Low-Energy Theorem for Non-Abelian Compton Effect and Magnetic-Moment Sum Rules

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Low-energy theorems are derived for the scattering of isovector-vector bosons on nucleons, the external boson mass having been continued to zero. The basic technique is that invented by Low to deal with the ordinary or Abelian Compton effect. These theorems are combined with forward-dispersion relations to yield two magnetic-moment sum rules. One of these sum rules is that derived by Cabibbo and Radicati; the other—to the best of the author's knowledge—is new. Neither sum rule admits of even approximate saturation by the (3,3) resonance.

1. INTRODUCTION

TWELVE years ago Low,[1] as well as Gell-Mann and Goldberger,[2] proved the theorem that conservation of electric charge coupled with the requirement of Lorentz invariance was sufficient to determine exactly the Compton amplitude on a spin-$\frac{1}{2}$ particle, up to and including terms linear in the frequency of the incident photon. Very recently this theorem has been used by Drell and Hearn[3] to write down an amusing sum rule for the anomalous magnetic moments of spin-$\frac{1}{2}$ particles.

The object of the present paper is twofold. We first explore the possibility of generalizing Low's theorem to the case where the electromagnetic field is replaced by an isovector-vector field whose conserved source charges[4] generate the non-Abelian group $SU(2)$, in contrast to the electromagnetic case where the electric charge generates the Abelian group $U(1)$. The quanta of such a field would, in general, be characterized by a nonvanishing mass; however, the low-frequency limit can still be attained by continuing the *external masses* to zero. (With the external masses continued to zero the process under study may aptly be described as a "non-Abelian Compton effect.")

Such a field may, for example, be the Yang-Mills field.[5] The existence of very profound and hitherto unresolved problems associated with the Yang-Mills theory prompts us to state, however, that it is not our intention in this paper to get involved in the details of any gauge theory of vector bosons. With this preset limitation on our work, it is obvious that we cannot present a straight generalization of Low's work. We are able, however, to isolate a part of the scattering amplitude for which a low-energy theorem can be proved. This is the part which depends only on the conserved source currents of the field and is more carefully delineated in Sec. 2A below. It will be obvious to the reader that this part of the amplitude can exist, and our manipulations with it are valid, in a variety of theories invariant under the isotopic-spin group.

The second contribution of this paper is prompted by a question which the reader may well ask: Does this extension of Low's theorem to non-Abelian fields serve any useful purpose? We answer this question in the affirmative by deriving two sum rules for the magnetic moments of baryons, exact to all orders in the strong interactions (though only to the lowest order in the electromagnetic coupling). This derivation is rendered possible by identification of the neutral component of the conserved isotopic-spin current, within a trivial scale factor, with the isovector part of the electromagnetic current. These sum rules are

$$\frac{(1+\kappa_p-\kappa_n)^2}{4M^2}-2\left[\left(\frac{\partial G_E{}^V}{\partial q^2}\right)_{q^2=0}+\frac{1}{8M^2}\right]=\frac{1}{4\pi^2\alpha}\int_0^\infty\frac{d\omega}{\omega}[(\sigma_{3/2}-2\sigma_{1/2})_A+(\sigma_{3/2}-2\sigma_{1/2})_P],\quad(1.1)$$

and

$$\frac{1+\kappa_p-\kappa_n}{2M}=\frac{1}{4\pi^2\alpha}\int_0^\infty d\omega[(\sigma_{3/2}-2\sigma_{1/2})_A-(\sigma_{3/2}-2\sigma_{1/2})_P].\quad(1.2)$$

Here κ_p and κ_n are the anomalous magnetic moments of the proton and neutron, respectively (in units of nucleon magnetons), $G_E{}^V$ is the electric-isovector Sachs form factor, $\alpha^{-1}=137$, and $\sigma_{3/2}$ and $\sigma_{1/2}$ are the cross sections for the absorption of an isovector photon on a proton in isotopic spin states $\frac{3}{2}$ and $\frac{1}{2}$, respectively. The suffixes A and P specify the helicity of the incident photon with respect to the spin of the target proton, A for antiparallel and P for parallel.

The first of the above sum rules is readily recognizable as the sum rule of Cabibbo and Radicati,[6] which may now be said to be liberated from the infinite-momentum limiting procedure on which the deduction of Cabibbo

[1] F. E. Low, Phys. Rev. 96, 1428 (1954).

[2] M. Gell-Mann and M. L. Goldberger, Phys. Rev. 96, 1433 (1954). These authors use standard Ward-Takahashi differentiation techniques, which have a tendency to become extremely cumbersome. We prefer the technique invented by Low.

[3] S. D. Drell and A. C. Hearn, Phys. Rev. Letters 16, 908 (1966).

[4] These are spatial integrals of the temporal components of the solenoidal isospin current of hadrons.

[5] C. N. Yang and R. L. Mills, Phys. Rev. 96, 191 (1954).

[6] N. Cabibbo and L. A. Radicati, Phys. Letters 19, 697 (1966).

and Radicati was predicated. The second sum rule, to the best of our knowledge, is new.

Questions pertaining to the convergence of the second sum rule, as well as detailed comparison with experiment, lie beyond the proper scope of this paper. While we hope to return to these questions in a later paper, we feel obliged to point out that the absence of an energy denominator in the integrand does not necessarily imply that the sum rule diverges. The integrand is the difference of two cross sections which are expected to approach each other, each of which being expected to approach zero[7] as $\omega\rightarrow\infty$. The integrand might well approach zero faster than $1/\omega$ and render the sum rule convergent. A quantitative estimation is possible within the framework of a "Regge-pole model" and will be undertaken in the sequel to this paper, as promised above.

2. THE LOW-ENERGY THEOREM

A. Preliminary Considerations

We focus our attention on the retarded part of the collision amplitude for scattering of vector bosons on nucleons:

$$R=(\epsilon_f^{\mu})^*(\zeta_f^{\alpha})^*R_{\mu\nu}{}^{\alpha\beta}\epsilon_i^{\nu}\zeta_i^{\beta}, \qquad (2.1)$$

where ϵ indicates the boson polarization vector, ζ the corresponding vector in charge space; μ and ν are Minkowski indices ($=0$, 1, 2, or 3), and α, β are isotopic indices ($=1$, 2, or 3). The tensor $R_{\mu\nu}{}^{\alpha\beta}$ is defined by

$$R_{\mu\nu}{}^{\alpha\beta}(k',k)=\frac{-i}{(2\pi)^3}\frac{1}{(4k_0'k_0)^{1/2}}\int e^{i(\mathbf{k}'\cdot\mathbf{x}-\mathbf{k}\cdot\mathbf{y})}d^4x d^4y \times\theta(x_0-y_0)\langle p'|[J_\mu{}^\alpha(x),J_\nu{}^\beta(y)]|p\rangle, \qquad (2.2)$$

k' and k being the final and initial boson four-momenta, and p' and p the final and initial nucleon momenta. The J's are the conserved isospin currents which generate the vector field.

The collision amplitude differs from R in that it contains some equal-time commutators which arise in the standard Lehmann-Symanzik-Zimmermann (LSZ) reduction of the matrix element $\{\langle k',p'\,\mathrm{out}|k,p\,\mathrm{in}\rangle-\langle k',p'\,\mathrm{in}|k,p\,\mathrm{in}\rangle\}$. A proper investigation of these equal-time commutators takes us deeper and deeper into the most slippery details of vector-boson theories. In this paper we therefore choose to bypass these complications and study only the properties of R. It is for this object that we establish a low-energy theorem; we shall see in Secs. 4 and 5 that such a theorem may be regarded as sufficient for the applications we consider.

[7] We have in mind a generalized Pomeranchuk theorem which would forbid both spin and isospin exchange at infinite energy. See L. L. Foldy and R. F. Peierls, Phys. Rev. **130**, 1585 (1963).

In the following we shall find it convenient to introduce a tensor $M_{\mu\nu}{}^{\alpha\beta}$ defined as follows:

$$R_{\mu\nu}{}^{\alpha\beta}=[1/(2\pi)^3](4k_0'k_0)^{-1/2}(2\pi)^4 \times\delta^4(p'+k'-p-k)M_{\mu\nu}{}^{\alpha\beta}, \qquad (2.3)$$

$$M_{\mu\nu}{}^{\alpha\beta}(k',k)=-i\int e^{ik'\cdot x}d^4x\,\theta(x_0) \times\langle p'|[J_\mu{}^\alpha(x),J_\nu{}^\beta(0)]|p\rangle \qquad (2.4)$$

and shall often make use of the crossing property of the amplitude, viz.,

$$M_{\mu\nu}{}^{\alpha\beta}(k',k)=\epsilon M_{\nu\mu}{}^{\beta\alpha}(-k,-k'). \qquad (2.5)$$

$\epsilon=+1$ for the dispersive part, $\epsilon=-1$ for the absorptive part.

We shall also need the equal-time commutation relations of the current operators.

$$[J_0{}^\alpha(x),J_0{}^\beta(y)]_{x_0=y_0}=i\epsilon^{\alpha\beta\gamma}J_0{}^\gamma(x)\delta^3(x-y), \qquad (2.6)$$

$$[J_0{}^\alpha(x),J_n{}^\beta(y)]_{x_0=y_0}=i\epsilon^{\alpha\beta\gamma}J_n{}^\gamma(x)\delta^3(x-y) +\Omega_n{}^{\alpha\beta}(x,y). \qquad (2.7)$$

The second term on the right-hand side of Eq. (2.7) is the now familiar Schwinger term,[8] $\Omega_n{}^{\alpha\beta}(x,y)$ being an operator involving gradients of delta functions, but whose precise form is not always known. Adler and Callan[9] have pointed out that within the framework of a quark model, or the σ model of Gell-Mann and Lévy, $\Omega^{\alpha\beta}$ is symmetric in α and β. Proceeding on the assumption that commutators gleaned from such models may represent a higher degree of truth than the models themselves, Adler and Callan conjecture that in the real world one may indeed have

$$\Omega_n{}^{\alpha\beta}(x,y)=\Omega_n{}^{\beta\alpha}(x,y). \qquad (2.8)$$

In the rest of this section we shall accept Eq. (2.8) as correct,[10] relegating to Sec. 5 a discussion of the extent to which our results depend on (2.8).

Finally we express M in terms of its isotopic projections

$$M_{\mu\nu}{}^{\alpha\beta}=M_{\mu\nu}{}^{(\alpha\beta)}+M_{\mu\nu}{}^{(\alpha,\beta)}, \qquad (2.9)$$

where[11]

$$M_{\mu\nu}{}^{(\alpha\beta)}=\delta^{\alpha\beta}S_{\mu\nu}, \qquad (2.10)$$

$$M_{\mu\nu}{}^{(\alpha,\beta)}=\tfrac{1}{2}[\tau^\alpha,\tau^\beta]A_{\mu\nu}, \qquad (2.11)$$

$S_{\mu\nu}$ and $A_{\mu\nu}$ being the charge-symmetric and charge-antisymmetric parts of the amplitude. We shall establish

[8] J. Schwinger, Phys. Rev. Letters **3**, 296 (1959).

[9] S. L. Adler and C. G. Callan (unpublished). See also K. Johnson and F. E. Low, Progr. Theoret. Phys. (Kyoto) (to be published).

[10] We feel it necessary to point out that Eq. (2.8) is not guaranteed within the framework of the Yang-Mills theory, and also record our view that this difficulty need not be taken seriously.

[11] In writing down equations such as (2.10) and (2.11) it is understood that the right-hand side is sandwiched between the appropriate nucleon spinors. This convention will be followed not only in charge space but also in spin space.

a low-energy theorem for the charge-antisymmetric part of the amplitude only.

B. Explicit Results

Equations (2.4)–(2.8) imply the divergence conditions

$$k'^{\mu}M_{\mu\nu}{}^{(\alpha,\beta)}=i\epsilon^{\alpha\beta\gamma}\langle p'|J_{\nu}{}^{\gamma}(0)|p\rangle, \quad (2.12)$$

$$M_{\mu\nu}{}^{(\alpha,\beta)}k^{\nu}=i\epsilon^{\alpha\beta\gamma}\langle p'|J_{\mu}{}^{\gamma}(0)|p\rangle, \quad (2.13)$$

$$k'^{\mu}M_{\mu\nu}{}^{(\alpha,\beta)}k^{\nu}=\tfrac{1}{2}i\epsilon^{\alpha\beta\gamma}(k'+k)^{\lambda}\langle p'|J_{\lambda}{}^{\gamma}(0)|p\rangle. \quad (2.14)$$

Following Low,[1] we note that since $M_{00}{}^{(\alpha,\beta)}$ involves only matrix elements of charge densities whose spatial integrals are constants of the motion, we can evaluate $M_{00}{}^{(\alpha,\beta)}$ exactly up to, and including, terms linear in the frequency by retaining only unexcited intermediate states on the right-hand side of Eq. (2.4). Of course this evaluation is rendered easy by the fact that the spectrum includes only one such state, namely, the one-nucleon state. Once we have determined $M_{00}{}^{(\alpha,\beta)}$, we can evaluate the quantity $k_m'M_{mn}{}^{(\alpha,\beta)}k_n$ through the identity

$$k_m'M_{mn}{}^{(\alpha,\beta)}k_n=k_0'M_{00}{}^{(\alpha,\beta)}k_0+k'^{\mu}M_{\mu\nu}{}^{(\alpha,\beta)}k^{\nu}-k_0'M_{0\nu}{}^{(\alpha,\beta)}k^{\nu}-k'^{\mu}M_{\mu0}{}^{(\alpha,\beta)}k_0 \quad (2.15)$$

(m, n are 3-space indices) or, alternatively,

$$k_m'A_{mn}k_n=k_0'A_{00}k_0+k'^{\mu}A_{\mu\nu}k^{\nu}-k_0'A_{0\nu}k^{\nu}-k'^{\mu}A_{\mu0}k_0. \quad (2.16)$$

Equation (2.16) may be used to determine A_{mn} to the requisite order. We can write

$$A_{mn}=A_{mn}{}^{(u)}+A_{mn}{}^{(e)}, \quad (2.17)$$

where the superscripts specify the parts of A_{mn} which emerge from unexcited and excited intermediate states on the right-hand side of Eq. (2.4). By constructing a complete set of (true) tensors that can be constructed from the vectors $\boldsymbol{\sigma}$, $\mathbf{k}'$, and $\mathbf{k}$, and taking account of the fact that $A_{mn}{}^{(e)}$ is free of singularities in the zero-frequency limit, one can convince oneself that $A_{mn}{}^{(e)}$ may be written as[11]

$$A_{mn}{}^{(e)}=A_1{}^{(e)}\delta_{mn}+\tfrac{1}{2}A_2{}^{(e)}[\sigma_m,\sigma_n]+O(k_0^2). \quad (2.18)$$

Since $A_{mn}{}^{(u)}$ admits of direct evaluation, we can combine Eqs. (2.12)–(2.14) and (2.16)–(2.18) to uniquely determine $A_{mn}{}^{(e)}$, and hence A_{mn}.

Without further ado then, we quote the explicit results. All our results are in the laboratory frame, $\mathbf{p}=0$.

$$(2\pi)^3A_{00}=\frac{1}{4}\left(\frac{1}{k_0'}+\frac{1}{k_0}\right)-\frac{\mathbf{k}'\cdot\mathbf{k}}{4M^2k_0}+\frac{k_0}{4M^2}-k_0\left(\frac{\partial G_E{}^V(Z)}{\partial Z}\right)_{Z=0}+O(k_0^2), \quad (2.19)$$

$$(2\pi)^3A_{mn}{}^{(u)}=a_{mn}{}^{(u)}(k',k)-a_{nm}{}^{(u)}(-k,-k'), \quad (2.20)$$

$$a_{mn}{}^{(u)}(k',k)=(16M^2k_0)^{-1}[(2\mathbf{k}-\mathbf{k}')_m-(1+\kappa_p-\kappa_n)i(\mathbf{k}'\times\boldsymbol{\sigma})_m][(\mathbf{k})_n+(1+\kappa_p-\kappa_n)i(\mathbf{k}\times\boldsymbol{\sigma})_n]+O(k_0^2), \quad (2.21)$$

$$(2\pi)^3A_1{}^{(e)}=-k_0\left[\left(\frac{\partial G_E{}^V(Z)}{\partial Z}\right)_{Z=0}+\frac{1}{8M^2}\right]+O(k_0^2), \quad (2.22)$$

$$(2\pi)^3A_2{}^{(e)}=(1+\kappa_p-\kappa_n)/4M+O(k_0^2). \quad (2.23)$$

Equations (2.19)–(2.23) exhaust the content of the low-energy theorem we set out to prove. For forward scattering of (massless) vector bosons with purely spatial polarizations (i.e. $\epsilon_f{}^0=\epsilon_i{}^0=0$) these equations may be put together to yield the neat result

$$(2\pi)^3(\epsilon_f{}^m)^*M_{mn}{}^{(\alpha,\beta)}\epsilon_i{}^n=\tfrac{1}{2}[\tau^\alpha,\tau^\beta]\left[\boldsymbol{\varepsilon}_f{}^*\cdot\boldsymbol{\varepsilon}_i\left\{\frac{1}{2}\left(\frac{1+\kappa_p-\kappa_n}{2M}\right)^2-\left(\frac{\partial G_E{}^V(Z)}{\partial Z}\right)_{Z=0}-\frac{1}{8M^2}\right\}k_0+i\boldsymbol{\sigma}\cdot(\boldsymbol{\varepsilon}_f{}^*\times\boldsymbol{\varepsilon}_i)\frac{1+\kappa_p-\kappa_n}{4M}\right]+O(k_0^2). \quad (2.24)$$

3. DISPERSION RELATIONS FOR FORWARD SCATTERING OF MASSLESS ISOVECTOR-VECTOR BOSONS

In this section we write down, without supplying any proofs, dispersion relations for forward scattering of massless isovector-vector bosons on nucleons. A heuristic proof would merely be a repetition of the argument advanced for photons by Gell-Mann, Goldberger, and Thirring[12].

For purely spatial polarization of the vector bosons, the retarded forward amplitude may be exhibited in full generality as

$$(2\pi)^3M_{mn}{}^{\alpha\beta}=\delta^{\alpha\beta}\{S_1(\omega)\delta_{mn}+S_2(\omega)\tfrac{1}{2}[\sigma_m,\sigma_n]\}+\tfrac{1}{2}[\tau^\alpha,\tau^\beta]\{A_1(\omega)\delta_{mn}+A_2(\omega)\tfrac{1}{2}[\sigma_m,\sigma_n]\}, \quad (3.1)$$

[12] M. Gell-Mann, M. L. Goldberger, and W. Thirring, Phys. Rev. 95, 1612 (1954).

where $\omega \equiv k_0$. The four amplitudes in Eq. (3.1) satisfy the crossing relations

$$S_1(\omega)=S_1(-\omega)=S_1^*(-\omega^*)\,, \tag{3.2}$$

$$S_2(\omega)=-S_2(-\omega)=-S_2^*(-\omega^*)\,, \tag{3.3}$$

$$A_1(\omega)=-A_1(-\omega)=-A_1^*(-\omega^*)\,, \tag{3.4}$$

$$A_2(\omega)=A_2(-\omega)=A_2^*(-\omega^*)\,. \tag{3.5}$$

The dispersion relations, if valid without subtractions, therefore read as follows:

$$S_1(\omega)=\frac{2}{\pi}\int_0^\infty \frac{\mathrm{Im}S_1(\omega')\omega' d\omega'}{\omega'^2-\omega^2-i\epsilon}=\frac{1}{\pi}\int_0^\infty \frac{\sigma(\omega')_A+\sigma(\omega')_P}{\omega'^2-\omega^2-i\epsilon}\omega'^2 d\omega'\,, \tag{3.6}$$

$$S_2(\omega)=\frac{2\omega}{\pi}\int_0^\infty \frac{\mathrm{Im}S_2(\omega')d\omega'}{\omega'^2-\omega^2-i\epsilon}=\frac{\omega}{\pi}\int_0^\infty \frac{\sigma(\omega')_A-\sigma(\omega')_P}{\omega'^2-\omega^2-i\epsilon}\omega' d\omega'\,, \tag{3.7}$$

$$A_1(\omega)=\frac{2\omega}{\pi}\int_0^\infty \frac{\mathrm{Im}A_1(\omega')d\omega'}{\omega'^2-\omega^2-i\epsilon}=\frac{\omega}{2\pi}\int_0^\infty \frac{[\sigma_{3/2}(\omega')-2\sigma_{1/2}(\omega')]_A+[\sigma_{3/2}(\omega')-2\sigma_{1/2}(\omega')]_P}{\omega'^2-\omega^2-i\epsilon}\omega' d\omega'\,, \tag{3.8}$$

$$A_2(\omega)=\frac{2}{\pi}\int_0^\infty \frac{\mathrm{Im}A_2(\omega')\omega' d\omega'}{\omega'^2-\omega^2-i\epsilon}=\frac{1}{2\pi}\int_0^\infty \frac{[\sigma_{3/2}(\omega')-2\sigma_{1/2}(\omega')]_A-[\sigma_{3/2}(\omega')-2\sigma_{1/2}(\omega')]_P}{\omega'^2-\omega^2-i\epsilon}\omega'^2 d\omega'\,. \tag{3.9}$$

Here $\sigma(\omega)$ is the total cross section for absorption of a circularly polarized neutral boson on a proton, $\sigma_{3/2}(\omega)$ and $\sigma_{1/2}(\omega)$ are the partial cross sections for absorption in isospin $\frac{3}{2}$ and $\frac{1}{2}$ states, respectively; the suffixes A and P indicate whether the helicity of the incident boson is antiparallel or parallel to the spin of the target proton. In deriving Eqs. (3.6)–(3.9) we have made use of the unitarity conditions and expressed all cross sections in terms of neutral boson absorption cross sections by performing elementary isotopic rotations. Equations (3.6) and (3.7) have been written down mainly for the sake of completeness and are not used in the next section.

4. MAGNETIC-MOMENT SUM RULES

It is now a trivial matter to combine the dispersion relations of Sec. 3 with the low-energy theorem of Sec. 2 to write down sum rules. Before we write down these sum rules, we take note of the fact that all our considerations are geared to the case of massless vector bosons whose coupling to baryons is normalized by the commutation relations (2.6) to be unity at zero momentum transfer, i.e., $G_E^V(0)=1$. We now formally identify the neutral component of the conserved isospin current with the isovector part of the electromagnetic current[13] by making the scale transformation $G_E^V(0)\rightarrow e$. [Our units are such that $e^2=4\pi/137$]. It is important to bear in mind, however, that this identification is legitimate if, and only if, we are prepared to neglect all effects which arise in order e^3 in the current.

[13] The notation of Sec. 2 B tacitly assumed the proportionality of these currents.

The sum rules therefore read as follows:

$$\alpha\left[\frac{(1+\kappa_p-\kappa_n)^2}{4M^2}-2\left(\frac{\partial G_E^V(Z)}{\partial Z}\right)_{Z=0}-\frac{1}{4M^2}\right]+O(\alpha^2)=\frac{1}{4\pi^2}\int_0^\infty \frac{(\sigma_{3/2}-2\sigma_{1/2})_A+(\sigma_{3/2}-2\sigma_{1/2})_P}{\omega}d\omega\,, \tag{4.1}$$

$$\alpha\left[\frac{1+\kappa_p-\kappa_n}{2M}\right]+O(\alpha^2)=\frac{1}{4\pi^2}\int_0^\infty [(\sigma_{3/2}-2\sigma_{1/2})_A-(\sigma_{3/2}-2\sigma_{1/2})_P]d\omega\,, \tag{4.2}$$

where we have suppressed the frequency dependence of the cross sections for absorption of isovector photons.

In writing down Eqs. (4.1)–(4.2) we have assumed, of course, that the retarded amplitudes $A_1(\omega)$ and $A_2(\omega)$ satisfy unsubtracted dispersion relations. This assumption is discussed further in the next section.

It is worth noticing that neither of the above sum rules admits of even approximate saturation by the (3,3) resonance. The left-hand side of Eq. (4.1) is negative whereas the right-hand side receives a positive contribution from the (3,3) resonance; for Eq. (4.2) the signs are exactly reversed.

5. CONCLUDING REMARKS

In this section we briefly summarize some aspects of this paper and, hopefully, shed some light on the underlying assumptions.

We started with the retarded part of the collision amplitude for the scattering of an isovector-vector boson by a nucleon. We studied only this part of the amplitude to avoid getting involved with some troublesome equal-time commutators and with the hope that this part of the amplitude may satisfy unsubtracted dispersion relations. We do not know any *a priori* criteria which will tell us whether a given amplitude will satisfy an unsubtracted dispersion relation; we merely choose to use the retarded amplitude.[14]

In deriving our low-energy theorem for the charge antisymmetric part of the amplitude, we used a conjecture of Adler and Callan which, in effect, told us to ignore the Schwinger terms in the divergence of this part of the amplitude. For the charge-symmetric part these terms persist and so no theorem was derived. The question naturally arises: Can one derive any theorem without using the Adler-Callan conjecture? We have investigated this question in some detail and our answers are as follows:

For $A_1(\omega)$ a low-energy theorem always exists, and is identical to that quoted in Eq. (2.24). Thus the Cabibbo-Radicati sum rule is independent of the Adler-Callan conjecture.

For $A_2(\omega)$ the low-energy theorem uses the Adler-Callan conjecture in an essential way. Thus the sum rule (4.2), it it converges, provides a test of this conjecture.

Note added in proof. Since this paper was written, we have found a more elegant and general derivation which enables us to handle also amplitudes such as $S_2(\omega)$, and derive more sum rules. [See M. A. B. Bég, Phys. Rev. Letters **17**, 333, (1966)]. The alternate derivation given in this reference makes no appeal to the Adler-Callan conjecture, the only input being a no-substraction-ansatz. However, if one believes that the absence of Schwinger terms is a necessary condition for the validity of a no-substraction ansatz, [See, e.g., I. J. Muzinich, Phys. Rev. (to be published)], one has made indirect use of the Adler-Callan conjecture in deriving Eq. (4.2)—in accord with the last paragraph of the present paper.

[14] The authors of Ref. 3 choose to use the full amplitude. We feel that a very intriguing alternative is provided by an amplitude in which the ordered product of current operators is modified just enough to make it formally covariant. [See L. S. Brown, Phys. Rev. **150**, 1338 (1966).] Some delicate points arise in this context; we hope to report on them in the near future.

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Crossing Relations and Born Terms for the Processes: $\pi N \to \pi N$, $\pi N^* \to \pi N$, and $\pi N^* \to \pi N^*$†

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The calculation of Born terms for the three unitarily coupled reactions, $\pi N \to \pi N$, $\pi N^* \to \pi N$, and $\pi N^* \to \pi N^*$, is considered from two viewpoints: (1) the usual method involving perturbation theory and the invariant amplitudes; (2) a more general method based on the crossing relations of Trueman and Wick. Our purpose is to facilitate the inclusion of the πN^* channel in relativistic multichannel calculations in which Born terms are used, and also to give a method for calculating Born terms that is independent of perturbation theory. Using this method we obtain a simple expression for the πN-πN Born term of total (orbital) angular momentum $J(l)$ corresponding to the exchange of a baryon of arbitrary mass and spin. A few remarks regarding the removal of kinematic singularities are made.

I. INTRODUCTION

THERE exist certain general requirements or symmetries which must be reflected in the structure of any relativistic theory of strong interactions. In this category are the geometrical symmetries and crossing. For spinless particles the latter is simply the statement that there exists one analytic function which can describe any one of several different reactions according to the region in which it is evaluated. When particles with spin are considered, more than one amplitude is needed for the description of any given reaction. In this case, crossing symmetry means that there is a linear relationship between the amplitudes of one reaction and the amplitudes for a crossed reaction. The relations between two such sets of amplitudes are referred to as crossing relations.

Trueman and Wick, as well as Muzinich,[1] have recently derived crossing relations for two-body

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[1] T. L. Trueman and G. C. Wick, Ann. Phys. (N. Y.) **26**, 322 (1964); hereafter referred to as TW; see also I. J. Muzinich, J. Math. Phys. **5**, 1481 (1964).