Conduction-Electron Spin Resonance

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We have calculated (using a simple theory) the paramagnetic-resonance absorption by conduction electrons in a thin metallic sample, thus extending the earlier work of Dyson. Our results are for a metal sample of arbitrary thickness, with a static magnetic field $\mathcal{H}_0$ at arbitrary angle with respect to the sample surface, and under either classical or anomalous skin-effect conditions. The electromagnetic field is assumed to be incident normally on both sides of the sample, but the relative phase and/or amplitude on the two sides can be arbitrary (asymmetric excitations). Under anomalous skin-effect conditions, as the field $\mathcal{H}_0$ is rotated from parallel to normal, the shape of the spin-resonance line is modified. For asymmetric boundary conditions on the electromagnetic field, the line decreases in intensity (to zero under certain conditions). For a symmetric excitation, or when the electromagnetic field is incident on one side only, the line shows a slight decrease in intensity and a slight narrowing as the field is rotated. Numerical results are presented.

I. INTRODUCTION

WHEN electromagnetic (EM) radiation is incident on a metallic sample the quantity of experimental interest is generally the power absorbed by the metal as a function of some external parameter; for example, an applied static magnetic field. For flat plate-like samples, and for frequencies much below the plasma frequency, it is easy to show that when the EM wave is incident on one side of the sample the power absorbed by the specimen (for samples thick compared to a skin depth) is proportional to the real part of the surface impedance, $Z(0)$.[1] The quantity

$$Z(0)=\frac{4\pi}{c}\frac{\hat{n}\cdot[\mathbf{E}(0)\times\mathbf{H}(0)]}{|\mathbf{H}(0)|^2}, \qquad (1)$$

where $\mathbf{E}(0)$ and $\mathbf{H}(0)$ are the values of the electromagnetic $\mathbf{E}$ and $\mathcal{H}$ fields at the front surface of the specimen and $\hat{n}$ is the unit vector normal to the face of the sample. Dyson[2] has calculated the paramagnetic contribution to the surface impedance Z in the neighborhood of conduction-electron spin resonance (CESR), i.e., near

$$\omega_S\equiv 2\mu|\mathcal{H}_0|/\hbar=\omega. \qquad (2)$$

Here μ is the effective electron magnetic moment, $\mathcal{H}_0$ is the external static magnetic field, and ω is the applied microwave frequency.

In making his calculation, Dyson made three simplifying assumptions: (1) The conduction electrons were taken to be an isotropic gas of noninteracting electrons colliding with impurities and moving under the influence of the applied electric and magnetic fields. (2) The field $\mathcal{H}_0$ was assumed normal to the surface of the sample. (3) Normal skin-effect conditions prevail.[3]

Azbel' *et al.*[4] have made a more general calculation dropping the above restrictions. However, they assumed that the exciting electromagnetic field was only incident on one side of the sample. Their calculation is sufficiently general and sufficiently complicated that much of the physics is not easily extracted. Line shapes or widths for CESR are not shown or discussed. In the present paper we present a simple calculation of CESR dropping restrictions 2 and 3, keeping 1, but allowing for the fact that the exciting fields may be incident from both sides of the slab samples with arbitrary symmetry.

Under so-called "anomalous conditions,"[5] in the presence of a static magnetic field, it is not in general possible to write down a closed form expression for the surface impedance of a metal. This problem has in fact been solved only approximately. For the magnetic field parallel to the surface of the sample (the so-called Azbel'-Kaner geometry), Azbel' and Kaner[6] have given approximate expressions for the surface impedance in the neighborhood of cyclotron resonance. Despite our inability to solve the skin-effect problem, it is still possible to find the effect of the electronic spin on the surface impedance of the metal in terms of the surface impedance Z_0, in the absence of any spin contribution. The quantity Z_0 plays the role of an unknown complex number, which enters the theory in a simple way, i.e., as an over-all multiplicative factor. Our solution is affected because of the fact that the magnetization induced by the fields internal to the metal is small. To lowest order in the electronic susceptibility it can be calculated using the EM fields prescribed by the solution to the skin-effect problem. In the skin-effect

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[1] M. Born and E. Wolf, *Principles of Optics* (Pergamon Press, Inc., New York, 1959), pp. 617–619.

[2] F. J. Dyson, Phys. Rev. **98**, 349 (1953).

[3] In general, normal-skin-effect conditions imply that the distance over which the EM field varies is large compared to distances over which the electrons (due to their orbital motion) may carry current information. We will be concerned with experiments in the microwave regime with a static magnetic field present. Since the cyclotron frequency $\omega_c\equiv e|\mathcal{H}_0|/mc$ the applied frequency ω and the spin-resonance frequency are all approximately equal. It is easily shown that the condition for the skin effect to be normal reduces to the condition $\omega_c\tau\ll 1$.

[4] M. Ya Azbel', V. I. Gerasimenko, and I. M. Lifschitz, Zh. Eksperim. i Teor. Fiz. **35**, 691 (1958) [English transl.: Soviet Phys.—JETP **8**, 480 (1959)].

[5] A. B. Pippard, in *Low-Temperature Physics*, edited by C. DeWitt (Gordon and Breach, Science Publishers, New York, 1962), pp. 58 and 129.

[6] M. Ya Azbel' and E. A. Kaner, Zh. Eksperim. i Teor. Fiz. **30**, 811 (1956) [English transl.: Soviet Phys.—JETP **3**, 772 (1956)].

problem the current is not small and is produced by fields which must be computed self-consistently.

A further simplification comes about because the spin relaxation time is extremely long (compared to transport mean free times). Since the spin "lives" for a long time, it is intuitively clear that the induced magnetization will vary on a scale of distance much greater than the scale of distance over which the EM fields vary. The detailed variation of the fields over the cyclotron orbits of the electrons is unimportant. The magnetic properties of the medium in the long spin-relaxation-time limit depends only on certain integrals of the field which can be simply related to the impedance Z_0. From the same considerations it is clear that the electron motion may still be thought of as a diffusion.

We will show that to a good approximation anomalous conditions[3] ($\omega_c\tau>1$) introduce two essential modifications of the usual CESR results. The unknown surface impedance Z_0 acts only as a complex multiplicative factor which mixes in amounts of the real and imaginary parts of the function characterizing the spin contribution to the magnetization. In addition, the diffusion constant describing the electron motion is anisotropic with respect to the direction of the field, $\mathcal{H}_0$. The effective thickness of the sample will depend on the orientation of $\mathcal{H}_0$ with respect to the surface of the sample.

This calculation was motivated by a desire to understand the CESR line shape observed by Walsh[7] in experiments on extremely pure thin samples of potassium at low temperatures ($\omega_c\tau\gg1$). In these experiments the sample enclosed in an insulating container is placed against the wall of a cavity. It is, in effect, excited by a microwave field which may be either symmetric or asymmetric. As the static magnetic field $\mathcal{H}_0$ was tilted away from the plane of the sample, Walsh observed an over-all decrease in line intensity. In addition, for small tilt angles, he found a shoulder developing on the line. For thin samples, we will show that as the field $\mathcal{H}_0$ is rotated from parallel to normal, the shape of the CESR is modified. For completely antisymmetric boundary conditions on the EM field, the line rapidly decreases and broadens very slightly. For symmetric and/or one-sided excitations the line shows a slight decrease in intensity and a slight narrowing. These features agree qualitatively with the experimental results. However, we find, within the framework of this model, no evidence for the shoulder found in the experimental data.

Recently there have been a series of transmission experiments which study the CESR.[8] These experiments, by Schultz and co-workers, have only been analyzed under normal-skin-effect conditions. Our results show that their analysis, with a very slight reinterpretation, is applicable in the anomalous regime as well.

II. CALCULATION

The Maxwell equations for the field inside the sample (assuming a time dependence of the form $e^{-i\omega t}$) are

$$\mathbf{H}=\mathbf{B}-4\pi\mathbf{M}\,, \tag{3a}$$

$$\nabla\times\mathbf{H}\equiv\nabla\times\mathbf{B}-4\pi\nabla\times\mathbf{M}=\frac{4\pi}{c}\mathbf{j}\,, \tag{3b}$$

$$\nabla\times\mathbf{E}=i\omega/c\mathbf{B}\,. \tag{3c}$$

At metallic densities and microwave frequencies the neglect of the displacement current is well justified. For weak rf fields there will be a linear nonlocal relationship between $\mathbf{M}$ and $\mathbf{H}$ of the form

$$M_i(\mathbf{r})=\int\chi_{ij}(r,r')H_j(r')d^3r'\,. \tag{4}$$

The magnitude of $|M/H|\cong\chi$ is small. All expressions will only be accurate to lowest order in this quantity. The function $\chi_{ij}(r,r')$ is the nonlocal susceptibility which will in general depend on the shape of the sample. The scale of distance over which it varies is characterized by the lifetime of the spins and the typical velocity with which the electrons move. That is to say,

$$\frac{\partial M}{\partial r}\sim\frac{1}{\delta_{\mathrm{Mag}}}M\,, \tag{5}$$

with δ_{Mag} of the order of $V_F(T\tau)^{1/2}$, where T is a phenomenological spin lattice relaxation time, τ a transport mean free time, and V_F is the Fermi velocity of the carriers.

The scale of distance over which the rf magnetic field varies is of the order of the skin depth δ. Under classical skin-effect conditions

$$\delta\equiv\delta_c=(c/\omega_p)(1/\omega\tau)^{1/2}\,, \tag{6}$$

where ω_p is the electron plasma frequency. Under anomalous skin-effect conditions, in the absence of magnetic field,

$$\delta\cong\delta_c^{2/3}\Lambda^{1/3}\,, \tag{7}$$

where Λ is an electron mean free path. In all cases of experimental interest even in the presence of magnetic fields $\delta/\delta_{\mathrm{Mag}}\ll1$. This fact allows us to analyze the finite-slab problem. Terms of order $\chi(\delta/\delta_{\mathrm{Mag}})$ will be dropped. Thus we may neglect $\nabla\times\mathbf{M}$ in Eq. (3b) and the set of Maxwell equations written in terms of B are independent of the magnetization.

If $\mathbf{H}_0(z)\equiv\mathbf{B}_0(z)$ is the magnetic field for spinless electrons, then $\mathbf{H}=\mathbf{H}_0-4\pi\mathbf{M}$, and $\mathbf{B}(z)=\mathbf{B}_0(z)+O(\chi\delta/\delta_{\mathrm{Mag}})$. Substituting into Eq. (1) we find that

$$Z(0)=Z_0\left(1+4\pi\frac{\mathbf{H}_0^*(0)\cdot\mathbf{M}(0)}{|H_0(0)|^2}\right), \tag{8}$$

[7] W. M. Walsh, Jr., L. W. Rupp, Jr., and P. H. Schmidt, Phys. Rev. 142, 414 (1966).
[8] S. Schultz and C. Latham, Phys. Rev. Letters 15, 148 (1965).

where

$$Z_0=\frac{4\pi}{c}\frac{\hat{n}\cdot(\mathbf{E}(0)\times\mathbf{H}_0^*(0))}{|H_0(0)|^2},\tag{9}$$

is the surface impedance for spinless electrons.

For an EM field varying only in a direction perpendicular to the surface of a slab-like sample (the z direction) the high-frequency magnetization $\mathbf{M}(z)$ is given by [see Dyson, Eq. (20)]

$$\mathbf{M}(z)=\frac{i}{2}\chi\omega_S\int_0^\infty dt'e^{-t'/T}\int_0^L G(z,z',t')\,,$$
$$\times[\mathbf{s}\cdot\mathbf{H}_0(z')]\mathbf{s}^*e^{+i\alpha t'}dz'.\tag{10}$$

The quantity $\alpha=\omega-\omega_S$, L is the slab thickness and $\mathbf{s}\equiv\langle u_1|\boldsymbol{\sigma}|u_2\rangle$. The states $|u_{1,2}\rangle$ are the usual free-electron spin eigenstates, parallel and antiparallel to the field $\mathcal{H}_0$ and $\boldsymbol{\sigma}$ is the set of 2×2 Pauli matrices.[9] $G(z,z',t)$ is the Green's function which characterizes the orbital motion of the electrons in the static magnetic field $\mathcal{H}_0$. It is precisely the probability that an electron at $z=z'$ at $t=0$ will have arrived at z at time t.

Because of the exponentially decaying factor in the time integration it is clear that the main contribution to the integral in Eq. (10) comes from times of the order of T, the magnetic relaxation time. Since $T\gg\tau$ we can think of the motion of the electrons (even for the case $\omega_c\tau\gg1$) as a diffusion. When $\omega_c\tau\gg1$ there is a well-defined cyclotron motion about a guiding center and a diffusion of the guiding center itself. The field $\mathbf{H}_0(z')$ in Eq. (10) is evaluated at a point on the electron's orbit. In practice, since the scale of spatial variation of $G(z,z',t')$ is of the order of δ_{Mag}, we can evaluate z' at the orbit center, making errors at most of the order of $r_c/\delta_{\mathrm{Mag}}\ll1$. These small terms will be dropped. In the experimentally interesting regime we can think of G as the Green's function for the one-dimensional diffusion equation as in the Dyson case even for $\omega_c\tau\gg1$. In the large $\omega_c\tau$ limit it is really a diffusion of the guiding centers.

Since the scale of variation of $H_0(z')$ is the skin depth δ, we may rewrite Eq. (10) dropping terms of order $\delta/\delta_{\mathrm{Mag}}$ as,

$$\mathbf{M}(z)=\frac{i}{2}\chi\omega_S\int_0^\infty dt'e^{-t'/T}e^{i\alpha t'}G(z,0,t')$$
$$\times\int_0^L dz'\mathbf{s}\cdot\mathbf{H}_0(z')\mathbf{s}^*.\tag{11}$$

The sample thickness is always large compared to δ, so that the integral on z' in Eq. (11) is given by

$$\int_0^L \mathbf{H}_0(z')dz'=\frac{ic}{\omega}(\mathbf{n}\times\mathbf{E}_0(0)).\tag{12}$$

[9] We have dropped an off-resonance term in Eq. (10) since it will not contribute near the center of the CESR line.

Since

$$\mathbf{n}\times\mathbf{E}_0(0)=\frac{c}{4\pi}Z_0\mathbf{H}_0(0)\tag{13}$$

we may rewrite Eq. (11) as

$$\mathbf{M}(z)=-\frac{c^2}{8\pi}\left(\frac{\omega_S}{\omega}\right)Z_0\chi\mathbf{s}\cdot\mathbf{H}_0(0)\mathbf{s}^*$$
$$\times\int_0^\infty e^{it'(\alpha+i/T)}G(z,0,t')dt'.\tag{14}$$

The Green's function $G(z,z't)$ for the one-dimensional diffusion equation with boundary condition

$$\frac{\partial G(z,z',t)}{\partial z}=0\tag{15}$$

at $z=0$ and $z=L$ is

$$G(z,z',t')=\frac{1}{L}\sum_{n=-\infty}^{\infty}\cos(\mu_n z)\cos(\mu_n z')e^{-1/3v_F\Lambda\mu_n^2t'},\tag{16}$$

with

$$\mu_n=n\pi/L.$$

The boundary condition Eq. (15) specifies that there be no net electron current at the surface $z=0$ and $z=L$.

The Green's function $G(z,z',t')$ is still the correct one for an electron moving in a magnetic field $\mathcal{H}_0$, which makes an angle $\pi/2-\theta$ with respect to the z axis. Now, the Green's function is to be interpreted as the probability of finding the electron's guiding center at z at t' if it started out at z' at time $t=0$. However, one must take for the mean free path Λ in the z direction an effective mean free path[10]

$$\Lambda^2=\Lambda_0^2\left[\sin^2\theta+\frac{1}{[1+(\omega_c\tau)^2]}\cos^2\theta\right].\tag{17}$$

The quantity

$$\Lambda_0=V_F\tau\tag{18}$$

is the mean free path in the absence of a magnetic field. For large $\omega_c\tau$ and for propagation perpendicular to the magnetic field, $\Lambda\approx r_c$ where r_c is the cyclotron orbit.

Inserting Eq. (16) in Eq. (14) performing the integral on t' and using the fact that

$$\sum_{n=-\infty}^{\infty}\frac{1}{A^2-\pi^2n^2}=\frac{1}{A}\cot A\,,\tag{19}$$

$$\sum_{n=-\infty}^{+\infty}\frac{(-1)^n}{A^2-\pi^2n^2}=\frac{\csc A}{A}\equiv\frac{\cot(A/2)-\cot A}{A},\tag{20}$$

[10] For a complete discussion of the diffusion of particles in a magnetic field see W. P. Allis, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1956), Vol XXI, p. 395 (particularly Eqs. 13.7–13.8).

we find that,

$$\mathbf{M}(0)=[N(\cot 2W)/W]\mathbf{s}\cdot\mathbf{H}_0(0)\mathbf{s}^*, \tag{21}$$

$$\mathbf{M}(L)=[N(\cot W-\cot 2W)/W]\mathbf{s}\cdot\mathbf{H}_0(0)\mathbf{s}^*, \tag{22}$$

where

$$N=\left(\frac{c^2}{8\pi}\right)\left(\frac{\omega_S}{\omega}\right)\left(\frac{a^2T}{L}\right)Z_0\chi, \tag{23}$$

$$a^2=\frac{3L^2}{2V_F\Lambda T}=\frac{L^2}{\delta^2_{\mathrm{Mag}}}, \tag{24}$$

and

$$W^2=\tfrac{1}{2}a^2(iT\alpha-1). \tag{25}$$

The transmission experiments of Schultz and Latham[8] measure $\mathbf{M}(L)$ directly, since

$$4\pi\mathbf{M}(L_-)\left[\frac{4\pi}{c}Z_0\right]=\mathbf{H}(L_+). \tag{26}$$

A brief derivation of this result is given in the Appendix. A more detailed discussion of Eq. (26) will be presented in a paper by Schultz and Dunnifer[11] who analyze the transmission experiment in the classical skin-effect regime. A brief discussion of this point is also given in Ref. (12). The subscripts minus and plus indicate that we evaluate things slightly to the left $(-)$ or to the right $(+)$ of the right-hand boundary. The important point to make is that anomalous conditions modify the classical results in an extremely simple way. Only Z_0 and Λ are changed, the functional form of the line is unaltered.

In order to calculate the power absorbed for a *one-sided excitation* we have only to substitute Eq. (24) into the real part of Eq. (8). For samples thick compared to a skin depth δ but not necessarily thick compared to δ_{Mag}, the value of $\mathbf{M}(L)$ is irrelevant. There is no magnetic field at L to couple to. If we excite from the other side of the sample as well, then an $\mathbf{H}$ field exists in the skin at L and there is a contribution to the absorption of the form $\mathbf{M}(L)\cdot\mathbf{H}(L)$.

If the two incident fields are similarly polarized so that

$$\mathbf{H}_0(L)=-b\mathbf{H}_0(0)$$

for some scalar b, then the power absorbed in the CESR line is, using Eqs. (21) and (22), proportional to

$$P=|H_0(0)|^2\frac{c^4}{(4\pi)^2}\left(\frac{T}{L}\right)a^2f\chi$$
$$\times\mathrm{Re}\left[Z_0^2\left\{\frac{(1+b)^2\cot 2W-2b\cot W}{W}\right\}\right], \tag{27}$$

where

$$f=\frac{|\mathbf{s}\cdot\mathbf{H}_0(0)|^2}{|\mathbf{H}_0(0)|^2}.$$

[11] S. Schultz and J. Dunnifer (to be published).

Equations (21)–(22) and (26) are the principal results of this calculation. The one-side result $b\rightarrow 0$ is

$$P_{\mathrm{os}}\sim\frac{\cot 2W}{W}. \tag{28}$$

The symmetric case $b=-1$ is

$$P_{\mathrm{sy}}\sim\left(\frac{\cot W}{W}\right). \tag{29}$$

Equations (27)–(28) along with the definitions Eqs. (24) and (25) are identical with Dysons results Eqs. (71)–(77) if one neglects terms of order $\delta/\delta_{\mathrm{Mag}}$.

It is clear from Eqs. (27) and (28) that the symmetric case with thickness L is equivalent to the one-sided case with thickness $\frac{1}{2}L$. Physically this must be so since the electromagnetic field is the same in both cases in the region $z\leq\frac{1}{2}L$ (small skin depth) and the diffusion in the symmetric case is unchanged if the region to the right is replaced by a boundary (reflecting wall at $z=\frac{1}{2}L$).

The general case can be regarded as a mixture of terms due to thickness L and thickness $\frac{1}{2}L$. For $a\geq 4$, i.e., large imaginary W in the neighborhood of the resonance cot W approaches an asymptotic value of $-i$. It is independent of W so that the thickness does not enter. For $a\lesssim 1$ the boundary conditions, the tilt of the field, and the anomalous skin effect all influence the shape of the line.

In Fig. 1 we show a series of curves of the derivative of power absorption for an antisymmetric field excitation. These figures are plotted with a ratio of sample thickness to spin diffusion distance in the absence of a field, i.e., $(3L^2/2V_F\Lambda_0T)^{1/2}$, of $\frac{1}{2}$. This ratio is denoted by

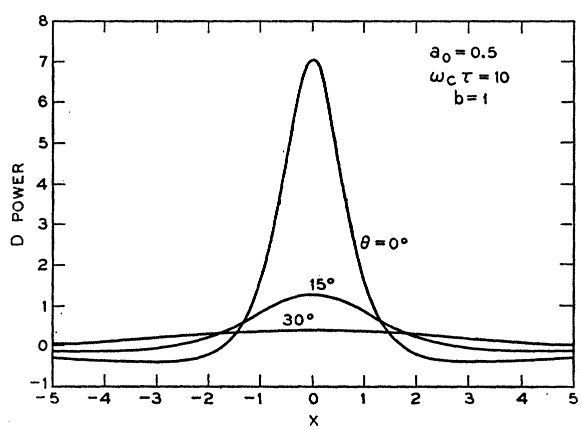

FIG. 1. Plot of the derivative of relative power absorbed by the spin system in the vicinity of CESR as a function of magnetic field x where $x\equiv(\omega_S/\omega-1)$. The power absorbed is plotted in arbitrary units. The microwave field is incident antisymmetrically $(b=1)$. The impedance $Z_0=\sqrt{i}$. The ratio of sample thickness to spin diffusion distance in the absence of a field is $a_0=0.5$ and $\omega_c\tau=10$. The curves are for various angles of field tilt relative to the surface.

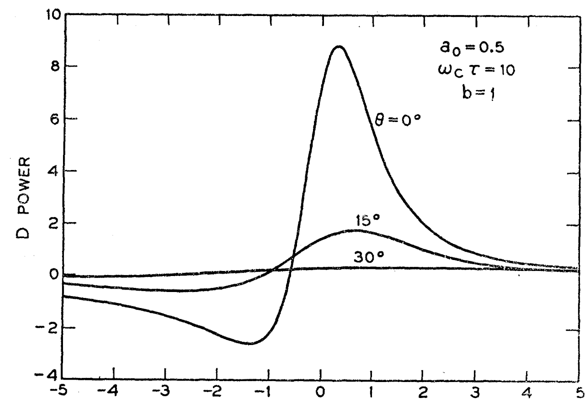

FIG. 2. Plot of the derivative of relative power absorbed to the spin system in the vicinity of CESR. All parameters are the same as in Fig. 1 except for Z_0^2, which now is assumed to have equal real and imaginary parts.

a_0 in the figures. We choose the surface impedance so that Z_0^2 is pure imaginary and assume an $\omega_c\tau$ of 10. In order to compute a series of figures corresponding to field rotation we have assumed that Z_0 is independent of field orientation. *A priori* this need not be true. Experimentally, however, the changes in Z_0 in the neighborhood of CESR in isotropic materials like potassium are thought to be considerably less than 1%.[12] As the field is rotated from the plane of the sample out to 90° the line decreases in intensity and broadens drastically. The effective sample thickness relative to the diffusion distance changes, as we rotate the field. For angles larger than about 30° the line intensity is almost zero. This is the point at which the effective diffusion distance becomes equal to the sample thickness. For fields in the plane the electrons are held in the skin and the intensity is large. For tilted fields they escape into the bulk and then over to the other side of the sample and the line intensity decreases rapidly. In effect power is going into the spin system at one side of the sample and because of the opposite phase of the field this power is reradiated back into the field at the other side.

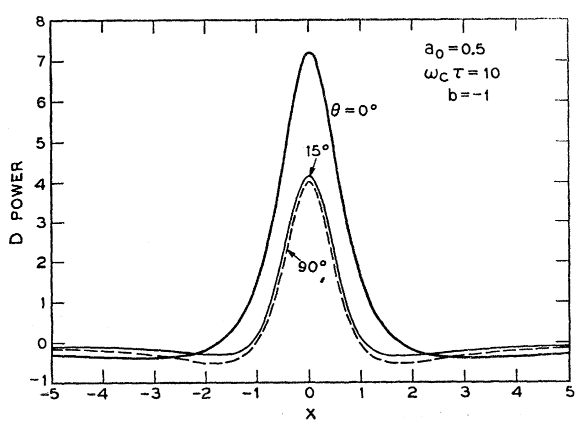

FIG. 3. Plot of the derivative of relative power absorbed to the spin system in the vicinity of CESR. All parameters are the same as in Fig. 1 except now the microwave field is incident symmetrically.

[12] W. Walsh and S. Schultz (private communication).

In Fig. 2 all parameters are kept the same except for the surface impedance squared, taken to have equal real and imaginary parts. The line shape is changed (we now have a dispersion mixed in with the absorption) but the decrease in intensity and broadening of the line occurs as in the previous sequence of figures.

In Figs. 3 and 4 we show a similar set of figures with symmetric boundary conditions on the EM fields. The CESR line in this case decreases in intensity slightly. In addition, there is a slight narrowing of the line. The symmetric fields are equivalent to a one-side excitation. When $\mathcal{H}_0$ is in the plane of the sample, the electrons are "tied" to the surface and the line intensity is relatively strong. As the field is tilted they can escape from the surface and the line intensity decreases. However, once the spin mean free path gets larger than the sample thickness (angles greater than 30°) the spins see the field on the other side and there is no longer a decrease in intensity.

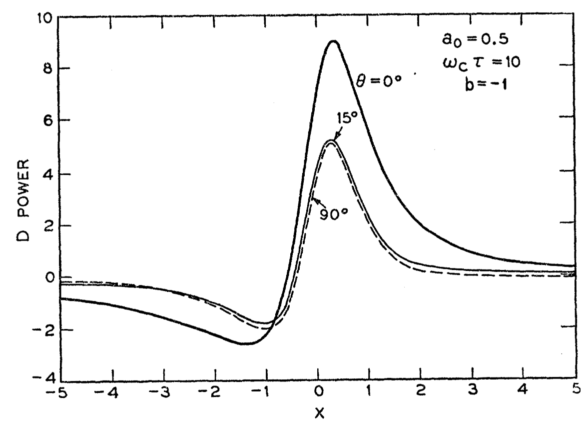

FIG. 4. Plot of the derivative of relative power absorbed to the spin system in the vicinity of CESR. All parameters are the same as in Fig. 3 except for Z_0^2, which now is assumed to have equal real and imaginary parts.

III. CONCLUSIONS

We have extended the calculations of Dyson to include realistic boundary conditions on the EM field. These boundary conditions are critical in determining the CESR line shapes for samples of intermediate thickness. With the purity of materials now becoming available, experiments involving parameters of the order of those considered here are being done by Walsh[7] and by Schultz and Latham.[8] The structure reported by Walsh in the K CESR line does not show up in this calculation. The inclusion of relaxation effects at the boundary, i.e., the introduction of another time into the problem along with the intermediate thickness may in fact be necessary to explain the shape of this type of line.

ACKNOWLEDGMENTS

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APPENDIX

Consider the derivation of Eq. (26). The magnetization produced by the field in the skin at $z=0$ will carry along an rf magnetic field with it (very small electric field). The continuity of the tangential components of **E** and **H** at the second boundary will violate the outgoing wave boundary condition [i.e., $E(L_+)=H(L_+)$] for the transmitted wave. The way to fix up the boundary condition is to add to the slowly varying field, associated with the magnetization, a rapidly varying skin-effect-like solution. This extra field will enable us to satisfy the outgoing wave boundary condition.[11,13]

Formally we can split our solutions up into a rapidly varying part (skin effect) plus a slowly varying part (magnetization wave). [In this analysis all polarization factors which multiply things by constants of order unity are left out.] The rapidly varying part satisfies

$$\nabla\times\mathbf{H}^{(0)}=\frac{4\pi}{c}\boldsymbol{\sigma}\cdot\mathbf{E}^{(0)}, \tag{A1}$$

$$\nabla\times\mathbf{E}^{(0)}=\frac{i\omega}{c}\mathbf{H}^{(0)}, \tag{A2}$$

where $\boldsymbol{\sigma}$ is the nonlocal conductivity operator. The slowly varying part satisfies the inhomogeneous equation

$$\nabla\times\mathbf{H}^{(1)}=\frac{4\pi}{c}\boldsymbol{\sigma}\cdot\mathbf{E}^{(1)}, \tag{A3}$$

$$\nabla\times\mathbf{E}^{(1)}=\frac{i\omega}{c}[\mathbf{H}^{(1)}-4\pi\mathbf{M}(r)], \tag{A4}$$

where $\mathbf{M}(r)$ is a prescribed function given by Eq. (22) in the text. In Eq. (A3) σ may be thought of as the local conductivity operator since the fields in (A3) and (A4) are slowly varying.

The continuity conditions on **E** and **H** at the boundary $z=L$ are

$$\mathbf{E}^{(1)}(L_-)+\mathbf{E}^{(0)}(L_-)=\mathbf{E}(L_+), \tag{A5}$$

$$\mathbf{H}^{(1)}(L_-)+\mathbf{H}^{(0)}(L_-)=\mathbf{H}(L_+). \tag{A6}$$

The outgoing-wave boundary conditions imply $E(L_+)=H(L_+)$ so that (within unimportant polarization factors)

$$E^{(1)}(L_-)+E^{(0)}(L_-)=H^{(1)}(L_-)+H^{(0)}(L_-). \tag{A7}$$

For the slowly varying wave $H^{(1)}$ it can be shown using Eq. (A4) that

$$H^{(1)}(L_-)=-4\pi M(L_-). \tag{A8}$$

Using (A3) it is a simple matter to show that

$$\frac{E^{(1)}(L_-)}{H^{(1)}(L_-)}\cong Z_l^2\left(\frac{k_m}{k_0}\right), \tag{A9}$$

where

$$Z_l^2=-\omega(\omega+i/\tau)/\omega_p^2, \tag{A10}$$

and k_m is a typical wave vector characterizing the spatial variation of the magnetization wave [see Eqs. (14) and (10) in the text]. The quantity k_0 is the free-space wavelength, ω/c. The ratio k_m/k_0 is of order one and the quantity $Z_l^2\ll 1$. The actual magnitude of k_m/k_0 is, as we will show, unimportant.

For the skin-effect field

$$\frac{E^{(0)}(L_-)}{H^{(0)}(L_-)}\equiv Z_0\left(\frac{4\pi}{c}\right)\equiv Z_0', \tag{A11}$$

where, as in the text, Z_0 is unknown but $Z_0'\ll 1$. Substituting (A8), (A9), and (A10) into (A7) we find that (leaving out the arguments L_-)

$$H^{(0)}=-H^{(1)}\left[\frac{Z_l^2k_0/k_m-1}{Z_0'-1}\right], \tag{A12}$$

so that

$$H^{(0)}+H^{(1)}=H^{(1)}\left[1-\frac{Z_l^2(k_0/k_m)-1}{Z_0'-1}\right], \tag{A13}$$

$$\cong -H^{(1)}Z_0'=4\pi M(L_-)Z_0'=H(L_+).$$

This completes the derivation of Eq. (26) in the anomalous as well as the classical regime.

[13] N. S. Vander Ven and R. T. Schumacher, Phys. Rev. Letters **12**, 695 (1964).