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$SU(6)_W$ and Tensor Forces*

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The one-meson exchange potentials in MB, MM, BB, and $B\bar{B}$ states may be represented approximately by sums of central and tensor potentials, where M and B denote the meson and baryon supermultiplets of dimensions 36 and 56. This formulation leads to a clarification and generalization of a model developed recently by the author, in which the relative forces in different channels are assumed equal to the relative magnitudes of the Born-approximation scattering amplitudes at the physical threshold. It is shown that the tensor forces dominate the $SU(6)_W$-symmetric meson bootstrap model, in which the M are MM bound states. A generalization of a theorem proved previously implies that the tensor forces cancel in the MM, MB, and $B\bar{B}$ states corresponding to the larger $SU(3)$ representations, and in all BB states. Actual nucleon-nucleon forces are very sensitive to symmetry breaking; the implications of the theorem for these forces are discussed and compared with phenomenological analyses of experiment.

I. INTRODUCTION

THE first proposed applications of the group $SU(6)$ to hadron physics had the property that the three components of the total intrinsic spin vector were generators of the group. Because of this, the forces were spin-independent. Thus, the theory was in conflict with the known strong spin dependence of strong interactions. This dilemma was solved by various modifications of the $SU(6)$ theory; the solution is stated most simply in terms of the group $SU(6)_W$. The intrinsic spin components are not generators of $SU(6)_W$, and spin-dependent forces are allowed.

In this paper we will assume $SU(6)_W$-symmetric MMM and MBB interactions, where M and B denote the odd-parity mesons and the baryons, identified with the representations $35\oplus 1$ and 56. Mass differences within the M and B multiplets are neglected. We are concerned with the M exchange (one-meson exchange) forces existing in MM, MB, BB, and $B\bar{B}$ states; the vertices are evaluated in the limit of small momentum transfer between the real particles. Two types of forces exist, central and tensor forces. We will assume that the tensor forces are dominant for determining the observed spectrum of hadrons. This assumption is the opposite of the spin-independence assumption of the early $SU(6)$ theories.

One of the main purposes of the paper is to clarify and extend two recent works of the author on $SU(6)_W$-symmetric M exchange forces in the states MM.[1,2] The first work (denoted by R1) shows that the M may be bootstrapped as MM bound states, and the second (denoted by R2) discusses the even-parity meson resonances predicted by the model. The forces in R1 and R2 are assumed proportional to the threshold values of the scattering amplitudes in Born approximation. The tensor force-central force picture allows one to weaken this threshold assumption.

The forms of the potentials are listed in Sec. II. A plausibility argument for the dominance of tensor potentials is given. It is shown in Sec. III that the tensor forces do dominate in the meson bootstrap model of R1. In Sec. IV, the theorem of R2 is generalized; this theorem shows that in the limit of exact $SU(6)_W$ symmetry, the tensor forces cancel in MM, MB, and $B\bar{B}$ states corresponding to the larger $SU(3)$ representations, and in all BB states. The effect of symmetry breaking on the nuclear forces is discussed.

II. THE POTENTIALS

Three types of meson-exchange processes occur in the model, P (pseudoscalar meson) exchange, and magnetically and electrically coupled V exchange. We consider the vertex ZZM, where the real initial and final particles Z both belong either to the 56-fold baryon multiplet or the 36-fold meson multiplet. The spins of the two Z particles may be different. In the limit of small momenta of the Z, the three types of vertices may be written in the forms,

$$i(G/m)(\mathbf{S}\cdot\mathbf{q}), \tag{1a}$$

$$i(F/m)(\mathbf{S}\cdot\mathbf{q}\times\mathbf{e}), \tag{1b}$$

$$fe_0, \tag{1c}$$

where the three components of $\mathbf{S}$ are Hermitian operators connecting spin states of the initial and final Z, $\mathbf{q}$ is the momentum transfer, m is the Z mass, $\mathbf{e}$ and e_0 are the space and time components of the polarization four-vectors of the exchanged V mesons, and G, F, and f are coupling constants corresponding to P, magnetic and electric vertices.[3]

We write the P and magnetic V exchange potentials (U_P and U_m) for the process $Z_1+Z_2\rightarrow Z_1+Z_2$ in configuration space, where Z_1 and Z_2 each represents either the meson, baryon, or antibaryon multiplet. The contributions of a particular P and V meson to these

* Supported in part by the National Science Foundation.
[1] R. H. Capps, Phys. Rev. **148**, 1332 (1966). The symbol R1 will be used to refer to this paper.
[2] R. H. Capps, Phys. Rev. Letters **16**, 1066 (1966). The symbol R2 will be used to refer to this paper.

[3] The relativistic MMM interactions are listed in R1. It is easy to show that these interactions may be written in the form of the above Eqs. (1a) through (1c) in the limit of small momenta of the real mesons.

potentials are,

$$U_P=(G_1G_2/m^2)(\mathbf{S}_1\cdot\nabla)(\mathbf{S}_2\cdot\nabla)e^{-\mu r}/r\,, \qquad (2a)$$

$$U_m=(F_1F_2/m^2)(\mathbf{S}_1\times\nabla)\cdot(\mathbf{S}_2\times\nabla)e^{-\mu r}/r\,, \qquad (2b)$$

where μ is the meson mass and r is the interparticle distance. The coupling constants depend on the quantum numbers of the real particles and of the exchanged P and V mesons; the relevant indices have been suppressed. If the sum over the nine virtual P and V mesons is taken, the resulting potentials are matrices in the space of the different Z_1Z_2 channels. A positive diagonal element of the potential matrix corresponds to repulsion. Each of these potentials may be written as a sum of central and tensor parts, i.e.,

$$U_{P,m}=A_{P,m}{}^c\mathcal{V}^c+A_{P,m}{}^t\mathcal{V}^t. \qquad (3)$$

The coefficients A corresponding to particular P and V mesons are given by the equations,

$$A_P{}^c=A_P{}^t=G_1G_2\,, \qquad (4a)$$

$$A_m{}^c=2F_1F_2\,,\quad A_m{}^t=-F_1F_2\,. \qquad (4b)$$

The forms of the central and tensor potentials are,

$$\mathcal{V}^c=\tfrac{1}{3}m^{-2}(\mathbf{S}_1\cdot\mathbf{S}_2)[\mu^2r^{-1}e^{-\mu r}-4\pi\delta(\mathbf{r})]\,, \qquad (5)$$

$$\mathcal{V}^t=m^{-2}[3(\mathbf{S}_1\cdot\mathbf{r})(\mathbf{S}_2\cdot\mathbf{r})r^{-2}-\mathbf{S}_1\cdot\mathbf{S}_2]\times[r^{-2}+\mu r^{-1}+\tfrac{1}{3}\mu^2]r^{-1}e^{-\mu r}. \qquad (6)$$

We now consider V-exchange forces of the electric type. It is shown in R1 that these forces must be modified if the forward and backward one-meson exchange amplitudes are to be $SU(6)_W$-symmetric. If the static value of the vertices [Eq. (1c)] is used, this modification takes the form of a simple subtraction to the propagator. One makes the replacement,

$$(\mu^2+q^2)^{-1}\rightarrow(\mu^2+q^2)^{-1}-(\mu^2)^{-1}. \qquad (7)$$

The V-exchange potentials that result are purely central and are of the form,

$$f_1f_2[r^{-1}e^{-\mu r}-\mu^{-2}4\pi\delta(\mathbf{r})]. \qquad (8)$$

If the modification of Eq. (7) were not made, the delta function term would not occur in Eq. (8).

We now assume that the interaction constants satisfy $SU(6)_W$ symmetry.[4] The resulting symmetry of the Born-approximation amplitudes can be established by using the method of R1. In momentum space, the forward amplitudes vanish and the backward amplitudes are all proportional to the kinematic factor $q^2/(\mu^2+q^2)$. The coefficients of this factor are operators in spin space and internal space that satisfy the $SU(6)_W$ symmetry.

The delta function term in the central potential may be regarded as the static limit of a core of finite range. One cannot make an accurate evaluation of the relative strengths of the central and tensor potentials for any particular process, since the extent to which recoil effects smear out this delta function is not known. However, since the magnitude of the delta function is such that the volume integral of the central potential vanishes, it is reasonable to hypothesize that the strongest meson-exchange forces are the tensor forces.

In R1 and R2 the M exchange amplitudes were examined at the physical threshold. Those terms linear in the initial and final momentum are P-wave amplitudes, while those terms quadratic in either the initial or final momentum are linear combinations of S-wave amplitudes and S-D transition amplitudes. The potential picture of the present paper allows one to weaken the threshold assumption. The potential is static, in that the spin dependence of the vertices is evaluated at threshold. However, the meson propagators are treated relativistically in the potential method. Furthermore, the method makes possible the treatment of higher partial waves, although we discuss these waves only briefly in this paper (in Sec. V). Our assumption of dominant tensor forces implies the assumption of R2, that the strongest forces in states of even parity correspond to S-D transitions.

The static potentials can represent only a first approximation to the actual interactions. Therefore, the goals of this model are only to compare the coefficients of the potentials in different states, and to compute the relative components of different two-particle channels in the states assumed to resonate.

III. THE MESON BOOTSTRAP MODEL

In this section, we analyze the meson bootstrap model of R1 in terms of the central and tensor force picture. We review briefly the method and results of R1, in order that this section be self-contained. The 36 meson states (P and V nonets) are taken to be degenerate, and exact $SU(6)_W$ symmetry is assumed for the MMM vertices. All P-wave MM amplitudes are computed in Born approximation at the threshold energy. These amplitudes are matrices in the various PP, PV, and VV channels. The forces are assumed to be measured by the threshold amplitudes. (This is called here the threshold approximation.) The largest positive eigenvalues of the amplitude matrices (most attractive forces) correspond to a P singlet and octet and a V singlet and octet. It is assumed that these states are bound and may be identified with the 36 original M states. The assumption that the relative constants coupling different two-particle channels to a particular bound-state pole are proportional to the relative components of the channels in the eigenamplitude leads to ratios of MMM constants consistent with the $SU(6)_W$-symmetric values assumed originally.

Each threshold P-wave amplitude may be written as a sum of two terms, which result from the central and tensor forces. The amplitudes of R1 are separated into these two parts in the Appendix. The threshold ap-

[4] The $SU(6)_W$ symmetry is defined and discussed by H. J. Lipkin and S. Meshkov, Phys. Rev. Letters 14, 670 (1965); Phys. Rev. 143, 1269 (1966).

proximation is equivalent to assuming a particular relative importance of the central and tensor force parts. In order to allow a different weighting of the potentials, we present the results with the central force terms multiplied by the parameter x. It is shown in the Appendix that the eigenfunctions of the threshold amplitude matrices are independent of x. The eigenvalues are listed as functions of x in Table I. (The normalization of the amplitudes and eigenvalues is not important, and is explained in R1.) The notation $(a,b)^i$ denotes a multiplet of $SU(3)$ and spin multiplicities a and b, whose $I_z=Y=0$ members are of charge-conjugation parity i. If $x=1$, these eigenvalues are equal to those of R1.

Because of the r^{-2} and μr^{-1} terms in Eq. (6), the fractional decrease in strength of the tensor potential with increasing r is greater than that of the central potential. The threshold approximation for P waves weighs the two potentials according to the Born approximation, with the radial wave function ψ given by $\psi=Cr$, where C is a constant. If a more realistic wave function were used, the constant C would be replaced by a function that decreases with increasing r. With such a wave function the relative importance of the central potential would be smaller. If the delta function terms of the central potentials were spread out over a finite range, this would decrease the effective strength of these potentials further. (The unmodified delta function plays no role in any states other than S states.) Hence, the most appropriate value for the parameter x of Table I is between 0 and 1.

It is seen from Table I that the states $(1,1)^+$, $(8,1)^+$, $(8,3)^-$, and $(1,3)^-$, identified with the P and V nonets, correspond to the largest eigenvalues for any value of x in the range $0<x<1$. The fact that the eigenvalue corresponding to the vector singlet is less than the others is expected, because the $SU(6)_W$-symmetric direct coupling of this particle is $\frac{2}{3}$ that of the others, as discussed in R1. We conclude that the meson bootstrap model remains self-consistent if the threshold approximation is relaxed.

The above arguments show that the tensor forces dominate in this P-wave bootstrap model, since the eigenfunctions and the ordering of the eigenvalues (force strengths) are the same for pure tensor forces as for any reasonable superposition of the central-type and tensor-type amplitudes.

IV. THE TENSOR-FORCE THEOREM

It was shown in R2 that if only $SU(6)_W$-symmetric one meson exchange forces are considered, the S-D amplitudes (tensor forces) vanish for MM and MB states of certain $SU(3)$ representations. If the M and B are considered as composites of spin $\frac{1}{2}$, $SU(3)$ quarks and antiquarks, the tensor forces vanish for every state that does not contain any quark and corresponding antiquark simultaneously. The theorem depends only on the $SU(6)_W$ symmetry of the MMM and MBB vertices, and not on the validity of any quark model. The results

TABLE I. Quantum numbers of multiplets with nonzero eigenvalues of the Born-approximation amplitude matrix.

Eigenvalue	States				
$5/3+\frac{1}{3}x$	$(1,1)^+$	$(8,1)^+$	$(8,3)^-$		
$\frac{5}{6}+\frac{2}{3}x$	$(1,3)^-$				
$\frac{1}{6}+\frac{1}{3}x$	$(1,5)^+$	$(8,5)^+$	$(8,3)^-$	$(8,5)^-$	$(8,7)^-$
$-\frac{1}{6}+\frac{2}{3}x$	$(1,5)^-$				
$-\frac{5}{6}+\frac{1}{3}x$	$(1,3)^+$	$(8,3)^+$	$(8,1)^-$	$(8,3)^-$	$(8,5)^-$
$-5/3+\frac{2}{3}x$	$(1,1)^-$				

of this theorem, extended to include BB and $B\bar{B}$ states, are given in Table II.

The fact that all tensor forces cancel in BB states follows from even simpler considerations, i.e., from the fact that the spin and W-spin operators are identical for B states. We hypothesize that the experimental absence of strongly bound BB states results from the lack of tensor forces.

Nuclear forces are very sensitive to the breaking of the $SU(6)_W$ symmetry, because nucleons are coupled particularly strongly with pions, and the pion mass is much smaller than that of any of the other mesons. In order to understand the symmetry-breaking effects, we consider the nucleon-nucleon forces in more detail. It is seen from Eqs. (4a) and (4b) that if the coupling constants at the two vertices are identical, the contributions to the tensor force of P exchange and magnetic V exchange are of opposite sign. In the exact symmetry limit the tensor force contributions from π and ρ exchange cancel, and those of η, X, ω, and φ exchange cancel.

Because of the large ρ-π mass difference, the ρ-exchange contribution to the tensor force cannot cancel the long-range part of the π-exchange contribution. If the coupling constants of Eqs. (1a) through (1c) are given by $SU(6)_W$ symmetry, then the ratio of the ρ and π contributions to the tensor force approaches (-1) as r approaches zero. The prediction that the tensor forces vanish in NN states is not new; several papers give results that imply such a cancellation.[5] However, the usual analytical procedure has been to use scattering data directly to test predictions of $SU(6)_W$ symmetry (or a related symmetry). Because of the large ρ-π mass difference, this procedure is unreasonable for any phenomenon affected appreciably by the long-range part of the potential. The most one can hope for is that the

TABLE II. $SU(3)$ representations for which tensor forces may exist.

Type of state	Representations
MM	**1, 8**
MB	**1, 8, 10** (but not **10***)
$B\bar{B}$	**1, 8, 10, 10*, 27**
BB	None

[5] See, for example, P. B. Kantor, T. K. Kuo, Ronald F. Peierls, and T. L. Trueman, Phys. Rev. 140, B1008 (1965).

symmetry predictions for relative coupling constants will be satisfied approximately.

Gammel and Thaler obtained evidence for a diminution at small distances of the tensor force in the proton-proton state several years ago.[6] A more recent analysis of NN data by Bryan and Scott, in which the potential is assumed to be a sum of various one-meson exchange contributions, leads to a value of about $F^2 \sim 20$ for the magnetic ρNN coupling constant.[7] This compares favorably with the value of (~ 14) obtained from $SU(6)_W$ symmetry and the known magnitude of the πNN interaction. Analyses such as those of Ref. 7 omit the coupling of the NN channels to other channels, such as the NN^* channels, and cannot be expected to yield accurate values for the interaction constants of the heavier mesons. The important fact is that the magnetic ρNN interaction is of the same order as the πNN interaction.

If $SU(6)_W$ symmetry is approximately valid, one does not expect a large isospin-independent tensor potential at either short or long range, since the masses of the isoscalar mesons η, X, ω, and φ are of the same order. This prediction is confirmed by the recent analysis of Ball, Scotti, and Wong,[8] (as well as by some of those listed in Ref. 7). In Ref. 8, the X is neglected, the η assumed weakly coupled, and the tensor $(\sigma_{\mu\nu}q_\nu)$ interactions of the ω and φ taken as zero. The contribution to the tensor potential resulting from the η and that resulting from the Dirac (γ_μ) coupling of the ω and φ are of opposite signs, and are smaller than the π and ρ contributions to the isovector-exchange tensor potential.

We now turn to the $B\bar{B}$, MM, and BB states. In the $B\bar{B}$ states the P and V_m (magnetic V) contributions to the tensor forces are additive, rather than subtractive. However, the P and V_m contributions cancel separately for all states of the $SU(3)$ representations **64**, **35**, and **35***. In the cases of MM and MB states the P and V_m contributions cancel each other for the large representations, but not for the small representations. One may understand most of these effects by noting that the ZZP and ZZV_m interactions transform oppositely under the operation of particle-antiparticle conjugation.

In the original $SU(6)$-symmetric baryon bootstrap model, in which the B are P-wave bound states of the type BM, produced by B-exchange forces, no use was made of the group $SU(6)_W$.[9] The validity of this model requires that some combination of P mesons and magnetically coupled V mesons be coupled to the baryons, the interaction constants being proportional to the matrix elements of the axial-vector (spin one) generators of $SU(6)$. The relative contributions of the P and V_m interactions are not fixed; one may write the meson states corresponding to angular momentum one in the symbolic form $\psi = aP + (1-a^2)^{1/2}V_m$.[10] If one considers the meson-exchange potentials to BB states that result from such an interaction, the central potentials are independent of a, and lead to $SU(6)$-symmetric S-states potentials.[11] On the other hand, the tensor potentials depend on a; they vanish only if $a = (\frac{1}{3})^{1/2}$, the value prescribed by $SU(6)_W$.

V. CONCLUDING REMARKS

The assumption that the tensor forces are more important than the central forces has been shown to be valid in the meson bootstrap model based on MMM vertices. If this assumption is valid generally, the results of the model are not very sensitive to the breaking of the symmetry of the MMM and MBB vertices. In fact, the electrically coupled V mesons do not contribute to the tensor potentials, so the results of the "tensor dominant" model are not affected at all if the electric V interaction constants deviate from the values prescribed by the symmetry, or if the vertices are not modified as in Eq. (7). Generally speaking, the largest tensor potentials occur in those states in which the contributions of P exchange and magnetically coupled V exchange are of the same sign. Only in rare cases may the sign of one of these contributions be changed by the symmetry breaking.

Because of the tensor force theorem of Sec. IV, the tensor-dominant hypothesis predicts two important features of the observed hadron spectrum correctly, i.e., the lightest hadron states correspond to the smaller of the $SU(3)$ representations allowed in the octet model and to the baryon numbers 0 and ± 1.

Two-particle scattering processes at high energy and small momentum-transfer are often considered ideal phenomena for the study of $SU(6)$ and the collinear symmetry $SU(6)_W$. However, the forward amplitudes vanish in our meson exchange model. One should not be surprised if peripheral processes are not the best place to look for $SU(6)_W$ symmetry.

The meson exchange force is not the only force in hadron physics that may lead to $SU(6)_W$ symmetry in the collinear directions. It is well-known that baryon-exchange forces are crucial for P-wave MB states. If the masses of the M and B particles are μ and m, the baryon-exchange forces may be written in particularly simple forms if $(\mu/2m)$ is zero or one. Experimentally, this ratio

[6] J. L. Gammel and R. M. Thaler, Phys. Rev. **107**, 291 (1957).

[7] Ronald A. Bryan and Bruce L. Scott, Phys. Rev. **135**, B434 (1964). A similar conclusion has been reached in other analyses of this type; many of these are summarized by John W. Durso and Peter Signell, Phys. Rev. **135**, B1057 (1964). The magnetic constant F^2 quoted above is approximately equal to the constant $[g_V + (2M/m_V)f_V]^2$ of Bryan and Scott, or to the constant $(g_v + g_T)^2$ of Durso and Signell.

[8] J. S. Ball, A. Scotti and D. Y. Wong, Phys. Rev. **142**, 1000 (1966).

[9] R. H. Capps, Phys. Rev. Letters **14**, 31 (1965); J. G. Belinfante and R. E. Cutkosky, *ibid.* **14**, 33 (1965).

[10] R. H. Capps, in *Proceedings of the Third Coral Gables Conference on Symmetry Principles at High Energy, 1966* (W. H. Freeman and Company, San Francisco, California, 1966), p. 222.

[11] The hypothesis that these central potentials produce S-wave multiplets corresponding to representations of $SU(6)$ has been explored by R. H. Capps, Phys. Rev. Letters **14**, 842 (1965).

is closer to zero; this is the well-known static limit. The baryon-exchange potential cannot be written as a simple potential in configuration space. However, in the static limit, the baryon-exchange vertices may be written in the form of Eqs. (1a) through (1c) if the virtual baryons are in positive-energy states, the spin operators act in the space of the baryons, and $\mathbf{q}$ refers to the meson momentum. The force affects only P waves in this limit.

In the bootstrap model of Sec. III, the M are bound states produced by M-exchange forces in MM states. The baryons may be assumed to be MB bound states, produced mainly by B-exchange forces.[9] In the present model, the next most important forces in MM and MB states of odd orbital parity are the M-exchange tensor forces that induce P-wave–F-wave transitions. This type of force may help produce Regge recurrances for some, but not all, of the M and B particles. For example, there is a tensor force that corresponds to the antisymmetric-octet, PP-VV process, where the total intrinsic spin of the VV state is two. The P-wave–P-wave part of this force helps produce the V octet in the bootstrap model of Sec. III. This same tensor potential affects the $j=3$, P-F transition amplitude, where the PP state is the F-wave state. There may be a resonance produced in this state that is identifiable with the Regge recurrence of the V octet. On the other hand, only PV states contribute to the bound V singlet in the bootstrap model. No P-F transitions involving these states correspond to total angular momentum 3.

APPENDIX

This Appendix contains the analysis discussed in Sec. III; the tensor force and central force parts of the P-wave $MM\rightarrow MM$ amplitudes in the Born approximation at threshold are separated. For each amplitude, characterized by a particular internal [$SU(3)$] symmetry and particular values of the initial and final total intrinsic spins, there are four angular momenta. These are the initial orbital angular momentum and total intrinsic spin ($\mathbf{L}$ and $\mathbf{S}$), and the final values of these variables ($\mathbf{L}'$ and $\mathbf{S}'$). We may substitute the momenta $\mathbf{k}$ and $\mathbf{k}'$ for $\mathbf{L}$ and $\mathbf{L}'$, since we are considering P-wave amplitudes at threshold. The angular momentum analysis of these amplitudes involves coupling $\mathbf{k}$ and $\mathbf{S}$, and contracting with a similar combination of $\mathbf{k}'$ and $\mathbf{S}'$. An alternate coupling scheme involves combining $\mathbf{k}$ and $\mathbf{k}'$, and contracting with a combination of $\mathbf{S}$ and $\mathbf{S}'$. It is pointed out in R1 that no $\mathbf{k}\times\mathbf{k}'$ terms exist, so that the combination of $\mathbf{k}$ and $\mathbf{k}'$ transforms under rotations partly like a scalar and partly like a tensor. These two parts are the central and tensor force parts, respectively. The contributions of central or tensor potential to the states of different total angular momentum j are proportional to the appropriate 6-j coefficients of $SU(2)$.[12] The central potential ($\mathbf{k}\cdot\mathbf{k}'$ term) contributes only if the magnitudes of $\mathbf{S}$ and $\mathbf{S}'$ are equal, and gives a contribution that is independent of j. The separation may be made easily from the amplitudes listed in Sec. IV of R1. We list the results below.

There are no central or tensor forces contributing in the P-wave states of the $SU(3)$ representations **10**, **10***, and **27**, as pointed out in R1. We consider next the states $\mathbf{8}^+$, i.e., $SU(3)$ octet states with $I_z=Y=0$ members of even charge-conjugation parity. The states involved are VV states of total intrinsic spin one, and PV states. All states are of total intrinsic spin one, and the relative contributions of the central and tensor potentials to the amplitude matrix corresponding to a particular total angular momentum are the same for each matrix element. These relative contributions may be read off from the eigenvalues of Table I, i.e.,

$$U_1=5/3+\tfrac{1}{3}x,\quad U_3=-\tfrac{5}{6}+\tfrac{1}{3}x,\quad U_5=\tfrac{1}{6}+\tfrac{1}{3}x,$$

where the central force term is multiplied by x, and U_n is the eigenvalue corresponding to the total angular momentum multiplicity n.

The $SU(3)$ singlet cases are even simpler. All the 1^+ states are VV states of total intrinsic spin one, and all the 1^- states are PV states. In both cases the relative contributions of the central and tensor potentials may be read off from Table I.

The remaining case, the $\mathbf{8}^-$ case, is the most complicated, since it involves states of all the types (PP), (PV), $(VV)_0$, and $(VV)_2$, where the subscript is the total intrinsic spin. The PV states are of the nonet D-type, and the other states are of F-type, as discussed in R1. If the central force contributions to the Born amplitude matrices for these states [Eqs. (23) of R1] are multiplied by x, the results are,

$$j=3,\quad U(VV)_2=\tfrac{1}{6}+\tfrac{1}{3}x,$$

$$j=0,\quad U(VP)=-\tfrac{5}{6}+\tfrac{1}{3}x,$$

$$j=2,\qquad U=\begin{array}{c} \\ (VV)_2 \\ (VP) \end{array}\!\!\begin{array}{cc} (VV)_2 & (VP) \\ \left(\begin{array}{c} -7/12+\tfrac{1}{3}x \\ \; \end{array}\right. & \left.\begin{array}{c} (\tfrac{3}{16})^{1/2} \\ -\tfrac{1}{12}+\tfrac{1}{3}x \end{array}\right) \end{array},$$

$$j=1,\qquad U=\begin{array}{c} \\ (VV)_2 \\ (VV)_0 \\ (VP) \\ (PP) \end{array}\!\!\begin{array}{c} \begin{array}{cccc} (VV)_2 & (VV)_0 & (VP) & (PP) \end{array} \\ \left[\begin{array}{cccc} 7/12+\tfrac{1}{3}x & (5/36)^{1/2} & (\tfrac{15}{16})^{1/2} & (5/12)^{1/2} \\ & \tfrac{1}{12}x & 0 & (1/48)^{1/2}x \\ & & 5/12+\tfrac{1}{3}x & 0 \\ & & & \tfrac{1}{4}x \end{array}\right] \end{array}.$$

It is easy to show that the eigenfunctions of these matrices are independent of x, and that the eigenvalues are those listed in Table I.

[12] Convenient formulas for the 6-j symbols are listed by A. R. Edmonds, *Angular Momentum in Quantum Mechanics* (Princeton University Press, Princeton, New Jersey, 1957), Table V.