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Dynamics at Infinite Momentum*

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Old-fashioned perturbation theory is applied to a relativistic theory in a reference frame with infinite total momentum. It is found that many undesirable diagrams disappear. The contribution of the remaining diagrams is described by a new set of rules with properties intermediate between those of Feynman diagrams and old-fashioned diagrams, e.g., energy denominators become covariant, and Feynman parameters appear naturally. The new rules are used to derive some integral equations.

I. INTRODUCTION

THIS article is concerned with the use of recent technical developments in the theory of current algebras to construct a dynamical formalism which combines some of the best features of Feynman diagrams and old-fashioned perturbation theory.

The Feynman rules provide a perturbation theory in which the Lorentz invariance of the S matrix is kept visible at every step. However, this is accomplished only at the cost of manifest unitarity, by lumping together intermediate states with different numbers of particles and antiparticles. Thus, when we try to sum Feynman diagrams to obtain integral equations like the Bethe-Salpeter equation, it proves very difficult to justify the omission of any particular diagrams, since there is no one-to-one relation between internal lines and intermediate states.

For this reason, even after 1949 there continued a subterranean interest in the use of old-fashioned perturbation theory (that is, energy denominators for intermediate states, instead of propagators for internal lines) to attack relativistic problems. Occasional outcroppings of this activity included the Tamm-Dancoff method and the relativistic Faddeev equations. However, the obvious difficulty with all such attempts is that the error made in truncating sums over intermediate states depends on the Lorentz frame in which the calculation is carried out, and there never seemed to be any reason for choosing one Lorentz frame rather than another. Furthermore, the vacuum fluctuations and other topological complexities encountered in relativistic theories made it difficult to derive useful integral equations by summing series of old-fashioned diagrams.

In the last year, difficulties of a very similar sort were met in the use of current algebras to derive sum rules, and were surmounted by the use of a Lorentz frame in which the total momentum P of the system is allowed to approach infinity.[1] It seemed to me natural to ask: What happens to the individually noncovariant old-fashioned diagrams in a relativistic theory if we let $P\to\infty$?

The answer is that every individual diagram either approaches a finite limit or vanishes, the vanishing diagrams being just the ones, like vacuum fluctuations, that caused the worst trouble heretofore. Thus at last there is some rationale for doing calculations of the Tamm-Dancoff type in a particular class of Lorentz frames, those with $P\to\infty$.

When the P factors are cancelled out of the surviving diagrams, we are left with a new set of rules for perturbation theory. In place of the invariant propagators for each internal line which occur in the Feynman rules, or the energy denominators for each intermediate state which occur in the old-fashioned rules, we find in the new rules invariant s denominators for each intermediate state. In simple cases these s denominators can be recognized as the result of combining propagators by use of Feynman parameters, but the work of combining the propagators is done automatically by the new rules. Also, in place of the integrals over four-dimensional momenta in the Feynman rules, or over three-dimensional momenta in the old-fashioned rules, the new rules require integrals over *two*-dimensional transverse momenta and Feynman parameters, the latter appearing naturally here in place of the longitudinal momentum components.

Some first attempts at using these new rules to derive useful integral equations are presented in Sec. V.

II. THE OLD RULES

We will restrict our attention in this article to an interaction Hamiltonian H' of the form

$$H'=\int d^3x\,\mathcal{H}(x)\,, \qquad (1)$$

where $\mathcal{H}(x)$ is a scalar, constructed as a polynomial in one or more neutral scalar causal fields, without derivatives. For example, we could take $\mathcal{H}=g\phi^3$, or $\mathcal{H}=g\phi^4$, etc. In such a theory the S matrix for a transition $\alpha\to\beta$ may be written

$$S_{\beta\alpha}=\delta_{\beta\alpha}-(2\pi)^4 iM_{\beta\alpha}\delta^4(P_\beta-P_\alpha)\prod_n{}^{(\alpha,\beta)}(2\pi)^{-3/2}(2\omega_n)^{-1/2}\,, \qquad (2)$$

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[1] S. Fubini and G. Furlan, Physics 1, 229 (1965); S. Adler, Phys. Rev. Letters 14, 1051 (1965); R. F. Dashen and M. Gell-Mann, in *Proceedings of the Third Coral Gables Conference on Symmetry Principles at High Energy* (W. H. Freeman and Company, San Francisco, California, 1966).

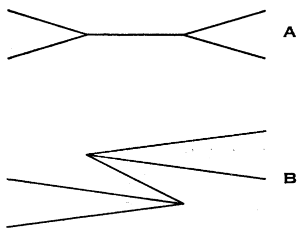

FIG. 1. Two old-fashioned diagrams for scattering in a theory with $\mathcal{H}=g\phi^3$. Under the new rules only A contributes.

where P^μ is the total four-momentum, and the product runs over all particles in initial and final states, with $\omega_n\equiv(\mathbf{p}_n^2+m_n^2)^{1/2}$. The matrix element $M_{\beta\alpha}$ is a Lorentz-invariant[2] function of the incoming and outgoing momenta, the factors $(2\omega)^{-1/2}$ being needed in Eq. (2) because the states α, β are defined with a conventional noncovariant norm, e.g., $\langle\mathbf{p}'|\mathbf{p}\rangle=\delta^3(\mathbf{p}'-\mathbf{p})$.

The results of old-fashioned perturbation theory (i.e., the Born series) may be summarized in a set of graphical rules[3] for constructing $M_{\beta\alpha}$:

(a) Draw all possible ordered diagrams for the transition $\alpha\rightarrow\beta$. (That is, draw each Nth order Feynman diagram $N!$ times, ordering the N vertices in every possible way in a sequence running from right to left, with lines for the particles in the initial state α and the final state β, respectively, entering on the right and leaving on the left. For examples, see Figs. 1 and 2.) Label each line with a three-dimensional momentum $\mathbf{p}$.

(b) For every internal line include a factor[3]

$$(2\pi)^{-3}(2\omega)^{-1}, \tag{3}$$

where $\omega\equiv(\mathbf{p}^2+m^2)^{1/2}$. [The external-line factors are already included in Eq. (2).]

(c) For every vertex except the last (leftmost), include a factor $(2\pi)^3$ times a momentum-conservation δ^3 function. [The $(2\pi)^3\delta$ factor for the last vertex is already included in Eq. (2).] Also, include all appropriate coupling constant factors.

(d) For every intermedite state γ (i.e., a set of lines between any two vertices) include an energy denominator

$$[E_\alpha-E_\gamma+i\epsilon]^{-1}, \tag{4}$$

where $E\equiv\sum\omega$ is the total energy of the state.

(e) Integrate the product of these factors over all internal momenta, and sum the result over all diagrams. This gives $M_{\beta\alpha}$.

[2] For a proof that a Hamiltonian like H' gives a Lorentz-invariant S matrix, see S. Weinberg, in *Brandeis 1964 Summer Institute on Theoretical Physics* (Prentice-Hall, Inc., New York, 1965), p. 424. A more general nonperturbative discussion of the conditions on H' for S to be Lorentz invariant will be given in a forthcoming article.

[3] For a derivation of the old rules in a nonrelativistic context, see e.g., Sec. IV of S. Weinberg, Phys. Rev. **133**, B232 (1964). The only new features here are that particles can be created and destroyed, and that factors $(2\omega)^{-1/2}$ occur, once for each external line and twice for each internal line. These factors arise from the fields $\phi(x)$, and are needed to make the fields scalar.

Although $M_{\beta\alpha}$ is Lorentz-invariant (for $P_\beta=P_\alpha$), the contribution of each individual diagram to $M_{\beta\alpha}$ is in general not invariant. For instance, in a theory with $\mathcal{H}=g\phi^3$ the contribution of diagrams A and B (Fig. 1) to two-body scattering (with $p_1+p_2=p_1'+p_2'$) is

$$\langle\mathbf{p}_1'\mathbf{p}_2'|M_A|\mathbf{p}_1\mathbf{p}_2\rangle=(g^2/2\omega(\mathbf{p}_1+\mathbf{p}_2))\times[\omega(\mathbf{p}_1)+\omega(\mathbf{p}_2)-\omega(\mathbf{p}_1+\mathbf{p}_2)]^{-1}, \tag{5}$$

$$\langle\mathbf{p}_1'\mathbf{p}_2'|M_B|\mathbf{p}_1\mathbf{p}_2\rangle=(g^2/2\omega(\mathbf{p}_1+\mathbf{p}_2))\times[-\omega(\mathbf{p}_1)-\omega(\mathbf{p}_2)-\omega(\mathbf{p}_1+\mathbf{p}_2)]^{-1}, \tag{6}$$

where $\omega(\mathbf{p})\equiv(m^2+\mathbf{p}^2)^{1/2}$. These are not separately invariant, but their sum gives

$$\langle\mathbf{p}_1'\mathbf{p}_2'|M_A+M_B|\mathbf{p}_1\mathbf{p}_2\rangle=g^2[(\omega_1+\omega_2)^2-(\mathbf{p}_1+\mathbf{p}_2)^2-m^2]^{-1}=g^2[s-m^2]^{-1}, \tag{7}$$

and this *is* invariant. Similar remarks apply to diagrams A and B of Fig. 2.

Since individual old-fashioned diagrams make non-invariant contributions to $M_{\beta\alpha}$, it makes sense to ask how their contributions depend on the Lorentz frame in which they are evaluated.

III. THE LIMIT $P\rightarrow\infty$

Suppose we calculate $M_{\beta\alpha}$ in a reference frame in which the total momentum $\mathbf{P}$ is very large. (We take $\mathbf{P}_\beta=\mathbf{P}_\alpha=\mathbf{P}$, but do not necessarily assume that $E_\beta=E_\alpha$.) The momentum of the nth particle in the initial or final state may be written

$$\mathbf{p}_n=\eta_n\mathbf{P}+\mathbf{q}_n, \tag{8}$$

where $\mathbf{q}_n$ is transverse

$$\mathbf{q}_n\cdot\mathbf{P}=0, \tag{9}$$

and, since $\mathbf{P}$ is the total momentum of the state,

$$\textstyle\sum_n\eta_n=1, \tag{10}$$

$$\textstyle\sum_n\mathbf{q}_n=0, \tag{11}$$

the sums running over all particles in the state α or β. The observor is supposed to be moving, with respect to the center-of-mass frame, at a high velocity in the $-\mathbf{P}$ direction, and will see all particles moving with high velocities more or less in the $+\mathbf{P}$ direction. Hence we let $P\rightarrow\infty$, in a fixed direction, with $\mathbf{q}_n$ and η_n held fixed and with

$$\eta_n>0. \tag{12}$$

It will be shown that *as $P\rightarrow\infty$ the contribution of each diagram to $M_{\beta\alpha}$ either vanishes or approaches a finite limit.* This is not quite a trivial result, for the individual terms in $M_{\beta\alpha}$ are not Lorentz-invariant, and even the total $M_{\beta\alpha}$ is only invariant when $E_\beta=E_\alpha$.

We note first that the virtual particle momenta may be parametrized just as in Eqs. (8)–(11), with one crucial exception: The virtual momenta at $P=\infty$ are not related by a Lorentz transformation to any other set of finite momenta, say at $\mathbf{P}=0$, but rather are either variables of integration or are fixed by momentum-conservation δ functions. Hence when we use Eqs. (8)–(11) to define the internal $\mathbf{q}$'s and η's we sometimes find that some of the internal η's are *negative.*

The importance of this possibility becomes apparent when we compute the energy denominators (4). The energy of the nth particle in any state is

$$\omega_n=[\eta_n^2P^2+\mathbf{q}_n^2+m_n^2]^{1/2}$$
$$=|\eta_n|P+[\mathbf{q}_n^2+m_n^2]/2P|\eta_n|+O(P^{-2}), \quad (13)$$

so the total energy of the state is

$$E=\lambda P+\frac{1}{2P}\sum_n\frac{[\mathbf{q}_n^2+m_n^2]}{|\eta_n|}+O(P^{-2}), \quad (14)$$

where

$$\lambda\equiv\sum_n|\eta_n|. \quad (15)$$

Note that the total center-of-mass energy squared is

$$s=-(\sum_n p_n)^2=E^2-P^2=\lambda\sum_n\frac{[\mathbf{q}_n^2+m_n^2]}{|\eta_n|}, \quad (16)$$

so that Eq. (14) may be written

$$E=\lambda P+s/2\lambda P+O(P^{-2}). \quad (17)$$

Now, if all η are positive, then Eqs. (10) and (15) give

$$\lambda=1 \quad (\text{all } \eta_n>0). \quad (18)$$

This is true in particular of the initial state α, so that if some intermediate state γ also has all η_n positive, then the momentum P will cancel in the energy denominator, leaving us with

$$[E_\alpha-E_\gamma+i\epsilon]^{-1}\rightarrow 2P[s_\alpha-s_\gamma+i\epsilon]^{-1} \quad (\text{all } \eta_n>0). \quad (19)$$

On the other hand, Eqs. (10) and (15) show that an intermediate state γ with some negative η's will have

$$\lambda>1 \quad (\text{some } \eta_n<0), \quad (20)$$

so that in this case the coefficients of P in E_α and E_γ do not cancel, and we are left with

$$[E_\alpha-E_\gamma+i\epsilon]^{-1}\rightarrow P^{-1}(1-\lambda_\gamma)^{-1} \quad (\text{some } \eta_n<0). \quad (21)$$

We now count powers of P. The internal-line factors under the old rules (b) and (e) become P-independent:

$$\frac{d^3p_n}{(2\pi)^3(2\omega_n)}\rightarrow\frac{Pd\eta_n d^2q_n}{(2\pi)^3 2P|\eta_n|}=\frac{d\eta_n}{2|\eta_n|}\frac{d^2q_n}{(2\pi)^3}. \quad (22)$$

The vertex factors under the old rule (c) become

$$(2\pi)^3\delta^3(\Delta\sum\mathbf{p})=(2\pi)^3P^{-1}\delta(\Delta\sum\eta)\delta^2(\Delta\sum q), \quad (23)$$

where Δ refers to the difference in the sums before and after the interaction. For an Nth order diagram there are $N-1$ of these [one delta-function having been factored out in (2)] so they contribute a factor P^{-N+1} to the matrix element. If all η_n in all intermediate states are positive, then this factor will be just cancelled by the $N-1$ energy denominators (19), leaving us with a finite limit. On the other hand, if ν of the $N-1$ intermediate states contain negative η's we get from the energy denominators only $N-2\nu-1$ factors of P, and when multiplied by the factor P^{-N+1} from the vertex factors (23); this gives a matrix element which vanishes as $P^{-2\nu}$. We conclude therefore that *it is just those diagrams which have all internal as well as external η's positive which make finite contributions to $M_{\beta\alpha}$ as $P\rightarrow\infty$, other diagrams being smaller by powers of P^{-2}.*

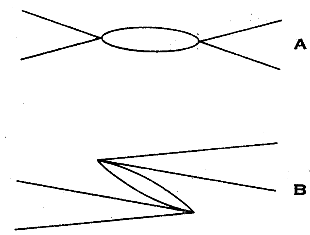

FIG. 2. Two old-fashioned diagrams for scattering in a theory with $\mathcal{H}=g\phi^4$. Under the new rules only A contributes.

Where some of the η_n are variables of integration, as in diagrams with loops, this means that as $P\rightarrow\infty$ the integral becomes restricted to a range such that all $\eta_n>0$. Possible complications in interchanging the order of integration and the limit $P\rightarrow\infty$ are being ignored here, though it is not entirely clear that this is always justified in the presence of ultraviolet divergences.

IV. THE NEW RULES

Since all P factors cancel in the leading diagrams, we can now forget about them and summarize our results in rules for what amounts to a new form of perturbation theory:

(a) Draw all ordered diagrams (defined as before) for the transition $\alpha\rightarrow\beta$. Label each line with an η and a two-vector $\mathbf{q}$. (Some diagrams will not contribute; see below.)

(b) For every internal line include a factor [see Eq. (22)]

$$\theta(\eta_n)/2(2\pi)^3\eta_n. \quad (24)$$

(c) For every vertex except the leftmost include a factor [see Eq. (23)]

$$(2\pi)^3\delta(\Delta\sum\eta)\delta^2(\Delta\sum\mathbf{q}), \quad (25)$$

where Δ refers to the difference in the sums before and after the interaction. Also include appropriate coupling constants.

(d) For every intermediate state γ include a factor [see Eq. (19)]

$$2[s_\alpha-s_\gamma+i\epsilon]^{-1}, \quad (26)$$

where s for any state is the usual total c.m. energy squared, given here by (16) and (18) as

$$s=\sum_n [\mathbf{q}_n^2+m_n^2]/\eta_n. \quad (27)$$

(e) Integrate the product of all these factors over all internal $\mathbf{q}$'s and η's, and sum the result over all diagrams. This gives $M_{\beta\alpha}$.

Momentum conservation has been imposed by requiring that the initial- and final-state $\mathbf{q}$'s and η's satisfy (10) and (11). If we also require energy conservation, i.e.,

$$s_\alpha=s_\beta=s\,, \quad (28)$$

then $M_{\beta\alpha}$ will be Lorentz-invariant, that is, it will depend only upon scalar products of external momenta, which can be expressed in terms of the q's and η's as

$$\begin{aligned}-p_{n\mu}p_m{}^\mu=\omega_n\omega_m-\eta_n\eta_m P^2-\mathbf{q}_n\cdot\mathbf{q}_m \\ \to(\eta_n/2\eta_m)(\mathbf{q}_m^2+m_m^2) \\ +(\eta_m/2\eta_n)(\mathbf{q}_n^2+\mathbf{m}_n^2)-\mathbf{q}_n\cdot\mathbf{q}_m \quad (29)\end{aligned}$$

[see Eq. (13)]. Hence, when (28) as well as (10) and (11) hold, all $\mathbf{q}$'s and η's in the total matrix elements can be eliminated in favor of the scalars $p_n\cdot p_m$, and the resulting formula for $M_{\beta\alpha}$ will then be valid in any Lorentz frame. The S matrix in an arbitrary Lorentz frame can then be calculated from Eq. (2).

The most important distinction between these new rules, and the old rules listed in Sec. II, is that the factors $\theta(\eta_n)$ under rule (b) eliminate some diagrams. This happens whenever a vertex has a number of lines coming in from the right but has no lines going out to the left, or vice versa, because η conservation would require that the sum of the η's of these lines would have to vanish, and this is forbidden by the requirement that all η's be positive. Therefore under rule (a) *we need not draw diagrams in which particles are created or destroyed out of the vacuum.* For instance, diagrams B of both Fig. 1 and Fig. 2 do not contribute to the matrix element. Also, there can be no vacuum fluctuation diagrams. In consequence, *the connectedness-structure of the new perturbation theory is the same as found when old-fashioned perturbation theory is applied to nonrelativistic problems*, that is, every incoming line in every diagram or part of a diagram is connected to some outgoing line (and vice versa) although it need not be connected to all of them or to all the other incoming lines.

As a first example, look at diagrams A and B of Fig. 1 for two-body scattering in a theory with $\mathcal{H}=g\phi^3$. The internal line in diagram B of Fig. 1 has η value -2, so 1B does not contribute at all under the new rules. The internal line in 1A has $\eta=+1$ and $\mathbf{q}=0$, so this diagram contributes a term, given by the new rules as

$$M_A=\frac{g^2}{s_\alpha-s_\gamma+i\epsilon}=\frac{g^2}{s-m^2+i\epsilon}.$$

Thus, under the new rules we get from A of Fig. 1 alone the same matrix element (7) as arose under the old rules from A of Fig. 1 plus B of Fig. 1.

To see what happens to loops in the new rules, look at diagrams A and B of Fig. 2 for two-body scattering in a theory with $\mathcal{H}=g\phi^4$. Diagram B of Fig. 2 obviously requires that one or both of the internal lines has $\eta<0$, so it does not contribute. The contribution of diagram A of Fig. 2 is given by the new rules as

$$\begin{aligned}M(s)&=\frac{1}{2(2\pi)^3}\int_0^1\frac{d\eta}{\eta(1-\eta)}\int d^2q \\ &\quad\times\left[s-\frac{\mathbf{q}^2+m^2}{\eta}-\frac{\mathbf{q}^2+m^2}{1-\eta}+i\epsilon\right]^{-1} \\ &=\frac{1}{2(2\pi)^3}\int_0^1 d\eta\int d^2q[s\eta(1-\eta)-\mathbf{q}^2-m^2+i\epsilon]^{-1}.\end{aligned}$$

This is logarithmically divergent, but we can get a finite result by the familiar trick of differentiating with respect to s:

$$\begin{aligned}M'(s)=-\frac{1}{2(2\pi)^3}\int_0^1\eta(1-\eta) \\ \times d\eta\int d^2q[s\eta(1-\eta)-\mathbf{q}^2-m^2+i\epsilon]^{-2}.\end{aligned}$$

The q integral is entirely trivial, and yields

$$M'(s)=+\frac{\pi}{2(2\pi)^3}\int_0^1\frac{\eta(1-\eta)}{s\eta(1-\eta)-m^2+i\epsilon}d\eta. \quad (30)$$

If we used the usual Feynman rules, we should begin by writing

$$M(s)=\frac{i}{(2\pi)^4}\int\frac{d^4k}{[(P-k)^2+m^2-i\epsilon][k^2+m^2-i\epsilon]}\,;\qquad s\equiv-P^2.$$

It would then be necessary to combine denominators by introducing a Feynman parameter α:

$$\begin{aligned}M(s)&=\frac{i}{(2\pi)^4}\int_0^1 d\alpha\int\frac{d^4k}{[(P-k)^2\alpha+k^2(1-\alpha)+m^2-i\epsilon]^2} \\ &=\frac{i}{(2\pi)^4}\int_0^1 d\alpha\int\frac{d^4k}{[(k-P\alpha)^2-s\alpha(1-\alpha)+m^2-i\epsilon]^2}.\end{aligned}$$

Shifting the k variable to $k'=k-P\alpha$, differentiating with respect to s, rotating the k'^0-contour, and doing the d^4k' integral, we would emerge with

$$M'(s)=\frac{\pi^2}{(2\pi)^4}\int_0^1 d\alpha\frac{\alpha(1-\alpha)}{[s\alpha(1-\alpha)-m^2+i\epsilon]}. \quad (31)$$

Comparing (31) with (30), we see that the results are the same, and that *the η parameter introduced by the new rules turns out to be nothing but the Feynman parameter α needed to combine denominators.* However, the new rules short-circuit the work required to give (31), because they yield formulas for the matrix element with denominators already combined, and with momentum-space integrals which are Euclidean and two-dimensional rather than Minkowskian and four-dimensional.

In more complicated examples, the contribution of individual diagrams to $M_{\beta\alpha}$ is not Lorentz-invariant even when $s_\alpha = s_\beta$, and to get Lorentz-invariant answers it is necessary to add up all ordered diagrams corresponding to a given Feynman diagram. (The reason why diagrams A of both Fig. 1 and Fig. 2 give Lorentz-invariant answers is just that the only other ordered diagrams which correspond to the same Feynman diagrams are, respectively, B of Fig. 1 and Fig. 2, and these perish by the $\eta > 0$ rule.) It should be kept in mind that the specification of a given **P** does not uniquely characterize a Lorentz frame, for apart from rotations about **P** there are two other combined rotations and boosts which also leave P^μ invariant. What we have done by letting $\mathbf{P} \to \infty$ is to find a *class* of Lorentz frames in which a great many unpleasant diagrams like B of Fig. 1 and Fig. 2 disappear; but this still leaves us with some freedom in the choice of Lorentz frame. This can be a nuisance, in that when truncating sums over intermediate states we sometimes lose Lorentz invariance, but it may also be a useful tool in proving exact theorems about the asymptotic and analytic properties of the S matrix.

V. INTEGRAL EQUATIONS

We have already remarked that the diagrams which survive under the new rules have essentially the same connectedness properties as found in nonrelativistic theories when we use the old rules. This opens up the possibility of writing integral equations for strong interactions, similar to those which have become familiar in multiparticle potential problems. As an example, the methods of Ref. 3, when applied to a theory with $\mathcal{H} = g\phi^3$, yield an integral equation shown schematically in Fig. 3. Putting some limit on the number of particles in intermediate states reduces this to a set of nonlinear integral equations which look superficially like linear equations with connected kernels. Note that no such equations could be derived using Feynman rules or old-fashioned rules, because the possibility of spontaneous-creation vertices could make the ID term of Fig. 3 disconnected.

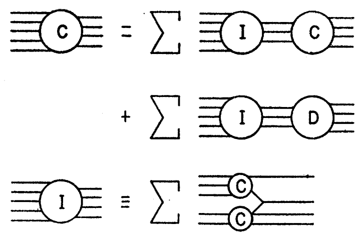

FIG. 3. The general connected relativistic integral equation in a theory with $\mathcal{H} = g\phi^3$. Here C is the connected part of the matrix element, D is the sum of all the disconnected parts, and I is the "irreducible kernel." The sums in the first equation run over the number of lines in the intermediate state. The sums in the second equation run over all divisions of the external lines on the left (right) into the two (three) sets shown. (If there were no particle creation or annihilation we could regard D and I as already known, and ID as the inhomogeneous term in the integral equation.)

Another possibly fruitful approach is suggested by the Low equation. When we let $P \to \infty$ (in the sense of Sec. III) this becomes an equation of similar form, but with integrations over **q**, η instead of **p**, and with an s denominator in place of an energy denominator. It is easy to reduce this new Low equation to a connected nonlinear equation for the connected part of $M_{\beta\alpha}$.

The above are exact integral equations, though approximations are needed to solve them. We can also write an approximate equation of the Bethe-Salpeter type:

$$\langle \mathbf{q}',\eta'|M|\mathbf{q},\eta\rangle = \langle \mathbf{q}',\eta'|V|\mathbf{q},\eta\rangle + \int d^2q'' \int_0^1 d\eta'' \frac{\langle \mathbf{q}'\eta'|V|\mathbf{q}''\eta''\rangle}{2(2\pi)^3\eta''(1-\eta'')} \times \langle \mathbf{q}''\eta''|M|\mathbf{q}\eta\rangle \left[s - \frac{\mathbf{q}''^2+m^2}{\eta''(1-\eta'')} + i\epsilon \right]^{-1}, \quad (32)$$

where

$$\langle \mathbf{q}',\eta'|M|\mathbf{q},\eta\rangle \equiv \langle \mathbf{q}',\eta'; -\mathbf{q}',1-\eta'|M|\mathbf{q},\eta; -\mathbf{q},1-\eta\rangle,$$

$$\langle \mathbf{q}',\eta'|V|\mathbf{q},\eta\rangle \equiv \frac{g^2}{|\eta'-\eta''|} \left\{ \theta(\eta'-\eta'') \left[s - \frac{\mathbf{q}''^2+m^2}{\eta''} - \frac{(\mathbf{q}'-\mathbf{q}'')^2+m^2}{\eta'-\eta''} - \frac{\mathbf{q}'^2+m^2}{1-\eta'} + i\epsilon \right] \right.$$
$$\left. + \theta(\eta''-\eta') \left[s - \frac{\mathbf{q}''^2+m^2}{1-\eta''} - \frac{(\mathbf{q}'-\mathbf{q}'')^2+m^2}{\eta''-\eta'} - \frac{\mathbf{q}'^2+m^2}{\eta'} + i\epsilon \right]^{-1} \right\},$$

$$s \equiv \frac{q^2+m^2}{\eta(1-\eta)}.$$

For s below the three-particle threshold, V is obviously nonsingular, and the solution of (32) will satisfy two-particle unitarity, with no need to perform a Wick rotation. However, the solution does not represent a sum of Feynman diagrams, and is therefore not Lorentz-invariant. It seems particularly convenient to write (32) in the Lorentz frame with $\mathbf{q}=0$, because then $\langle \mathbf{q}',\eta' | M | \mathbf{q},\eta \rangle$ depends only upon η, η', and $\mathbf{q}'^2$, and the angular integrations in (32) can be done immediately; however, it must be admitted that we have no reason to suppose that (32) is a better approximation when $\mathbf{q}=0$ than in any other Lorentz frame with $P=\infty$.

Note added in proof. (1) Equation (32) is similar to, though not identical with, an integral equation suggested recently by R. Blankenbecler and R. Sugar, Phys. Rev. **142**, 1051 (1966). (2) Even if some interval lines carry spin $\frac{1}{2}$ (or, perhaps, even in general) it is still time that each old-fashioned diagram makes a finite or zero contribution to the S-matrix as $P\to\infty$. I wish to thank F. Low for a discussion on this point.

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One-Meson Propagator at Large Momenta*

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A recently developed approximation method is used to calculate some contributions to the one-meson propagator at space-like momenta. An $SU(3)$-invariant interaction is assumed. The nonperturbative formula obtained for the propagator as $p^2\to-\infty$ allows one to test the idea of imposing a boundary condition, in this limit, for restricting the coupling constants. An experimentally plausible inequality is obtained.

1. INTRODUCTION

RECENTLY, a method was developed[1] for calculating strong-interaction dynamics by optimizing the information contained in low-order Feynman diagrams. This has yielded plausible results when applied to the two-pion system.[2] The present article is a preliminary test of the method's applicability in deriving essentially nonperturbative restrictions on coupling constants.

As the source for such restrictions, we shall impose a qualitative boundary condition on a particle propagator in momentum space. This is motivated by the behavior of, say, nucleon-nucleon total cross sections, which, at the highest currently available energies, fail to approach zero. Perturbation methods, however, would seem to predict a vanishing high-energy limit to any finite order in the coupling constants. It is therefore not impossible that one-particle propagators, if they could be observed directly, would likewise exhibit such a nonperturbative behavior at large four-momenta. Specifically, *we assume that the one-pion Green's function, considered at large space-like momenta, does not decrease as fast as predicted by perturbation calculations.* (The main reason for considering space-like rather than time-like momenta is that the approximation to be used here works best whenever the exact result is real and free from singularities.[1])

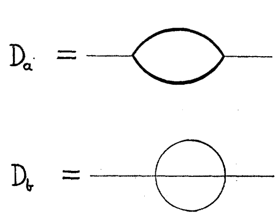

FIG. 1. The Feynman diagrams D_a, D_b considered in this article. The heavy lines represent baryons, the light lines mesons.

For definiteness we assume that only the strong interactions exist, that they are exactly invariant under $SU(3)$, and that there are no elementary particles apart from one baryon octet, containing the nucleons, and one meson octet, containing the pions. (In such a model, the other strongly interacting particles must come out as bound states or resonances.) The method used in this article does not depend in any essential way on such restrictive assumptions, but present-day experimental knowledge about coupling constants, as well as available Feynman-diagram calculations, make more general assumptions pointless.

The total interaction Lagrangian density will be taken as

$$\mathcal{L}_I = -g_0 \sum_{\alpha,\beta,\gamma} \Gamma_{\alpha\beta\gamma}\bar{\psi}^\alpha\gamma^5\psi^\beta\phi^\gamma - \tfrac{1}{4}\theta g_0^2 \sum_{\alpha,\beta}(\phi^\alpha)^2(\phi^\beta)^2, \quad (1.1)$$

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[1] M. Wellner, Phys. Rev. **132**, 1848 (1963).
[2] M. Alexanian and M. Wellner, Phys. Rev. **137**, B155 (1965); **140**, B1079 (1965).