Parity of Fermions: Tests and Ambiguities*

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Parity tests and ambiguities are discussed for fermion interactions. These include decays into spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ fermions, as well as fermion production from a polarized target. Complete tests for the several-step decay of a high-spin formation resonance are presented.

THIS article presents, through the use of invariance arguments, simple discussions of parity tests and of ambiguities in the following processes: the strong decay of a fermion F_J into a fermion $F_{1/2}$ plus a boson B_0; the strong decay of an F_J into an $F_{3/2}$ plus a B_0; and the production of an $F_{1/2}$ plus a B_0 from a polarized target.[1,2] Decay of a "formation" resonance into an $F_{3/2}$ is treated extensively.

DECAY INTO $F_{1/2}$

No parity information can be obtained from the decay angular distribution of a spin-J fermion (F_J) that yields a spin-$\frac{1}{2}$ fermion ($F_{1/2}$) plus a spinless boson (B_0). A decay matrix (M_+) describing decay of one parity must be multiplied by a pseudoscalar $\boldsymbol{\sigma}\cdot\hat{p}$ to obtain the decay matrix (M_-) required for the opposite parity. (The operator $\boldsymbol{\sigma}$ is associated with the spin of the final $F_{1/2}$, and $\hat{p}$ is a unit vector along the direction of decay momentum in F_J's rest frame.) Thus

$$M_-\equiv\boldsymbol{\sigma}\cdot\hat{p}\,M_+. \tag{1}$$

The initial state is describable by a density matrix ρ_i, so normalized that $\mathrm{Tr}\rho_i=1$. The angular distributions for the two parities are

$$\begin{aligned} I_+&=\mathrm{Tr}(M_+\rho_iM_+^\dagger)\\ I_-&=\mathrm{Tr}[(\boldsymbol{\sigma}\cdot\hat{p}M_+)\rho_i(M_+^\dagger\boldsymbol{\sigma}\cdot\hat{p})]\\ &=\mathrm{Tr}[(\boldsymbol{\sigma}\cdot\hat{p})^2M_+\rho_iM_+^\dagger]. \end{aligned}\tag{2}$$

Since $(\boldsymbol{\sigma}\cdot\hat{p})^2=I$, the M_+ and M_- transformations are here indistinguishable.[3]

The polarization of the outgoing $F_{1/2}$ is found by evaluating

$$I\mathbf{P}_+=\mathrm{Tr}(\boldsymbol{\sigma}\rho_{f+})=\mathrm{Tr}[\boldsymbol{\sigma}(M_+\rho_iM_+^\dagger)], \tag{3}$$

or, for the opposite parity,

$$I\mathbf{P}_-=\mathrm{Tr}[\boldsymbol{\sigma}(\boldsymbol{\sigma}\cdot\hat{p}M_+\rho_iM_+^\dagger\boldsymbol{\sigma}\cdot\hat{p})]. \tag{4}$$

By definition,[4] $M_+\rho_iM_+^\dagger$ or ρ_{f+} must equal $\frac{1}{2}I(1+\mathbf{P}_+\cdot\boldsymbol{\sigma})$; thus

$$\mathbf{P}_-=\boldsymbol{\sigma}\cdot\hat{p}(\mathbf{P}_+\cdot\boldsymbol{\sigma})\boldsymbol{\sigma}\cdot\hat{p}. \tag{5}$$

But $-i\boldsymbol{\sigma}\cdot\hat{p}$ is the same as the rotation operator $R(\pi)=\exp(-i\boldsymbol{\sigma}\cdot\hat{p}\pi/2)$; hence Eq. (5) may be written

$$\mathbf{P}_-=R(\pi)[\mathbf{P}_+\cdot\boldsymbol{\sigma}]R^{-1}(\pi). \tag{6}$$

The $F_{1/2}$ vector polarizations for the two decay parities thus differ by a rotation of 180° about $\hat{p}$.

DECAY INTO $F_{3/2}$

The angular distribution for decay of an F_J into $F_{3/2}$ is *not* parity-ambiguous in the same sense as that for decay into $F_{1/2}$. However, a parity determination from the angular distribution alone is sometimes impossible.

Two orbital angular momenta are possible for each parity in the strong decay into an $F_{3/2}$: $l_+=J-\frac{3}{2}$ and $l_+'=J+\frac{1}{2}$, or $l_-=J-\frac{1}{2}$ and $l_-'=J+\frac{3}{2}$.[6] If the transition matrices are separated into lower and higher l-wave contributions, $\mathfrak{M}^l$ and $\mathfrak{M}^{l'}$, they are related by[7]

$$\mathfrak{M}_-^{\,l}+\mathfrak{M}_-^{\,l'}=eT_{10}\mathfrak{M}_+^{\,l}+fT_{30}T_{20}^{-1}\mathfrak{M}_+^{\,l'}. \tag{7}$$

[The T_{L0} are spin-$\frac{3}{2}$ operators expressed in the helicity system, with $T_{10}\propto S_z=\mathbf{S}\cdot\hat{p}$. The e and f are complex numbers. Cf. Eqs. (3) and (5) of Ref. 7.] Neither of the "parity operators" T_{10} or $T_{30}T_{20}^{-1}$ is unitary, as is $\boldsymbol{\sigma}\cdot\hat{p}$ for spin $\frac{1}{2}$:

$$T_{10}\propto\begin{bmatrix}3&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-3\end{bmatrix};\quad T_{30}T_{20}^{-1}\propto\begin{bmatrix}1&0&0&0\\0&3&0&0\\0&0&-3&0\\0&0&0&-1\end{bmatrix}. \tag{8}$$

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[1] Examples of such treatment are to be found in H. A. Bethe and F. de Hoffman, *Mesons and Fields* (Row, Peterson and Company, Evanston, Illinois, 1965), Vol. II, p. 75 (the Dyson and Nambu proof of the Minami ambiguity); L. Wolfenstein and J. Ashkin, Phys. Rev. **85**, 947 (1952); R. K. Adair, Rev. Mod. Phys. **33**, 406 (1951).

[2] Here F_J designates a fermion of spin J and $B_{J'}$ a boson of spin J'.

[3] In π-N scattering, this deficiency of parity information in the angular distribution has been known as the "Minami ambiguity."

[4] The density matrix equals $(2J+1)^{-1}\sum_\mu\langle S_\mu\rangle^*S_\mu$, where the S_μ are a complete set of spin operators (L. Wolfenstein and J. Ashkin, Ref. 1).

[5] Cf. the special cases calculated by R. K. Adair, Ref. 1 ($J=\frac{1}{2}$), and by J. B. S. ($J=\frac{3}{2}, \frac{5}{2}$), [J. B. Shafer, J. J. Murray, and D. O. Huwe, Phys. Rev. Letters **10**, 179 (1963)], and a discussion of C. Zemach, Phys. Rev. **140**, B109 (1965).

[6] The + or − subscript designates the $J^P=\frac{1}{2}^+, \frac{3}{2}^-, \frac{5}{2}^+\cdots$ sequence or the $\frac{1}{2}^-, \frac{3}{2}^+\cdots$ sequence, respectively, (P being the $F_J-F_{3/2}$ *relative* parity). Angular-momentum conservation permits only the higher l' waves for $J^P=\frac{1}{2}^+$ and $\frac{1}{2}^-$.

[7] J. Button-Shafer, Phys. Rev. **139**, B607 (1965).

Thus, in general, the angular distribution

$$I=\mathrm{Tr}[(\mathfrak{M}^l+\mathfrak{M}^{l'})\rho_i(\mathfrak{M}^l+\mathfrak{M}^{l'})^\dagger]/\mathrm{Tr}\rho_i \qquad (9)$$

differs for even and odd parities.[8]

Although the angular distribution does not involve a Minami-type ambiguity, it does not yield enough information to determine the F_J parity (as well as two partial amplitudes) if J is $\leqslant\frac{5}{2}$.

Neither of the (nonunitary) parity operators can be equivalent to a rotation operator that acts on $F_{3/2}$ polarization.[9]

PARITY TESTS FOR FORMATION RESONANCES

Decays of fermions into an $F_{3/2}$ have recently been analyzed in *"formation"* experiments.[10] The two tests utilized may be considerably extended.

The process to be discussed is

$$F_J \xrightarrow[(S)]{(1)} F_{3/2} \xrightarrow[(S)]{(2)} F_{1/2} \xrightarrow[(W)]{(3)} f_{1/2}. \qquad (10)$$

A spinless boson is understood to accompany each final fermion. The numbers indicate the step of decay; the letters, the strength of decay. The decay of a *final-state* resonance F_J in this sequence has been treated theoretically, with and without the use of T_{LM} spin operators.[7,11]

A brief discussion of the T_{LM} tensors will be helpful. These are of great utility for spin-state description, as they make possible the formulation of a complete set of independent spin-parity tests. Each $\langle T_{LM}\rangle$ characterizing a particle's state combines with a $Y_{LM}(\theta,\phi)$ or a $\mathfrak{D}_{MM'}{}^L(\phi,\theta,0)$ in its decay distribution.[12] A system of spin J requires $(2J+1)^2$ parameters for the description of its spin state. For an $F_{3/2}$, the normalization and vector-polarization terms ($\langle T_{00}\rangle=\langle I\rangle$, $\langle T_{10}\rangle\propto\langle S_z\rangle$, etc.) plus 12 additional quantities such as $\langle T_{20}\rangle\propto\langle 3S_z^2-S^2\rangle$, $\langle T_{21}\rangle\propto\langle S_z(S_x+iS_y)\cdots\rangle$, $\langle T_{22}\rangle\propto\langle(S_x+iS_y)^2\rangle$, and $\langle T_{30}\rangle\propto\langle S_z^3\cdots\rangle$ are required. The $\langle T_{2M}\rangle$, which are second-rank tensor polarizations, correspond to *alignment* of spin. They are quantities similar to moments of inertia or to the nuclear electric-quadrupole moment.

For the "formation resonance" produced from a $B_0+F_{1/2}$ system, angular-momentum conservation in production permits only even-L, $M=0$ $\langle T_{LM}\rangle$ if the incident-beam direction is the z axis.[13] (Only the $m_J=\pm\frac{1}{2}$-spin states are occupied.)

The derivations of Ref. 7 may be readily extended to treat the *formation* resonance. The initial $\langle T_{LM}\rangle\equiv t_{LM}$ and the helicity amplitudes A_λ [contained in $\mathfrak{M}$, Eq. (7)] are used to form the density matrix for the outgoing spin-$\frac{3}{2}$ particle:

$$[\rho_{(3/2)}]_{\lambda\lambda'}=A_\lambda A_{\lambda'}{}^*\sum_{L_e}^{2J-1} n_{L,\lambda-\lambda'}{}^{(2\lambda)}\,t_{L0}\times\mathfrak{D}_{0,\lambda-\lambda'}{}^L(0,\theta,0)\,, \qquad (11)$$

where L_e is even. The $n_{L,\lambda-\lambda'}{}^{(2\lambda)}$ quantities each contain a Clebsch-Gordan coefficient; they may be expressed in terms of $n_{L0}{}^{(1)}$ by use of recursion relations.[7]

For initial spin $J=\frac{5}{2}$,

$$A_+=(1/20)^{1/2}\begin{bmatrix}2a+(\surd6)c & 0 & 0 & 0\\ 0 & (\surd6)a-2c & 0 & 0\\ 0 & 0 & (\surd6)a-2c & 0\\ 0 & 0 & 0 & 2a+(\surd6)c\end{bmatrix};$$

$$A_-=(1/28)^{1/2}\begin{bmatrix}2(\sqrt3)b+(\sqrt2)d & 0 & 0 & 0\\ 0 & (\sqrt2)b-2(\sqrt3)d & 0 & 0\\ 0 & 0 & -(\sqrt2)b+2(\sqrt3)d & 0\\ 0 & 0 & 0 & -2(\sqrt3)b-(\sqrt2)d\end{bmatrix}; \qquad (12)$$

here a, b, c, and d designate the p- through g-wave amplitudes. For a formation resonance of spin $\frac{5}{2}$,

$$t_{00}=1.000;\qquad t_{20}=-0.478;\qquad t_{40}=0.309;\ \text{all other } t_{LM}=0.^{14} \qquad (13)$$

The angular distribution for decay (1) is [$\mathrm{Tr}\rho_{(J)}$ being 1][15]

$$I(\theta)=\mathrm{Tr}\rho_{(3/2)}=\sum_{L_e}^{4} C_L t_{L0} Y_{L0}(\theta)\,, \qquad (14)$$

[8] There is one special case when these are indistinguishable: when the spin J and the partial amplitudes are such that $|e\mathfrak{M}_+{}^l|^2=|f\mathfrak{M}_+{}^{l'}T_{20}{}^{-1}|^2(3/7)$, the $\mathfrak{M}_-$ terms give incoherent contributions proportional to the identity and thus similar to $\mathfrak{M}_+$ contributions. (Interference terms from $\mathfrak{M}_-$ and $\mathfrak{M}_+$ are always similar.)

[9] Any rotation operator is unitary; the parity operators here are not. For the special case of Ref. 8, a unitary combination of parity operators exists, but is not equivalent to any $R_p(\phi)$.

[10] A "formation" resonance is an s-channel resonance involving all particles produced. See Ref. 17.

[11] S. M. Berman and M. Jacob, Stanford Linear Accelerator Center Report No. SLAC-43, 1965 (unpublished); C. Zemach, Ref. 5.

[12] N. Byers and S. Fenster, Phys. Rev. Letters **11**, 52 (1963). The $\langle T_{LM}\rangle$ are referred to by Byers and Fenster as "multipole parameters." All $\langle T_{LM}\rangle$ with $0\leqslant L\leqslant 2J$ and $-L\leqslant M\leqslant L$ are allowed. Extensive discussion of the T_{LM} tensors and the Byers-Fenster method are given by J. D. Jackson (see Ref. 19) and by R. H. Dalitz [lectures given at Varenna, 1964, Clarendon Laboratory, Oxford (unpublished)]. (The latter also discusses the $\boldsymbol{\sigma}\cdot\hat{p}$ operator briefly.)

[13] The incident-beam direction is the only possible choice in a "formation" experiment because the decay must be referred to axes from a prior system.

[14] These are formed by taking

$$\mathrm{Tr}[\rho_{(J)}T_{LM}]=\tfrac{1}{2}[(T_{LM})_{1/2,1/2}+(T_{LM})_{-1/2,-1/2}],$$

where $(T_{LM})_{mm'}\equiv C(JLJ;m'M)$ with $m-m'=M$.

[15] Cf. Eq. (16) of Ref. 7. The values of the $n_{L0}{}^{(1)}$ coefficients required for $J=\frac{5}{2}$ are $n_{00}{}^{(1)}=(4\pi)^{-1/2}$, $n_{20}{}^{(1)}=-1.07(4\pi)^{-1/2}$, and $n_{40}{}^{(1)}=0.925(4\pi)^{-1/2}$.

where each C_L is a function of $|a|^2$, $|c|^2$, and $2\,\mathrm{Re}a^*c$ or $|b|^2$, $|d|^2$, and $2\,\mathrm{Re}b^*d$. With the three C_L from $I(\theta)$ data of a $J=\frac{5}{2}$ formation resonance, amplitude solutions can be found for either parity.

If some estimate of $|c|$ (or $|d|$) relative to $|a|$ (or $|b|$) can be made, however, a parity determination may be possible. Equation (16) of Ref. 7 with $c, d=0$ and $J=\frac{5}{2}$ yields the production distributions presented by Minami[16,17]:

$$I_+(\theta)=\tfrac{1}{2}[1+0.800P_2(\cos\theta)], \quad (15)$$

$$I_-(\theta)=\tfrac{1}{2}[1+0.409P_2(\cos\theta)-0.976P_4(\cos\theta)]. \quad (16)$$

Decay (2) can be analyzed for F_J parity information. The distribution of $\hat{F}_{1/2}$ (in $F_{3/2}$'s rest frame)$\cdot\hat{F}_{3/2}$ (in the resonance rest frame) will have the form[18]

$$\mathscr{I}(\theta,\psi)\propto I(\theta)[1-\langle T_{20}\rangle(\theta)(\sqrt{5})P_2(\cos\psi)], \quad (17)$$

with $\cos\psi=\hat{F}_{1/2}\cdot\hat{F}_{3/2}$. If the θ of decay (1) and the higher l' wave are ignored [Eqs. (22) and (23) of Ref. 7], then

$$\begin{aligned}\mathscr{I}_+'(\psi)&\propto\{1+[(2J-3)/4J]P_2(\cos\psi)\}\\ \mathscr{I}_-'(\psi)&\propto\{1-[(2J+5)/(4J+4)]P_2(\cos\psi)\};\end{aligned} \quad (18)$$

for $J=\frac{5}{2}$ these equations are[19]:

$$\mathscr{I}_+'(\psi)\propto[1+0.200P_2] \quad\text{and}\quad \mathscr{I}_-'(\psi)\propto[1-0.714P_2]. \quad (19)$$

Transformations of $\langle T_{2m}\rangle$ along $\hat{F}_{3/2}$ to $\langle T_{20}\rangle$ along other axes give different P_2 coefficients.[20] With the incident beam as polar axis, these are 0.800 and -0.114 for even and odd parity, respectively.[19] With the production normal as polar axis, these coefficients become -0.700 and 0.786 for even and odd parity.[21] (Some caution should be exercised in interpreting average $F_{3/2}$ alignment along a single axis if the formation resonance has any background.) A *complete* analysis is unaffected by the choice of coordinates.

COMPLETE PARITY TESTS FOR F_J(FORMATION) $\rightarrow F_{3/2}$

The above tests [Eqs. (14) and (19)] treat only two "profiles" of a probability distribution. A complete analysis of the distribution involves the full examination of decay (2) for each θ interval in decay (1).

The following [from Eq. (19), Ref. 7] give the expected θ dependence of the $F_{3/2}$'s (real) second-rank tensor polarizations[22]: [The first- and third-rank polarizations are not observable in decay (2).]

$$\begin{aligned}
I\langle T_{20}\rangle&=\mathrm{Tr}[\rho_{(3/2)}T_{20}]=2\pi(\tfrac{1}{5})^{1/2}\\
&\quad\times\sum_{L_e}^{2J-1}[2A_3{}^2n_{L0}{}^{(3)}-2A_1{}^2n_{L0}{}^{(1)}]t_{L0}Y_{L0}(\theta),\\
I\langle T_{21}\rangle&=2\pi(\tfrac{2}{5})^{1/2}\sum_{L_e}^{2J-1}(-A_1A_3{}^*-A_{-3}A_{-1}{}^*)\\
&\qquad\times n_{L1}{}^{(3)}t_{L0}D_{01}{}^L(0,\theta,0),\\
I\langle T_{22}\rangle&=2\pi(\tfrac{2}{5})^{1/2}\sum_{L_e}^{2J-1}(A_{-1}A_3{}^*+A_{-3}A_1{}^*)\\
&\qquad\times n_{L2}{}^{(3)}t_{L0}D_{02}{}^L(0,\theta,0),\\
I\langle T_{l,-m}\rangle&=(-)^mI\langle T_{l,m}\rangle^*=(-)^mI\langle T_{l,m}\rangle.
\end{aligned} \quad (20)$$

For $J=\frac{5}{2}$, the first of these becomes

$$I\langle T_{20}\rangle=\frac{2\pi}{\sqrt{5}}\sum_{L_e}^{4}\left\{\begin{matrix}[(2a^2+3c^2+(2\sqrt{6})\,\mathrm{Re}a^*c)(1-\tfrac{1}{8}L(L+1))-(3a^2+2c^2-(2\sqrt{6})\,\mathrm{Re}a^*c)](\tfrac{1}{5})\\ \text{or}\\ [(6b^2+d^2+(2\sqrt{6})\,\mathrm{Re}b^*d)(1-\tfrac{1}{8}L(L+1))-(b^2+6d^2-(2\sqrt{6})\,\mathrm{Re}b^*d)](1/7)\end{matrix}\right\}n_{L0}{}^{(1)}t_{L0}Y_{L0}(\theta). \quad (21)$$

In these equations, amplitudes have been abbreviated (A_3 instead of $A_{3/2}$, and A^2 instead of $|A|^2$); and $D_{0M'}{}^L$ has replaced $[(2L+1)/4\pi]^{1/2}\mathfrak{D}_{0M'}{}^L$.

The analysis of the above tensor polarizations may be made by comparing the data with[7]

[16] S. Minami, Nuovo Cimento 31, 258 (1964).

[17] R. W. Birge, R. P. Ely, G. E. Kalmus, A. Kernan, J. Louie, J. S. Sahouria, and W. M. Smart, Proceedings of the Athens Topical Conference on Recently Discovered Resonant Particles, June, 1965 (unpublished); R. Armenteros, M. Ferro-Luzzi, D. W. G. Leith, R. Levi-Setti, A. Minten, R. D. Tripp, H. Filthuth, V. Hepp, E. Kluge, H. Schneider, R. Barloutaud, P. Granet, J. Meyer, and J.-P. Porte, Phys. Letters 19, 338 (1965); also R. B. Bell, R. W. Birge, Y.-L. Pan, and R. T. Pu, Phys. Rev. Letters 16, 203 (1966).

A brief reanalysis of CERN (Armenteros *et al.*) data has recently appeared; this takes account of higher l waves for just the two distributions examined by experimenters. [G. F. Wolters and D. J. Holthuizen, Phys. Letters 19, 701 (1966)].

[18] The customary Byers-Fenster distribution for decay into $F_{1/2}$ yields the expression in brackets. Here the notation $\langle T_{lm}\rangle$ is reserved for $F_{3/2}$ and t_{LM} for F_J.

[19] Cf. J. D. Jackson, in *Grenoble Université, Ecole d'été de physique théorie, Les Houches*, edited by C. DeWitt and M. Jacob (Gordon and Breach, Science Publishers, Inc., New York, 1965), p. 325; C. Zemach also presents the $\hat{F}_{1/2}\cdot\hat{F}_{3/2}$ distribution (Ref. 5).

[20] The T_{LM} transform according to

$$RT_{LM}R^{-1}=\sum_{M'}\mathfrak{D}_{M'M}{}^L(\alpha,\beta,\gamma)T_{LM'},$$

where R is the rotation operator and α, β, and γ are the Euler angles.

[21] A simple method is to retain the usual $\hat{z}=\hat{F}_{3/2}$ representation and to calculate the expectation value, $\mathrm{Tr}[\rho_{(3/2)}T_{20}]$, of

$$T_{20}(\hat{n})=T_{20}(\hat{y})=(1/3\sqrt{5})(3S_y{}^2-S^2)$$

$$=\frac{1}{2\sqrt{5}}\begin{pmatrix}-1&0&-\sqrt{3}&0\\0&1&0&-\sqrt{3}\\-\sqrt{3}&0&1&0\\0&-\sqrt{3}&0&-1\end{pmatrix}.$$

Alignment along the normal was first calculated by R. Barloutaud and R. D. Tripp and was presented in Armenteros *et al.*, Ref. 17.

[22] The $n_{L0}{}^{(3)}$, $n_{L1}{}^{(3)}$, and $n_{L2}{}^{(3)}$ follow from Eqs. (43), (45), and (46) of Ref. 7.

$$\mathcal{I}(\theta;\psi,\zeta)=(1/4\pi)I(\theta)\{1-\langle T_{20}\rangle(\theta)\sqrt{5}(3\cos^2\psi-1)/2 +2(15/2)^{1/2}\,\mathrm{Re}\langle T_{21}\rangle(\theta)\cos\zeta\sin\psi\cos\psi -(15/2)^{1/2}\,\mathrm{Re}\langle T_{22}\rangle(\theta)\cos2\zeta\sin^2\psi\}\,. \quad (22)$$

Histograms of $I(\theta)$ and $I\langle T_{2m}\rangle(\theta)$ may be compared with the following expressions:

$$I(\theta)=\sum_{L_e}\sigma_L Y_{L0}(\theta)(4\pi)^{1/2},$$

$$I\langle T_{20}\rangle(\theta)=\sum_{L_e}\tau_L Y_{L0}(\theta)(4\pi)^{1/2},$$

$$I\langle T_{21}\rangle(\theta)=\sum_{L_e}\mu_L' D_{01}{}^L(0,\theta,0)=\sum_{L_e}\mu_L Y_{L1}(\theta,0)(4\pi)^{1/2},$$

$$I\langle T_{22}\rangle(\theta)=\sum_{L_e}\nu_L' D_{02}{}^L(0,\theta,0)=\sum_{L_e}\nu_L Y_{L2}(\theta,0)(4\pi)^{1/2}. \quad (23)$$

The coefficients σ_L, τ_L, μ_L, and ν_L depend on spin, parity, and amplitudes. They are given in Table I for $J=\frac{5}{2}$ decay with the higher l' amplitude neglected. Figure 1 displays $I\langle T_{21}\rangle(\theta)$ and $I\langle T_{22}\rangle(\theta)$ for $J=\frac{5}{2}$ and $\frac{3}{2}$.

After analyzing the data for the $I\langle T_{2m}\rangle(\theta)$, one may evaluate parity (and spin) by taking a ratio of certain moments.[23] The following is valid *with any amount* of higher l' wave:

$$I\langle T_{22}\rangle\text{moment}/I\langle T_{21}\rangle\text{moment} =\langle\langle T_{22}\rangle D_{02}{}^{L*}\rangle/\langle\langle T_{21}\rangle D_{01}{}^{L*}\rangle =\nu_L/(-\mu_L)=\Gamma(J+\tfrac{1}{2})/[(L+2)(L-1)]^{1/2}, \quad (24)$$

where $\Gamma=+1$ or -1 for "even" ($\frac{3}{2}^-$, $\frac{5}{2}^+$, etc.) or "odd" parity, respectively. [Equation (24) is similar to Eq. (31) of Ref. 7.] If $J=\frac{5}{2}$, two independent tests are possible (for $L=2$ and $L=4$).

Parity tests may be possible in decay (3) of the formation-resonance decay scheme. The odd-l polarizations resulting from the *formation*-resonance decay, $F_J\rightarrow F_{3/2}$, are

$$I\langle T_{10}\rangle=I\langle T_{30}\rangle=I\langle T_{33}\rangle=0\,,$$

$$I\langle T_{11}\rangle=-2\pi(2/15)^{1/2}\sum_{L_e}^{2J-1}(A_1A_3{}^*-A_{-3}A_{-1}{}^*)(\sqrt{3}) \times n_{L1}{}^{(3)}t_{L0}D_{01}{}^L(0,\theta,0)\,, \quad (25)$$

$$I\langle T_{31}\rangle=4\pi(1/35)^{1/2}\sum_{L_e}^{2J-1}(-A_1A_3{}^*+A_{-3}A_{-1}{}^*) \times n_{L1}{}^{(3)}t_{L0}D_{01}{}^L(0,\theta,0)\,,$$

$$I\langle T_{32}\rangle=2\pi(2/7)^{1/2}\sum_{L_e}^{2J-1}(A_{-1}A_3{}^*-A_{-3}A_1{}^*) \times n_{L2}{}^{(3)}t_{L0}D_{02}{}^L(0,\theta,0)\,.$$

These reduce to expressions proportional to $\mathrm{Im}a^*c$ or $\mathrm{Im}b^*d$. A ratio of an $I\langle T_{32}\rangle$ moment (for $L=2,4\cdots$) to either an $I\langle T_{11}\rangle$ or an $I\langle T_{31}\rangle$ moment may yield parity (and spin) information.

[23] The "moment" of a distribution of defined as the coefficient of some orthonormal function.

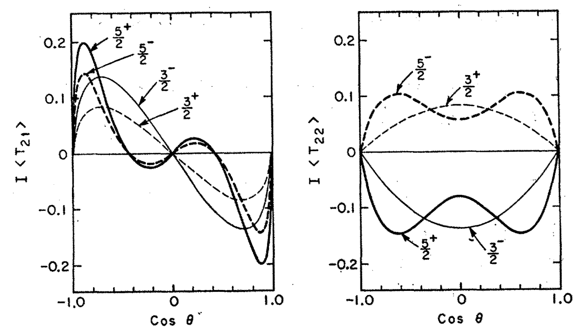

FIG. 1. Tensor polarization components of $F_{3/2}$ resulting from the decay F_J (formation resonance) $\rightarrow F_{3/2}$. The angle θ is that of the $F_{3/2}$ relative to the incident beam. The labels indicate J^P (parity relative to $F_{3/2}$) of the F_J resonance. The higher l' amplitude is neglected here. The ratio of each $I\langle T_{22}\rangle$ moment to the corresponding $I\langle T_{21}\rangle$ moment yields $(J+\frac{1}{2})(-)^P$. (See Table I for the two coefficients or moments of each $J=\frac{5}{2}$ curve. For $J^P=\frac{3}{2}^-$, $\mu_2=0.100$ and $\nu_2=-0.100$; for $J^P=\frac{3}{2}^+$, $\mu_2=0.060$ and $\nu_2=0.060$.)

The $I\langle T_{lm}\rangle$ of Eq. (25) may be analyzed by determining the polarization of $F_{1/2}$ from the angular distribution of its weak decay. See Eq. (27) of Ref. 7 (or Addendum to University of California Radiation Laboratory No. UCRL-16857).[24]

In conclusion, the following can be said about $F_J\rightarrow F_{3/2}$ decay:

(1) A "formation" resonance generally yields considerably less spin-parity information than a "final-state" resonance. [The number of nonzero t_{LM} parameters is $\frac{1}{2}(2J+1)$ for the former, but may be $\frac{1}{2}(2J+1)^2$ for the latter.]

(2) Parity cannot be tested in (formation) decay (1) if the higher l wave is taken into account and if $J\leqslant\frac{5}{2}$.

(3) Parity analysis does not require initial-state vector polarization, as F_J alignment yields an excellent test in the strong decay (2) (even with higher l wave).

(4) Spin-parity information may be obtained from the weak decay (3), especially for the final-state resonance.

(5) If complete angular dependences of decay are investigated, it is unlikely that the spin-parity conclu-

TABLE I. Coefficients for $F_{3/2}$ distributions $J=\frac{5}{2}$, lower l wave only [Eq. (23)].

	σ_L	τ_L	μ_L	ν_L
		Even parity		
$L=0$	0.500	−0.0446	0.000	0.000
$L=2$	0.179	−0.0574	0.0685	−0.103
$L=4$	0.000	−0.0765	0.0700	−0.0496
		Odd parity		
$L=0$	0.500	0.159	0.000	0.000
$L=2$	0.0914	0.0081	0.0488	0.0732
$L=4$	−0.163	−0.0911	0.0500	0.0352

[24] Janice Button-Shafer, Lawrence Radiation Laboratory Report Addendum to UCRL-16857 (unpublished).

sions can be affected by the choice of coordinate system.

The above descriptions are complete and relativistic. For a more extensive discussion, see Ref. 24.

$F_{1/2}$ PRODUCTION FROM A POLARIZED TARGET

Invariance arguments may be used to determine parity effects in the distribution and polarization of an $F_{1/2}$ from a polarized $F_{1/2}'$ in the process[25]

$$B_0+F_{1/2}'(\text{polarized}) \rightarrow B_0'+F_{1/2}. \tag{26}$$

A simple treatment may be made in analogy to the above discussion of the decay $F_J \rightarrow F_{1/2}$.

The transition matrix for the process of Eq. (26) is

$$M_+=g+h\boldsymbol{\sigma}\cdot\hat{n}, \tag{27}$$

(where $\hat{n}$ is the normal to the production plane and g and h are complex amplitudes), if the intrinsic parity $P(F_{1/2})$ is *even* relative to $P(B_0)\times P(F_{1/2}')\times P(B_0')$. If the parity $P(F_{1/2})$ is relatively odd, then a "parity operator" $\boldsymbol{\sigma}\cdot\hat{k}$ changes M_+ to a pseudoscalar form:

$$M_-=(g+h\boldsymbol{\sigma}\cdot\hat{n})(\boldsymbol{\sigma}\cdot\hat{k}). \tag{28}$$

The vector $\hat{k}$ may be any combination of initial and final momenta in the c.m. frame.

The *angular distribution* of the outgoing $F_{1/2}$ is, with P_t defined as target polarization and $\cos\phi\equiv\hat{n}\cdot\mathbf{P}_t/P_t$,

$$\begin{aligned}I_+(\phi)&=\text{Tr}[M_+\tfrac{1}{2}(1+\mathbf{P}_t\cdot\boldsymbol{\sigma})M_+{}^\dagger]\\&=|g|^2+|h|^2+2\,\text{Re}g^*hP_t\cos\phi.\end{aligned} \tag{29}$$

In a separate experiment that produces $F_{1/2}$ from an *unpolarized* target, the cross section I_0 and polarization I_0P_{F0} are found. Thus Eq. (29) may be rewritten:

$$I_+(\phi)=I_0(1+P_{F0}P_t\cos\phi). \tag{30}$$

[25] These have been discussed with different language by S. M. Bilenky, Nuovo Cimento **10**, 1049 (1958); and A. Bohr, Nucl. Phys. **10**, 486 (1959).

If the relative $F_{1/2}$ parity is *odd* rather than even, the angular distribution becomes

$$I_-(\phi)=\text{Tr}(M_+\boldsymbol{\sigma}\cdot\hat{k}\,\rho_i-\boldsymbol{\sigma}\cdot\hat{k}M_+{}^\dagger); \tag{31}$$

but as discussed above [Eqs. (5,6)], $-i\boldsymbol{\sigma}\cdot\hat{k}=R_k(\pi)$ and thus

$$I_-(\phi)=\text{Tr}\{M_+[R(\pi)\rho_iR^{-1}(\pi)]M_+{}^\dagger\}. \tag{32}$$

This means that the $\mathbf{P}_t$ in the initial density matrix will appear to be rotated (directed along $-z$ instead of $+z$). The differential cross section becomes

$$I_-(\phi)=I_0(1-P_{F0}P_t\cos\phi). \tag{33}$$

[We check that P_{F0} has not changed: $IP_{F0-}=\text{Tr}(\boldsymbol{\sigma}\cdot\hat{n}M_+\boldsymbol{\sigma}\cdot\hat{k}\tfrac{1}{2}\boldsymbol{\sigma}\cdot\hat{k}M_+{}^\dagger)=2\,\text{Re}g^*h.$] Evidently the relative parity of $F_{1/2}$ will be manifested in the sign of the $\cos\phi$ term.[26]

The *polarization* of the outgoing $F_{1/2}$ from a polarized target depends on its relative parity. If events are selected so that the scattering normal is parallel to $\mathbf{P}_t$, then for even parity

$$\begin{aligned}I\mathbf{P}_F\cdot\hat{P}_t\equiv I\hat{P}_F\cdot\hat{z}&=\text{Tr}[\sigma_zM_+\tfrac{1}{2}(1+P_t\sigma_z)M_+{}^\dagger]\\&=I_0(P_{F0}+P_t);\end{aligned} \tag{34}$$

for odd parity,

$$\begin{aligned}I\mathbf{P}_F\cdot\hat{P}_t&=\text{Tr}[\sigma_zM_+\boldsymbol{\sigma}\cdot\hat{k}\tfrac{1}{2}(1+P_t\sigma_z)\boldsymbol{\sigma}\cdot\hat{k}M_+{}^\dagger]\\&=I_0(P_{F0}-P_t).\end{aligned} \tag{35}$$

Again the parity operator is equivalent to a rotation of the initial density matrix and this rotation causes a sign change in P_t. Thus Eqs. (34) and (35) yield a further test for the $F_{1/2}$ parity.

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[26] One could also write $M_-=(\boldsymbol{\sigma}\cdot\hat{k})(g+h\boldsymbol{\sigma}\cdot\hat{n})$. The fact that $\boldsymbol{\sigma}\cdot\hat{k}$ precedes M_+ causes I_- to have the same form as I_+, but "rotates" P_{F0} to $-2\,\text{Re}g^*h/I_0$; actually redefining M_- has changed the sign of h. Equation (33) again is obtained.