# Possible One-Channel Castillejo-Dalitz-Dyson Poles in $\pi N$ and $\pi\pi$ Scattering*

D. ATKINSON AND M. B. HALPERN
*Department of Physics, University of California, Berkeley, California*
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We explore interchannel coupling as a qualitative guide in dynamical calculations, and as an ingredient in models of $\pi N$ and $\pi\pi$ scattering. Our first model is the $SU(6)$-symmetric static baryon bootstrap of Capps, Belinfante, and Cutkosky. We find, on the assumption of a successful bootstrap, that the $P_{11}$ and $P_{33}$ $\pi N$ waves each contain a one-channel Castillejo-Dalitz-Dyson (CDD) pole. If these CDD poles survive $SU(6)$ symmetry breaking, we can understand and correlate a large number of heretofore puzzling features of these waves, for example, the well-established zero in the $P_{11}$ amplitude, and the observed $P_{33}$ high-energy phase shift. Secondly, we assume that the observed nonet of $2^+$ particles is the Regge recurrence of a nonet of $0^+$ extinct bound states, and that these latter can be calculated in a dynamical $0^-0^-$ (3-channel) calculation. We find, in exact $SU(3)$ symmetry, that certain waves contain one-channel CDD poles; for example, the $T=J=0$ $\pi\pi$ wave contains one. On the assumption of $SU(6)$, however, *every* extinct bound state appears to bring down a CDD pole, in which case there would be *no* observable consequence of the $0^+$ nonet. In these models, we try to avoid quantitative group theory, emphasizing general principles that we hope may be valid in a larger context.

## 1. INTRODUCTION

NEAREST singularity arguments, and the analysis of the relative strength of cross-channel forces (by means of crossing matrices), are relatively well understood as qualitative guides in $S$-matrix theory.[1] In this paper, we wish to explore another simple consideration—that of interchannel coupling[2–5]—as a guide in calculations, and as a way of understanding certain features of $\pi N$ and $\pi\pi$ scattering. The technique of "crankshaft analysis"[5] will be employed in this study.

In Sec. 2, we shall show that in the $SU(6)$ static baryon bootstrap model of Capps, Belinfante, and Cutkosky,[6] both the $P_{11}$ (nucleon) and $P_{33}$ ($N^*$ or **33** resonance) waves of $\pi N$ scattering contain one-channel Castillejo-Dalitz-Dyson (CDD) poles. Moreover, we shall argue that $SU(6)$ is only being used in a semi-quantitative way to obtain rough interchannel couplings. If these CDD poles are really present,[7] one obtains a model that "explains" many heretofore puzzling features of $\pi N$ scattering. In particular:

(1) It is tempting to identify the well-known zero[8] of the $P_{11}$ amplitude, at about 175 MeV above threshold, with the CDD pole in this wave. This would then explain the failure of dynamical calculations to obtain the zero.[9,10]

(2) The real parts of the $P_{33}$ and $P_{11}$ phase shifts are predicted to be asymptotic to $\pi$ and 0, respectively, at very high energies. In fact, the latest observations show the $P_{33}$ phase beginning to level off at about 160° near 1150 MeV.[11] The nucleon wave ($P_{11}$) is generally believed[11] to be leveling off near 140° around 1050 MeV—although there has been some confusion as to whether it actually turns over before reaching 90°.[12] Our model predicts that the phase shift will eventually turn over, and tend to zero. Note that if there is no CDD pole, the phase must go down even further (to $-\pi$).

(3) The known phase shifts $S_{11}$ and $D_{33}$ (coupled to $P_{11}$ and $P_{33}$, respectively, by the MacDowell symmetry) can be fitted in a dynamical calculation if the $P_{11}$ and $P_{33}$ poles are included[13]—and evidently cannot be understood without these poles.

(4) It is known that, if one subtracts off the $P_{11}$ inelasticity due to that part of the $\pi\pi N$ channel in which the two pions are in a relative $S$ state (the $\sigma N$ channel), then the remaining inelasticity (due to $\pi N^*$, for example) is small. Our model not only yields a small inelasticity due to $\pi N^*$, etc., but associates this directly with the very existence of the CDD pole. Although the Capps model does not include the $\sigma N$ channel, it can be incorporated in a satisfying way: It is known[10] that the inelasticity in $P_{11}$ (mainly $\sigma N$) is incapable of producing the zero at 175 MeV. In our model, the function of the $\sigma N$ channel is to create the Roper resonance,[14] *after* the CDD pole has caused the phase shift to change

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[1] See, for example, G. F. Chew, *S-matrix Theory of Strong Interactions* (W. A. Benjamin Company, Inc., New York, 1961).

[2] E. J. Squires, Nuovo Cimento 34, 1751 (1964).

[3] H. Munczek, Phys. Letters 13, 92 (1964).

[4] M. Bander, P. W. Coulter, and G. L. Shaw, Phys. Rev. Letters **14**, 270 (1965).

[5] D. Atkinson, K. Dietz, and D. Morgan, Ann. Phys. (N. Y.) **37**, 77 (1966).

[6] R. H. Capps, Phys. Rev. Letters **14**, 31 (1965); J. G. Belinfante and R. E. Cutkosky, *ibid.* **14**, 33 (1965); R. H. Capps, Phys. Rev. **139**, B421 (1965); J. G. Koerner and R. H. Capps, *ibid.* **139**, B1388 (1965).

[7] That is, if the CDD poles of the Capps model remain on the first sheet as $SU(6)$ is smoothly broken.

[8] See, for example, L. D. Roper, R. M. Wright, and B. T. Feld, Phys. Rev. **138**, B190 (1965).

[9] G. D. Doolen, T. Kanki, and A. Tubis, Phys. Rev. **142**, 1072, (1966); **142**, 1082 (1966). These papers contain an adequate referencing.

[10] P. W. Coulter and G. L. Shaw, Phys. Rev. **141**, 1419 (1966).

[11] B. H. Brandsen, P. J. O'Donnell, and R. G. Moorehouse, Phys. Letters **19**, 420 (1965).

[12] P. R. Auvil *et al.*, Phys. Letters **12**, 76 (1964); B. H. Brandsen *et al.*, Phys. Rev. **139**, B1566 (1965).

[13] See Ref. 10. Their inclusion of an elementary nucleon pole is equivalent to a CDD pole. This will be discussed further in Sec. 2.

[14] L. D. Roper, Phys. Rev. Letters **12**, 340 (1964).

sign. We shall emphasize that a CDD pole naturally represents another channel which is important in the binding, but which produces a *small* inelasticity (like $\pi N^*$), and not one that gives a large inelasticity (like $\sigma N$). That is, $\pi N^*$ and not $\sigma N$ seems the most desirable second channel to include in multichannel attempts to avoid the CDD pole.

(5) Within the framework of the model, it seems possible to understand how it is that the existing reciprocal-bootstrap calculations[9] have succeeded, insofar that they have done so, even though they ignore CDD poles. We shall discuss various calculations, but their common feature seems to be a judicious use of cutoffs and parameters to facilitate approximate agreement with experiment (generally only in a region relatively remote from the CDD poles). In particular, if the $P_{33}$ CDD pole lies somewhere below the nucleon force pole, one understands qualitatively why the calculated $P_{33}$ phase shift has always agreed with experiment to higher energies than has been the case for $P_{11}$.

On the basis of nearest singularity arguments alone, a one-channel CDD pole would not be expected in the lowest channel ($\pi N$). A CDD pole in the $\pi N$ channel indicates that a relatively large part of the nucleon binding is done in the sum of all other participating channels. In the Capps model, this is certainly the case. [Moreover, this does not seem to depend quantitatively on $SU(6)$, but is a feature of any large group. For example, $SU(6)_W$ has the same features.] Of course, this effect is artificially enhanced in the Capps model by the degeneracy of the thresholds (and consequent irrelevance of nearest singularity arguments). As the threshold degeneracy is lifted, one certainly expects the $\pi N$ channel to do more of the binding. We shall not claim to prove that the CDD poles of the Capps model remain on the physical sheet as the symmetry is progressively broken (although we know of no mechanism whereby they would necessarily be forced from it); but rather, we shall enlarge upon those features (just enumerated) of a model in which the CDD poles do remain.

In Sec. 3, we discuss $\pi\pi$ scattering as one channel in a coupled ($\pi\pi, K\bar{K}, \eta\eta$) system, under the assumption that an $SU(3)$ nonet of $0^+$ extinct bound states (EBS)[15–17] arise in the calculation. The well-established $2^+$ nonet[15,18] would be the first Regge recurrence of these "particles." Analysis of the coupled system reveals one-channel CDD poles in various waves. In particular, the $T=J=0$ $\pi\pi$ wave, with two EBS (which we call $P$ and $P'$) has one CDD pole. It should be emphasized that these EBS, in that they are "poles" with zero residues, have very few properties in any ordinary sense. However, they do influence high-energy phase shifts,[16,17] so our crankshaft analysis is peculiarly suited to the predynamical discussion of waves which contain EBS. For example, this unitary symmetry model predicts that the real part of the $T=J=0$ $\pi\pi$ phase shift will tend to $-\pi$ at high energies.

On the other hand, if we assume $SU(6)$ symmetry, and put the $0^+$ EBS's into the $35_D$-plet that arises in the reduction

$$35\otimes 35 = 1\oplus 35_D\oplus 35_F\oplus 189\oplus 280\oplus \overline{280}\oplus 405$$

then we find that every EBS comes down with a one-channel CDD pole (thus there are two CDD poles in $\pi\pi$ scattering, for example). Again, the result depends only on the largeness of $SU(6)$, rather than on quantitative details. It has been shown[17] that, when a CDD pole and an EBS occur in the same wave, they completely obliterate one another. If the $SU(6)$ limit has any application to this system, then there would be no observable consequences of any of the EBS, and all the phase shifts would tend to zero asymptotically. In particular, this could remove one of Chew's reasons for feeling that the $T=J=0$ $\pi\pi$ phase shift starts off negatively near threshold.

## 2. CDD POLES IN $\pi N$ SCATTERING

As mentioned in the Introduction, there are several peculiar features of the $P_{11}$ and $P_{33}$ waves of $\pi N$ scattering. Perhaps the most peculiar is the zero of the $P_{11}$ amplitude, at about 175 MeV above threshold. To our knowledge, this zero has never come out of a dynamical calculation. In fact, the only known dynamically based fitting of the $P_{11}$ wave (up to about 500 MeV)[10] involves the introduction of the nucleon as an elementary particle pole (in the $N$ function), which is entirely equivalent to the use of a CDD pole.[19] Several years ago, Rothleitner and Stech[20] proposed an empirical test for a CDD pole in the nucleon wave. This was subsequently discussed by Squires.[2]

Our point in this section is that in the $SU(6)$ static baryon bootstrap model of Capps, Belinfante, and Cutkosky,[6] both the $P_{11}$ and $P_{33}$ waves do in fact contain one CDD pole each. This is because, in the model, the nucleon is relatively little bound in the $\pi N$ system: most of the binding arises from the sum of all the other channels. Certainly, $SU(6)$ breakage will enhance the $\pi N$ at the expense of the other channels. However, because of the resultant agreement with experiment (and for the other reasons mentioned in the Introduction), we feel that the interchannel coupling may in fact dominate the nearest singularity effect to the extent that the CDD poles remain on the first sheet. At any rate, we shall never pretend to *prove* the existence of CDD poles in actual partial-wave amplitudes. We

[15] G. A. Ringland and E. J. Squires, Phys. Letters **16**, 86 (1965).
[16] G. F. Chew, Phys. Rev. Letters **16**, 60 (1966).
[17] D. Atkinson and M. B. Halpern, Phys. Rev. (to be published).
[18] S. L. Glashow and R. H. Socolow, Phys. Rev. Letters **15**, 329 (1965).
[19] D. Atkinson and D. Morgan, Nuovo Cimento **41**, 559 (1966). In fact, as it must, Coulter and Shaw's real part of the $P_{11}$ phase asymptotes to zero.
[20] J. Rothleitner and B. Stech, Z. Physik **180**, 375 (1964).

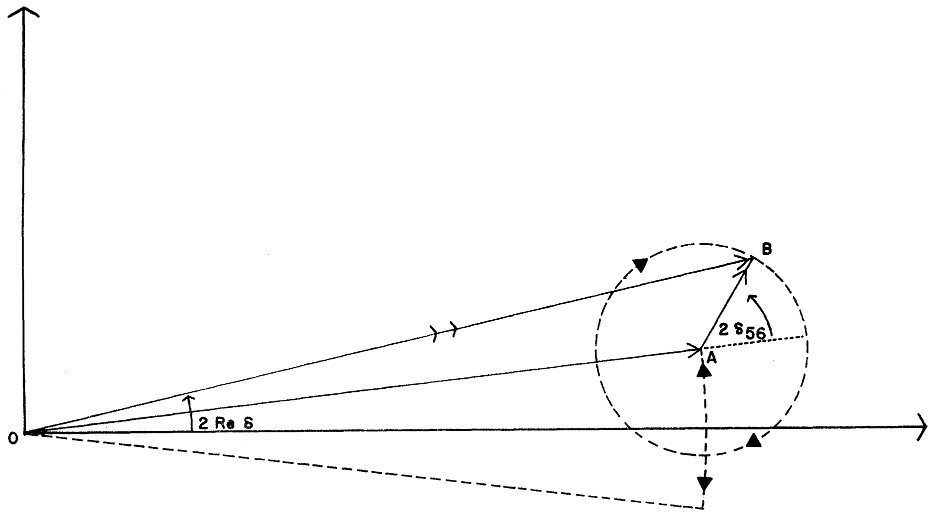


FIG. 1. Vector diagram useful in crankshaft analysis; OA is the resultant of the inactive vectors, AB is the **56** vector, and the modulus of OB is $\eta$, the inelasticity.

simply discuss models which *contain* CDD poles arising naturally from interchannel coupling, and which seem to describe the actual phase shifts. Speculations about which other channels the CDD poles mainly represent—i.e., which other channels are important in the binding—will be made below.

The $SU(6)$ static baryon bootstrap assigns the baryons (octet+decuplet) to a 56-plet; and it is hoped that the 56-plet arises self-consistently in ($35\otimes56$) scattering with the mesons. We assume that this bootstrap is successful, that is, the baryons are in fact generated. This means that we will not limit the left-hand-cut contributions to those arising from one-baryon exchange. We assume that realistic forces have been included, sufficient to generate the 56-plet in the direct channel. Note that, in the $SU(6)$ limit, the (average mass) 56 is a *bound state*, being below the (average mass) $56\otimes35$ threshold.

Our technique for deducing the existence of CDD poles in the Capps model is crankshaft analysis, for which we need only expand the physical $S$ matrices in eigen $S$ matrices of $SU(6)$. For example, the expansion of the $P_{11}$ wave of the $\pi N$ scattering is

$$\langle\pi N|S|\pi N\rangle_{P_{11}}=(13/90)\langle56|S|56\rangle+(17/192)\langle70|S|70\rangle +(103/288)\langle700|S|700\rangle +(1179/2880)\langle1134|S|1134\rangle. \quad (2.1)$$

If we neglect any channel not included in $35\otimes56$, we find that the $S$ matrices on the right are separately unitary, and we obtain the $P_{11}$ wave in terms of the eigenphases (suppressing the index $P_{11}$):

$$\eta e^{2i\,\mathrm{Re}\,\delta}=\frac{13}{90}e^{2i\delta_{56}}+\frac{17}{192}e^{2i\delta_{70}}+\frac{103}{288}e^{2i\delta_{700}}+\frac{1179}{2880}e^{2i\delta_{1134}}, \quad (2.2)$$

where $\eta$ is the usual inelasticity (the modulus of the $P_{11}$ $S$-matrix element). We suppose that the eigenphases satisfy Levinson's theorem individually. The assumption that the nucleon arises dynamically in the coupled-channel calculation, while there are no other bound states in the wave, implies

$$\delta_{56}(\infty)=-\pi,\quad \delta_{70}(\infty)=\delta_{700}(\infty)=\delta_{1134}(\infty)=0. \quad (2.3)$$

We can now deduce the high-energy behavior of $\mathrm{Re}\delta$ itself.

Equation (2.2) is represented in Fig. 1 as a sum of vectors with appropriate phases. For simplicity, the figure only shows the **56** vector and the resultant of the other vectors. The **70**, **700**, and **1134** vectors (and so their resultant), gyrate a little, but do not go all the way around, while the **56** vector describes a complete circle (clockwise), as the energy changes from threshold to infinity. In that *the "active"* **56** *vector is much smaller than the resultant of the "inactive" vectors* (it is about 15% of the whole), its rotation will not succeed in swinging the over-all resultant vector around. Thus

$$\mathrm{Re}\delta(\infty)=0.$$

Then an application of Levinson's theorem to the inelastic $\pi N$ system shows that, because the nucleon bound state is in the channel, there must also be a CDD pole. One might think, in that no one of the "inactive" vectors has magnitude greater than $\frac{1}{2}$, it would be possible that, for example, the large **700** vector could swing around to $-\pi$, cancelling most of the **1134** vector, and, at that moment, the **56** vector could sneak around the origin. In fact, however, the inactive vectors will never swing around far enough for this to happen, for that would correspond to a large inelasticity in the $\pi N$ $P_{11}$ wave, due to say the $\pi N^*$ channel. This inelasticity is known to be small. The only significant inelasticity which has been seen experimentally is in the $\sigma N$ channel, associated with the "Roper resonance." The $\sigma N$ channel is not included in our model: we shall return to its effects below.

It is worth emphasizing that the rather small weighting of the **56** wave—which causes the CDD pole—is an example of a general principle: most physical scattering states in $35\otimes 56$ are heavily weighted toward the larger irreducible representations.[21] (In the case of the $N^*$, as we shall see, one of the inactive vectors will actually have a weight larger than $\frac{1}{2}$.) A little thought will convince the reader that what has happened is roughly as follows: At the $SU(3)$ level, the crossing of two octets gives

$$8\otimes 8=1\oplus 8_F\oplus 8_D\oplus 10\oplus \overline{10}\oplus 27\,.$$

In $SU(6)$, the calculation of the relevant Clebsch-Gordan coefficient involves, for example, picking an 8 and asking to which $SU(6)$ representation (**56, 70, 700, 1134**) it should be assigned. One finds one $8^2$ in **56**, one in **70** and in **700**, and two in **1134**. In the case of 27, one finds none in **56** nor in **70**, one in **700**, and two in **1134**. Roughly speaking then, the larger representations gain weight for "statistical" or combinatorial reasons, in that a given $SU(3)$ multiplet tends to occur more often in a larger than in a smaller $SU(6)$ multiplet. Another way of saying this is that 56 is small, as compared with the full product $56\times 35$, i.e.,

$$\frac{56}{35\times 56}=\frac{1}{35}$$

is very small. In particular, it is much smaller than the corresponding quotient

$$\frac{8}{8\times 8}=\frac{1}{8}$$

in $SU(3)$. Presumably this sort of combinatorial weight increase would be found in the application of any large group, and is more or less independent of $SU(6)$ itself.[21] We shall have more to say about this general principle later.

The features of our model for the nucleon wave then are these: We find a CDD pole, which we are strongly tempted to identify with the puzzling zero of the amplitude at 175 MeV above threshold; and, correspondingly, we find that the real part of the $P_{11}$ phase shift will be asymptotic to 0, not $-\pi$. Moreover, *the inactivity of the* **70, 700,** *and* **1134** *vectors corresponds to a small inelasticity due to those competing channels* ($\pi N^*$, *etc.*) *that are included in the model.*[22] If the $\sigma N$ channel is subtracted away from the experimental findings, this is roughly what one sees. In fact, one feels that it is the $\pi N^*$ channel which is at least the next strongest competitor to $\pi N$ for nucleon binding. Certainly this is true in the Capps model. For example, expanding the baryon octet ($B_8$) backwards, in terms of the baryon octet-pseudoscalar octet ($B_8P_8$), and baryon decuplet-pseudoscalar octet ($B_{10}P_8$), etc., Capps has given

$$|B_8\rangle=\left(\frac{18}{45}\right)^{1/2}|B_8P_8\rangle+\left(\frac{4}{9}\right)^{1/2}|B_{10}P_8\rangle+\left(\frac{2}{15}\right)^{1/2}|B_8V_8\rangle+\left(\frac{1}{45}\right)^{1/2}|B_8V_1\rangle. \quad (2.4)$$

It is clear from this that the nucleon is spending, in fact, slightly more time as $\pi N^*$ than as $\pi N$. An expansion of the $\pi N^*$ $P_{11}$ wave, analogous to Eq. (2.1), is in complete agreement with this: one finds the coefficient of the **56** wave in this expansion to be 16/45 [as compared with 13/90 in Eq. (2.1)].

Of course, the CDD pole in the Capps model may move smoothly off onto the second sheet as $SU(6)$ is broken [although the known large breaking of $SU(6)$ is no guarantee that this will happen]. However, we should consider (1) the strength of these arguments (the smallness of the **56** weight), (2) their hopeful generality [in that $SU(6)$ is not needed quantitatively], (3) the fact that the degenerate threshold assumption, although bad from the point of view of low energies, should be reasonable for the prediction of high-energy phase shifts (because in this regime the effects of the mass differences would be minimized), and, most important, (4) the interesting correspondence of the model with observation. On the basis of all these considerations, we conjecture that the predictions of crankshaft analysis survive the symmetry breaking, and that interchannel coupling is the mechanism that produces the well-known zero in $P_{11}$ (and controls the asymptotics).

A little more discussion of the model is essential. As noted above, the interchannel coupling arguments run counter to the simplest nearest singularity arguments, which would suggest that the $\pi N$ channel, being the closest to the nucleon bound-state pole, should dominate its binding. Thus one would feel that the $\pi N$ channel ought to be free of CDD poles. According to our degenerate-threshold crankshaft analysis, however, the nucleon binding is shared among all the channels, and a CDD pole is induced in any single channel. One way of understanding this intuitively is to note that, from the standpoint of *any* single channel, it is the sum of the rest of the channels that does the bulk of the binding, and yet the other channels refuse to give the single channel much information about the work they are doing (i.e., the inelasticity is small). This information loss must be made good, so to speak, by the knowledge of the CDD parameters in the one-channel calculation. We see the $P_{11}$ wave then as rather like the "weak-coupling model" of Ref. 5. Of course, symmetry breaking will certainly reduce the amount of binding done by some of the higher lying channels

[21] J. C. Carter, J. J. Coyne, and S. Meshkov, Phys. Rev. Letters **14**, 523 (1965). That the same phenomenon is found in $SU(6)_W$ is evident from J. C. Carter, J. J. Coyne, and S. Meshkov, Phys. Rev. Letters **15**, 373 (1965). The $N^*$ half of an $SU(6)_W$ reciprocal bootstrap has been discussed by R. Gatto and G. Veneziano, Phys. Letters **19**, 512 (1965).

[22] This is easily seen by computing the inelasticity $\eta$. This then is the simple physical interpretation of the inactivity of **70, 700,** and **1134.**

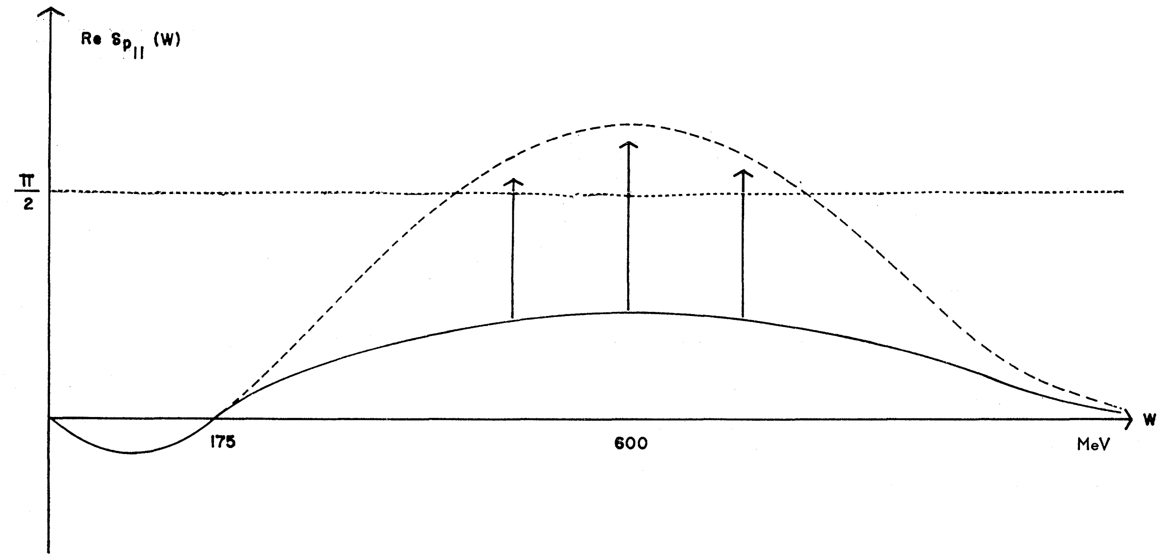


FIG. 2. Conjectured influence of the $\sigma N$ channel on $P_{11}$. The solid line is the phase without $\sigma N$, and the elevated dashed line includes the effect of the $\sigma N$ channel.

(their efficacy will be reduced by their distance from the nucleon pole). We are only proposing that enough of the crankshaft argument survives to leave the CDD pole on the physical sheet. We shall discuss how much of the binding may be done in the other channels (of which the $\pi N^*$ is the most important) in Secs. II D and II E.

There is a good deal of inelasticity in the $P_{11}$ wave; and, as mentioned above, it is primarily associated with the state $\sigma N$ (the two pions in a relative $S$ state). In our model, the function of this channel is simply to produce the Roper resonance, by pushing up the already positive phase shift (see Fig. 2). In fact, the recent calculation by Coulter and Shaw,[10] including the phenomenological inelasticity, but excluding the CDD pole —see their Fig. 5(a)—shows that the inelasticity does just this: that is, it is not adequate to produce the resonance by itself. It is precisely because of its large inelasticity that we think the $\sigma N$ channel has little to do with the nucleon or the CDD pole. This channel *does* tell the $\pi N$ channel about its activity, via the inelasticity factor. By contrast, in the models adduced in the past,[4,5] the one-channel CDD poles are generated when the "binding" channels couple weakly (in some sense) to the channel of interest (and produce, therefore, a small inelasticity).

### A. 33 Wave

The expansion of the $\pi N$ $P_{33}$ $S$-matrix element is

$$\eta e^{2i\,\mathrm{Re}\delta_{33}} = \frac{4}{45}e^{2i\delta_{56}} + \frac{11}{18}e^{2i\delta_{700}} + \frac{3}{10}e^{\delta_{1134}}. \qquad (2.5)$$

This time, as was said, one $SU(6)$ vector (the **700**) has magnitude greater than $\frac{1}{2}$. The assumption of a dynamical $N^*$ in the **56** wave, and at most resonances in the **700** and **1134** waves, gives

$$\delta_{56}(\infty) = -\pi, \quad \delta_{700}(\infty) = \delta_{1134}(\infty) = 0.$$

(Recall that $N^*$ is a bound state, not a resonance, in this model.) Since the **700** vector is longer than the other two vectors combined, there is no way for the **56** to communicate its full rotation to the $P_{33}$ wave itself. Hence the real part of the $P_{33}$ phase must go to zero at infinity, and since there is one bound state ($N^*$), it follows that there must be a CDD pole as well. This time, the mass splitting in the physical case is important in this respect: As the splitting is increased, the $N^*$ pole moves toward the $\pi N$ threshold, and is then forced off the physical sheet by elastic unitarity. There is no such mechanism to force the CDD pole from the physical sheet. Of course, it is always possible that it does migrate through the cut; but there are experimental reasons for believing that it stays on the physical sheet, for, in that case, we obtain

$$\mathrm{Re}\delta_{33}(\infty) = \pi,$$

since there is one CDD pole and no bound state. This, as mentioned in the Introduction, is quite close to existing experiment. Presumably, in that the amplitude exhibits no zero in the physical region, the CDD pole is elsewhere. We shall speculate briefly on its possible location below.

It should also be emphasized that, although Coulter and Shaw have calculated the $P_{33}$ phase up to about 175 MeV in agreement with experiment, it is clear—see their Fig. 3a—that their phase is already beginning to turn around at 200 MeV, preparatory to returning to zero (as any CDD-pole free phase must do); at higher energies there is no agreement with the available experimental data, which show the phase still continuing to climb towards 180° at 1050 MeV.[11]

### B. MacDowell Symmetry

It should also be mentioned that the static model neglects the MacDowell symmetry.[23] Thus, in the fully relativistic case, the $P_{11}$ wave is linked to the $S_{11}$ wave, and the correct Levinson theorem is really

$$\delta_{P_{11}}(\infty)+\delta_{S_{11}}(\infty)=-\pi(n_B-n_C), \qquad (2.6)$$

where $n_B$ and $n_C$ are the total numbers of bound states and CDD poles in both channels. The static model in effect gives $\delta_{S_{11}}\equiv 0$; but, in fact the $S_{11}$ wave is quite active, two resonances having been observed.[11,24] Moreover, it seems that the real part of the phase shift attains 100° at about 1050 MeV, although it is beginning to level off. We have seen that one-channel CDD poles arise only from bound states, or from resonances that are bound states before symmetry breaking (and when the bulk of the binding comes from other channels, of course). Thus we feel that the rather high-lying $S_{11}$ resonances (425 and 612 MeV above the $\pi N$ threshold, as compared with 158 MeV for the 33) will not be able to induce any CDD poles. On this basis, one feels that the $S_{11}$ phase shift will turn over, and

$$\delta_{S_{11}}(\infty)=0.$$

Thus Eq. (2.6) reduces in this case to its static counterpart, and the prediction for $\delta_{P_{11}}$ is not altered by the relativistic considerations. In this connection it is very striking to note that in the calculation of Coulter and Shaw[10] involving a CDD pole (pseudo-elementary nucleon)—see their Fig. 6—both the $P_{11}$ and the $S_{11}$ phase shifts agree well with experiment. Without the CDD pole, both fail badly, except at the lowest energies.

In the case of the $P_{33}$ wave, the MacDowell symmetry involves also the $D_{33}$ wave. A CDD pole in the $P_{33}$ amplitude (i.e., a pseudo-elementary $N^*$) would probably also push up the $D_{33}$ phase, as happened for $S_{11}$, (in that, in both cases, the pseudo-elementary insertion lowers the effective pole residues) and this would improve agreement with experiment. It would certainly be interesting to see the detailed results of such a calculation.

### C. Larger Groups

One feels, from the discussion above of combinatorial weight gaining, that, if larger and larger groups were used to include more and more channels, one would eventually find CDD poles in any finite subset of channels. One might regard such CDD poles as being, in some sense, elementary. However, we prefer to take the argument as one against the validity, or utility, of larger and larger groups, rather than one for "real" CDD poles. The point is that there will eventually be competition between the interchannel coupling arguments and those deriving from symmetry breaking or nearest singularity considerations. It seems likely that if the group is very big, this splitting countertendency must dominate, so that the crankshaft analysis will be vitiated. One might very well have expected, *a priori*, that the dividing line (above which nearest singularity arguments tend to dominate) would fall somewhere between $SU(3)$ and $SU(6)$, and probably rather closer to the former. However, we feel that the agreement with experiment resulting from the assumption that the $SU_6$ CDD poles remain on the first sheet indicates that $SU_6$ may at least be accurate for the gross features explored by the crankshaft analysis. In particular, $SU_6$ may be giving very crudely the right interchannel couplings between $\pi N$ and $\pi N^*$, etc.; certainly this would be our interpretation if the CDD poles are in fact found to exist.

### D. Discussion of Existing Dynamical Calculations

If the CDD poles discussed above are in fact present, one expects some difficulty in performing dynamical one-channel calculations, because the $N/D$ equations contain arbitrary parameters. However, as has been pointed out, our CDD poles are essentially one-channel phenomena, and they can be circumvented in principle by considering a larger number of channels. (For a "real" CDD pole, such a simple step would not suffice.)

However, many one-channel calculations of $N$ and $N^*$ have been performed[9,25] (the reciprocal bootstrap) with no CDD poles, and some of the results are in rough correspondence with experiment. A very interesting problem, then, is (assuming our CDD poles exist) just how much agreement we might expect from a CDD pole-free approach.

We would like to discuss several indications that a CDD pole is in fact representing (in a one-channel calculation) a good deal of nucleon binding activity in other channels. One such indication is the aforementioned calculation of Coulter and Shaw.[10] They found, with representative forces (and a cutoff adjusted to give the correct nucleon mass), a $\pi N$ coupling constant twice too large. The introduction of the phenomenological $\pi N$ inelasticity did not materially improve this: The main effect was an upward boost of the (negative) $P_{11}$ phase shift, although not enough to produce the zero. To obtain the zero, and to fit the phase shift out to about 500 MeV, they were forced to include the nucleon pole itself in the $N$ function, with the correct mass and coupling. This elementary pole in the $N$ function is entirely equivalent to a CDD pole in the $D$ function. As it must then, the real part of their phase asymptotes to zero.

[23] S. W. MacDowell, Phys. Rev. **116**, 774 (1959).
[24] P. Bareyre *et al.*, Phys. Letters **18**, 342 (1965).
[25] G. F. Chew, Phys. Rev. Letters **9**, 233 (1962).

A somewhat more quantitative estimate of the weight of the CDD pole can be obtained in Chew's[25] original static model. Chew took the $N$ function to be

$$N_{11} \approx \gamma_{33}/(\omega+\omega_{33})$$

and assumed a once-subtracted $D$ function. He determined the resultant linear divergence of the $D$ function integral by requiring a zero at the correct nucleon mass. This resulted in good agreement for the coupling.

In order to obtain the zero of $\delta_{P_{11}}$, at 175 MeV, Schwarz[26] has fitted the data, using Chew's $D$ function, plus a CDD pole:

$$D(\omega)=1-\frac{\omega+\omega_{33}}{\pi}\int^{\Lambda}\frac{d\omega'}{\omega'+\omega_{33}}\frac{N(\omega')\rho(\omega')}{\omega'-\omega}+\frac{0.35(\omega+\omega_{33})}{\omega+\omega_{175}}. \quad (2.7)$$

To obtain the correct nucleon mass, one must set the cutoff $\Lambda$ at 10 pion masses. It is of interest to ask, keeping $\Lambda$ fixed at this value, whether the nucleon remains if the CDD pole is omitted. The answer, in this model, is that it does appear, but as a low-lying resonance. According to Schwarz, this model is optimal in that other models (with slightly different CDD parametrizations) result in a nucleon much farther removed in the second sheet. Another way of looking at Eq. (2.7) is to notice that, at the nucleon pole ($\omega=0$), the CDD pole cancels about 35% of the unit term. However, the 35% is not directly connected to the diagonalization coefficients, which would be obtained by actually performing the real multichannel calculation, and then making the (energy-dependent) diagonalization. That is, writing the calculated $S$ matrix as[5,27]

$$S=US_DU^T, \quad (2.8)$$

where $S_D$ is the diagonalized matrix, one has, for example,

$$S_{\pi N}=u_{11}^2(\omega)S_1+u_{12}^2(\omega)S_2+\cdots+u_{1n}^2(\omega)S_n. \quad (2.9)$$

[The diagonalized $S$-matrix elements are, of course, no longer $SU(6)$ waves.] One could then tell, by an energy-dependent crankshaft analysis, whether or not a CDD pole would be induced in the $\pi N$ channel (with diagonalization cuts all drawn below the lowest threshold branch point). Evidently, the multichannel calculation that would yield Eq. (2.7) would give diagonalization coefficients $u_{ij}(\omega)$ appropriate to the generation of one CDD pole. (Unfortunately, without the multichannel calculation, one has no way of obtaining the relevant $u_{ij}$.) Although these coefficients are connected qualitatively to the "35%" [c.f. the discussion after Eq. (2.7)], the relationship is not immediate, and the percentage does not tell us much more about the $u_{ij}(\omega)$ than that they will yield a CDD pole. (Any nonzero percentage tells us the same.) In particular, we emphasize that the 35% should *not* be rigidly compared with the 85% non-$\pi N$ binding in the Capps model. These are essentially unrelated percentages. The 35% does indicate of course that the CDD pole might be expected to be rather important—which is indeed borne out by the detailed considerations, as we have seen.

If one *had* obtained the correct $u_{ij}(\omega)$, the energy interplay of vectors leading to the CDD pole would presumably be less extreme, by virtue of nearest singularity arguments, than in the (degenerate threshold) Capps model. That is, one certainly expects the $\pi N$ channel to do an appreciable part of the binding in the real situation. We say merely "appreciable" because the percentage of binding is difficult to discuss in the real situation—in that the relative importance in the crankshaft analysis of the various channels is energy dependent.

One can see from Eq. (2.7) more or less how Chew's original cutoff prescription helps to circumvent the CDD problem. By specifying the $D$ function at $-\omega_{33}$ (the location of the $N^*$ force pole), and requiring a zero at the nucleon mass, he has relieved the dynamics of a good deal of its responsibility, and is essentially asking it only to describe the curvature of $D$ between these two points. Thus one tends to obtain good accuracy near the force pole, near the nucleon pole, and, in that the CDD pole is 315 MeV to the right of the nucleon pole, even into the low-energy scattering region. However, agreement with experiment becomes steadily worse as one approaches the sphere of influence of the CDD pole.

One obtains a similar qualitative interpretation of the $P_{33}$ calculation, on the reasonable assumption that the CDD pole lies below the nucleon-exchange short cut in the energy plane. That is, by determining $D$ at the $N$ force pole, and requiring the correct $N^*$ mass, one would expect a reasonable width for the $N^*$, and a reasonable low-energy phase shift (since the low-energy region is a long way from the assumed CDD pole location). Another reason for feeling that the CDD pole may in fact be below the nucleon short cut (that is, relatively remote from the physical region) is that low-energy $P_{33}$ calculations have always agreed with experiment to higher energies than have those for $P_{11}$. It is hard to believe that the CDD pole would lie high in the physical region, because it would have to be above 1050 MeV, and in general one feels that these one-channel CDD poles stay fairly close to their associated particles. Of course at very high energies, the CDD pole will dominate the phase shift, regardless of its position.

We feel that, in general, cutoff parameters[28] in any bootstrap may easily help circumvent the CDD problem, just as we imagined its happening in the original

[26] J. Schwarz, Berkeley Report, 1966 (unpublished).
[27] R. C. Hwa and D. Feldman, Ann. Phys. (N. Y.) **21**, 453 (1963).

[28] See, for example, E. Abers and C. Zemach, Phys. Rev. **131**, 2305 (1963); J. S. Ball and D. Wong, *ibid.* **133**, B179 (1964); K. Y. Lin and R. E. Cutkosky, *ibid.* **140**, B205 (1965).

Chew model. To our knowledge, there are only a few calculations for which a complete bootstrap (masses and couplings) are claimed. In general, these employ the Balász[29] method; but we feel that the well-known sensitivity of these calculations to the matching point indicates that this point itself is being used effectively as a parameter. That is, we feel the parameter is being used to simulate part of the CDD pole information in the vital regions, just as in the case of cutoffs earlier. Not surprisingly, these calculations never yield the zero (nor the observed asymptotics). Another recent calculation of note is that of Doolen, Kanki, and Tubis,[9] who avoid the Balász matching-point difficulties by feeding in experimental data in order to determine the Balász pole coefficients. Demanding agreement with experiment at certain points is, in essence, exactly what was done previously with cutoffs and parameters, and will clearly help to minimize the effect of CDD poles. In general, any time one knows the value of the amplitude at or near a set of points (in one way or another), and, although his calculation will not yield these values, he demands them anyway, this procedure will tend to circumvent the influence of the CDD poles, at least in the vicinity of these "known" input points.

As a final comment on the reciprocal bootstrap, it is worth repeating that Coulter and Shaw were unable to bootstrap the nucleon, or to reproduce any but the lowest energy $P_{11}$ phase shift, even when they included the phenomenological inelasticity. In the end, to fit the phase shift out to 500 MeV, they were forced to insert in fact a CDD pole.

Finally, it should be mentioned that the recent calculation by Dashen of the neutron-proton mass difference[30] involved the assumption of no CDD poles in an essential way. The same applies to the Dashen-Frautschi octet enhancement calculations.[31] That the inclusion of phenomenological $D$ functions (with or without CDD poles) can easily upset Dashen's conclusions, both in magnitude and sign, has been noted by Shaw and Wong.[32] When these authors constructed a $D$ function with CDD poles, they chose to associate them with the Roper resonance.[33] Their choice was guided by an early model,[4] but, as mentioned above, the CDD ambiguity arises most naturally even in this model for small interchannel coupling (as is in general true for these models). In fact, however, this interchannel coupling is large, as we have stressed above. For this reason and, of course, because of the Capps model, we have instead associated the CDD pole with the nucleon.

[29] See, for example, V. Singh and B. M. Udgaonkar, Phys. Rev. **130**, 1177 (1963); P. Narayanaswamy and L. K. Pande, *ibid.* **136**, B1760 (1964).

[30] R. Dashen, Phys. Rev. **135**, B1196 (1964).

[31] R. Dashen and S. Frautschi, Phys. Rev. **137**, B1318 (1965); **137**, B1331 (1965); **140**, B698 (1965).

[32] G. L. Shaw and D. Y. Wong, Phys. Rev. **147**, 1028 (1966).

[33] Shaw and Wong supposed that the one-channel $N$ and $D$ functions (in the $\eta$ method) had two "CDD branch points" (associated with the Roper resonance). It is easy to show, however, that there is a different $N/D$ decomposition in which both inverse square-root branch points have been multiplied out, and replaced by a single pole in the $D$ function (our form). See also J. B. Hartle and C. E. Jones, Phys. Rev. **140**, B90 (1965).

### E. Prognosis

It is difficult to say from our model just which of the channels included in our analysis are best added to the $\pi N$ channel, in an attempt to avoid the CDD ambiguity. As mentioned above, the $\pi N^*$ channel seems to be doing a large part of the binding in the Capps model. For this reason, and from nearest-singularity considerations, we feel that this is the most logical next candidate for inclusion in a many-channel calculation, although we are by no means certain that it will suffice. It is worth cautioning the dynamicist, however, that our analysis applies to a calculation done with the correct forces (recall that it cost us nothing to postulate their inclusion), and that ostensibly reasonable cutoff procedures and parametrizations can lead to a rough "circumvention" of the CDD ambiguity in $n$ channels, just as in the one-channel case. One channel that we feel it will *not* be necessary to include explicitly is $\sigma N$. As discussed above, the inclusion of the inelasticity factor induced by this channel will probably suffice. According to Fulco, Shaw, and Wong,[34] it will not do much good to include the $K\Sigma$ channel in the $P_{33}$ calculation [even though, at the $SU(3)$ level, the $N^*$ is as much in a $K\Sigma$ as in a $\pi N$ state].

To our knowledge, there exists only one attempt thus far to calculate $\pi N$ and $\pi N^*$ together.[35] It is unclear whether the approximations in this work are trustworthy; among other things, one worries about approximating the unusual physical-region short cut[36] (caused by the large mass of $N^*$) by poles on the left.[37] These authors have not yet calculated their high-energy phase shifts. In this connection, it should be emphasized that, although the approximations in a calculation of this sort will be essentially low-energy ones, the high-energy limit of the *real* part of the phase shift is dictated only by the bound state and CDD-pole structure of the amplitude, and thus is worth extracting (despite the low-energy approximations).

In conclusion then, we feel it may indeed be very fruitful to re-examine in detail many of these troublesome one-channel $\pi$-$N$ bootstrap calculations (reciprocal bootstrap, neutron-proton mass difference, $F/D$ ratio, etc.) this time in the framework of two or more channels, presumably beginning by adding $\pi N^*$.

## 3. EXTINCT BOUND STATES IN $0^-0^-$ SCATTERING

Chew[16] has conjectured that the Pomeranchuk ($P$), and second-rank Pomeranchuk ($P'$) Regge trajectories

[34] J. R. Fulco, G. L. Shaw, and D. Y. Wong, Phys. Rev. **137**, B1242 (1965).

[35] R. Brunet and R. W. Childers, Argonne Report, 1966 (unpublished).

[36] R. F. Peierls, Phys. Rev. Letters **6**, 641 (1961).

[37] B. Kayser, University of California, Berkeley Report, 1965 (unpublished).

cross zero angular momentum at negative total energy squared. Two $J^P=0^+$ "particles" would be generated, with imaginary masses. To avoid a conflict with unitarity in the crossed channel, one hopes that the residues at the particle "poles" will vanish. We call these entities "extinct bound states" (EBS).[17]

In the spirit of Ringland and Squires,[15] we suppose the existence of an $SU(3)$ nonet of $0^+$ EBS. The first Regge recurrence of this nonet would be the well-known $2^+$ nonet.[15,18] We suppose that the recurrences of $P$ and $P'$ are $f_0$ and $f_0'$, respectively, both isotopic singlets. We call the isovector EBS $\pi'$, corresponding to the $A_2$, and the isospinor $K'$, corresponding to the $2^+K^*$ resonance. We suppose that the $P$ is predominantly an $SU(3)$ singlet and $P'$ the isoscalar member of an octet, but mixing is to be expected, as in the case of $f_0$ and $f_0'$, and the subsequent arguments will allow for this. We shall assume that the nonet can be generated dynamically in a three-channel, $SU(3)$ symmetric, $ND^{-1}$ calculation of $0^-0^-$ scattering ($\pi\pi, K\bar{K}, \eta\eta$). Thus we exclude the Gell-Mann mechanism[38] for the extinction of the $0^+$ states. (This mechanism would be operative, for example, if the $0^+$ states were produced mainly in the nucleon-antinucleon channel.)

It is important to note that, as extinct bound states, this nonet would not be observable in any ordinary sense.[39] In fact, because of their zero "coupling," they have very few properties at all. As emphasized earlier,[16,17] their most outstanding property is that, as actual zeros of the $D$ function, they do influence asymptotic phase shifts. Our crankshaft-analysis technique then is ideally suited to discuss such properties.

In a recent paper[17] the authors have discussed the possibility of a dynamical calculation of EBS's in the $ND^{-1}$ framework. It was shown there that the masses of such states cannot be determined in a one-channel calculation and that the one-channel problem which is equivalent to a many-channel dynamical system often contains CDD poles. It will be our purpose in this section to guess, with crankshaft analysis, in which channels the extinct bound states will bring down CDD poles. The analysis mirrors that given in Sec. 2, except that here $S$-matrix elements are expanded in terms of $SU(3)$ eigenphase shifts.

We first discuss $\pi\pi$ elastic scattering with $J=T=0$, under the assumption of $SU(3)$ symmetry. In this case, the $S$-matrix expansion in eigenphases is

$$\eta e^{2i\,\mathrm{Re}\delta}=\frac{3}{8}e^{2i\delta_1}+\frac{3}{5}e^{2i\delta_{8S}}+\frac{1}{40}e^{2i\delta_{27}}. \qquad (3.1)$$

Here $\delta_1$ refers to the unitary singlet, $\delta_{8S}$ to the symmetric octet, and $\delta_{27}$ to the 27-plet that are obtained when the direct product $8\otimes 8$ is reduced. Since there is a bound state in both the singlet and the octet for $T=J=0$ (each one a mixture of the "physical" $P$ and $P'$), it follows that

$$\delta_1(\infty)=-\pi=\delta_{8S}(\infty), \qquad (3.2)$$

where the eigenphase shifts are defined to be zero at threshold and it is supposed that there are no CDD poles.

In Eq. (3.1), the coefficient of the $8_S$ eigen-$S$-matrix element is greater than $\frac{1}{2}$, so that a simple crankshaft analysis assures us that

$$\mathrm{Re}\delta_{\pi\pi}(\infty)=\delta_{8S}(\infty)=-\pi. \qquad (3.3)$$

However, the physical $S$-matrix element has two EBS ($P$ and $P'$), so that there must be one CDD pole in the $\pi\pi$ $S$-wave. Evidently, from the weights of the **1** and $8_S$ waves, it is the $P$ that brings down the CDD pole, while the $P'$ is "dynamical" in this channel.

A similar expansion for $K\bar{K}\to K\bar{K}$, with $T=J=0$ (taking account of Bose statistics in the unitary limit) is

$$\eta e^{2i\,\mathrm{Re}\delta}=\tfrac{1}{2}e^{2i\delta_1}+\tfrac{1}{5}e^{2i\delta_{8S}}+\tfrac{3}{10}e^{2i\delta_{27}}. \qquad (3.4)$$

In this case, it is probable that the term $e^{2i\delta_1}$ will force

$$\mathrm{Re}\delta_{K\bar{K}}(\infty)=-\pi. \qquad (3.5)$$

A very special cancellation would be required to make this phase shift tend to zero at infinite energy; and it is strictly impossible for it to go to $-2\pi$. Thus we expect one single-channel CDD pole in the $K\bar{K}\to K\bar{K}$ $T=J=0$ channel (since this also contains the $P$ and $P'$ EBS). This time the $P'$ is bringing down the CDD pole.

An off-diagonal $S$-matrix element which also contains $P$ and $P'$ in its $T=J=0$ wave is $\pi\pi\to K\bar{K}$. The expansion coefficients, the sum of which is not unity, are approximately equal in magnitude for the contributing **1**, $\mathbf{8}_S$, and **27** waves, thus it is difficult to state what will happen. The phase shift could go to 0, $-\pi$, or $-2\pi$, at infinite energy, corresponding to two, one, or no CDD poles; but perhaps one CDD pole would again be the most likely eventuality.

A rough way of seeing when CDD poles are induced, and with which particles they are associated, is afforded by the inverse expansions:

$$|P_1\rangle=(\tfrac{3}{8})^{1/2}|\pi\pi\rangle+(\tfrac{1}{2})^{1/2}|K\bar{K}\rangle-(\tfrac{1}{8})^{1/2}|\eta\eta\rangle, \qquad (3.6a)$$

$$|P_8\rangle=-(\tfrac{3}{5})^{1/2}|\pi\pi\rangle+(\tfrac{1}{5})^{1/2}|K\bar{K}\rangle-(\tfrac{1}{5})^{1/2}|\eta\eta\rangle. \qquad (3.6b)$$

Here $|P_1\rangle$ is the unitary scalar, $|P_8\rangle$ the isoscalar member of the octet. We can say that the $P_8$ is made mainly from $\pi\pi$, the $P_1$ mainly from $K\bar{K}$. In the sense that the $\pi\pi$ CDD pole is thus associated with the $P$ (mostly singlet) we might expect, very roughly speaking, a one-channel $\pi\pi$-$\pi\pi$ calculation, with no CDD pole, to yield the $P'$, but not the $P$. (A CDD pole would be needed to generate both $P'$ and $P$.) The same statement can be made for $K\bar{K}\to K\bar{K}$, but exchanging the roles of $P$ and $P'$ (in this channel the $P$ is dynamical).

[38] M. Gell-Mann, in *Proceedings of the International Conference on High-Energy Physics at CERN, 1962* (CERN, Geneva, 1962), p. 533.

[39] It is tempting to speculate that the $0^+$ EBS's might be the tadpole octet of S. Coleman and S. L. Glashow [Phys. Rev. **134**, B671 (1964)].

These conclusions agree with those made in the foregoing paragraphs. From Eq. (3.6) one would conclude that the $\eta\eta$ channel is so weakly coupled to both EBS that two CDD poles would be required in a calculation of $\eta\eta \to \eta\eta$. This conclusion is validated by a crankshaft analysis, but we omit the details.

Even though the $P'$ appears "dynamical" in the $\pi\pi$ system, it is not possible to determine its "mass" in a one-channel calculation.[17] Evidently, in order to determine both the $P$ and $P'$ masses, and to avoid all CDD ambiguities, a two-channel $(\pi\pi,K\bar{K})$ calculation would be necessary. The $(\eta\eta)$ channel could probably be omitted.

As in the meson-baryon application discussed in Sec. 2, one has to consider the possibility that, as the symmetry-breaking interaction is "turned on," the CDD poles that have been inferred in the symmetric situation may disappear from the physical sheet. This can be associated with the emergence of "diagonalization branch points" from the right-hand cut of one of the eigenamplitudes.[5,27] The more seriously the symmetry is broken, the more likely is it that this should happen, especially when the interchannel couplings are strong.

Attention should be called to a general principle operative in waves containing both $P$ and $P'$. When two particles differ only in their $SU_3$ quantum numbers (that is, are in the same $SU_2$ wave) it is almost certain that at least one CDD pole will be induced by the pair. For there to be no CDD pole, it would be necessary, for example, for the $\mathbf{8}_S$ vector to be longer than the **1** vector in some energy range, during which the $\mathbf{8}_S$ encircled the origin, and then for the **1** to grow to more than one-half, at the expense of the $\mathbf{8}_S$, in a succeeding energy range, after which the **1** also encircled the origin.

We now turn to the consideration of the isovector $\pi'$ and the isospinor $K'$. The $K'$ appears in $\pi K$ scattering, in the $T=\frac{1}{2}$ $S$ wave. Only the $\mathbf{8}_S$ and **27** states contribute to this wave, with respective weights 9/10 and $\frac{1}{10}$. Thus there is a strong prediction of no CDD pole, and a phase shift tending to $-\pi$. The $\pi'$ EBS contributes to the $T=1$ $S$ wave of $K\bar{K}\to K\bar{K}$. This time the $\mathbf{8}_S$ and **27** weights are $\frac{3}{5}$ and $\frac{2}{5}$. In this case also the EBS is dynamical: i.e., there is no CDD pole, and the phase shift goes to $-\pi$.

It should be emphasized that, whenever an EBS and a CDD pole occur together, they completely obliterate one another.[17] The final trace of the EBS (the depression of the infinite-energy phase shift to $-\pi$) is gone, and it is as if the EBS had never arisen at all.

Finally, it is worth mentioning that if the $0^+$ nonet is assumed to arise as part of a 35-plet in $SU_6$ (in $35\otimes35$ scattering), the larger group combinatorial effect is such as to force *each* EBS to bring down a CDD pole. Of course $SU_6$ arguments are always rather more suspect than $SU_3$. However, if one believes that all of these CDD poles survive the breaking, then there would be *no observable consequence* of any EBS. All amplitudes would be indistinguishable from a set with no EBS's and no CDD poles. (In the absence of bound states, all phases would go to zero at infinity.) In particular, this could remove one of Chew's arguments for expecting a negative $\pi\pi$-$\pi\pi$ $T=J=0$ phase shift.

### $\rho$ Meson

Another interesting application of our methods is to the $\rho$ meson in $T=J=1$ $\pi\pi$ scattering. One knows that the $\pi\omega$ channel is also important to the correct binding of the $\rho$,[34,40] the one-channel $\pi\pi$ calculation being notoriously inadequate. One can hope to understand this qualitatively on the basis of our simple models. To bring the $\omega$ meson into the model, we go to $SU(6)$, and consider the $\rho$ as a member of a dynamical 35-plet in $35\otimes35$ scattering.[41] In this case, the combinatorial effect is not extreme. In the $\pi\pi$ wave, one finds **35**, **280**, and $\overline{\mathbf{280}}$ with equal coefficients $(\frac{1}{3})$. Thus one expects a CDD pole in the $\pi\pi$ $T=J=1$ wave, presumably due to the $\pi\omega$ (etc.?) channels. Thus the known difficulty in obtaining the correct width in one-channel $\rho$-meson calculations could be another confirmation of the general principles we have discussed. There is evidence that a three-channel $(\pi\pi,K\bar{K},\pi\omega)$ calculation is not adequate to produce the correct width,[34] although the $\pi\omega$ channel does assist in reducing the width. Fulco, Shaw, and Wong suggest in fact that no small number of nearby channels will adequately reduce the width; if this is true, our $SU_6$ calculation, though a step in the right direction, may conceivably not include enough channels.

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[40] F. Zachariason and C. Zemach, Phys. Rev. **128**, 849 (1962).
[41] To do this, we attach the orbital angular momentum first to one meson, then to the other, and symmetrize.