Particle Mixing and $J^P=2^+$ Meson Decays

R. J. RIVERS*
International Atomic Energy Agency, International Centre for Theoretical Physics, Trieste, Italy
(Received 9 May 1966; revised manuscript received 23 June 1966)

Particle mixing for meson fields of arbitrary spin is considered in a field-theoretic context. Expressions for the field renormalization constants and mixing parameters are given in terms of the physical masses, assuming Okubo-type octet breaking. Broken-$SU(3)$ coupling constants are evaluated for the 2^+ meson decays and the predicted partial widths compared with experiment. Satisfactory agreement is found.

I. INTRODUCTION

STATEMENTS about the validity of $SU(3)$ for resonance decays are statements about the validity of assuming exact $SU(3)$ coupling constants and *physical* masses for the decaying particles and their decay products. That is, the assumption is made that the $SU(3)$ symmetry-breaking interaction (neglecting electromagnetic effects) renormalizes the $SU(3)$ multiplet masses to the physical masses without renormalizing either the fields or the vertices. This is a strong assumption, but since the mass renormalization due to the $SU(3)$-breaking interactions is relatively small ($\Delta m/m\approx 10\%$), it is expected to give good qualitative results, as indeed it does.

In addition to renormalizing the fields, the $SU(3)$-breaking interactions will induce mixing between fields with the same medium-strong quantum numbers. The first aim of this paper is to obtain expressions for the field renormalization constants and mixing parameters for mesons of arbitrary spin. This we do in Sec. II by a generalization of the field-theoretic approach of Coleman and Schnitzer.[1]

In Sec. III we derive the equations to be satisfied by the mixing parameters of these meson fields of arbitrary spin. Assuming Okubo-type[2] octet breaking, we examine the consistency of these equations and their sufficiency for determining the effective parameters in the theory.

Assuming that we can neglect vertex renormalization (for which we have no method of calculation), we examine the renormalized coupling constants in Sec. IV, in particularly for the $J^P=2^+$ mesons. The relations between these coupling constants and the unrenormalized coupling constants are discussed and the predicted partial decay widths are compared with those of the unrenormalized $SU(3)$ and with experiment.

II. RENORMALIZATION AND MIXING

Let us consider meson fields of arbitrary spin l described by tensor fields $A^i{}_{(\mu)}(x)$ [where i is an $SU(3)$ index and (μ) is an abbreviation for $\mu_1\mu_2\cdots\mu_l$]. The matrix propagator is of the form

$$\langle 0|T(A^i{}_{(\mu)}(x)A^j{}_{(\nu)}(y))|0\rangle = -i\int d^4p(2\pi)^{-4}e^{-ip\cdot(x-y)}\Delta^{ij}{}_{(\mu)(\nu)}(p^2)\,, \quad (2.1)$$

where $\Delta^{ij}{}_{(\mu)(\nu)}(p^2)$ has a spectral representation of the form

$$\Delta^{ij}{}_{(\mu)(\nu)}(p^2)=\int dm^2(p^2-m^2+i\epsilon)^{-1} \times\{(-1)^l P_{l(\mu)(\nu)}(m^2)\rho_l{}^{ij}(m^2) +\sum_{r=0}^{l-1}P_{r(\mu)(\nu)}(m^2)\rho_r{}^{ij}(m^2) +\sum_{r=1}^{L}T_{r(\mu)(\nu)}(m^2)\sigma_r{}^{ij}(m^2)\}\,. \quad (2.2)$$

All operators are symmetric, $P_l(p^2)$ being the normal spin-l projection operator and $P_r(p^2)$, $(r=0, 1, \cdots, l-1)$ being spin-r projection operators in $2l$ indices associated with the divergence of the fields. The projection operators $T_r(p^2)$ [$L=\frac{1}{4}l^2$ if l is even and $\frac{1}{4}(l^2-1)$ if l is odd] are constructed from trace operators and spin projection operators and are associated with the traces of the fields.[3] That is, if we have current conservation the spectral functions ρ_r $(r=0, 1, \cdots, l-1)$ and all but one σ_r are identically zero, whereas current tracelessness requires all the σ_r to be identically zero. We do not assume either divergence-free or traceless currents.

The major assumption that we make is that the $SU(3)$-breaking interaction leaves unaffected the vacuum expectation values of the meson-field equal-time commutators and their first time derivatives.

Let the unperturbed spectral functions be $\rho_r{}^0$ $(r=0, 1, \cdots, l)$ and $\sigma_r{}^0$ $(r=1, 2, \cdots, L)$. Then, since $P_l(m^2)$ contains terms in m^{-2s} for $s=0, 1, \cdots, l$,[4] we obtain $(l+1)$ relations between the perturbed and unperturbed

* Permanent address: Imperial College, London, England.

[1] S. Coleman and H. J. Schnitzer, Phys. Rev. **134**, 863 (1964).

[2] S. Okubo, Progr. Theoret. Phys. (Kyoto) **27**, 942 (1962); M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).

[3] The relevant projection operators for spin-2 fields (the case of most interest to us) are given in R. J. Rivers, Nuovo Cimento **34**, 386 (1964).

[4] The spin-l projection operator $P_l(p^2)$ is a sum of products of l spin-one projection operators. Since $P_1(p^2)=g_{\mu\nu}-p_\mu p_\nu/p^2$, $P_l(p^2)$ contains terms in p^{-2S} for $s=0, 1, \cdots, l$.

spectral functions. These can be written in form

$$\int dm^2 m^{-2s}\{(\rho_l-\rho_l^0)+\sum_{r=0}^{l-1} a_r^{(s)}(\rho_r-\rho_r^0)+\sum_{r=1}^{L} b_r^{(s)}(\sigma_r-\sigma_r^0)\}^{ij}=0, \quad (2.3)$$

for $s=0, 1, \cdots, l$ where $a_r^{(s)}$ and $b_r^{(s)}$ are numerical coefficients, the values of which are obtained from the explicit form of the projection operators.

The only spectral functions with poles are ρ_l and ρ_l^0. Normalizing the unperturbed fields up to the superstrong interactions, we have

$$\begin{aligned}\rho_l^{0ij}(m^2)&=\sum_r \delta_r^{ij}\delta(M_r^2-m^2)+\tau_l^{0ij}(m^2),\\ \rho_l^{ij}(m^2)&=\sum_r Z_r^{ij}\delta(m_r^2-m^2)+\tau_l^{ij}(m^2),\end{aligned} \quad (2.4)$$

where

$$\delta_r^{ij}=\delta_r^i\delta_r^j. \quad (2.5)$$

The masses M_r are the $SU(3)$ masses of the particles considered, m_r are their physical masses [assuming $SU(2)$ invariance], and τ_l^0, τ_l have no poles. The Z_r^{ij} are relative renormalization matrices which, for the case of no mixing, are the ratios of the field renormalization constants with and without symmetry breaking, and for the case of mixing contain all the mixing parameters.

Substituting (2.4) into (2.3) we obtain

$$\int dm^2 m^{-2s}\{(\tau_l-\tau_l^0)+\sum_{r=0}^{l-1} a_r^{(s)}(\rho_r-\rho_r^0)+\sum_{r=1}^{L} b_r^{(s)}(\sigma_r-\sigma_r^0)\}^{ij}+\sum_r (Z_r/m_r^{2s}-\delta_r/M_r^{2s})^{ij}=0, \quad (2.6)$$

for $s=0, 1, \cdots, l$.

We now make the assumption that the integral over the continuum can be neglected in (2.6) for the case $s=l$, in which case the power of m in the integrand is at its least and hopefully causes the greatest damping in the integration. This approximation is improved if we have restrictions on the currents (e.g., partially or fully divergence-free) that cause some spectral functions to vanish identically.

We therefore approximate (2.6) for $s=l$ by

$$\sum_r (Z_r/m_r^{2l})=\sum_r(\delta_r/M_r^{2l}) \quad (2.7)$$

[where the $SU(3)$ indices have been omitted for clarity].

For the special case of a single field this gives

$$Z=(m/M)^{2l}. \quad (2.8)$$

If, in the limit of small mass-breaking, we take

$$\begin{aligned}m/M&=1+\Delta,\\ Z&=1+\Delta Z,\end{aligned} \quad (2.9)$$

we have

$$\Delta Z\approx 2l\Delta. \quad (2.10)$$

That is, the extent to which the (relative) field renormalization differs from unity is proportional to the mass breaking, the coefficient of proportionality being twice the spin.[5]

Let us now consider Eq. (2.7) in the case of particle mixing. Suppose we have fields 1 and 2, with $SU(3)$ masses M_1 and M_2 that, on switching on the $SU(3)$-breaking interaction are renormalized and mixed, the resultant physical fields having masses m_1 and m_2.

The 2×2 propagator submatrix has as its pole term, using (2.2) and (2.4)

$$\Delta_{(\mu)(\nu)}(p^2)_{\text{pole}}=\sum_{r=1,2}(-1)^l\frac{P_{l(\mu)(\nu)}(m_r^2)Z_r}{p^2-m_r^2+i\epsilon}. \quad (2.11)$$

Let us define $D(p^2)$ by

$$D(p^2)=\sum_{r=1,2} Z_r(p^2-m_r^2+i\epsilon)^{-1}. \quad (2.12)$$

This can be inverted to give[1]

$$D^{-1}(p^2)=(p^2-\sum_r M_r^2\delta_r+\alpha p^2+\beta), \quad (2.13)$$

where

$$\alpha=1-(\sum_r Z_r)^{-1}, \quad (2.14)$$

and

$$\beta=\sum_r M_r^2\delta_r-(\sum_r Z_r/m_r^2)^{-1} \quad (2.15)$$

are real symmetric matrices. Let us take

$$\alpha=\begin{pmatrix} b-1 & f\\ f & d-1\end{pmatrix},\quad \beta=\begin{pmatrix}(1-a)M_1^2 & e\\ e & (1-c)M_2^2\end{pmatrix}. \quad (2.16)$$

Then

$$D^{-1}(p^2)=\begin{pmatrix} bp^2-aM_1^2 & fp^2+e\\ fp^2+e & dp^2-cM_2^2\end{pmatrix}. \quad (2.17)$$

From (2.12) we see that (2.17) gives Z_1 and Z_2 as

$$Z_1=\frac{1}{[bd-f^2](m_1^2-m_2^2)}\times\begin{pmatrix} dm_1^2-cM_2^2 & -(fm_1^2+e)\\ -(fm_1^2+e) & bm_1^2-aM_1^2\end{pmatrix}, \quad (2.18)$$

[5] Thus the partially renormalized coupling constants obtained by renormalizing the fields alone will satisfy the sum rules obtained from λ_8-breaking in the vertex function (in the absence of mixing) to first order in mass-breaking.

and

$$Z_2=\frac{1}{[bd-f^2](m_1^2-m_2^2)}\times\begin{pmatrix}-dm_2^2+cM_2^2 & fm_2^2+e\\ fm_2^2+e & -bm_2^2+aM_1^2\end{pmatrix}. \quad (2.19)$$

Alternatively, from (2.7), (2.12) and (2.17) we see that we can express Z_1 and Z_2 in terms of a renormalized mixing angle β, in immediate analogy with unrenormalized $SU(3)$, as

$$Z_1=Z_1^0\begin{pmatrix}\cos^2\beta & -I^l\cos\beta\sin\beta\\ -I^l\cos\beta\sin\beta & I^{2l}\sin^2\beta\end{pmatrix}, \quad (2.20)$$

$$Z_2=Z_2^0\begin{pmatrix}\sin^2\beta & I^l\cos\beta\sin\beta\\ I^l\cos\beta\sin\beta & I^{2l}\cos^2\beta\end{pmatrix}, \quad (2.21)$$

where

$$Z_i^0=m_i^{2l}/M_1^{2l}, \quad (i=1,2), \quad (2.22)$$

and

$$I=M_1/M_2. \quad (2.23)$$

The consistency of (2.18) and (2.19) with (2.21) and (2.22) under the restriction of first-order mass breaking is discussed in the next section.

III. CONSISTENCY CONDITIONS

We now consider the restrictions imposed on the parameters in (2.16) and the masses as a consequence of (2.7) and (2.11).

The requirement that $D(p^2)$ has poles at m_1^2 and m_2^2 (2.11) gives

$$m_1^2m_2^2(bd-f^2)=(acM_1^2M_2^2-e^2) \quad (3.1)$$

and

$$(m_1^2+m_2^2)(bd-f^2)=(bcM_2^2+adM_1^2+2ef). \quad (3.2)$$

Equation (2.7), taken in conjunction with (2.18) and (2.19) gives three more conditions. These are

$$f\Delta_{l-1}+e\Delta_l=0, \quad (3.3)$$

$$(m_1^2-m_2^2)(bd-f^2)=M_1^{2l}(d\Delta_{l-1}-cM_2^2\Delta_l), \quad (3.4)$$

and

$$(m_1^2-m_2^2)(bd-f^2)=M_2^{2l}(b\Delta_{l-1}-aM_1^2\Delta_l), \quad (3.5)$$

where

$$\Delta_l=m_1^{-2l}-m_2^{-2l}. \quad (3.6)$$

Let us suppose that, before $SU(3)$ breaking, field 1 described an $I=Y=0$ member of an octet and field 2 a unitary singlet. The assumption that the symmetry-breaking part of the Lagrangian transforms like part of an octet[2] determines M_1^2, a and b. For simplicity, we adopt an inverse mass-squared law. If m_3 is the physical mass (neglecting $e-m$ differences) of the isospin triplet and m_4 the physical mass of the isospin doublets we take

$$M_1^2=m_3^2(1-2\epsilon)=m_4^2(1+\epsilon), \quad (3.7)$$

whence[6]

$$a=(1+2\epsilon)^{l-1}, \quad b=(1+2\epsilon)^l. \quad (3.8)$$

We consider Eqs. (3.1) to (3.5) for particular values of l.

A. $l=0$

Equations (3.1) to (3.5) give

$$f=0,\quad b=d=1, \quad (3.9)$$

[as required by (2.14)] and

$$m_1^2m_2^2=acM_1^2M_2^2,\quad m_1^2+m_2^2=aM_1^2+cM_2^2, \quad (3.10)$$

where aM_1^2 is given by (3.7) and (3.8) in terms of m_3^2 and m_4^2. Thus cM_2^2 is determined. The parametrization is thus consistent and can be seen to be equivalent to the normal mixing theory.

B. $l=1$

Equations (3.1) to (3.5) give

$$e=0,\quad a=c=1, \quad (3.11)$$

[as required by (2.15)] and

$$m_1^2m_2^2(bd-f^2)=M_1^2M_2^2,\quad (m_1^2+m_2^2)(bd-f^2)=bM_2^2+dM_1^2, \quad (3.12)$$

where b,M_1^2 are given in terms of m_3^2 and m_4^2 by (3.7) and (3.8). The parametrization is thus consistent. Since in this paper we are mainly concerned with $J^P=2^+$ decays we shall not discuss spin-1 mixing in any detail.

C. $l=2$

Substituting for e from (3.3) into (3.1) and (3.2) we have three homogeneous equations in d,f^2, and cM_2^2 with coefficients that, by (3.7) and (3.8), are functions of the physical masses m_1, m_2, m_3, and m_4. This gives rise to the determinantal mass relation

$$\begin{vmatrix}(\Delta_1/\Delta_2)^2-m_1^2m_2^2 & m_1^2m_2^2b & -aM_1^2\\ 2\Delta_1/\Delta_2-(m_1^2+m_2^2) & (m_1^2+m_2^2)b-aM_1^2 & -b\\ -(m_1^2-m_2^2) & (m_1^2-m_2^2)b-M_1^4\Delta_1 & M_1^4\Delta_2\end{vmatrix}=P=0. \quad (3.13)$$

[6] The "unphysical" octet mass m_u ($m_u^2=aM_1^2/b$) is given by $3m_u^{-2}=4m_4^{-2}-m_3^{-2}$, and $b^{-1}=(m_u^2/M_1^2)^l$.

If we define Δ_i by $m_i^2=M_1^2(1+\Delta_i)$ for $i=1$, 2 Eq. (3.13) gives[7]

$$P=0(\epsilon^2\Delta_i,\epsilon\Delta_i^2,\Delta_i^3) \tag{3.14}$$

and, insofar as we are taking only first-order octet breaking, is satisfied identically to the order required.

Suppose m_1, m_2, m_3, and m_4 are taken from experiment. Let us provisionally put $M_1^2=M_2^2$. We can then solve Eqs. (3.1) to (3.5) for c, d, e, f to give values c_0, d_0, e_0, f_0.

Insertion of these values in (2.18) and (2.19) will give the renormalized angle β as defined in (2.20) and (2.21). Since Eq. (3.13) is not satisfied exactly the values of β obtained from (2.18) and (2.19) will differ slightly.

This angle is to be compared with the unrenormalized mixing angle α given by

$$\sin^2\alpha=(m_1^2-M_1^2/a)(m_1^2-m_2^2)^{-1}. \tag{3.15}$$

We are justified in taking Eq. (3.13) to be approximately satisfied if for given m_1 and m_2 the uncertainty in β is of the same magnitude as the uncertainty in α arising from the experimental errors in m_1 and m_2.

IV. THE $J^P=2^+$ NONET

The known[8] $J^P=2^+$ nonet are the $f'(M=1500\pm20$, $\Gamma=80)$, the $f(M=1253\pm20$, $\Gamma=118\pm16)$, the $A_2(M=1324\pm9$, $\Gamma=90\pm10)$ and the $K^*(M=1405\pm8$, $\Gamma=95\pm11)$. These masses approximately satisfy the Schwinger-type mass formula[9]

$$(m_f^2-m_8^2)(m_{f'}^2-m_8^2)=-8/9[m_{K^*}^2-m_{A_2}^2]^2, \tag{4.1}$$

where

$$m_8^2=M_1^2/a. \tag{4.2}$$

In calculating the partial decay widths of the 2^+ nonet we shall make the further approximation that we can neglect vertex renormalization and can consider all the coupling constant renormalization to arise from the field renormalization. In many ways we shall adopt the nomenclature of Glashow and Socolow[10] in order to facilitate comparison between our results and those of unbroken $SU(3)$.

A. $2^+\rightarrow 0^-0^-$ DECAYS

The 0^- nonet consists of the π, κ, η, X particles. From Eq. (2.7) we see that the 0^- particles are not renormalized by the symmetry-breaking interaction, and from (2.18) to (2.21) we see that a mixing angle can be defined in the usual way.[11] This mixing angle is small ($\theta\approx\pm10°$) but the effects of mixing can be appreciable. For those cases where the decay products contain η we first calculate decay fractions on the assumption that the η is a pure member of an octet. When the decay fraction is appreciable we estimate the effects of mixing by taking a mixing angle of $+10°$ and relate the η and X coupling constants by taking the octet and singlet together as a nonet

$$P_9=P_8+3^{-1/2}X_u\mathbf{1}, \tag{4.3}$$

where P_8 is the pseudoscalar octet in which η has been replaced by $\eta\cos\theta-X\sin\theta$, and

$$X_u=X\cos\theta+\eta\sin\theta. \tag{4.4}$$

We take account of the f', f mixing as follows: Let us consider an identical decay of f', f (i.e., the same decay products). In the absence of symmetry-breaking interactions let us take $G_{f'}$ and G_f to be the respective coupling constants for the unperturbed f' and f.

We can define unrenormalized but mixed coupling constants g_i^0 $(i=f', f)$ by the equations

$$Z_i^0g_i^{02}=G^TZ_iG, \quad (i=f', f), \tag{4.5}$$

where

$$G=\begin{pmatrix}G_{f'}\\G_f\end{pmatrix}. \tag{4.6}$$

This gives [from (2.20) and (2.21)]

$$g_{f'}^0=G_{f'}\cos\beta-G_f I\sin\beta, \tag{4.7}$$

$$g_f^0=G_{f'}\sin\beta+G_f I\cos\beta. \tag{4.8}$$

We note that taking $M_1^2\neq M_2^2$ does not increase the number of effective parameters.

The renormalized coupling constants for f, $f'\rightarrow P_1+P_2$ are thus

$$g_{iP_1P_2}=Z_i^{0\frac{1}{2}}g_{iP_1P_2}^0 \quad (i=f, f'), \tag{4.9}$$

where, from (2.20) and (2.21),

$$Z_i^{0\frac{1}{2}}=m_i^2/M_1^2. \tag{4.10}$$

The unrenormalized $SU(3)$ coupling constants are g_{iPP0} in which β has been replaced by α and I put equal to unity.

If $g_{A_2}^0$ and $g_{K^*}^0$ are the A_2 and K^* coupling constants in the absence of symmetry breaking, the renormalized coupling constants g_{A_2} and g_{K^*} are given by

$$g_{A_2P_1P_2}=Z_{A_2}^{1/2}g_{A_2P_1P_2}^0, \tag{4.11}$$

and

$$g_{K^*P_1P_2}=Z_{K^*}^{1/2}g_{K^*P_1P_2}^0, \tag{4.12}$$

where

$$Z_{A_2}^{1/2}=m_{A_2}^2/M_1^2, \quad Z_{K^*}^{1/2}=m_{K^*}^2/M_1^2. \tag{4.13}$$

[7] The consistency condition for general spin $l>2$ is obtained from (3.13) by replacing Δ_2, Δ_1 by Δ_l, Δ_{l-1}, respectively.

[8] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **37**, 633 (1965). The uncertainty in the f' mass is that quoted by S. U. Chung *et al.*, Phys. Rev. Letters **15**, 325 (1965).

[9] J. Schwinger, Phys. Rev. **135**, B816 (1964).

[10] S. L. Glashow and R. H. Socolow, Phys. Rev. Letters **15**, 329 (1965).

[11] J. J. Sakurai, Phys. Rev. Letters **9**, 472 (1962).

TABLE I. Decays of $J^P=2^+$ nonet into two 0^- mesons.

Decay mode[a]	Rate in terms of F and G[b]	Predicted rate (MeV)[c]			
$f\to\pi+\pi$	$161Z_f^0(2F\sin\beta+G\cos\beta)^2$	*110*	*110*	*110*	*110*
$f\to K+\bar{K}$	$21.2Z_f^0(F\sin\beta-G\cos\beta)^2$	8.2	7.9	7.6	7.0
$f\to\eta+\eta$[d]	$1.5Z_f^0(2F\sin\beta-G\cos\beta)$	0.5	0.4	0.4	0.4
$A_2\to\eta+\pi$[d]	$205Z_{A_2}F^2$	8.2	9.6	10.0	12.0
$A_2\to\eta+\pi$[e]	$114Z_{A_2}F^2$	4.5	5.1	5.5	6.6
$A_2\to K+\bar{K}$	$113Z_{A_2}F^2$	4.5	5.1	5.5	6.6
$K^*\to K+\pi$	$775Z_{K^*}F^2$	40	47	49	58
$K^*\to K+\eta$[d]	$24Z_{K^*}F^2$	1.3	1.5	1.7	1.9
$f'\to\pi+\pi$	$300Z_{f'}^0(2F\cos\beta-G\sin\beta)^2$	*1.0*	*2.3*	*4.0*	*9.0*
$f'\to K+\bar{K}$	$107Z_{f'}^0(F\cos\beta+G\sin\beta)^2$	48	49	51	53
$f'\to\eta+\eta$[d]	$17.3Z_{f'}^0(2F\cos\beta+G\sin\beta)^2$	14	15	16	17
$f'\to\eta+\eta$[e]	$17.3Z_{f'}^0(0.91F\cos\beta+G\sin\beta)^2$	6.9	7.3	7.6	7.7

[a] All the charge states are included.
[b] The phase space is evaluated in units of 10^{-3} BeV^3.
[c] Input values are italicized. Of the two values of F/G consistent with the input data we have taken $F/G>0$ since the alternative solution is in total disagreement with experiment.
[d] The η is here taken to be a pure octet member.
[e] An $\eta-X$ mixing angle of $+10°$ has been taken and the η,X coupling constants related by Eqs. (4.3) and (4.4).

We take

$$\begin{aligned} m_f^2&=1.56\text{ BeV}^2,\\ m_{f'}^2&=2.30\text{ BeV}^2,\\ m_{K^*}^2&=2.00\text{ BeV}^2,\\ m_{A_2}^2&=1.75\text{ BeV}^2,\end{aligned}\tag{4.14}$$

which, with (3.1) to (3.8) give

$$\begin{aligned} M_1^2&=1.91\text{ BeV}^2,\\ a&=0.91,\\ b&=0.83,\\ c_0&=1.17,\\ d_0&=1.38,\\ e_0&\approx\mp0.27\text{ BeV}^2,\\ f_0&\approx\pm0.29.\end{aligned}\tag{4.15}$$

These values give[12]

$$\begin{aligned}\alpha&=31.2°,\\ \beta&\approx22.7°,\end{aligned}\tag{4.16}$$

and

$$\begin{aligned} Z_f^{0\frac{1}{2}}&=0.82, & Z_{f'}^{0\frac{1}{2}}&=1.20,\\ Z_{A_2}^{1/2}&=0.92, & Z_{K^*}^{1/2}&=1.05.\end{aligned}\tag{4.17}$$

We take the effective $SU(3)$ interaction Lagrangian in the absence of symmetry breaking as

$$\mathcal{L}_{\text{eff}}=6^{1/2}F\,\text{Tr}(T_8\{P_8,P_8\})+G_0f_0\,\text{Tr}(P_8P_8),\tag{4.18}$$

where T_8 is the unperturbed 2^+ octet and f_0 the unperturbed singlet. F and G_0 are the two independent coupling constants. Writing $I^2G=G_0$ [from (2.20) and (2.21)], all the $2^+\to0^-0^-$ partial decay widths can be expressed in terms of the two parameters F and G. Table I gives the predicted partial decay widths using $\Gamma(f\to\pi+\pi)$ and $\Gamma(f'\to\pi+\pi)$ as input for different values of the latter.

From (4.11) and (4.12) we see that the renormalization depresses the ratio of A_2 to K^* partial widths by a factor of $Z_{A_2}/Z_{K^*}\approx0.77$. Although this deviation is significant, it is hard to distinguish experimentally between the renormalized and unrenormalized predictions since the only decays in which there is no $\eta-X$ uncertainty are $A_2\to K+\bar{K}$ and $K^*\to K+\pi$. Both at present have experimental errors larger than 20%, in addition to the former partial width being small.

For f and f' decays the deviations from unrenormalized $SU(3)$ are less transparent since we have a two-parameter fit. The one obvious difference is that the depression of the mixing angle by renormalization will not give the observed strong suppresssion of $f'\to\pi+\pi$ decays (relative to $f\to\pi+\pi$ decays) if we assume the Okubo nonet ansatz[13] that the unperturbed octet and singlet form a degenerate nonet

$$T_9=T_8+3^{1/2}f_0\mathbf{1},\tag{4.19}$$

and in addition forbids terms proportional to Tr T_9 in the effective interaction Lagrangian. This assumption gives $I=1$, $G=2\sqrt{2}F$, whereas for the input values used in Table I the ratio G/F varies from 3.1 to 4.2.

We can eliminate F and G and obtain several relations between renormalized and unrenormalized partial widths. Qualitatively we find it difficult to reduce the $f\to K+\bar{K}$ branching fraction below 7% [in contrast to unrenormalized $SU(3)$] and we require less suppression of $f'\to\pi+\pi$ than is necessary in unrenormalized $SU(3)$. For given $f\to\pi+\pi$ and $f'\to\pi+\pi$ partial widths, the $f'\to K+\bar{K}$ partial width is increased by 40–60% on renormalization to give rather better agreement with experiment than the unrenormalized $SU(3)$.

B. $2^+\to1^-0^-$ DECAYS

From Eq. (2.7) we see that the 1^- nonet [ρ,ω,ϕ, $K^*(890)$] is renormalized. From Eqs. (2.18), (2.19) and (3.12) we see that we do not alter the number of effective parameters by taking $d=1$. Taking

$$\begin{aligned} m_{K^*}^2&=0.794\text{ BeV}^2,\\ m_\rho^2&=0.582\text{ BeV}^2,\\ m_\omega^2&=0.613\text{ BeV}^2,\\ m_\phi^2&=1.039\text{ BeV}^2,\end{aligned}\tag{4.20}$$

we can calculate M_1^2 and M_2^2 for the 1^- nonet (denoted by N_1^2 and N_2^2 to avoid confusion). We get

$$\begin{aligned} N_1^2&=0.708\text{ BeV}^2,\\ N_2^2&=0.693\text{ BeV}^2,\end{aligned}\tag{4.21}$$

[12] The calculated values of β from (2.18) and (2.19) are 22.2° and 23.2°, respectively (we take β and α to be positive).

[13] S. Okubo, Phys. Letters 5, 165 (1963).

from (3.12). This gives [from (2.18)][14]

$$(Z_\omega^{11})^{1/2}=0.434\,,$$
$$Z_{K^*}^{1/2}=m_{K^*}/N_1=1.06\,, \qquad (4.22)$$

and

$$Z_\rho^{1/2}=m_\rho/N_1=0.907\,.$$

Except for the case of $K^*\to\omega+K$ we define g_{TVP}^0 in analogy with g_{TPP}^0.

For these cases the renormalized coupling constants are given by

$$g_{TVP}=Z_T^{1/2}Z_V^{1/2}g_{TVP}^0 \qquad (4.23)$$

when T is A_2 or K^* and

$$g_{TVP}=Z_T^{0\frac{1}{2}}Z_V^{1/2}g_{VTP}^0 \qquad (4.24)$$

when T is f or f'.

For $K^*\to\omega+K$,

$$g_{K^*\omega K}=Z_{K^*}^{1/2}(Z_\omega^{11})^{1/2}g_{K^*\phi_0K}^0 \qquad (4.25)$$

where $g_{K^*\phi_0K}^0$ is the $SU(3)$ coupling constant of K^* and K to the $I=Y=0$ member of the unperturbed vector octet.

In the absence of symmetry breaking we take the effective Lagrangian to be

$$\mathcal{L}_{\text{eff}}=H\,\text{Tr}(T_8[V_8,P_8]) \qquad (4.26)$$

where V_8 is the unperturbed vector meson octet.

In Table II we give the predicted values of $2^+\to1^-0^-$ decays using $\Gamma(A_2\to\rho+\pi)$ as input.

Because of the vector meson renormalization the deviations from unrenormalized $SU(3)$ can be greater than in pseudoscalar decays. For example $Z_{K^*}Z_{K^*(890)}/Z_{A_2}Z_\rho\approx1.75$. This enables us to make predictions for K^* and A_2 decays that are in better agreement with experiment than the prediction of unrenormalized $SU(3)$. However, the $f'\to K^*+K$ decay is renormalized by a factor of $Z_{K^*(890)}Z_{f'}^0\cos^2\beta/Z_{A_2}Z_\rho\cos^2\alpha\approx2.6$ relative to the $A_2\to\rho+\pi$ decay. This leads to an $f'\to K^*+K$ partial width rather larger than suggested by experiment. We note however that the phase space is rather sensitive to the mass values that are taken.

TABLE II. Decays of $J^P=2^+$ nonet into one pseudoscalar and one vector meson.

Decay mode[a]	Form in terms of H[b]	Predicted rate (MeV)[c]		
$A_2\to\rho+\pi$	$54.4Z_{A_2}Z_\rho H^2$	*90*	*80*	*70*
$K^*\to K^*(890)+\pi$	$17.4Z_{K^*}Z_{K^*(890)}H^2$	51	46	40
$K^*\to\rho+K$	$4.5Z_{K^*}Z_\rho H^2$	9.7	8.6	7.6
$K^*\to\omega+K$	$3.4Z_{K^*}Z_\omega^{11}H^2$	1.7	1.5	1.3
$f'\to K^*+K$	$14.4Z_{f'}^0Z_{K^*(890)}\cos^2\beta H^2$	46	41	36

[a] All charge states are included.
[b] Phase space is evaluated in units of 10^{-3} BeV[5].
[c] The input value is italicized.

[14] We do not need to calculate any other matrix elements of $Z\omega$ and $Z\phi$ because we are not considering any decays with ϕ in the decay products and the pure $SU(3)$ 1^- singlet does not couple to the 2^+ octet-0^- octet.

V. COMPARISON WITH EXPERIMENT

A. A_2 Decays

The decay fractions are not yet well determined experimentally. Deutschmann *et al.*[15] gave

$$\Gamma(A_2\to\eta+\pi)/\Gamma(A_2\to\rho+\pi)=0.0\pm0.03\,; \qquad (5.1)$$

in a later paper Deutschmann *et al.*[16] give

$$\Gamma(A_2\to\eta+\pi)/\Gamma(A_2\to\rho+\pi)<0.03\,. \qquad (5.2)$$

This is consistent with the work of Chung *et al.*[17] who give

$$\Gamma(A_2\to\rho+\pi)/\Gamma(A_2)=0.91_{-0.10}^{+0.04}\,,$$

and

$$\Gamma(A_2\to\eta+\pi)/\Gamma(A_2)=0.03\pm0.03\,, \qquad (5.3)$$

but not in good agreement with the results of Trilling *et al.*[18] which are

$$\Gamma(A_2\to\rho+\pi)/\Gamma(A_2)\approx0.70\,,$$

and

$$\Gamma(A_2\to\eta+\pi)/\Gamma(A_2)\approx0.20\,, \qquad (5.4)$$

or with the results of Aderholtz *et al.*[19] who give

$$\Gamma(A_2\to\eta+\pi)/\Gamma(A_2\to\rho+\pi)=0.3\pm0.2\,. \qquad (5.5)$$

We see from Tables I and II that $\eta-X$ mixing as given in (4.3) and (4.4) give predictions in reasonable agreement with (4.23), (4.24) and (4.25).

The $A_2\to K+\bar{K}$ decay fraction is given by Deutschmann *et al.*[16] as

$$\Gamma(A_2\to K+\bar{K})/\Gamma(A_2\to\rho+\pi)=0.03\pm0.02\,, \qquad (5.6)$$

and by Chung *et al.*[17] as

$$\Gamma(A_2\to K+\bar{K})/\Gamma(A_2)=0.055\pm0.015\,. \qquad (5.7)$$

The predicted decay fractions are seen to be in agreement with these results.

B. f Decays

There is no information on $f\to\eta+\eta$. The decay fraction for $f\to K+\bar{K}$ is given by Chung *et al.*[14] as

$$\Gamma(f\to K+\bar{K})/\Gamma(f)<0.04 \qquad (5.8)$$

which is smaller than that predicted in Table I.

[15] M. Deutschmann *et al.*, Phys. Letters **12**, 356 (1964).
[16] M. Deutschmann *et al.*, Phys. Letters **20**, 82 (1966).
[17] S. U. Chung, O. I. Dahl, L. M. Hardy, R. I. Hess, L. D. Jacobs, J. Kirz, and D. H. Miller, Phys. Rev. Letters **15**, 325 (1965).
[18] G. H. Trilling, J. L. Brown, G. Goldhaber, S. Goldhaber, J. A. Kadyk, and J. MacNaughton (unpublished). See G. Goldhaber, in *Proceedings of the Second Coral Gables Conference on Symmetry Principles at High Energy, 1965*, edited by B. Kursunoğlu, A. Perlmutter, and I. Sakmar (W. H. Freeman Company, San Francisco, California, 1966).
[19] Aachen-Berlin-Birmingham-Bonn-Hamburg-London (I.C.)-München Collaboration, Phys. Rev. **138**, 897 (1965).

C. K^* Decays

Chung *et al.*[17] give

$$\Gamma(K^*\to K^*(890)+\pi)/\Gamma(K^*\to K+\pi)=\tfrac{1}{3}\pm\tfrac{1}{3}$$
$$\text{and}\quad \Gamma(K^*\to\rho+K)/\Gamma(K^*\to K+\pi)<\tfrac{1}{12}. \quad (5.9)$$

Segar *et al.*[20] give

$$\Gamma(K^*\to K^*+\pi)/\Gamma(K^*\to K+\pi)=0.30\pm0.10$$
$$\text{and}\quad \Gamma(K^*\to\rho+K)/\Gamma(K^*\to K+\pi)<0.10, \quad (5.10)$$

whereas the Brussels-CERN Collaboration[21] gives

$$\Gamma(K^*\to K^*+\pi)/\Gamma(K^*\to K+\pi)=0.70\pm0.15$$
$$\text{and}\quad \Gamma(K^*\to\rho+K)/\Gamma(K^*\to K+\pi)<0.10. \quad (5.11)$$

Badier *et al.*[22] give

$$\Gamma(K^*\to K+\pi):\Gamma(K^*\to K^*(890)+\pi):$$
$$\Gamma(K^*\to K+\rho):\Gamma(K^*\to K+\omega):\Gamma(K^*\to K+\eta)$$
$$=(37\pm19):(41\pm14):(13\pm5):(7\pm4):(2\pm2). \quad (5.12)$$

The predictions of Tables I and II are in good agreement with these last results.

D. f' Decays

There is no information of $f'\to\pi+\pi$ except that it is small. Barnes *et al.*[23] give

$$\Gamma(f'\to\bar{K}^*+K)/\Gamma(f'\to\bar{K}+K)\approx0.25, \quad (5.13)$$

with which the theoretical predictions disagree, the lowest ratio [for $\Gamma(f'\to\pi+\pi)=9.0$ MeV and $\Gamma(A_2\to\rho+\pi)=70$ MeV] being 0.68.

VI. CONCLUSION

In determining mixing parameters and field renormalization constants for mesons of arbitrary spin in broken $SU(3)$ the major assumption that we have made is that the vacuum expectation values of the equal-time commutators and their first time derivatives are unaffected by the $SU(3)$ symmetry-breaking interactions. This leads to relations between perturbed and unperturbed spectral functions that have been approximated by neglecting the continua for those cases where the integral over the continuum seems most likely to be damped strongly by high powers of the integrated variable in the denominator of the integrand.

This approximation leads to spin-dependent field renormalization constants and to consistency conditions between the mixing parameters of meson fields of spin greater than one that are exactly satisfied to first order mass breaking. Taken in conjunction with Okubo-type octet breaking, we find the equations are sufficient to determine all the effective parameters in the theory.

The further assumption that vertex renormalization can be neglected enables us to obtain broken $SU(3)$ coupling constants in terms of exact $SU(3)$ coupling constants, renormalized mixing angles and simple renormalization constants. In the absence of mixing to first order in mass-breaking the broken $SU(3)$ coupling constants are linear in the mass-breaking of the interacting particles and hence satisfy the usual first order sum rules obtained from λ_8 breaking.

When applied to $J^P=2^+$ nonet decays the renormalized theory seems to be an improvement over the unrenormalized theory for K^* decays and $f\to K+\bar{K}$ decays. It suffers in predicting rather too large a partial width for $f'\to K^*+\bar{K}$ on existing experimental evidence. The best over-all picture is given by using as input the values $\Gamma(f\to\pi+\pi)=110$ MeV, $\Gamma(A_2\to\rho+\pi)=70$ MeV, $\Gamma(f'\to\pi+\pi)=4$–9 MeV, and $\eta-X$ mixing to be $+10°$.

The unrenormalized $SU(3)$ results can be easily obtained from the renormalized decay amplitudes by replacing the renormalized mixing angle by the usual unrenormalized angle and putting all renormalization constants equal to unity. The deviations in decay widths can be as large as 100%, depending on the processes considered.

ACKNOWLEDGMENTS

Most of this work was done at Imperial College, London. The author would like to thank Professor P. T. Matthews and Professor G. Feldman for helpful discussion. He is also grateful to Professor Abdus Salam and the International Atomic Energy Agency for the hospitality extended to him at the International Centre for Theoretical Physics, Trieste.

[20] A. M. Segar (unpublished) quoted by A. H. Rosenfeld, in *Proceedings of the Oxford Conference on Elementary Particles, 1965* (Rutherford High Energy Laboratory, Harwell, England, 1966).

[21] Brussels—CERN Collaboration (unpublished) quoted by A. H. Rosenfeld, in *Proceedings of the Oxford Conference on Elementary Particles, 1965* (Rutherford High Energy Laboratory, Harwell, England, 1966).

[22] J. Badier *et al.*, Phys. Letters **19**, 612 (1965).

[23] V. E. Barnes *et al.*, Phys. Rev. Letters **15**, 322 (1965).