Hot-Carrier Magnetoresistance

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The characteristics of hot-carrier magnetoresistance for small magnetic fields are studied with the assumptions: (a) The constant-energy surfaces are spherical and carriers are scattered by acoustic and optical phonons; and (b) the constant-energy surfaces are ellipsoidal and the scattering is due to acoustic, optical, and intervalley phonons. It is shown that for a spherical constant-energy-surface model the magnetoresistance has a small negative value decreasing with increasing field, and the magnetoresistance coefficient is independent of the field if the scattering is by acoustic phonons alone. On the other hand, for predominant optical-phonon scattering the magnetoresistance and also the magnetoresistance coefficient are positive and decrease with increasing field. In the case of many-valley band structure as in n-type germanium, the characteristics of magnetoresistance and magnetoresistance coefficient for predominant acoustic-phonon scattering are the same as for a spherical constant-energy-surface model, but the sign is positive and the values are larger. The variation of magnetoresistance and magnetoresistance coefficient for predominant optical-phonon scattering when the electric field is in the [100] direction is similar to that for a spherical constant-energy-surface model, but the values are higher and the characteristic varies in details. For the [111] direction of the field, on the other hand, the magnetoresistance decreases with the field, but the magnetoresistance coefficient increases with the field. The values of magnetoresistance calculated from theory, assuming acoustic- and optical-phonon scattering, are found to agree closely with the experimental results. The inclusion of the effects of intervalley scattering leads to a further improvement in the agreement.

I. INTRODUCTION

It has been well established during the last decade that the transport characteristics of semiconductors are altered at high electric fields. The variation of conductivity mobility with the strength of the electric field has been extensively studied and discussed in the literature. Theoretical studies on high-field galvanomagnetic properties have been done by Sodha and Eastman,[1] Conwell,[2,3] Nag *et al.*,[4,5] Budd,[6,7] Das and Nag,[8] and Matz and Garcia-Moliner.[9] Some experimental results on the Hall mobility of p-type germanium have been published by Nag *et al.*[10] and Yamamoto *et al.*[11] However, the characteristics of high-field or the so-called hot-carrier magnetoresistance for semiconductors like n-type germanium at temperatures of 300°K, as far as the authors are aware, have not yet been studied in detail.[12] Some calculations for magnetoresistance assuming only acoustic-phonon scattering and high magnetic fields were made by Nag *et al.*[4] and some numerical values of the magnetoresistance coefficient for n-type germanium have been published by Das.[13] Since the magnetoresistance is more sensitive to the carrier distribution function and scattering mechanisms than conductivity mobility, it is of interest to make a detailed examination of the characteristics of hot-carrier magnetoresistance.

The theory of hot-carrier conduction involves the solution of the Maxwell-Boltzman transport equation, which is complicated for high electric fields. Three methods of approximation have been used to obtain the solution of the equation under these conditions. In one method, only collisions between the carriers and lattice are considered and the transport equation is solved using the diffusion approximation and retaining the first two terms only.[14,15] In the second method the symmetric part of the distribution function is assumed to be Maxwellian with an unknown parameter, the carrier temperature resulting from intercarrier collisions. This is determined from the energy-balance condition, assuming an expression for mobility as at low fields.[16,17] In the third method[18,19] the distribution function is assumed to be Maxwellian but displaced in the momentum space. The carrier temperature and the displacement are obtained by using the momentum-balance condition together with the energy-balance condition. Theoretically, these methods are applicable for three different ranges of carrier concentration. The first

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[12] An analytical study of hot-electron magnetoconductivity, assuming isotropic effective mass and acoustic-phonon and impurity scattering, has been published [H. F. Budd, Phys. Rev. **140**, A2170 (1965)] since this paper was submitted. The effect of intervalley scattering has also been outlined for silicon with the electric field in the [$\bar{1}$10] direction and magnetic field in the [111] direction. In view of the assumptions, these results are likely to be applicable to n-type germanium only at low temperatures.
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[19] R. Barrie and R. R. Burgess, Can. J. Phys. **40**, 1056 (1962).

method is applicable for low concentration, the second method for intermediate concentration, and the third for large carrier concentration. In this paper, the theory of hot-carrier magnetoresistance is worked out using the first method, i.e., assuming low carrier concentration. Though nearly identical conductivity mobility is obtained from the three methods, different magnetoresistance characteristics may be obtained. The values of magnetoresistance given by the other two methods have also been worked out and will be presented in a separate paper.

From the studies of hot-carrier conductivity mobility of n-type germanium at 300°K it has been found that to obtain agreement between theory and experiment one is required to take into account the effects of acoustic phonons, optical phonons, intervalley scattering, and the many-valley band structure. However, the improvement in the results obtained from the inclusion of all these effects is mainly quantitative in nature. In the case of magnetoresistance, on the other hand, the characteristics are radically altered when the effects of the different scattering mechanisms and the complexities of the band structure are considered. To illustrate these differences the characteristics of magnetoresistance are discussed in this paper for the assumptions: (a) the constant-energy surface is spherical and only acoustic-phonon scattering occurs; (b) the constant-energy surface is spherical and both acoustic- and optical-phonon scattering occur; (c) the constant-energy surfaces are ellipsoidal and only acoustic-phonon scattering occurs; and (d) the constant-energy surfaces are ellipsoidal and the scattering is by acoustic, optical, and intervalley phonons.

It is, of course, true that any agreement between theory and experiment for n-type germanium or silicon at a temperature of 300°K could only be expected from assumption (d). However, the results obtained from (b) give an indication of the expected characteristics for p-type germanium and silicon and for some particular directions of n-type germanium and silicon. The results, obtained from (a) and (c), may be of some importance in considering the characteristics for low temperatures.

The general theory of magnetoresistance is introduced in Sec. II. The detailed characteristics for the above-mentioned assumptions are discussed in Sec. III and IV. In Sec. V, the numerical results calculated by the present authors from the theory discussed in this paper are compared with the experimental results.

II. GENERAL CONSIDERATIONS

In this section the expression for the current density is given in a general form assuming that the constant-energy surfaces are ellipsoidal and both optical- and acoustic-phonon scattering occur. The current density for the different assumptions may be obtained as special cases from this general expression.

The current density $\mathbf{J}$ in a semiconductor may be written as

$$\mathbf{J}=\frac{e}{\hbar}\int f\times(\boldsymbol{\nabla}_{\mathbf{K}}E)d\mathbf{K}, \tag{1}$$

where e and E are, respectively, the charge and energy of a carrier, $\mathbf{K}$ is the wave vector, f is the distribution function of the carriers, $d\mathbf{K}$ is a volume element in $\mathbf{K}$ space, and $2\pi\hbar$ is Planck's constant.

In the absence of any field the distribution function of the carriers in semiconductors is Maxwellian at the lattice temperature. When a field is applied the distribution function is perturbed, but for low fields the perturbation is in the form of a small directional term added to the Maxwellian distribution. For high fields the perturbation is more complicated, but f may be written as

$$f=f_0+\mathbf{K}\cdot\mathbf{f}_1. \tag{2}$$

The higher order terms may be assumed to be much smaller than f_0 and $\mathbf{f}_1$. In Eq. (2) f_0 is non-Maxwellian and in the general case, assuming acoustic- and optical-phonon scattering and ellipsoidal constant-energy surfaces, may be written as[8,14,15,20]

$$f_0=N_0\exp\left[-\int(W+r)\left\{q+\frac{p\left[1+\left(\frac{e\tau}{m_c}\right)^2|\mathbf{M}|(\mathbf{F}\cdot\mathbf{B})^2\Big/(\mathbf{F}\cdot\mathbf{M}\cdot\mathbf{F})\right]}{\Omega\left[1+\left(\frac{e\tau}{m_c}\right)^2|\mathbf{M}|(\mathbf{M}^{-1}\cdot\mathbf{B})\cdot\mathbf{B}\right]}\right\}^{-1}dW\right], \tag{3}$$

where

$$W=E/kT;\quad \tau=\tau_{am}/\Omega;\quad \Omega=1+\tau_{am}/\tau_0;\quad q=\tfrac{1}{2}(\tau_{ae}/\tau_0W)(\theta_0/T)^2;$$
$$r=\theta_0\tau_{ae}/\tau_0W(2n_0+1)T;\quad p=\frac{2}{3}\frac{\tau\times\tau_{ae}}{kT}e^2\mathbf{F}\cdot\mathbf{M}\cdot\mathbf{F}.$$

The normalization constant is N_0, $\mathbf{M}$ is the tensor of the reciprocal effective mass multiplied by the conductivity effective mass m_c, and $|\mathbf{M}|$ is the determinant of its matrix; n_0 and θ_0 are the optical phonon concentration and temperature; $\mathbf{B}$ is the magnetic field; $\mathbf{F}$ is the total electric field; τ_{am} and τ_{ae} are, respectively, the momentum- and energy-relaxation time due to the acoustic phonons; and τ_0 is the momentum-relaxation time due to the optical phonons; k is Boltzmann constant; T is the lattice temperature.

[20] O. Tsutsumi, J. Phys. Soc. (Japan) 19, 1290 (1964).

However, $\mathbf{f}_1$ is related to f_0 by the same equation as at low electric fields given by Shibuya[21]

$$\mathbf{f}_1=-\left(\frac{e\tau}{m_c}\right)\frac{h}{kT}\mathbf{M}\cdot\left[\frac{\mathbf{F}+\left(\frac{e\tau}{m_c}\right)(\mathbf{M}\cdot\mathbf{F})\times\mathbf{B}+\left(\frac{e\tau}{m_c}\right)^2|\mathbf{M}|(\mathbf{F}\cdot\mathbf{B})(\mathbf{M}^{-1}\cdot\mathbf{B})}{1+\left(\frac{e\tau}{m_c}\right)^2|\mathbf{M}|(\mathbf{M}^{-1}\cdot\mathbf{B})\cdot\mathbf{B}}\right]\frac{df_0}{dW}. \quad (4)$$

In obtaining (3) and (4) the following assumptions have been made: (a) The scattering is isotropic and there is equipartition of energy of the phonons; (b) the energy of the carriers is much higher than the optical-phonon energy; (c) the field is high enough to make W/p negligible compared to unity.

One may evaluate $\mathbf{J}$ by substituting f from (2), (3), and (4) and performing the integration. The magnetoresistance R_m may then be obtained from the following relation:

$$R_m=(J_{x0}-J_{xB})/J_{x0}, \quad (5)$$

where J_{x0} and J_{xB} denote the current densities in the direction of the applied electric field, respectively, in the absence and presence of the magnetic fields. The above equations may also be used for the case of many-valley band structure by including the intervalley scattering term in τ and taking into account the different concentration in the valleys when summing their contribution to the total current.

III. MAGNETORESISTANCE CHARACTERISTICS FOR SPHERICAL ENERGY SURFACE

We first assume that the constant-energy surface is spherical, the electric field F_x is applied in the x direction and the magnetic field B_z is applied in the z direction. The generated Hall field F_y would than be in the y direction. We consider first only the effect of acoustic phonons. Results for this case are also given in Ref. 12. However, to illustrate the approximations made in the later analysis and to bring out the distinctions in characteristics of hot-carrier magnetoresistance for different scattering mechanisms this analysis is briefly presented here.

A. Characteristics for Acoustic-Phonon Scattering

The distribution function f_0 obtained from (3) and (4) in this case is

$$f_0=(N/I_n)\exp\{-[\tfrac{1}{2}W^2+(\mu_{ac}B_z)^2W]\times(1/p_{ac})(1+F_y^2/F_x^2)^{-1}\}, \quad (6)$$

where $\mu_{ac}=\frac{3}{4}\sqrt{\pi}\times$low-field acoustic mobility; $p_{ac}=(\frac{1}{3})(\mu_{ac}^2F_x^2)/c_l^2$; c_l=longitudinal acoustic velocity; N is the concentration of carriers;

$$I_n=\tfrac{3}{2}c\int_0^\infty W^{1/2}dW;\quad c=(4\pi/3N_0)(2kTm/h^2)^{3/2}. \quad (7)$$

The current density J_x in the direction of the applied field is obtained from (1) by substituting f from (6) and (4) and eliminating F_y by using the condition that J_y, the current density in the y direction is equal to zero. One obtains

$$J_x=Ne\mu_{ac}F_x(I_1/I_n)[1+(\mu_{ac}B_z)^2(I_2/I_1)^2], \quad (8)$$

where

$$I_1=c\int_0^\infty f_0\frac{W^2}{p_{ac}}\left(1-\frac{F_y^2}{F_x^2}\right)dW, \quad (9)$$

$$I_2=c\int_0^\infty f_0\frac{W^{3/2}}{p_{ac}}\left(1-\frac{F_y^2}{F_x^2}\right)dW. \quad (10)$$

Now for magnetic fields such that $(\mu_{ac}B_z)^2\ll1$, one may neglect terms involving $(\mu_{ac}B_z)$ higher in order than $(\mu_{ac}B_z)^2$. The exponential function in (6) may also be written as

$$(f_0/N_0)=[\exp(-W^2/p_{ac})]\times[1-(\mu_{ac}B_z)^2W/p_{ac}+(W^2/2p_{ac})(F_y^2/F_x^2)]. \quad (11)$$

All the integrals I_1, I_2, and I_n may then be analytically evaluated and, retaining the terms up to $(\mu_{ac}B_z)^2$ and noting that $(F_y/F_x)=(\mu_{ac}B_z)I_2/I_1$, one obtains

$$R_m=\frac{(\mu_{ac}B_z)^2}{(2p_{ac})^{1/2}}\left[2\left(\frac{\Gamma(2)}{\Gamma(3/2)}-\frac{\Gamma(5/4)}{\Gamma(3/4)}\right)-\left(\frac{\Gamma(5/4)}{\Gamma(3/2)}\right)^2\left(\frac{\Gamma(5/2)}{\Gamma(3/2)}-\frac{\Gamma(7/4)}{\Gamma(3/4)}\right)\right]$$
$$=(-0.0050/(p_{ac})^{1/2})(\mu_{ac}B_z)^2. \quad (12)$$

It is seen from (12) that if the energy surface is spherical and the scattering is due to acoustic phonons only, the magnetoresistance is negative but very nearly equal to zero in the hot-carrier region. The magnitude of the magnetoresistance is also inversely proportional to the applied field. The magnetoresistance coefficient ξ on the other hand, is found to be independent of the applied field:

$$\xi=R_m/(\mu_HB_z)^2=-0.0068, \quad (13)$$

since

$$\mu_H=\frac{\mu_{ac}I_2}{I_1}=\frac{\Gamma(5/4)}{\Gamma(3/2)}\frac{\mu_{ac}}{(2p_{ac})^{1/4}}. \quad (14)$$

[21] M. Shibuya, Phys. Rev. 95, 1385 (1954).

B. Characteristics for Predominant Optical-Phonon Scattering

It is now assumed that in addition to acoustic-phonon scattering, optical-phonon scattering occurs and that energy loss occurs more through collisions with optical phonons than through collisions with the acoustic phonons. In this case the distribution function f_0 is given by

$$f_0=N_0\exp\left[-\int\frac{W+r}{q+p_{\mathrm{op}}\{1+(\mu_{\mathrm{op}}B_z)^2/W\}^{-1}(1+F_y^2/F_x^2)}dW\right],\tag{15}$$

where $\mu_{\mathrm{op}}=\mu_{\mathrm{ac}}/\Omega$ and $p_{\mathrm{op}}=p_{\mathrm{ac}}/\Omega$. The current density is given by (8) in which the integrals I_n, I_1, and I_2 are the same as (7), (9), and (10), but f_0/N_0 within the integral has to be replaced by the exponential function of (15).

As in the earlier case, one may assume that $(\mu_{\mathrm{op}}B_z)^2\ll 1$ and write f_0/N_0 in the form

$$(f_0/N_0)=[W^{-s}\{1+(p_{\mathrm{op}}/p_1)(F_y^2/F_x^2)(rW/p_1)-(p_{\mathrm{op}}/p_1^2)(\mu_{\mathrm{op}}B_z)^2W\}\exp\{-(rW/p_1+W^2/2p_1)\}],\tag{16}$$

where

$$s=(p_{\mathrm{op}}r/p_1^2)(\mu_{\mathrm{op}}B_z)^2;\quad p_1=q+p_{\mathrm{op}}.$$

On using (16) and assuming that $(p_1/2r^2)$ is much less than unity the magnetoresistance is found to be given by

$$R_m=B_z^2[\epsilon\{q/p_1+0.385(p_{\mathrm{op}}/p_1)(S(2,3)/S(1,2))\}-\mu_{H0}^2(q/p_1+0.5p_{\mathrm{op}}/p_1)],\tag{17}$$

where

$$S(m)=\sum\frac{(-1)^n}{n!}\left(\frac{p_1}{2r^2}\right)^n\Gamma(2n+m);\quad S(m,l)=S(m)+\frac{p_1}{r^2}S(l),$$

$$\epsilon=\mu_{\mathrm{op}}^2\left(\frac{r}{p_1}\right)\frac{S(1,2)}{S(2,3)},\tag{18}$$

$$\mu_{H0}^2=\mu_{\mathrm{op}}^2\left(\frac{r}{p_1}\right)\left[\frac{S(3/2,5/2)}{S(2,3)}\right]^2,\tag{19}$$

It is seen from (17) that R_m depends on the field through a complicated relation. Calculations for the parameters of n-type germanium indicate that R_m is positive and its magnitude decreases with increase in the field. However, in contrast to the case of acoustic-phonon scattering the magnetoresistance coefficient is also dependent on the field. The magnetoresistance coefficient is given by

$$\xi=[(\epsilon/\mu_{H0}^2)\{q/p_1+0.385(p_{\mathrm{op}}/p_1)(S(2,3)/S(1,2))\}-(q/p_1+0.5p_{\mathrm{op}}/p_1)].\tag{20}$$

The exact nature of variation of ξ with the field may be obtained from detailed calculation using (20). Calculations with the parameter values of n-type germanium indicate that ξ decreases with the electric field. It is of interest to note here that the contribution of the term $(\mu_{\mathrm{op}}B_z)^2$ in f_0 even for low values of B_z is very significant for magnetoresistance, though it may be neglected in the calculation of Hall mobility. If $(\mu_{\mathrm{op}}B_z)^2$ is neglected in f_0, R_m/B_z^2 is found to vary less rapidly with the field.

IV. MAGNETORESISTANCE CHARACTERISTICS FOR ELLIPSOIDAL ENERGY SURFACE

We now assume that the energy surfaces are ellipsoidal and there are equivalent ellipsoids on the axes of symmetry. Specifically, four ellipsoids lying on the ⟨111⟩ axes as occur in germanium are considered. Results for similar band structure in other semiconductors may be obtained following the procedure outlined here. The scattering is assumed to be due to acoustic and optical phonons.

At low fields for the above-mentioned band structure one obtains both transverse and longitudinal magnetoresistance. Also, the magnetoresistance in any arbitrary direction may be obtained knowing the magnetoresistance coefficients ξ_{100}^{001} and ξ_{100}^{100}.[22] The characteristics at high fields are, however, more complicated. In general, since the distribution function f depends on the direction of the applied electric field and involves the magnetic field it is not possible to find the magnetoresistance coefficient in any direction from ξ_{100}^{100} and ξ_{100}^{001}. Hence, for the hot-carrier condition with anisotropic band structure the magnetoresistance coefficient for any arbitrary direction has to be individually calculated. Generalized expressions for the magnetoresistance coefficients for any arbitrary direction of applied electric and magnetic fields have been given by Das.[13] However, the expressions are too complicated for the discussion of the distinctive features

[22] R. A. Smith, *Semiconductors* (Cambridge University Press, New York, 1961), p. 363.

of the characteristics. If, however, the electric field is assumed to be applied in the [100] or [111] direction, simple analytic expressions for ξ may be obtained, and the particular characteristics for hot carrier condition may be made evident. The characteristics for other directions of electric field are also not significantly different. Hence, the discussion in this section will be confined to the above-mentioned two directions of the electric field.

A. Characteristics for Electric Field in the [100] Direction

Transverse Magnetoresistance

It is assumed that the electric field F_x is applied in the x direction and the magnetic field B_z in the z direction. In general, under hot-electron conditions even in the absence of a magnetic field a transverse voltage is produced in semiconductors with many-valley band structure due to Sasaki effect. However, this voltage is absent when the electric field is in the [100] or [111] direction. The transverse electric field is, therefore, assumed to be due to the Hall effect only and is F_y in the y direction.

We first assume that the electric field is applied in the [100] direction and the magnetic field in the [001] direction. In this case all four valleys are identically heated and contribute equally to the total current. Assuming only acoustic-phonon scattering and evaluating the integrals with the approximations outlined in Sec. III A, one obtains from (1), (3), and (4)[23]

$$\begin{aligned} R_m &= [(\alpha_t\mu_{ac}B_z)^2/(2p_{ac})^{1/2}]\left[2\left(\frac{\Gamma(2)}{\Gamma(3/2)}-\frac{\Gamma(5/4)}{\Gamma(3/4)}\right)\right. \\ &\qquad \left. -\alpha_t^2\left(\frac{\Gamma(5/4)}{\Gamma(3/2)}\right)^2\left(\frac{\Gamma(5/2)}{\Gamma(3/2)}-\frac{\Gamma(7/4)}{\Gamma(3/4)}\right)\right], \\ &= [(\mu_{ac}B_z)^2/(p_{ac})^{1/2}]\times 0.089, \end{aligned} \tag{21}$$

$$\begin{aligned} \xi &= \left[\frac{2}{\alpha_t^2}\left(\frac{\Gamma(3/2)}{\Gamma(5/4)}\right)^2\left(\frac{\Gamma(2)}{\Gamma(3/2)}-\frac{\Gamma(5/4)}{\Gamma(3/4)}\right)\right. \\ &\qquad \left. -\left(\frac{\Gamma(5/2)}{\Gamma(3/2)}-\frac{\Gamma(7/4)}{\Gamma(3/4)}\right)\right], \\ &= 0.1970, \end{aligned} \tag{22}$$

where $\alpha_t^2=(m_c^2/m_{xx}m_{yy}-m_c^2/m_{xy}^2)=0.785$; m_c/m_{xx}, m_c/m_{yy}, etc., are the components of $\mathbf{M}$.

Since the value of α_t^2 is independent of the direction of the transverse magnetic field, (21) and (22) give the values of R_m and ξ for the electric field in the [100] direction and all directions of the magnetic field in the transverse plane. The qualitative characteristics of the variation of R_m and ξ are identical to that for the spherical-energy-surface model, but the sign is positive and the values are larger.

If one assumes predominant optical-phonon scattering as in Sec. III B one obtains

$$\begin{aligned} R_m &= \alpha_t^2B_z^2\left[\epsilon\left(\frac{q}{p_1}+0.385\frac{p_0}{p_1}\frac{S(2,3)}{S(1,2)}\right)\right. \\ &\qquad \left. -(\alpha_t\mu_{H0})^2\left(\frac{q}{p_1}+0.5\frac{p_0}{p_1}\right)\right], \end{aligned} \tag{23}$$

$$\begin{aligned} \xi &= \left[\frac{\epsilon}{(\alpha_t\mu_{H0})^2}\left(\frac{q}{p_1}+0.385\frac{p_0}{p_1}\frac{S(2,3)}{S(1,2)}\right)\right. \\ &\qquad \left. -\left(\frac{q}{p_1}+0.5\frac{p_0}{p_1}\right)\right]. \end{aligned} \tag{24}$$

As α_t^2 is less than unity, the values of ξ are increased from the values obtained by assuming spherical energy surface. Calculations assuming the parameters of n-type germanium indicates that the value of R_m is also increased.

It should be noted that Hall mobility, considering the anisotropy in the band structure and for the electric field in the [100] direction, has been shown to be modified from the value obtained from spherical-energy-surface model by a factor $(\alpha_t)^2$ at all fields. But as is evident from (21) through (24) even for this direction of the electric field R_m and ξ vary with the strength of the electric field obeying a law different from that obtained from the spherical constant-energy-surface model.

Longitudinal Magnetoresistance

We now assume that the magnetic field B_x is applied along the [100] direction i.e., in the direction of the electric field. One then obtains for acoustic-phonon scattering alone

$$R_m=\left[\frac{(\alpha_lB_x)^2}{(2p_{ac})^{1/2}}\right]\left[2\left(\frac{\Gamma(2)}{\Gamma(3/2)}-\frac{\Gamma(5/4)}{\Gamma(3/4)}\right)\right], \tag{25}$$

where

$$\alpha_l^2=\left(\frac{m_c^2}{m_{yy}m_{zz}}-\frac{m_c^2}{m_{yz}m_{xy}}\right)-m_c^3|\mathbf{M}|.$$

Since Hall mobility for the assumed direction of the electric field is independent of the direction of the transverse magnetic field one may express the magnetoresistance coefficient using the Hall mobility obtained in the preceding section as

$$\xi=\left(\frac{\alpha_l}{\alpha_t}\right)^2\left(\frac{\Gamma(3/2)}{\Gamma(5/4)}\right)^2\left[2\left(\frac{\Gamma(2)}{\Gamma(3/2)}-\frac{\Gamma(5/4)}{\Gamma(3/4)}\right)\right]. \tag{26}$$

[23] The contribution of the term $(m_{xx}/m_{xy})\mu_HB_z$ in f_0 (Eq. 3) has been neglected. The inclusion of this term alters the numerical factors slightly. For the cases considered in the following section the contribution of this term is smaller still.

One also obtains for predominant optical-phonon scattering

$$R_m=(\alpha_l B_x)^2\epsilon\left(\frac{q}{p_1}+0.385\frac{p_0}{p_1}\frac{S(2,3)}{S(1,2)}\right), \tag{27}$$

$$\xi=\left(\frac{\alpha_l}{\alpha_t}\right)^2\frac{\epsilon}{(\alpha_t\mu_{H0})^2}\left(\frac{q}{p_1}+0.385\frac{p_0}{p_1}\frac{S(2,3)}{S(1,2)}\right). \tag{28}$$

The above expressions show that the variation of R_m and ξ with the electric field is of the same nature as for the transverse magnetoresistance and magnetoresistance coefficient. For pure acoustic-phonon scattering R_m is positive and decreases inversely as the electric field while ξ is independent of the filed. But, for predominant optical-phonon scattering both R_m and ξ decreases with increase of field.

B. Characteristics for Electric Field in the [111] Direction

In this case, of the four valleys in germanium three are equally heated while the carriers in the fourth valley are less heated. Hence the expressions for R_m and ξ, though similar to those in Sec. IV A involve summation over the four valleys. When the magnetic field is applied in a transverse direction one obtains for predominant optical-phonon scattering

$$R_m=(\mu_{\text{op}}B_z)^2\left[\frac{\sum\alpha_{tv}^2\left(\frac{m_c}{m_{xx}}\right)_v\left(\frac{r}{p_{1v}}\right)^{3/2}\left\{\frac{q}{p_{1v}}+0.385\frac{p_{0v}}{p_{1v}}\frac{S(2,3)_v}{S(1,2)_v}\right\}}{\sum\left(\frac{r}{p_{1v}}\right)^{1/2}\frac{S(2,3)_v}{S(3/2)_v}\left(\frac{m_c}{m_{xx}}\right)_v}\right.$$
$$\left.-\left\{\frac{\left[\sum\left(\frac{r}{p_{1v}}\right)\frac{S(3/2,5/2)_v}{S(3/2)_v}\alpha_{tv}^2\right]^2}{\left[\sum\left(\frac{r}{p_{1v}}\right)^{1/2}\frac{S(2,3)_v}{S(3/2)_v}\left(\frac{m_c}{m_{yy}}\right)_v\right]}\right\}\left\{\frac{\left[\sum\left(\frac{r}{p_{1v}}\right)^{1/2}\left(\frac{m_c}{m_{yy}}\right)_v\frac{S(2,3)_v}{S(3/2)_v}\left(1-0.5\frac{p_{0v}}{p_{1v}}\right)\right]}{\sum\left(\frac{r}{p_{1v}}\right)^{1/2}\left(\frac{m_c}{m_{xx}}\right)_v\frac{S(2,3)_v}{S(3/2)_v}}\right\}\right], \tag{29}$$

where

$$p_{1v}=q+p_{0v};\quad p_{0v}=\frac{2}{3}\frac{\tau\times\tau_{ae}e^2F_x^2}{m_ckT}\left(\frac{m_c}{m_{xx}}\right)_v.$$

The subscript v in the above equation indicate values for different valleys. Since α_t^2, m_{xx}, and m_{yy} are different, the functions $S(m,l)_v$ are different for the different valleys. The detailed nature of variation of R_m and ξ may hence be obtained only from numerical computations. It is, however, evident that the general characteristic of hot-electron magnetoresistance that it decreases with increase in the field, is valid also in this case. The detailed computations presented in the following sections also show that it is positive, and that ξ increases with the field.

It may be noted that at low fields when the electric field is applied in the [111] direction the transverse magnetoresistance is independent of the direction of the transverse magnetic field. Consideration of the effective-mass tensor for the different valleys indicates that this result would be applicable also under hot-electron condition.

The expression for the longitudinal magnetoresistance is similar to that given for the [100] direction of the electric field. One obtains

$$R_m=(\mu_{\text{op}}B_x)^2\left[\frac{\sum\alpha_{lv}^2\left(\frac{m_c}{m_{xx}}\right)_v\left(\frac{r}{p_{1v}}\right)^{3/2}\frac{S(1,2)_v}{S(3/2)_v}\left(\frac{q}{p_{1v}}+0.385\frac{p_{0v}}{p_{1v}}\frac{S(2,3)_v}{S(1,2)_v}\right)}{\sum\left(\frac{r}{p_{1v}}\right)^{1/2}\frac{S(2,3)_v}{S(3/2)_v}\left(\frac{m_c}{m_{xx}}\right)_v}\right]. \tag{30}$$

As in the earlier case, R_m decreases with increase in the field.

The values of ξ may be obtained using (29) and (30) and noting that Hall mobility is given by

$$(F_y/F_x)=\left[\sum\left(\frac{r}{p_{1v}}\right)\frac{S(3/2,5/2)_v}{S(3/2)_v}\alpha_{tv}^2\right]\Big/\left[\sum\left(\frac{r}{p_{1v}}\right)^{1/2}\frac{S(2,3)_v}{S(3/2)_v}\left(\frac{m_c}{m_{yy}}\right)_v\right]. \tag{31}$$

Expressions for R_m and ξ for pure acoustic-phonon scattering may be obtained as in the earlier case. One finds that the values are increased, but no new significant features are exhibited.

V. DISCUSSION

The applicability of the theory developed in the preceding section may be tested for n-type germanium, for which experimental results were recently obtained by the authors.[24] The experimental plot of magnetoresistance R_m/B_z^2 is reproduced in Fig. 1 together with the results calculated from the present theory. For these calculations expressions for τ_{ae}, τ_{am}, and τ_0 given by Reik and Risken[15] were used and the values of the parameters were taken as follows: $\rho=5.35$ kg/m³; $c_l=5.54\times10^3$ m/sec; $T=300°$K; $m_l/m_0=1.64$; $m_t/m_0=0.0819$; $\Xi_1=11$ eV; $\Xi_0=30$ eV; $\theta_0=400°$K; $D_0=0.8\times10^9$ eV/cm.

It is evident from Fig. 1 that the agreement between theory and experiment may be considered fairly good. The qualitative agreement is excellent, while the quantitative agreement is within 20%.

Intervalley scattering and the anisotropy in scattering has been neglected in the present theory. The effect of anisotropy is to alter the mass ratio m_l/m_t wherever it occurs from the assumed value of 20 to lower values. Calculations made by us with a value of 15 indicates that the magnitude of magnetoresistance is not significantly altered.

Intervalley scattering affects the results in two ways. Firstly, the momentum and energy relaxation times are altered. For not too large fields the change in the energy relaxation time is not very significant in n-type germanium. The modification of momentum relaxation time may be expressed as an alteration in Ω given below:

$$\Omega=1+(\tau_{am}/\tau_0)\times[1+3(D_i/D_0)^2(\theta_0/\theta_i)(2n_i+1)/(2n_0+1)], \quad (32)$$

where D_i is the deformation potential for intervalley phonons, and θ_i and n_i are the corresponding temperature and concentration. The second effect of intervalley scattering is repopulation of carriers among the different valleys. In the absence of the magnetic field one may calculate the changed concentrations for the near-saturation region using the following equation[25]:

$$(Nv/N)=(r/p_1v)^{1/2}/\sum (r/p_1v)^{1/2}. \quad (33)$$

For lower fields, the carrier population may be obtained from the analysis of Paige.[26] If the effect of magnetic field on carrier repopulation is neglected one finds, using (32) and (33), that on inclusion of intervalley scattering effects the values of magnetoresistance are increased only about 3%. On the other hand, when the values of carrier concentration as given by Paige are used the theoretically calculated values of R_m/B_z^2 (shown in Fig. 1) agree with the experimental values to within 12%.

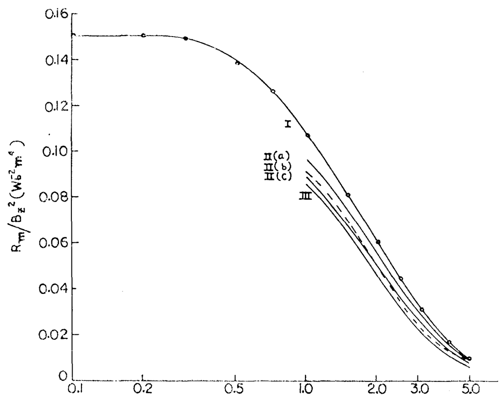

FIG. 1. Variation of magnetoresistance with electric field. I. Experimental values. II. Theoretical values including intervalley scattering according to (a) Ref. 23; (b) Eq. (35); and (c) Eq. (3). III. Theoretical values neglecting intervalley scattering.

The repopulation of the carriers in the presence of the magnetic field is rather difficult to determine. However, an estimate of its effect may be obtained assuming that (N_v/N) is primarily determined by the effective temperature in the different valleys. One may then write

$$(N_v/N)=(N_{v0}/N)[1+\tfrac{1}{2}(\alpha_{tv}\bar{\mu}_vB_z)^2(p_{0v}/p_{1v}) -\tfrac{1}{2}\sum (r/p_{1v})^{1/2}(p_{0v}/p_{1v})(\alpha_{tv}\bar{\mu}_vB_z)^2/\sum (r/p_{1v})^{1/2}], \quad (34)$$

where N_v and N_{v0} are the concentrations in a valley in the presence and absence of the magnetic field, $\bar{\mu}_v$ is an average mobility for a particular valley for the corresponding effective field. $\bar{\mu}_v$ is approximately equal to $\mu_{op}(q/p_{1v})^{1/2}$. The change in the carrier population by the magnetic field thus adds a term ΔR_m to R_m, where

$$\begin{aligned}\Delta R_m=\tfrac{1}{2}\{ &\sum (N_{v0}/N)[-(p_{0v}q/p_{1v}^2) +\sum (r/p_{1v})^{1/2}(p_{0v}q/p_{1v}^2)/\sum (r/p_{1v})^{1/2}]\\ &\times[(r/p_{1v})^{1/2}(S(2,3)_v/S(3/2)_v)\\ &\times(m_c/m_{xx})_v(\alpha_{tv})^2(\mu_{op}B_z)^2]\}\\ &\times[\sum (r/p_{1v})^{1/2}(S(2,3)_v/S(3/2)_v)\\ &\times(m_c/m_{xx})_v(N_{v0}/N)]^{-1}. \quad (35)\end{aligned}$$

The values of (R_m/B_z^2) when the above correction is made are also shown in Fig. 1. One finds that ΔR_m is negative and partially annuls the increment in (R_m/B_z^2) obtained by the inclusion of intervalley effects, as discussed earlier.

The over-all agreement between theory and experiment, when the effect of intervalley scattering is totally taken into account, is found to be about 15%. One may thus note that for n-type germanium at 300°K, the results for which are discussed here, intervalley

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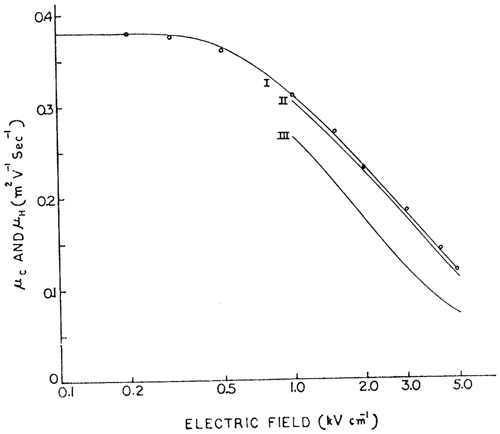

FIG. 2. Variation of mobilities with electric field. I. Experimental values of conductivity mobility of 5-Ω cm n-type germanium. II. Theoretical values of conductivity mobility. III. Theoretical values of Hall mobility.

scattering changes the magnetoresistance by a small amount. However, this result may be substantially modified at low temperatures where acoustic-phonon scattering is more predominant.

The values of magnetoresistance are found to be critically dependent on D_0. Calculations made for $D_0=0.5\times10^9$ eV/cm, the value derived by Reik and Risken[25] from the saturated drift velocity, gives values which are much lower than the experimental values. In fact, a value of $D_0=0.8\times10^9$ eV/cm gives the best fit with the experimental results.

It should be mentioned that conductivity, Hall mobility, and also ξ and $R_m/\mu_c^2B_z^2$ (μ_c= conductivity mobility) have been calculated using the parameter values which give good agreement between the theoretical and experimental values of magnetoresistance. These calculated values together with the experimental values of $R_m/\mu_c^2B_z^2$ and conductivity as obtained by the present authors are shown in Figs. 2 and 3. The theoretical values of conductivity are found to agree with the experimental values almost exactly. The values of $R_m/\mu_c^2B_z^2$ are also found to be almost independent of the field, as has been observed experimentally.[24] Since no data on Hall mobility or ξ are available, checking of the theoretical values of Hall mobility and ξ against experiments could not be done. However, considering the agreement in the values of R_m/B_z^2, μ_c, and $R_m/\mu_c^2B_z^2$ one may perhaps conclude that the interaction parameter for optical phonons in n-type germanium should be taken to be 0.8×10^9 eV/cm, rather than 0.5×10^9 eV/cm, obtained from the study of the saturation drift velocity. This value is also closer to the value obtained by Meyer[27] and to the value derived by Reik and Risken from the analysis of anisotropy data.

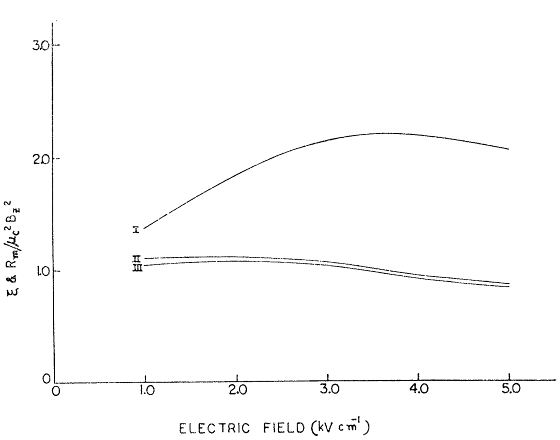

FIG. 3. Variation of ξ and $R_m/B_z^2\mu_c^2$ with electric field. I. Calculated values of ξ. II. Experimental values of $R_m/B_z^2\mu_c^2$. III. Theoretical values of $R_m/B_z^2\mu_c^2$.

VI. CONCLUSION

The present study indicates the following general features of hot-electron magnetoresistance. (a) The magnetoresistance coefficient is small and independent of the field if the scattering is due to acoustic phonons only. (b) Magnetoresistance as well as the magnetoresistance coefficient decreases with increase in the field if the scattering is predominantly due to optical phonons and the constant-energy surfaces are spherical. (c) For multivalley band structure as for spherical constant-energy-surface model the transverse magnetoresistance decreases with increase in the electric field for both [100] and [111] directions of the electric field. For predominant optical-phonon scattering the magnetoresistance coefficient decreases with increase in the field for the [100] direction but increases with increase in the field for the [111] direction. The values of the magnetoresistance or the magnetoresistance coefficient are increased from the values for spherical constant-energy-surface model. For acoustic-phonon scattering alone the dependence of magnetoresistance and magnetoresistance coefficient on the field are identical to that for spherical constant-energy-surface model, but the sign is positive and the values are higher. (d) The longitudinal magnetoresistance also decreases with increase in the electric field.

One also finds that the assumptions of isotropic-, acoustic-, optical-, and intervalley-phonon scattering and the many-valley band structure for n-type germanium lead to values agreeing closely with experiments if the optical-deformation potential is assumed to be 0.8×10^9 eV/cm. The contributions of anisotropy in scattering are also found to be not very significant.

ACKNOWLEDGMENTS

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