}{2}$ and two $\frac{3}{2}$ states in this energy region. Not shown in the diagram are three more states between 1 and 2 MeV which are reported[55] to be weakly excited in the $Ca^{44}(d,p)Ca^{45}$ reaction. The model predicts in this region a $\frac{9}{2}$, an 11/2, and a $\frac{7}{2}$ state. The shell-model computation of the levels of Ca^{45} with a $(1f_{7/2})^{-3}$ configuration[6] predicts only three states in the excitation energy range from 1 to 2 MeV. On the other hand, the complete inclusion of the $2p_{3/2}$ configuration in the shell-model calculation predicts in this region a level density which is about a factor of 2 higher than the observed one.[9] Restricting only one nucleon at a time to the $2p_{3/2}$ configuration in the shell-model calculation can only account for two of the three observed $l=1$ states.[10] In view of the large number of theoretical calculations now available, it should be desirable to have more experimental information on the Ca^{45} level spectrum.

Figure 11 shows the experimental and computed excitation spectra of Mn^{53}. Most of the levels in Mn^{53} have recently been found in the $Cr^{52}(p,\gamma)Mn^{53}$ (Ref. 53) and $Fe^{56}(p,\alpha)Mn^{53}$ (Ref. 52) reaction studies. The ground-state spin of $\frac{7}{2}$ and the first excited state spin of $\frac{5}{2}$ (Ref. 23) are well reproduced by the model. Furthermore, the model predicts the correct level density up to 2.5 MeV. The shell model with $(1f_{7/2})^{-3}$ configuration predicts only three states, against six observed, between 1 and 2.5 MeV in excitation energy.

We may conclude that the preliminary predictions of the model are in good accord with the observed data on the nuclei in this group.

D. Nuclei of Group IV: Ti^{49}, Cr^{51}

This group is characterized by an odd nucleon count of seven above the closed $2s$-$1d$ shell. Figure 12 illus-

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51 R. Stock (private communication, 1966).
52 G. Brown and S. E. Warren, Nucl. Phys. **77**, 365 (1966).
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54 P. M. Endt and C. van der Leun, Nucl. Phys. **34**, 1 (1962).
55 W. R. Cobb and D. B. Guthe, Phys. Rev. **107**, 181 (1957).
56 T. A. Belote, W. E. Dorenbusch, O. Hansen, and J. Rapaport (unpublished).

trates the change in the positions of the computed energy levels with the deformation for Cr^{51} as an example. Because of the similarity of the rotational constant of the two nuclei in this group (~190 keV), the dependence of the level positions on the deformation is about the same for Ti^{49} and Cr^{51}. Only one additional nucleon is needed to fill the $1f_{7/2}$ shell. Therefore a small deformation is expected for nuclei in this group and the model predicts a ground-state spin of $\frac{7}{2}^-$ and a relatively high excitation energy for the first excited state.

A summary of the experimental situation in Ti^{49} (Refs. 23, 27, 36, 45, 57, 58) is given in Fig. 13. The energy gap between the ground state and the first excited state is well reproduced by the Coriolis coupling model. The first excited state is seen only in inelastic proton scattering[45] and could correspond to the $\frac{9}{2}^-$ state predicted at 1.15 MeV. The $\frac{3}{2}^-$ and $\frac{1}{2}^-$ states[23,36,58] at 1.39 and 1.73 MeV and two additional states near 2.5 MeV with probable spin assignments of $\frac{7}{2}^-$ and $\frac{3}{2}^-$ (Ref. 36) are accounted for by the model. The two states at 2.45 and 2.62 MeV have positive parity[36] and are beyond the scope of this model. With one exception a predicted negative parity counterpart can be found for all the remaining states below 2.6 MeV.

The appearance of low-lying $\frac{3}{2}^-$ and $\frac{1}{2}^-$ states with large $l=1$ reduced neutron widths is puzzling on the basis of the shell model with pure $1f_{7/2}$ configurations.[6] On the other hand, in the Coriolis coupling model, these states are based mainly on Nilsson level 17 and large $l=1$ reduced neutron widths are expected.

The observed spectrum of Cr^{51} (Refs. 23, 27, 50, 59, 60) indicates the presence of a doublet near 0.78-MeV excitation energy and a probable spin of $\frac{3}{2}^-$ has been assigned to one of its members. This cannot be understood in terms of the shell model with the $1f_{7/2}$ configuration only, which predicts the first $\frac{3}{2}^-$ state at 1.7 MeV.

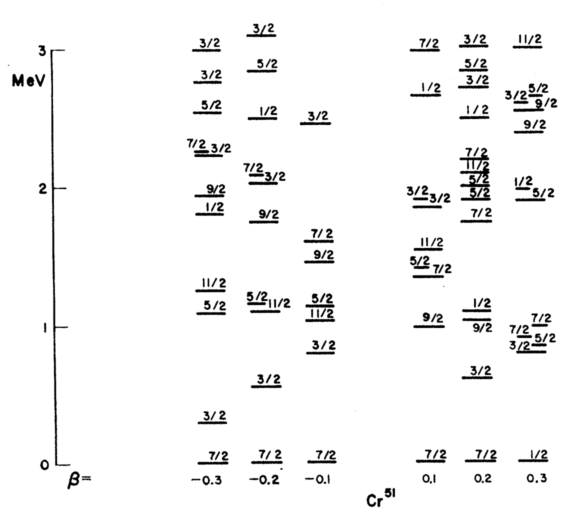

FIG. 12. The dependence of the computed spectrum of Cr^{51} on the deformation. For oblate and for sufficiently small prolate deformation the ground-state spin is $\frac{7}{2}$.

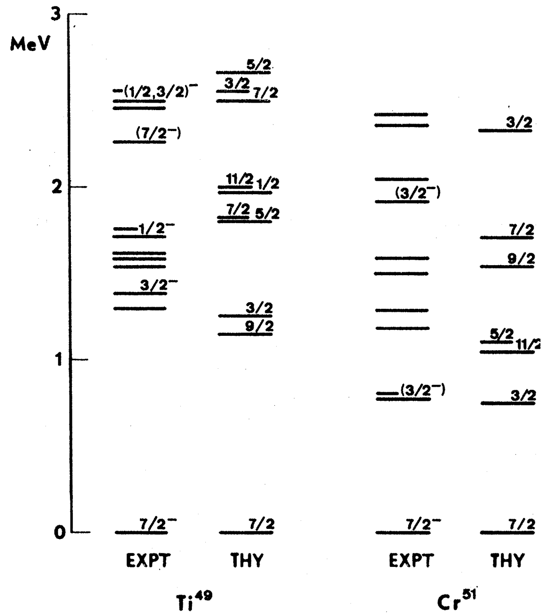

FIG. 13. The experimental and the theoretical level spectra of the two nuclei in Group IV. The deformations for Ti^{49} and Cr^{51} are taken to be 0.15 and −0.12, respectively.

On the other hand, the Coriolis coupling model predicts a $\frac{3}{2}^-$ state at about 0.78-MeV excitation energy both for negative and for positive deformation. The measured lifetime of 10.8 nsec for one member of the doublet obtained from $V^{51}(p,n)Cr^{51}$ reaction[59] is two orders of magnitude slower than the Weisskopf estimate for an $E2$ transition to the ground state. In view of the fact that the $E2$ transitions are enhanced throughout the entire $1f_{7/2}$ shell, an $M2$ character for this transition is reasonable if the transition proceeds directly to the ground state. This would suggest a possible $\frac{3}{2}^+$ assignment for the other member of the doublet and consequently an oblate deformation would be probable. If, however, the long-lived state is the upper member of the doublet, the cascade decay of this state through the lower lying state of presumably $\frac{3}{2}^-$ spin is possible and in that case the observed lifetime argues for a decay exclusively to the lower member of the doublet by a dipole transition. This would then be consistent with a $\frac{1}{2}^\pm$ assignment for the upper member (the other possible assignments of $\frac{3}{2}^\pm$ and $\frac{5}{2}^\pm$ would be difficult to justify because it would require a very high inhibition factor for the transition to the ground state). It is interesting to point out that a $(\frac{3}{2}^-,\frac{1}{2}^-)$ doublet can be

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obtained for positive deformation in the Coriolis coupling model without appreciably affecting the density of levels up to 2-MeV excitation energy.

In Fig. 13 we present the computed level spectrum of Cr^{51} for a deformation $\beta=-0.12$ which should be considered preliminary pending further investigation. Besides the $\frac{3}{2}^-$ state at 0.78 MeV, the next four excited states between 1.0 and 1.6 MeV are also accounted for in this model.

V. CONCLUSION

Of all the consequences of this model, the most important one is the correct prediction of the ground-state spin of all odd nuclei in the $1f_{7/2}$ shell. Thus the argument that the observed ground-state spin of $\frac{7}{2}$ in odd nuclei in this mass region suggests a spherical-shell model, is no longer tenable. The Coriolis coupling model, which is still basically a single-particle model, also accounts for the anomalous $\frac{5}{2}$ ground-state spins of Ti^{47}, Cr^{49}, and Mn^{51}.

Another important conclusion is that this treatment reproduces well the experimental information available to date on the level spectra below 2.5 MeV in excitation energy for all nuclei in this region. Specifically such striking features as the large number of low-lying states in the Sc isotopes, the ground-state triplets in V^{47} and V^{49}, the $\frac{5}{2}^-$ and $\frac{7}{2}^-$ doublets followed by a gap of about 1 MeV in nuclei of group III and the low-lying $\frac{3}{2}^-$ state in Cr^{51} are well accounted for. The model usually predicts the correct number of levels for nuclei in the upper-half of the $1f_{7/2}$ shell. For nuclei in the lower half of the $1f_{7/2}$ shell the presence of additional positive parity states arising from core excitation of the $2s$-$1d$ shell in the observed spectrum is suggested from the calculated spectra.

The actual sign and value of the deformation must await further experimental study of the nuclei in the $1f_{7/2}$ shell since it depends to some degree on the values chosen *a priori* for the parameters of the deformed single-particle potential and for the core overlap integral. Nevertheless a certain systematic behavior of the deformations is already apparent from our preliminary study. With the exception of Cr^{51}, where the actual sign of the deformation cannot be ascertained on the basis of the present experimental data, nuclei in Groups I, III, and IV are prolate while for nuclei in Group III the possibility of an oblate shape appears to be slightly favored. Further experimental tests of the various predictions of these models regarding the spins and locations of the excited states would help to answer these open questions and assist in improving the computed level spectra.

The importance of the proper treatment of band mixing, due to the Coriolis coupling, in the wave functions and the level spectra is well exhibited in the composition of the ground-state wave functions of the $1f_{7/2}$ nuclei shown in Fig. 3 and in the large shifts of the perturbed levels with respect to the unperturbed ones as shown in Fig. 4 for Sc^{47}. Without the Coriolis coupling, it would be impossible to obtain even the correct ground-state spin. Although in simplicity and in principle the Coriolis coupling model is only analogous to the elementary single-particle shell model without the residual interaction, in practice this simple model appears to account for the low-lying spectra of all odd-even nuclei in the $1f_{7/2}$ shell better than the much more sophisticated shell-model computations including residual interactions. On the basis of the present experimental data there seems to be no serious discrepancy between the observations and the predictions of this model.

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