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Experimental Test of the Josephson Frequency-Voltage Relation*†

D. N. Langenberg, W. H. Parker, and B. N. Taylor‡
Department of Physics and Laboratory for Research on the Structure of Matter,
University of Pennsylvania, Philadelphia, Pennsylvania

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The Josephson frequency-voltage relation in superconduction's $\nu = 2eV/h$ has been tested by measuring frequency-voltage ratios in Josephson junctions using three different Josephson-effect phenomena: (1) microwave emission from evaporated-film tunnel junctions, (2) microwave-induced quantum voltages in evaporated-film tunnel junctions, and (3) microwave-induced current steps in the I-V characteristics of evaporated-film tunnel junctions and of "weak-link" point-contact junctions. The use of the "harmonic effects" associated with each phenomenon to increase the accuracy of measurement, and the factors limiting the accuracy in each case, are discussed. The measured frequency-voltage ratios for the three methods were the same to within their respective experimental errors. The highest accuracy was obtained using microwave-induced current steps. The measured frequency-voltage ratio in this case was equal to the currently accepted value of $2e/h$ to within an experimental accuracy of 0.006% (60 ppm). The frequency-voltage ratio was also found to be independent of the type of junction used, temperature, magnetic field, harmonic number, voltage polarity, microwave power, and frequency to within the 10-ppm precision of the measurements.

THERE now exists a considerable body of experimental and theoretical information about phenomena which are direct consequences of long-range order and macroscopic quantization in super fluids. For superconductors, these phenomena include the quantization of magnetic flux and the dc and ac Josephson effects. (Analogous effects are also observed in superfluid helium.) The existence of these phenomena can be shown theoretically to follow quite simply and generally from assumptions of long-range order and macroscopic quantization, independent of any microscopic model of the superfluid.[1,2] In the case of the ac Josephson effect, the theory predicts the existence of an ac supercurrent between two weakly coupled superconductors maintained at a potential difference V, with a frequency ν given by the Josephson frequency-voltage relation $\nu = e^*V/h$, where h is Planck's constant and e^* is the charge of the "particle" which carries the supercurrent.[3] A connection with the microscopic theory is normally made by taking e^* to be $2e$, the charge of the Cooper pair. This choice has some experimental support: the quantum of magnetic flux has been shown to be equal to the theoretically predicted value $hc/2e$ to within 3%[4] and studies of the radiation emitted by the ac supercurrents in tunnel junctions have shown that the frequency-voltage ratio is $2e/h$ to an accuracy of about 1%.[5] However, experiments of the latter type have a potential accuracy considerably greater than 1%. Because of the fundamental part played by the Josephson frequency-voltage relation in the present general picture of superconductivity, we felt it was important to make full use of this potential to test the relation as rigorously as possible. In this paper, we report the results of such a test. We find that within our present experimental accuracy of 0.006% (60 ppm), the Josephson frequency-voltage ratio is indeed equal to $2e/h$, and that, within the 10 ppm precision of the measurements, the ratio is independent of all experimental parameters checked.

Three similar but operationally different experimental methods can be used to measure the Josephson frequency-voltage ratio; they are discussed in some detail below. In each case it is necessary to make an absolute measurement of a dc potential difference between two weakly coupled superconductors and a microwave frequency. In the present experiments the microwave frequency was in X band (8.0–12.4 GHz) and was easily measured to an accuracy of better than 10 ppm using a Hewlett-Packard frequency counter and microwave-transfer oscillator. The over-all accuracy of the results was limited solely by the accuracy of the voltage measurements. The factors determining this accuracy were the calibration of the potentiometer and standard cell used, the stability of the voltage to be measured, and the presence of spurious voltages in the measuring circuit (e.g., thermoelectric voltages). Such spurious voltages are usually a serious source of difficulty in accurate measurements of small dc voltages, but pose no serious problem in our experiments because of a unique property of Josephson junctions, the dc Josephson effect. This effect provides a zero reference voltage at the junction when the junction is operating in the

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† For a preliminary account of this work, see B. N. Taylor, D. N. Langenberg, and W. H. Parker, Bull. Am. Phys. Soc. **11**, 191 (1966).

‡ Present address: RCA Laboratories, Princeton, New Jersey.

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superconducting dc Josephson state. (We have experimentally shown that the dc voltage across junctions biased in the dc Josephson mode is ≤ 5 nV, the limit of resolution of our present equipment; this was done by observing that no detectable change in galvanometer deflection occurred when the junction current was reversed. In addition, we have shown that the voltage across junctions biased in the dc Josephson mode is constant to within 5 nV for all values of current from zero up to the maximum dc Josephson current.) Any nonzero voltage then appearing in the measuring circuit must be spurious and can readily be corrected for, if it remains sufficiently constant during a measurement. In our experiments, the spurious voltages were typically 40 to 80 nV and remained constant to within 5 nV for the duration of several measurements. The voltage measurements were made using a calibrated Rubicon Model 2773 potentiometer with a Guildline photocell amplifier and galvanometer. The uncertainty due to the potentiometer system was of the order of ± 5 nV (determined by our ability to interpolate between the 10-nV steps of the last dial), or 60 ppm (determined by the accuracy of calibration of the instrument), whichever was larger. The precision of the voltage measurement was limited to about 10 ppm by the 5-nV resolution and linearity of the potentiometer, the stability of the standard cell, and the incidental frequency modulation on the microwave source, used to induce the effects discussed below. According to the Josephson frequency-voltage relation, the voltage corresponding to a frequency in X band is of order 20 μV ($2e/h = 484$ MHz/μV), so that a measurement of a voltage directly related to an X-band frequency could be expected to have an accuracy of only 250 ppm. It was necessary, therefore, to use the "harmonic effects" associated with each of the three methods in order to make use of the full accuracy of the potentiometer.

Block diagrams illustrating the principal features of the three methods are shown in Fig. 1. The methods are:

A. Radiation Emission

Josephson tunnel junctions which displayed natural-mode structure in their I-V characteristics (i.e., current steps at essentially constant voltages) were biased at a high-order mode or step and the frequency of the radiation emitted in X band was measured.[5] Since this frequency was the nth subharmonic of the fundamental frequency corresponding to the bias voltage (cf. Ref. 5), the quantity derived from the frequency and voltage measurements was actually $n\nu/V$. (The modified Josephson frequency-voltage relation corresponding to this situation is $\nu = 2eV/nh$, where V is the bias voltage at the nth mode and ν is the frequency of the detected radiation.) Typical values for n in these experiments ranged between 1 and 9. Radiation was observed with the junction biased at modes with $n > 9$, but the frequency of this radiation had a voltage dependence which differed markedly from that of the Josephson radiation. (The properties of this "non-Josephson" radiation have been studied and will be reported elsewhere.[6]) This effect limited the maximum voltage to be measured to approximately 200 μV. This method has therefore not yielded as accurate a test of the Josephson frequency-voltage relation as the third method described below.

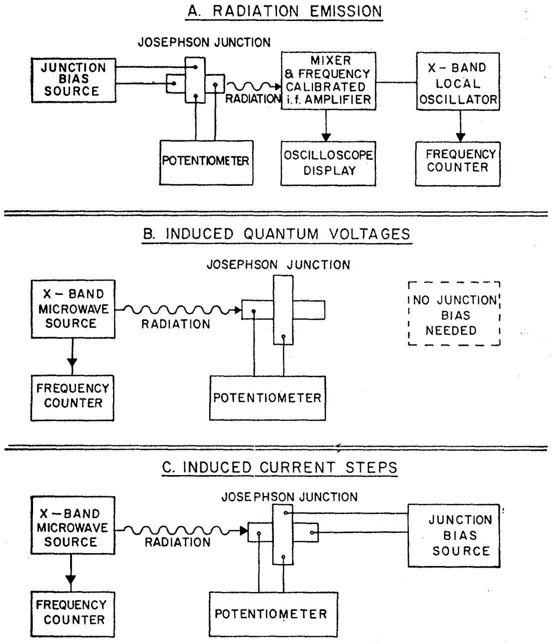

FIG. 1. Schematic block diagrams showing the three different methods used to test the Josephson frequency-voltage relation.

B. Induced Quantum Voltages

Josephson tunnel junctions were irradiated with X-band microwaves and the dc quantum voltages induced across the junctions were measured.[7] The equation relating the frequency of the applied radiation to the induced quantum voltages is again $\nu = 2eV/nh$. In this case, n corresponds to the quantum number of the induced voltage V, and ν is the frequency of the applied radiation. In these experiments, values for n ranged between 1 and 11. The voltages corresponding to the larger values of n were rather unstable, i.e., the junctions tended to switch spontaneously from one quantum voltage to another. For this reason, the larger voltages were quite difficult to measure and this caused a corresponding increase in uncertainty.

C. Induced Current Steps

Josephson junctions were irradiated with X-band microwaves and steps in the current at discrete voltages

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[6] D. N. Langenberg, W. H. Parker, and B. N. Taylor, Phys. Letters **22**, 259 (1966).

[7] For a discussion of these quantum voltages, see D. N. Langenberg, D. J. Scalapino, B. N. Taylor, and R. E. Eck, Phys. Letters **20**, 563 (1966).

were induced in the junction I-V curve as originally observed by Shapiro.[8] These induced steps were more stable than the induced quantum voltages, so that more accurate voltage measurements could be made. The steps were observed to be vertical (i.e., the voltage at each step was constant over the full range of current on the step) to within the 5-nV resolution of our equipment. The equation $\nu=2eV/nh$ again gives the relation between the frequency of the applied microwaves and the voltage at the nth current step. In our experiments, values of n from 1 to 88 were used (note that for the larger n values, V was of order 2 *mV*). With this method, we were able to check whether the results depended on details of the experimental situation. All of the available experimental parameters were varied and several types of junctions were used. Data was obtained for a large number of different n values, for both voltage polarities, for values of magnetic field between 0 and 10 G, for $0.3<T/T_c<0.9$ (on an evaporated-film tin-tin-oxide–tin junction), for a 10-dB variation of incident microwave power (approximately 0.1 to 1 mW), and for a frequency range of about 9 to 12 GHz. Data were obtained from a number of evaporated thin-film tin-tin-oxide–tin junctions, from an evaporated tin–tin-oxide–lead junction which contained a superconducting bridge, and from niobium-tantalum "weak-link" point-contact junctions.[9] Since stable current steps could be induced at very high values of n, this method yielded results with an accuracy limited only by the uncertainties in the calibration of the potentiometer system (60 ppm). No dependence of the results on any of the experimental parameters or junction types was found to the 10-ppm precision of the measurements.

Results obtained using all three methods are summarized in Table I and compared with the current "best" value of $2e/h$.[10] We conclude that, to within the uncertainties stated in the table, the Josephson frequency-voltage ratio is equal to $2e/h$ in several different types of junctions and under a wide variety of experimental conditions. It should be reiterated that the stated uncertainty of 60 ppm for the third method (induced current steps) is attributable entirely to calibration uncertainty in our present voltage-measuring system and not to any fundamental experimental difficulties. It appears that this method (and also the others, with improvements in experimental technique) can be used to achieve a direct determination of e/h with an accuracy significantly better than has been achieved using other methods. We are currently engaged in making this determination.

TABLE I. Comparison of the Josephson frequency-voltage ratios obtained in the present work with 2 times the value of e/h derived by Cohen and DuMond in their 1963 least-squares adjustment of the fundamental constants (Ref. 10). The experimental values have been corrected for the difference between the absolute volt and the legal volt as maintained by the National Bureau of Standards (1 NBS volt $=1.000012\pm0.000004$ absolute volt). All of the uncertainties indicated represent approximately a 70% confidence level.

Method	Josephson frequency-voltage ratio, in MHz/μV
A. Radiation emission	483.62 ± 0.05
B. Induced quantum voltages	483.59 ± 0.15
C. Induced current steps	483.59 ± 0.03
$2(e/h)=483.610\pm0.005$ MHz/μV	

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[8] S. Shapiro, Phys. Rev. Letters **11**, 80 (1963); S. Shapiro, A. R. Janus, and S. Holly, Rev. Mod. Phys. **36**, 223 (1964).

[9] It has been shown by several workers that weak links can exhibit properties very similar to those of Josephson tunnel junctions. See, for example, P. W. Anderson and A. H. Dayem, Phys. Rev. Letters **13**, 195 (1964); J. E. Zimmerman and A. H. Silver, Phys. Rev. **141**, 367 (1966).

[10] E. R. Cohen and J. W. M. DuMond, Rev. Mod. Phys. **37**, 537 (1965).