Strong and Electromagnetic Vertices*

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It is suggested that the form factors in the helicity decomposition of the effective coupling of a vector meson to baryons are proportional. This result is shown to follow, within the framework of an $SU(6)_W$-invariant theory, from a speculation closely related to this group. By assuming ρ-meson dominance of unsubtracted dispersion relations in the helicity decomposition of the isovector nucleon electromagnetic current, reasonable values of the isovector charge and magnetic moment are obtained simultaneously, thus confirming the above suggestion. A discussion is given of the problems arising in this approach concerning analyticity of the form factors and their behavior both asymptotically and at the threshold points $q^2=0$ and $4m^2$.

I. INTRODUCTION

Whenever a calculation is made in which dominance by exchanged vector mesons is assumed, the question arises as to what form the vector-meson–baryon coupling should take. The effective coupling of any vector meson (with C parity $=-1$) to a baryon can be written in momentum space in the form

$$\left[1-\frac{q^2}{4m^2}\right]^{-1}\bar{U}(p')\left[G_E\frac{P^\mu}{2m}+G_M\frac{r^\mu}{4m^2}\right]U(p), \qquad (1)$$

where $P=p'+p$, $q=p'-p$, $r^\mu=\epsilon^{\mu\nu\rho\lambda}P_\nu q_\rho\gamma_\lambda\gamma_5$, and $G_{E,M}$ are functions of the invariant q^2 (if the meson is off the mass shell), with their ratio providing the basic arbitrariness under consideration. Furthermore, this coupling may be re-expressed in the form

$$\bar{U}(p')\left[F_1\gamma^\mu+F_2 i\frac{\sigma^{\mu\nu}}{2m}q_\nu\right]U(p), \qquad (2)$$

where

$$G_E=F_1+\frac{q^2}{4m^2}F_2, \qquad (3)$$

$$G_M=F_1+F_2, \qquad (4)$$

with inverses

$$F_1=\left(1-\frac{q^2}{4m^2}\right)^{-1}\left(G_E-\frac{q^2}{4m^2}G_M\right), \qquad (5)$$

$$F_2=\left(1-\frac{q^2}{4m^2}\right)^{-1}(G_M-G_E). \qquad (6)$$

There are, of course, an infinite number of similar decompositions of the current, but the above two will be sufficient for the purposes of the present argument.

With these observations it is clear that a group-theoretic argument, or similar guiding principle, of genuine physical content has to be made if some relationship is to be obtained between two of these form factors; for the general prescription of pole models which "evaluates the coupling on the meson mass shell" will produce different results according to which particular choice of the above current forms is taken. [Specifically, this would produce the form equality of G_E and G_M with the current taken as in (1), or of F_1 and F_2 with the choice (2)]. That considerations of analyticity (and the related questions of subtractions and asymptotic form) do not resolve this question is clear, and will be taken up in detail for the specific case of the nucleon electromagnetic form factors in Sec. IV.

For a short time it seemed that perhaps an answer could be given within the framework of a higher symmetry[1] containing spin, $\tilde{U}(12)$, but it now seems clear that the highest symmetry (of those proposed to date) which can be applied to the three-point function[2] is that known as $SU(6)_W$, which allows exactly the original freedom of choice specified above.[3] In retrospect it is perhaps fortunate that the $\tilde{U}(12)$ prescription should have proved untenable, for this implies[1] the form equality of F_1 and F_2 in the expression (2) for the current, and leads in a natural way (through a vector-meson-dominance model) to a similar relationship[1] between the isovector Dirac and Pauli electromagnetic form factors of the nucleon which is known to be incorrect.[4]

The purpose of this paper is to clarify a speculation put forward earlier[3] for the resolution of this problem. This speculation (treated in detail in Sec. II) is closely related to the $SU(6)_W$ group of invariance of the strong-interaction three-point vertex, and leads naturally to the well-known equalities[4] between the nucleon electromagnetic form factors. In Sec. III a more detailed qualitative check is made on the general method by calculating the nucleon isovector charge and magnetic

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[1] R. Delbourgo, A. Salam, and J. Strathdee, Proc. Roy. Soc. (London) **A284**, 146 (1965); B. Sakita and K. C. Wali, Phys. Rev. Letters **14**, 404 (1965).

[2] K. J. Barnes, P. Carruthers, and Frank von Hippel, Phys. Rev. Letters **14**, 82 (1965); H. J. Lipkin and S. Meshkov, *ibid.* **14**, 670 (1965); K. J. Barnes, *ibid.* **14**, 798 (1965).

[3] K. J. Barnes, Phys. Rev. **139**, B947 (1965); **140**, B1355 (1965).

[4] T. Janssens, R. Hofstadter, E. B. Hughes, and M. R. Yearian, Phys. Rev. **142**, B922 (1966).

moment from unsubtracted dispersion relations. Finally, in Sec. IV, a detailed discussion is given of the problems of analyticity and asymptotic behavior which are relevant to these calculations.

II. SPECIFICATION OF THE VERTEX

It has become increasingly clear over the recent past that the symmetry $SU(6)_W$ is (for some mysterious reason) a reasonable one for treating strong-interaction three-point vertices, and that perhaps the easiest way to apply this prescription is to write down formally $U(6,6)$-invariant vertices and insert the two independent momentum spurions in all possible ways.[5] For the purposes of the present work it will be sufficient to consider the quark-meson vertex, and internal variables may be completely ignored. All that it is necessary to know is that the quark is represented by a Dirac spinor $U(p)$ in momentum space, and the mesons by an irreducible mixed spinor

$$\Phi=\Phi_R(\Gamma^R)=\theta 1+\varphi\gamma_5+F_{\mu 5}i\gamma^\mu\gamma_5+\varphi_\mu\gamma^\mu+\tfrac{1}{2}F_{\mu\nu}\sigma^{\mu\nu}, \quad (7)$$

where ϕ and its subsidiary field $F_{\mu 5}$ describe a pseudoscalar, while ϕ_μ and its subsidiary field $F_{\mu\nu}$ describe a vector meson.[1] (θ would represent a scalar meson but will later be eliminated from the analysis.) A general transformation of the three-parameter invariance group may be written in infinitesimal form as

$$U(p)\rightarrow U(p)+i\alpha_A\epsilon^{ABCD}\hat{p}_B\hat{p}_C'\sigma_{DE}U(p), \quad (8)$$

where $\hat{p}$ and $\hat{p}'$ are unit four-vectors along the directions of the incoming and outgoing quarks, exactly as in Sec. I, and the usual Dirac matrix notation has been conveniently extended into five dimensions by adding a fifth space-like direction. Thus

$$\gamma^A\equiv\{\gamma^0,\gamma^1,\gamma^2,\gamma^3,\gamma^5\},\quad \sigma^{AB}=\tfrac{1}{2}i(\gamma^A\gamma^B-\gamma^B\gamma^A),$$

and the multiplication table reads

$$\gamma^A\gamma^C=g^{AC}-i\sigma^{AC}, \quad (9)$$

$$\gamma^A\sigma^{CD}=i(g^{AC}\gamma^D-g^{AD}\gamma^C)-\tfrac{1}{2}\epsilon^{ACDEF}\sigma_{EF}, \quad (10)$$

$$\sigma^{AB}\sigma^{CD}=g^{AC}g^{BD}-g^{AD}g^{BC}-\epsilon^{ABCDE}\gamma_E +i(\sigma^{AD}g^{BC}-\sigma^{AC}g^{BD}+\sigma^{BC}g^{AD}-\sigma^{BD}g^{AC}), \quad (11)$$

where g^{AB} has diagonal elements $(1,-1,-1,-1,-1)$, and ϵ_{ABCDE} is totally antisymmetric with $\epsilon_{01235}=1$. It is perhaps worth pointing out that with this invariance alone, the meson field

$$\Phi=\theta 1+\phi_A\gamma^A+\phi_{AB}\sigma^{AB} \quad (12)$$

is reducible into fields and subsidiary fields as indicated by this notation; since $B\Phi^T B^{-1}$ transforms exactly like Φ, if B is defined within the Dirac algebra so that

$$B\gamma_A B^- =\gamma_A{}^T, \quad (13)$$

$$B^T=-B. \quad (14)$$

The introduction of transformations which have as generators products of the above "spin" generators with SU(3) generators[6] removes this degeneracy (and specifies the ratio of D and F couplings of mesons to baryons), and in the following work we assume that Φ is irreducible, as it would therefore be under this larger $SU(6)_W$ group. Thus the general quark-meson vertex which is "spin"-independent takes the form in momentum space

$$\bar{U}(p')\Lambda_R U(p)\Phi^R(q), \quad (15)$$

where, *a priori*, with all particles off the mass shells,

$$\Lambda_R=app'\Gamma_R pp'+bpp'\Gamma_R+b'\Gamma_R pp'+cp\Gamma_R p+c'p'\Gamma_R p' +dp\Gamma_R p'+ep'\Gamma_R p+h\Gamma_R+fpp'\Gamma_R p +f'p'\Gamma_R pp'+gpp'\Gamma_R p'+g'p\Gamma_R pp'+kp\Gamma_R +k'\Gamma_R p'+lp'\Gamma_R+l'\Gamma_R p, \quad (16)$$

and the a, b, b', $\cdots$, l', are real analytic functions of scalar variables, such that under the substitution $p\rightarrow p'$, a, d, e, k are invariant, while $b\leftrightarrow b'$ etc. Here the only principles applied have been that the resulting interactions should be charge-conjugation–invariant, and that the current is Hermitian. (Notice that Φ has been taken, as usual, to be self-adjoint, and that the incoming and outgoing quarks are assumed to belong to the same representation.) At this stage [and this is the $SU(6)_W$ result] it requires only somewhat lengthy but straightforward algebra to reach the conclusion that the insertion of the momentum spurions p and p' has destroyed the $\tilde{U}(12)$ prediction[1] of the form equality of F_1 and F_2 in the vector current [a result obtained by taking only the term $h\Gamma_R$ in Eq. (16)] and has restored the arbitrariness evident in the general forms (1) and (2). To remove this arbitrariness the following speculations are proposed:

(A) Λ_R is the sum (with, *a priori*, arbitrary functions of scalars as coefficients) of the unit operator and the "spin" generators.

(B) No scalar particles shall be generated.

Notice that these conditions are *not* group-theoretic statements (although the first seems closely related to the group invariance) and that they generalize in an obvious manner to the full $SU(6)_W$ group. It is perhaps pertinent to point out that it is by no means clear that this prescription [in particular condition (A)] can be applied without obtaining internal inconsistency, nor is it clear that (if it may be applied consistently) it will remove the basic arbitrariness under consideration. The hope, of course, is that this latter will indeed be

[5] R. Delbourgo, M. A. Rashid, Abdus Salam, and J. Strathdee, in *Proceedings of the International Conference on High Energy Physics and Elementary Particles* (International Atomic Energy Agency, Vienna, 1965), p. 455.

[6] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); Y. Ne'eman, Nucl. Phys. **26**, 222 (1961).

the case and that moreover the form equality of G_E and and G_M will be achieved in the vector current, so that a simple pole model will lead naturally to the experimentally[4] well verified form equality[7] of the corresponding electric and magnetic nucleon isovector form factors. As the simplest form for Λ_R, namely $h\Gamma_R$, leads to the $\tilde{U}(12)$ result and its undesirable consequences, condition (A) represents the only obvious simple alternative, specifically, that Λ_R lies in the set of symmetry generators.

In practice these conditions are most easily applied in the more concrete form

(A.1) $\Lambda=\gamma_0\Lambda^\dagger\gamma_0$;

(A.2) The commutators $[\Lambda,p]$ and $[\Lambda,p']$ are each zero;

(B.1) $\Lambda\equiv 0$ when $\Gamma_R=1$.

With these conditions the vertex reduces to the simple form

$$\bar{U}(p')(g\{q,[p',p'\Phi]\}+g'\{q,[p,\Phi p]\})U(p)\,, \quad (17)$$

where $\{A,B\}=AB+BA$. Written out in detail with the quarks on the mass shells, and making the Bargmann-Wigner[8] identifications

$$F_{\mu 5}=-iq_\mu\phi/\mu\,, \quad (18)$$

$$F_{\mu\nu}=(i/\mu)(q_\nu\phi_\mu-q_\mu\phi_\nu) \quad (19)$$

(where μ is the meson mass), the effective vertex finally takes the form

$$-4mq^2g\bar{U}(p')\left[\frac{P^\mu}{2m}\phi_\mu+\frac{\epsilon_{\mu\nu\rho\lambda}P^\nu q^\rho\gamma^\lambda\gamma_5}{2m\mu}\phi^\mu-\frac{i\phi\epsilon_{\mu\nu\rho\lambda}P^\rho q^\lambda\sigma^{\mu\nu}}{4m\mu}\right]U(p) \quad (20)$$

$$=-4mq^2g\bar{U}(p')\left[\frac{P^\mu}{2m}\phi_\mu+\frac{2m}{\mu}\frac{r^\mu}{4m^2}\phi_\mu+\frac{2m}{\mu}\left(1-\frac{q^2}{4m^2}\right)\phi\gamma_5\right]U(p)\,. \quad (21)$$

Although the latter form (21) is perhaps the most convenient expression of the results, the earlier form (20) has the advantage of showing directly the appearance of the "spin" generators in the vertex. This may be made even more explicit by rewriting the vertex in the Breit frame where $P^\mu=(2E,0,0,0)$ and $q^\mu=(0,0,0,-2k)$, and the resulting form is

$$-4mq^2g\bar{U}(p')\left[\frac{E}{m}\phi_0+\frac{2Eki}{m\mu}\times(-\phi_2\gamma_0\sigma_1+\phi_1\gamma_0\sigma_2+\phi\sigma_3)\right]U(p)\,. \quad (22)$$

In this frame (where $\phi_3=0$) the $SU(2)_W$ scalarity of the interaction shows clearly that the $SU(2)_W$ scalar is ϕ_0, while ϕ is the third component of the $SU(2)_W$ vector (W-S flip).[9] Moreover, the exact phase of the transverse parts of the vector field when considered as components of an $SU(2)_W$ vector is directly exhibited.

The point of view advocated here is that whatever further approximations are made to the effective vertex (e.g., the pole prescription of evaluation at $q^2=\mu^2$, except in perhaps certain kinematical factors) they should be made to this explicit form (21), and that in particular the direct proportionality of the P^μ and r^μ types of vector-meson coupling should be maintained. The author advocates strongly the philosophy that whatever *relative* momentum dependence is predicted by symmetry or related arguments) between the various types of effective coupling in some basic vertex [such as in (21) above], it should be maintained in any related process in which dominance is assumed by the particles whose couplings are so related (in particular for electromagnetic coupling). The prescription of evaluating the effective vertex at the meson pole and demanding pole dominance in an *arbitrarily* chosen form of the current (rather than the one indicated by the original form of the effective vertex) seems, to the present author, to entirely negate the whole philosophy employed in determining symmetry (or similar type) restrictions on the effective vertex in the first place. Indeed, for the case of the strong interactions, the above "philosophy" is no more than a strong restatement of what is meant by the effective vertex in the case where a vector meson is coupled to the baryons. For the electromagnetic case, this point of veiw is essentially the argument that (apart from the strength of the coupling) the photon interacts with the baryons in the same way as the neutral members of the SU(3) octet of vector mesons. In the simplest pole model, where the interaction of a photon with baryons is described by a direct transition of the photon into a neutral vector meson which then interacts with the baryon, this result depends only on the momentum dependence introduced at the photon-meson transition "vertex" being the same for the two helicity states. (Most models, of course, introduce no momentum dependence at all in such a transition.)

III. ISOVECTOR NUCLEON FORM FACTORS

It is now known to a very good approximation that the proton and neutron electromagnetic form factors obey the relationships[4,7]

$$G_E^N=0\,, \quad (23)$$

$$G_M^P/\mu_P=G_M^N/\mu_N\,, \quad (24)$$

$$G_E^P=G_M^P/\mu_P\,, \quad (25)$$

[7] K. J. Barnes, Phys. Letters, 1, 166 (1962).

[8] V. Bargmann and E. Wigner, Proc. Natl. Acad. Sci. U. S. 34, 211 (1948).

[9] H. J. Lipkin and S. Meshkov, Phys. Rev. 143, 1269 (1966).

where the notation is such that the electromagnetic current is written in the form

$$\frac{e}{(1-q^2/4m^2)}\bar{U}(p')\left(\frac{P^\mu}{2m}G_E+\frac{r^\mu}{4m^2}G_M\right)U(p), \quad (26)$$

and therefore $G_E(0)$ is the charge of the particle in units of e, while $G_M{}^{P,N}(0)$ is the total magnetic moment ($\mu_{P,N}$) of the particle in units of $e/2m$. These results, of course, follow[3] directly from vector pole dominance within the $SU(6)_W$ framework, provided that the prescription given in Sec. II is used, as has also been emphasized by Freund and Oehme.[10] Notice, however, that the calculation performed in Ref. (10) is in flat contradiction with the spirit of the present work. These authors start with the $\tilde{U}(12)$ result[1] for the strong vertex (in which the P^μ and r^μ terms differ in their q^2 dependence) and force the required proportionality of G_E and G_M for the electromagnetic vertex by demanding pole dominance in the form (1) of the current (chosen arbitrarily). In the present work, the choice of the form of the current in which pole dominance is demanded is determined by the arguments of Sec. II, and corresponds to the philosophy that the *relative* momentum dependence of the two types of coupling of a photon to the nucleons is the same as for the strongly interacting vector mesons which are being assumed to dominate the photon coupling.

The question now arises as to whether this simple model will stand up to a more detailed investigation. Consider the isovector nucleon form factors

$$G_{E,M}{}^V=G_{E,M}{}^P-G_{E,M}{}^N$$

which are expected to be dominated by the contribution from a single vector meson (the ρ-meson), and should therefore provide an excellent illustration of the present ideas. Qualitatively the above model will yield the form equality of $G_E{}^V$ and $G_M{}^V$ (in agreement with experiment) and will give for this dependence $(m_\rho{}^2-t)^{-1}$, where $t=q^2$. This dependence is by no means in exact agreement with experiment, but is certainly qualitatively satisfactory and even roughly quantitatively so in the region of small t where the model is expected to be valid. There is, however, one more quantitative test yet to be applied. If the results (23)–(25) hold for large momentum transfers, and $G_{E,M}{}^P$ show no new and unexpected structure, then it is expected that $G_{E,M}{}^V$ fall off[11] at least as fast as t^{-1}. Thus it should be possible to write *unsubtracted* dispersion relations for $G_{E,M}{}^V$ and hence calculate the isovector charge and magnetic moment in terms of the known parameters of the ρ-meson and π-N scattering.

Perhaps the best suited calculation for this purpose is the one first performed by Singh and Udgaonkar,[12] in that the physical input is large, so that hopefully the inherent inaccuracies of the approximations made are minimized. (Particularly those contributions to the far left-hand cut which are parametrized by the method of Balázs.[13]) As those authors demanded ρ-meson dominance in the form (2) of the current rather than the $G_{E,M}$ form, their method will be briefly reviewed here for the convenience of the reader, who is, however, referred to the original paper and the references contained therein for a more detailed discussion of the approach.

Unsubtracted dispersion relations are written for the form factors in the form

$$G_{E,M}{}^V(t)=\frac{1}{\pi}\int_4^\infty\frac{g_{E,M}(t')dt'}{t'-t}, \quad (27)$$

where the spectral functions $g_{E,M}=\mathrm{Im}(G_{E,M})$ are approximated through unitarity in the standard way[14] by

$$g_{E,M}=\frac{eF_\pi{}^*(t-4)^{3/2}}{8(t)^{1/2}}\Gamma_{E,M}. \quad (28)$$

Here the pion has been adopted as the unit of mass, F_π is the pion electromagnetic form factor (normalized to 1 when $t=0$) and $\Gamma_E=f^+/2m$, $\Gamma_M=f^-/4m\sqrt{2}$, where $f^\pm$ are the helicity amplitudes derived by a Jacob and Wick[15] formulation of the process $\pi\pi\to N\bar{N}$ in a $J=1$ state. If the amplitude for $\pi+\pi\to\pi+\pi$ in the $J=1$ channel is now taken in the form $A(t)=N(t)/D(t)$, where $D(t)$ has only the physical branch cut for $t>4$ and $N(t)$ has singularities only on the negative real axis, then standard methods[16] allow the identification

$$F_\pi(t)=D(0)/D(t) \quad (29)$$

in this approximation (no four-pion or heavier intermediate states). Moreover it is possible to write[14] a dispersion relation

$$f^\pm D=\frac{1}{\pi}\int_{-\infty}^{a}\frac{dt'D(t')\mathrm{Im}[f^\pm(t')]}{t'-t}, \quad (30)$$

where $a=4(1-t/4m^2)$, and the inelastic right-hand cut has been neglected. In the region $0<t<a$ the only contribution to $\mathrm{Im}[f^\pm(t)]$ is from the nucleon pole in the crossed channel, giving

$$\mathrm{Im}[f^+(t')]_N=\frac{-8\pi m^3f^2(t'-2)^2}{(4m^2-t')^{3/2}(4-t')^{3/2}}, \quad (31)$$

[10] P. G. O. Freund and R. Oehme, Phys. Rev. Letters **14**, 1085 (1965).
[11] K. W. Chen, J. R. Dunning, Jr., A. A. Cone, N. F. Ramsey, J. K. Walker, and Richard Wilson, Phys. Rev. **141**, B1267 (1966).
[12] V. Singh and B. M. Udgaonkar, Phys. Rev. **128**, 1820 (1962).
[13] L. A. P. Balázs, Phys. Rev. **125**, 2179 (1962).
[14] W. R. Frazer and J. R. Fulco, Phys. Rev. **117**, 1603 (1960).
[15] M. Jacob and G. C. Wick, Ann. Phys. (N. Y.) **7**, 404 (1959).
[16] *Dispersion Relations*, edited by G. R. Screaton. (Oliver and Boyd, Edinburgh, 1961), p. 227.

$$\mathrm{Im}[f^-(t')]_N=\frac{-4\sqrt{2}m^2f^2}{(4m^2-t')^{3/2}(4-t')^{3/2}}[(4m^2-t')(4-t')-(2-t')^2], \quad (32)$$

where $f=0.08$ is defined in terms of the coupling constant by $f^2=g_r^2/16\pi m^2$, and the rescattering correction and the effect of the inelastic processes are nonzero only[14] for $t<-11$.

Following Singh and Udgaonkar[12] the integral in (30) is written in two parts:

$$f^\pm(t)D(t)=I_N^\pm(t)+I_R^\pm(t), \quad (33)$$

where

$$I_N^\pm(t)=\frac{1}{\pi}\int_{-8}^{a}\frac{D(t')\mathrm{Im}[f^\pm(t')]_N}{t'-t}dt', \quad (34)$$

and

$$I_R^\pm(t)=\frac{1}{\pi}\int_{-\infty}^{-8}\frac{D(t')\mathrm{Im}[f^\pm(t')]}{t'-t}dt'. \quad (35)$$

This then allows the Balázs approximation[13] to be used in the form

$$\frac{1}{t'-t}=\frac{4}{0.14}\left[\frac{50(3.32+0.16t')}{t+196}-\frac{6.25(3.82+0.02t')}{t+21}\right] \quad (36)$$

to obtain the result

$$I_R^\pm(t)=\frac{\alpha^\pm}{t+21}+\frac{\beta^\pm}{t+196}, \quad (37)$$

where $\alpha^\pm$ and $\beta^\pm$ are unknown constants. Notice that this approximation is mathematically no more than a straight-line approximation to a hyperbola of small curvature in the region of interest, and that physically it corresponds to a two-pole parametrization of the far part of the cut, where the positions of the poles are known. This same approximation may also be employed in conjunction with the standard N/D treatment[16] of the π-π amplitude to yield

$$N(t)=a_0+(t-t_0)\left(\frac{a_1}{t+21}+\frac{a_2}{t+196}\right), \quad (38)$$

and

$$D(t)=1-\frac{(t-t_0)}{\pi}\int_4^\infty dt\left(\frac{t'-4}{t'}\right)^{1/2}\frac{N(t')}{(t'-t)(t'-t_0)}dt', \quad (39)$$

where a_0, a_1, a_2 are constants, and a subtraction has been made at $t=t_0$, chosen to be -4. (The nearby part of the left-hand cut of the π-π amplitude has been shown[13] to have only a weak effect, and is neglected here.) Substituting (38) into (39) gives

$$D(t)=1-(t-t_0)[a_0K(t,t_0)+a_1K(t,-21)+a_2K(t,-196)], \quad (40)$$

where

$$K(t,t_0)=\frac{1}{\pi}\int_4^\infty dt'\left(\frac{t'-4}{t'}\right)^{1/2}\frac{1}{(t'-t)(t'-t_0)} \quad (41)$$

are the well-known function defined by Chew and Mandelstam.[17]

The constants, a_0, a_1, a_2 may now be determined by observing that in the resonance region A has the form

$$A=\frac{\gamma(t-4)}{(t_R-t)-i\gamma[(t-4)^3/t]^{1/2}} \quad (42)$$

and ensuring that this yields the correct mass (767 MeV) and width (120 MeV) for the ρ meson, while $N(4)=0$ to give the required threshold behavior. (The physical values have been chosen exactly as in the original calculation of Singh and Udgaonkar[12] to allow easy comparison.) All that remains is to evaluate the constants $\alpha^\pm$ and $\beta^\pm$ in Eq. (37) by constructing $f^\pm(0)$ and $f'^\pm(0)=[(d/dt)f^\pm(t)]_{t=0}$ in terms of these constants and comparing with the values obtained from forward π-N scattering data. The data of Ball and Wong[18] and a more recent set of data produced by Hamilton's group[19] have been used for comparison, and these data are given in Table I. Furthermore, the present author has performed by hand the integrals in Eq. (34) which were machine integrated by the original authors, and this is believed to have introduced an additional error of some 5%. (For reasons which will be made clear in Sec. IV, the first and second entries in the final column Table III are expected to be in fair agreement, and it is believed that the discrepancy confirms that the extra error induced by these approximations is not too severe.) The resulting values for the constants $\alpha^\pm$ and $\beta^\pm$ are given in Table II.

Finally, approximating the ρ-meson resonance by a Dirac δ function,

$$\frac{1}{|D(t)|^2}=\frac{\pi\gamma[t_R(t_R-4)]^{1/2}}{N^2(t_R)}\delta(t_R-t), \quad (43)$$

TABLE I. Data from π-N scattering.

Source of data	$f^+(0)$	$f^-(0)$	$f^{+\prime}(0)$	$f^{-\prime}(0)$
Ball and Wong[a]	0.9568	0.09939	−0.2664	0.0334
Hamilton[b]	0.967	0.0300	−0.2839	0.0554

[a] Reference 18.
[b] Reference 19.

TABLE II. Values of $\alpha^\pm$ and $\beta^\pm$.

Data used	α^+	β^+	α^-	β^-
Ball and Wong[a]	32.9	− 45.0	33.8	−179
Hamilton[b]	41.0	−119	23.0	− 92.0

[a] Reference 18.
[b] Reference 19.

[17] G. F. Chew and S. Mandelstam, Phys. Rev. **119**, 467 (1960).
[18] J. S. Ball and D. Y. Wong, Phys. Rev. Letters **6**, 29 (1961).
[19] J. Hamilton, P. Menotti, G. C. Oades, and L. L. J. Vick, Phys. Rev. **128**, 1881 (1963).

TABLE III. The isovector nucleon charge and magnetic moments.

Data used	Form of current	Charge (e)	Moment ($e/2m$)
Ball and Wong[a]	F_1 and F_2	1.1	2.8
Ball and Wong	G_E and G_M	1.8	2.6
Hamilton[b]	G_E and G_M	1.3	5.2
Experimental value		1	4.7

[a] Reference 18.
[b] Reference 19.

the final result is obtained in the form

$$G_E{}^V=\lambda_C\frac{et_R}{t_R-t},\tag{44}$$

$$G_M{}^V=\lambda_M\frac{e}{2m}\frac{t_R}{t_R-t},\tag{45}$$

where

$$\lambda_C=\frac{\gamma(t_R-4)^2D(0)f^+(t_R)D(t_R)}{4mt_RN^2(t_R)},\tag{46}$$

and

$$\lambda_M=\frac{\gamma(t_R-4)^2D(0)f^-(t_R)D(t_R)}{4\sqrt{2}t_RN^2(t_R)}.\tag{47}$$

The final results are given with the experimental values, with Singh and Udgaonkar's[12] result shown for comparison, in Table III. It seems reasonable to conclude from the magnetic moment results (which are roughly *independent* of the choice of the form of current) that the later πN data of Hamilton is more reliable than the earlier data of Ball and Wong. Furthermore, the value of the isovector charge is strongly dependent on the form of current chosen, but the value obtained with the Hamilton data and the helicity form of the current is not at all unreasonable. [As has been repeatedly emphasized above, this is the form of the current suggested by the speculations in Sec. II, and which leads to the desired form equality exhibited in Eqs. (44) and (45).]

IV. DISCUSSION

It is instructive to see exactly why the above calculations give such widely differing results for the value of the isovector charge in the first two cases shown in Table I, when exactly the same physical input was used. If unsubtracted dispersion relations are assumed for $F_1{}^V$ and $F_2{}^V$, then with an obvious notation and the neglect of the isovector index,

$$F_i(t)=\frac{1}{\pi}\int_{t_0}^{\infty}\frac{f_i(t')dt'}{t'-t},\tag{48}$$

and therefore

$$G_M(t)=F_1(t)+F_2(t)=\frac{1}{\pi}\int_{t_0}^{\infty}\frac{f_1(t')+f_2(t')}{t'-t}dt',\tag{49}$$

and

$$G_E(t)=F_1(t)+\frac{t}{4m^2}F_2(t)=\frac{1}{\pi}\int_{t_0}^{\infty}\frac{f_1(t')+(t'/4m^2)f_2(t')dt'}{t'-t}-\frac{1}{4m^2}\frac{1}{\pi}\int_{t_0}^{\infty}f_2(t')dt'.\tag{50}$$

At first sight it may appear that such a calculation should differ from one assuming unsubtracted dispersion relations for G_E and G_M only by the subtraction of a constant of magnitude

$$\frac{1}{4m^2\pi}\int_{t_0}^{\infty}f_2(t')dt'\tag{51}$$

from $G_E(t)$. However, one immediately realizes that the assumption of unsubtracted relations for G_E and G_M implies directly that F_2 behaves asymptotically at least like t^{-2}, and therefore $tF_2(t)$ may be dispersed[20] in the form

$$tF_2(t)=\frac{1}{\pi}\int_{t_0}^{\infty}\frac{t'f_2(t')dt'}{t'-t}.\tag{52}$$

Evaluating this expression when $t=0$ gives at once the sum rule

$$\int_{t_0}^{\infty}f_2(t')dt'=0,\tag{53}$$

and the two methods are shown to coincide exactly in principle. In practice, however, it is clear that dominance of F_2 by a single pole cannot yield the sum rule required. Thus the two approaches will give results which approximately agree on G_M, while differing in their predictions of $G_E(0)$ and the t dependence of $G_E(t)$.

For the chosen example of the isovector nucleon form factors experiment clearly favors the $G_{E,M}$ form of the current (once single-pole dominance is assumed) for three reasons:

(i) $G_M{}^V$ and $G_E{}^V$ have the same shape[4] (given in this approximation by the meson propagator).

(ii) Reasonable static ($t=0$) values of $G_{E,M}{}^V$ can be obtained using unsubtracted dispersion relations and data from π-N scattering and the ρ meson.

(iii) $G_{E,M}$ (at least for the proton) fall off[11] as t^{-1}. Thus F_2 falls off at least as t^{-2} and satisfies the sum rule.

It therefore appears reasonable to interpret this as evidence in favor of the speculation proposed in Sec. II, so closely related to the symmetry group [SU(6)$_W$] whose application to the vertex yielded the good results of Eqs. (23) and (24).

[20] R. G. Sachs, Phys. Rev. 126, 2256 (1962); Phys. Rev. Letters 12, 231 (1964); A. P. Balachandran, P. G. O. Freund, and C. R. Schumacher, *ibid.* 12, 209 (1964).

The final question in this case is posed by the well-known equality[21] of G_E and G_M at $q^2=4m^2$, which theoretically ensures that F_1 and F_2 are analytic at that point, and physically ensures isotropy in the annihilation channel ($p+\bar{p} \to e^+ + e^-$) when the baryons are at rest. Equations (3) and (4) indicate that this equality is satisfied if F_1 and F_2 are first calculated and G_E and G_M then formed from them; but this leaves open the question if G_E and G_M are calculated directly. With the above dispersion relations for these quantities, one can at once discover

$$G_E(4m^2)-G_M(4m^2)=\frac{1}{\pi}\int_{t_0}^{\infty}\frac{g_E(t')-g_M(t')}{t'-4m^2}dt', \quad (54)$$

and using the identity

$$\mathrm{Im}(AB)=B\,\mathrm{Im}(A)+A^*\,\mathrm{Im}(B), \quad (55)$$

this leads to

$$\mathrm{Re}[G_E(4m^2)-G_M(4m^2)]=\frac{1}{4m^2\pi}\int_{t_0}^{\infty} f_2(t')dt'. \quad (56)$$

The right-hand side of this expression would, of course, be zero by the implied sum rule (53) in an exact calculation, but will not be so in practice because of the crudity of the single-pole approximation used. Thus the price that has been payed in order to obtain reasonable agreement with experiment using the single-pole model is the introduction of spurious poles at $q^2=4m^2$ in the F_1 and F_2 form factors. This should cause little concern, for the approximations used are specifically designed to produce a phenomenology for negative q^2 near zero, and not in the region of large positive momentum transfers. It will be of great interest to have definitive measurements of the proton form factors in the annihilation region[22] $q^2>4m^2$, not only because these should be sensitive to possible higher mass contributions than the presently well-established vector mesons, but also in order to determine whether the remarkable form equality of G_E^P and G_M^P is maintained in this region. In this connection it would be particularly instructive to have results near to the threshold ($q^2=4m^2$), where presumably the form factors (and the cross section) would have to be zero if the form equality of G_E^P and G_M^P is still valid while the threshold condition of their equality must apply. It should be noted, of course, that a completely different set of approximations would have to be employed in calculating the form factors from a dispersion theory in this region of momentum transfers [particularly if the $G_{E,M}$ form of the current is used, as Eq. (56) shows quite clearly].

The conclusion of this investigation is that, if the approximation of vector-pole dominance has to be made, then the form of coupling most likely to yield satisfactory physical results in that which has the P^μ and r^μ terms proportional. This particular form seems to have a close connection with the collinear symmetry groups (through the speculation in Sec. II), and certainly gives agreement with experiment in the one situation where a clear test is available. Although the present work has been primarily concerned with the electromagnetic vertex, it should be noted that the basic property is one of the strong coupling of vector mesons to baryons, and that the theoretical reason suggested for this property is a speculation based on the symmetry of this strong vertex. It seems clear that the adoption of the helicity form of the current rather than the more usual type (2) would cause strong momentum dependence effects in peripheral calculations in which a vector meson is exchanged, and might well provide an even more direct test of the ideas put forward in this work. The complications caused by absorption and the relatively unknown strong form factors seem to make this possibility somewhat remote at the present time.

[21] S. Bergia and L. Brown, in *Nucleon Structure, Proceedings of the International Conference at Stanford University, 1963*, edited by R. Hofstadter and L. I. Schiff (Stanford University Press, Stanford, California, 1964); W. Alles and S. Bergia, Nuovo Cimento **31**, 262 (1964); V. Barger and R. Cahart, Phys. Rev. **136**, B281 (1964).

[22] Experiments in progress at CERN [A. Zichichi *et al.* (unpublished)] and California Institute of Technology [A. V. Tollestrup *et al.* (unpublished)] have established that the cross section is small in the 6–7-BeV/c range, and give rough upper limits (of the order of 0.1) for the form factors. (Private communication from R. Hofstadter.)

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