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W-Spin Selection Rules*

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The relation between the additive quantum numbers of the W-spin operators and the multiplicative quantum numbers of reflection in a plane is discussed. We find two types of W-spin selection rules: those that are identical to the selection rules that come from the general principle of reflection invariance, and those that form new additional restrictions. The implications of the latter group for the baryons and mesons are discussed.

IT is well known[1] that selection rules that come from W-spin conservation include those that stem from reflection invariance (the latter will be called trivial selection rules). In a recent paper,[2] Lipkin and Meshkov established the connection between W spin and the reflection operators in quark dynamics. The purpose of the present paper is to develop further this approach to find the connection between W-spin conservation and the trivial selection rules, and to see when nontrivial selection rules have to be expected.

Let us repeat the derivation of the W spin for one-quark systems according to Ref. 2. One starts with one quark (or anti-quark) moving in the y-z plane. Defining the symmetry operation R_x which is a reflection in the y–z plane, one finds

$$R_x=Pe^{i\pi j_x}=P_{\text{int}}e^{i\pi s_x}, \tag{1}$$

where P_{int} denotes the intrinsic parity ($+1$ for quarks and -1 for anti-quarks). Because the quark has spin $\frac{1}{2}$, one can easily show that

$$R_x=P_{\text{int}}e^{i\pi s_x}=e^{i\pi P_{\text{int}}s_x}=2iP_{\text{int}}s_x. \tag{2}$$

Let us define now

$$w_x=P_{\text{int}}s_x. \tag{3}$$

Equation (2) tells us that w_x is a good quantum number for one-quark states. If we limit ourselves now to a collinear motion in the z direction, we can define in an analogous way

$$w_y=P_{\text{int}}s_y \tag{4}$$

which will aslo be a good quantum number. To these two we can add now a third conserved operator

$$w_z=s_z. \tag{5}$$

It is conserved because $s_z=j_z$ for any particle moving in the z direction. The three w_i operators form an $SU(2)$ algebra. This is the reason they are called the w-spin operators.

This treatment can be easily extended to a system of quarks and anti-quarks. If we have such a system in the y–z plane, then

$$R_x=\prod_j P_{\text{int}}{}^j e^{i\pi s_x{}^j}=\prod_j e^{i\pi w_x{}^j}=\exp(i\pi\sum_j w_x{}^j)=e^{i\pi W_x}, \tag{6}$$

where W_x is the sum of all $w_x{}^j$ (j referring to the individual quarks and anti-quarks). If this system is a collinear one, then we can define

$$R_y=e^{i\pi W_y} \tag{7}$$

in the same manner. The three W_i now form an $SU(2)$ algebra defined for the system of quarks and anti-quarks. However, it is no more generally true that $R_x=2iW_x$ or $R_y=2iW_y$. We learn from Eq. (6) that conservation of R_x or R_y means

$$\Delta W_{x,y}\equiv 0(\text{mod}2). \tag{8}$$

This means that $W_{x,y}$ conservation is a stronger assumption than $R_{x,y}$ conservation. Equation (8) is due to the fact that the W_i are additive quantum numbers, whereas the R_i are multiplicative ones.

The treatment of the system of quarks and anti-quarks in coplanar or collinear motion can be extended to any corresponding particle system, within a model in which particles are composed of quarks and anti-quarks in s states. In that case, the W-spin eigenvalues of the particle will be given by the sum of the w eigenvalues of the constituent quarks and anti-quarks. It is well known that the $0^-(P)$ and $1^-(V)$ meson nonets and the $\frac{1}{2}^+(B)$ and $\frac{3}{2}^+(B^*)$ baryon octet and decuplet can be incorporated in such a model. We can therefore try to apply W-spin considerations to these particles and their processes. Equation (8) now tells us that the trivial selection rules will arise from forbidding reactions in which the eigenvalues of the W operators of the incoming and outgoing particle systems differ by an odd number of units. However, in cases in which the difference is an even number of units, the assumption of W conservation will lead to nontrivial selection rules.

Using this criterion, we are now able to analyze all possible 3- and 4- particle vertices in order to find the nontrivial selection rules due to W_x or $\mathbf{W}$ conservation. It turns out that the only three-particle vertex of the above-mentioned particles where a nontrivial selection rule occurs is in the coupling of B^* to B and V_0 (V_0 is

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the vector particle with $S_z=0$ which is a W scalar). If W spin is conserved, then this 3-point function should vanish because it couples a W-spin $\frac{3}{2}$ object to $\frac{1}{2}$ and 0. This is a place where a $|\Delta\mathbf{W}|=2$ transition is forbidden. This selection rule forbids the vector meson to mediate the collinear $PB\rightarrow PB^*$ reactions, and therefore stands in contradiction to their peripheral character. Within the framework of $SU(6)_W$, we are led in this case to many connections between the various amplitudes[3] which are contradicted by experiment.[4]

If we add to the $B^*\nrightarrow BV_0$ selection rule the further property of unique coupling of the B^* to B and $V_{\pm 1}$ due to the explicit W-spin Clebsch-Gordan coefficients, we find that the electroproduction and photoproduction of N^* should be pure $M1$.[5] This prediction seems to be favored by experiment.

In the case of 4-particle vertices of both mesons and baryons, it turns out that there are four cases in which nontrivial selection rules can be derived. They occur in the coupling of $B\bar{B}^*$ to V_0V_0 and $B^*\bar{B}^*$ to PV_0, V_1V_0, and V_0V_0. None of these can be related to a physical feasible decay or scattering process.

An interesting case is the pure baryonic vertex. If no antibaryons are involved, then W_x conservation is the same as S_x conservation. This operator is conserved in both the collinear $SU(6)_W$ and the coplanar $SU(3)\otimes SU(3)$ [6] theories. Thus in both theories it turns out that many amplitudes are forbidden. The simplest test of the selection rule is now given by NN scattering, where the transition between incoming $S_x=1$ state to an outgoing state of $S_x=-1$ is forbidden (as noted in Ref. 6). This means that the selection rule forbids $\mathbf{S}\cdot\mathbf{L}$ and tensor forces, and is therefore in contradiction with present available scattering data. Kantor *et al.*[7] compiled the relevant data in their paper and have shown that the disagreement is quite big. This can be interpreted as a strong argument against W_x conservation in this process. However, the disagreement can also be due to an incorrect description of the particles as composites of s-wave quarks.

Using the selection rule of W_x conservation, one can produce many more polarization predictions, but the NN elastic scattering is by far the easiest case to be measured experimentally. Let us describe some of the predictions. In studying the processes $BB\rightarrow BB^*$, it turns out that 50% of the possible scattering amplitudes vanish by the requirement of W_x $(=S_x)$ conservation. This means, for instance, that if the target nucleon is polarized in the x direction then the outgoing B^* cannot be fully polarized in the opposite direction (i.e., cannot be an eigenstate of $S_x=-\frac{3}{2}$). Still a larger fraction of amplitudes vanishes in the case of $BB\rightarrow B^*B^*$. A typical prediction here is that one can never find the outgoing B^*'s in a state of total $S_x=\pm 3$.

Similar arguments can also be applied to $\bar{B}B$ scattering, but here one should remember that, for $\bar{B}$, $W_x=-S_x$. Thus, the analogous statement to S_x conservation in BB elastic scattering will here be that the transition between $\bar{B}(S_x=-\frac{1}{2})B(S_x=\frac{1}{2})$ to $\bar{B}(S_x=\frac{1}{2})$ $B(S_x=-\frac{1}{2})$ is forbidden. Now the transition between total $S_x=1$ to $S_x=-1$ is allowed. In the case of B^* production, we find again that 50% of the transitions are forbidden by W_x conservation. If the target nucleon is polarized in the x direction, it turns out that in $\bar{B}B^*$ production, B^* cannot have $S_x=-\frac{3}{2}$, whereas in $B\bar{B}^*$ production, $\bar{B}^*$ cannot have $S_x=\frac{3}{2}$. In the case of $\bar{B}B\rightarrow\bar{B}^*B^*$, the predictions can be stated in the following way: If one of the outgoing particles is fully polarized in one perpendicular direction (i.e., $S_x=\pm\frac{3}{2}$), then the other cannot be in an eigenstate that has an S_x value of the opposite sign.

In summary, let us repeat our main conclusions. We find that the assumption of W-spin conservation in a direction perpendicular to that of collinear or coplanar processes has a very simple relationship to reflection invariance in a simple quark theory. The relationship between the two is that of an additive to a corresponding multiplicative quantum number. That is the reason that W-spin invariance leads to nontrivial selection rules. We have discussed the predictions that are obtained. They should be tested whenever it is experimentally feasible. It would be of interest to get further data on NN elastic scattering at higher energies and momentum-transfer values, in order to see whether and when this theory can be applied.

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