Weak-Interaction Predictions of $R(11)$*

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Further consideration is given to $R(11)$ as a possible alternative to $SU(6)$; in particular, we look for predictions concerning weak-interaction currents. Because of indeterminacy in the definition of the physical baryons, a further condition is needed to obtain a significant prediction; time-reversal invariance furnishes such a condition. The prediction $3f^2+d^2=9/8$ then results for the parameters describing the axial-vector octet. Combination of this result with the experimental value of μ_p/μ_n yields $(G_A/G_V)_{n\to p}=1.17$, in complete agreement with the experimental value 1.18 ± 0.02.

I. INTRODUCTION

With the increasing popularity of $SU(6)$, the question arises whether some other group may not give equally good predictions. If one assumes that such an alternative group must contain $[SU(3)/Z_3]\otimes R(3)$, and not mix different trialities or statistics in the same irreducible representation, then the simplest possibility (the one with the fewest generators) turns out[1] to be $R(11)$. This group does require the existence of as yet unobserved spin-zero mesons and a second octet of $\frac{1}{2}^+$ baryons[1]; but there is now evidence at least for the octet of baryons.[2] Therefore, further investigation of this group seems warranted; we present here a determination of the weak-current and magnetic-moment predictions.

II. BARYON CURRENTS

The octet of stable baryons is assigned to the 32-dimensional spinor representation **32** of $R(11)$, which decomposes into two spin-$\frac{1}{2}$ $SU(3)$ octets. As usual, we assume that the baryon currents associated with weak and electromagnetic interactions belong to the regular representation **55** which occurs in the direct product of the baryon representation with its conjugate. Since **32** is self-conjugate, this means we are interested in the direct product decomposition

$$\mathbf{32}\otimes\mathbf{32}=\mathbf{462}_s+\mathbf{330}_a+\mathbf{165}_a+\mathbf{55}_s+\mathbf{11}_s+\mathbf{1}_a. \quad (2.1)$$

(Here s and a denote symmetric and antisymmetric parts of the product, respectively.) Note that the regular representation occurs only once [as it does for $SU(6)$], so that simple predictions may be possible. The decomposition of **55** in terms of $SU(3)\otimes R(3)$ multiplets is

$$\mathbf{55}\to(\mathbf{10,1})+(\mathbf{\overline{10},1})+(\mathbf{8,1})+(\mathbf{8,3})+(\mathbf{1,3}). \quad (2.2)$$

In analogy to the Gell-Mann scheme we shall assume that $(\mathbf{8,1})$ contains the time components of the vector weak-current and charge-current operators while $(\mathbf{8,3})$ contains the space components of the axial-vector weak-current and magnetic-moment operators in corresponding positions with respect to $SU(3)$.[3]

The highest weight $[0\,1\,0\,0\,0]$ occurring in **55** maps into the highest weight $[0\,3,0]$ in the $SU(3)\otimes R(3)$ submultiplet $(\mathbf{\overline{10},1})$. The baryonic current operator corresponding to this weight, that is, the top of the **55** appearing in (2.1), is

$$\mathbf{55}_{\rm top}=(\mathbf{\overline{10},1})_{\rm top}=(\sqrt{\tfrac{1}{8}})\{x[\mathbf{\Xi}_-^{-*}\mathbf{n}_-+\mathbf{\Xi}_+^{-*}\mathbf{n}_++\mathbf{\Xi}_-^{0*}\mathbf{p}_- +\mathbf{\Xi}_+^{0*}\mathbf{p}_+]+x^2[\mathbf{\Xi}_-'^{-*}\mathbf{n}_-'+\mathbf{\Xi}_+'^{-*}\mathbf{n}_+' +\mathbf{\Xi}_-'^{0*}\mathbf{p}_-'+\mathbf{\Xi}_+'^{0*}\mathbf{p}_+']\}, \quad (2.3)$$

where $x\equiv\exp(2\pi i/3)$ and the lower $\pm$ signs denote spin projections. The first (unprimed) and second (primed) baryon octets correspond to the two distinct "spinor" octet representations of $R(8)$. With respect to $SU(3)$, these octets become indistinguishable, and the physical octets will be assumed to be linear combinations of them.

The only weight in **55** which maps into the highest weight $[1\,1,2]$ of the submultiplet $(\mathbf{8,3})$ is $[1\,0\,0\,-1\,2]$; application of appropriate lowering operators to (2.3) (and a change of phase) yields the corresponding operator

$$(\mathbf{8,3})_{\rm top}=(\sqrt{1/24})\{\sqrt{2}\mathbf{\Xi}_-^{-*}(\mathbf{\Sigma}_+'^0+x\mathbf{A}_+') +\sqrt{2}\mathbf{\Xi}_-'^{-*}(\mathbf{\Sigma}_+^0+x^2\mathbf{A}_+)+(1-x)\mathbf{\Xi}_-'^{0*}\mathbf{\Sigma}_+^+ +(1-x^2)\mathbf{\Xi}_-^{0*}\Sigma_+'^+-\sqrt{3}i\mathbf{\Sigma}_-'^{-*}\mathbf{n}_++\sqrt{3}i\mathbf{\Sigma}_-^{-*}\mathbf{n}_+' +\sqrt{2}(x\mathbf{\Sigma}_-^{0*}+\mathbf{A}_-^*)p_+'+\sqrt{2}(x^2\mathbf{\Sigma}_-'^{0*}+\mathbf{A}_-'^*)\mathbf{p}_+\}, \quad (2.4)$$

where $\mathbf{A}\equiv(\sqrt{\tfrac{3}{4}})\mathbf{\Lambda}+(\sqrt{\tfrac{1}{4}})\Sigma^0$.

There are 4 weights in **55** which map into the highest weight $[1\,1,0]$ of the submultiplet $(\mathbf{8,1})$. The combination $[-1\,0\,1\,0\,0]+x[1\,0\,-1\,1\,0]+x^2[1\,0\,1\,-1\,0]$ of the corresponding operators cannot be raised by $SU(3)$ or $R(3)$ raising operators, and so it must be the operator at the top of $(\mathbf{8,1})$. [The fourth operator, corresponding to $[1\,0\,0\,0\,0]$, belongs to $(\mathbf{8,3})$.] Evaluating these operators by again applying $R(11)$ lowering operators

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[1] D. Joseph, Phys. Rev. **139**, B1406 (1965).
[2] See, e.g., R. H. Dalitz, in Proceedings of the Midwest Conference on Theoretical Physics, 1966 (Indiana University, to be published); E. W. Anderson, *et al.*, Phys. Rev. Letters **16**, 855 (1966).
[3] This is what we expect if the regular representation of $R(11)$ contains the nonrelativistic limits of these operators.

to (2.3), we obtain

$$\begin{aligned}(\mathbf{8,1})_{\text{top}}=(\sqrt{1/24})\{[&\sqrt{2}\boldsymbol{\Xi}_-^{-*}(\boldsymbol{\Sigma}_-^0-\mathbf{A}_-)\\&+\sqrt{2}\boldsymbol{\Xi}_+^{-*}(\boldsymbol{\Sigma}_+^0-\mathbf{A}_+)-\sqrt{2}(\boldsymbol{\Sigma}_-^{0*}-\mathbf{A}_-^*)\mathbf{p}_-\\&-\sqrt{2}(\boldsymbol{\Sigma}_+^{0*}-\mathbf{A}_+^*)\mathbf{p}_++\boldsymbol{\Xi}_-^{0*}\boldsymbol{\Sigma}_-^++\boldsymbol{\Xi}_+^{0*}\boldsymbol{\Sigma}_+^+\\&-\boldsymbol{\Sigma}_-^{-*}\mathbf{n}_--\boldsymbol{\Sigma}_+^{-*}\mathbf{n}_+]+[']\}\,,\end{aligned}\tag{2.5}$$

where the second bracket contains terms like those shown but with primes on all baryons.

III. PHYSICAL BARYON STATES

The only distinction between the two sets of baryons above (which will be denoted by $\mathbf{B}$ and $\mathbf{B}'$) is that they correspond to different representations of $R(8)\otimes R(3)$, a subgroup of $R(11)$ containing $[SU(3)/Z_3]\otimes R(3)$. That is, there is no additive quantum number in $R(11)$ which distinguishes between them; they differ only by belonging to different eigenvalues of the $R(8)$ Casimir operators. Since there seems to be little evidence for $R(8)$ symmetry in particle physics at the present time, we have no *a priori* reason for supposing the physical baryons to be identified with either one of these octets rather than with a linear combination of them. Therefore we shall assume that the low-lying physical octet of baryons B and a second octet of physical spin-$\frac{1}{2}$ baryons B' are related to the "mathematical" baryons $\mathbf{B}$ and $\mathbf{B}'$ as follows:

$$B=\alpha\mathbf{B}+\beta\mathbf{B}'$$
and
$$B'=\gamma\mathbf{B}+\delta\mathbf{B}'\,,\tag{3.1}$$
where
$$|\alpha|^2+|\beta|^2=1=|\gamma|^2+|\delta|^2$$
and
$$\alpha^*\gamma+\beta^*\delta=0\,.\tag{3.2}$$

Incidentally, we may note that the case $\alpha=\delta=1$, $\beta=\gamma=0$ is ruled out by the observation from (2.4) that axial-vector weak currents occurring in the $(\mathbf{8,3})$ would then not lead to decay of members of the B octet, but only connect the B octet to the (presumably heavier) B' octet.

As a simple mechanism for the breaking of $R(11)$ symmetry we might consider an $SU(3)\otimes R(3)$-invariant interaction of the form

$$H_1=g_1\mathbf{B}^*\mathbf{B}'+\text{H.c.}\tag{3.3}$$

Concerning the other $SU(3)\otimes R(3)$-invariant interactions among the $\mathbf{B}$ and the $\mathbf{B}'$, we may note that

$$H_2=g_2(\mathbf{B}^*\mathbf{B}+\mathbf{B}'^*\mathbf{B}')\tag{3.4}$$

leads to no symmetry breaking, since it merely renormalizes the masses equally, while

$$H_3=g_3(\mathbf{B}^*\mathbf{B}-\mathbf{B}'^*\mathbf{B}')\tag{3.5}$$

depends on $R(8)$ Casimir operators (to distinguish the octet associated with the positive coupling constant). Thus we can ignore H_2; and we shall assume for the present that H_3 is much less important than H_1. Evaluation of H_0+H_1, where H_0 is the $R(11)$-invariant part of the Hamiltonian, between "mathematical" states $|\mathbf{B}\rangle$ and $|\mathbf{B}'\rangle$ at rest leads to the Hermitian mass matrix

$$M=\begin{pmatrix}m_0 & g_1\\ g_1^* & m_0\end{pmatrix}.\tag{3.6}$$

Diagonalization of (3.6) leads to the eigenvectors (physical baryon states)

$$|B\rangle=\eta(\sqrt{\tfrac{1}{2}})(|\mathbf{B}\rangle-\xi|\mathbf{B}'\rangle)\,,$$
and
$$|B'\rangle=\eta'(\sqrt{\tfrac{1}{2}})(|\mathbf{B}\rangle+\xi|\mathbf{B}'\rangle)\tag{3.7}$$

with eigenvalues (masses)

$$m=m_0-|g_1|\,,$$
and
$$m'=m_0+|g_1|\,.\tag{3.8}$$

In (3.7), $\xi\equiv g_1^*/|g_1|$, while η and η' are arbitrary phase factors.

In general, one might expect time-reversal invariance to force the off-diagonal elements of a mass matrix such as (3.6) to be real. Here, however, the situation is unusual in that we have two identical octets; time-reversal can mix corresponding members of the two octets without observable consequences. The existence of a time-reversal transformation under which (3.6) is invariant for complex g_1 is most easily demonstrated by first defining the transformation for the physical baryons B and B'. For these, no mixing of the octets is permissible, since they have different masses. We assume an antilinear transformation of the usual form:

$$B_T=\zeta\sigma_2B$$
and
$$B_T'=\zeta'\sigma_2B'\,,\tag{3.9}$$

where σ_2 acts on the spin indices and ζ and ζ' are phase factors. If we write $\mathfrak{B}$ for the column formed by placing B over B', then (3.9) can be abbreviated to

$$\mathfrak{B}_T=Z\sigma_2\mathfrak{B}\,,\tag{3.10}$$

where Z is a diagonal matrix with elements ζ and ζ'. Equations (3.1) can be rewritten

$$\mathfrak{B}=\mathfrak{U}^*\boldsymbol{\mathfrak{B}}\,,\tag{3.11}$$

where $\boldsymbol{\mathfrak{B}}$ is formed from $\mathbf{B}$ and $\mathbf{B}'$ and $\mathfrak{U}$ is the unitary matrix which diagonalizes the mass matrix. Also, then, we have

$$\boldsymbol{\mathfrak{B}}=\mathfrak{U}^T\mathfrak{B}\,,\tag{3.12}$$

where T means the transpose, so that (3.10) and the antilinearity of the time-reversal operation lead to

$$\boldsymbol{\mathfrak{B}}_T=\mathfrak{U}^\dagger\mathfrak{B}_T=\Sigma_2\boldsymbol{\mathfrak{B}}\,,$$
where
$$\Sigma_2\equiv\mathfrak{U}^\dagger Z\sigma_2\mathfrak{U}^*.\tag{3.13}$$

We note that Σ_2 is unitary:

$$\Sigma_2^\dagger\Sigma_2=\Sigma_2\Sigma_2^\dagger=1\,. \tag{3.14}$$

Also,

$$(\mathfrak{B}_T)_T=Z\sigma_2Z^*\sigma_2^*\mathfrak{B}=ZZ^*\sigma_2\sigma_2^*\mathfrak{B}=-\mathfrak{B}\,, \tag{3.15}$$

and

$$(\boldsymbol{\mathfrak{B}}_T)_T=\Sigma_2\Sigma_2^*\boldsymbol{\mathfrak{B}}=\mathfrak{U}^\dagger Z\sigma_2\mathfrak{U}^*\mathfrak{U}^TZ^*\sigma_2^*\mathfrak{U}\boldsymbol{\mathfrak{B}}=-\boldsymbol{\mathfrak{B}}\,. \tag{3.16}$$

Thus the transformation defined by (3.10) or (3.13), with complex conjugation of c numbers for antilinearity, has all the properties that can be required of time reversal. Invariance of the Hamiltonian under this transformation corresponds to the condition $K\Sigma_2^\dagger M\Sigma_2K=M$ on the mass matrix M, where K denotes complex conjugation. It is easily verified that this last condition is satisfied:

$$\begin{aligned}K\Sigma_2^\dagger M\Sigma_2K&=K\mathfrak{U}^T\sigma_2Z^*\mathfrak{U}M\mathfrak{U}^\dagger Z\sigma_2\mathfrak{U}^*K\\&=K\mathfrak{U}^T\sigma_2Z^*M_dZ\sigma_2\mathfrak{U}^*K\\&=K\mathfrak{U}^TM_d\mathfrak{U}^*K=KM^*K=M\,,\end{aligned} \tag{3.17}$$

where M_d is the diagonal matrix with real elements m and m'. Note that we have not made use of the special form (3.6) for the mass matrix; Eq. (3.17) holds equally well for the general Hamiltonian $H_0+H_1+H_2+H_3$.

Although time-reversal invariance leads to no restrictions when applied to the baryon Hamiltonian, it does lead to a restriction when applied to the interaction of the baryons with mesons. Since the usual vector and pseudoscalar mesons belong to the **55**, we might expect them to couple to baryon combinations such as those in (2.3), (2.4), and (2.5). Actually, parity and orbital-angular-momentum considerations modify the spin-independent interactions which could be written down immediately (in fact, W-spin and S-spin meson multiplets turn out to be quite different); however, it seems reasonable to assume that some meson combination (i.e., something which interacts strongly) will couple to each of the baryonic **55** combinations. The assumption of time-reversal invariance then restricts the manner in which the terms in these combinations can change phase under time reversal; while an over-all change of phase of a combination such as (2.4) may be permissible (it could possibly be absorbed by the meson factor), different changes of phase by the different terms is not.

We shall assume a time-reversal transformation of the form (3.9) acting on the physical baryons B and B', in order to demonstrate that our results do not depend on any assumptions concerning the time-reversal transformation for the (presumably unobservable) "mathematical" baryons $\mathbf{B}$ and $\mathbf{B}'$. Directing our attention first to the expression (2.4), we see that it is made up of terms of the form

$$\begin{aligned}&\lambda\mathbf{B}'^*\mathbf{B}+\lambda^*\mathbf{B}^*\mathbf{B}'\\&\quad=(\lambda\alpha^*\beta+\lambda^*\alpha\beta^*)B^*B+(\lambda\gamma^*\delta+\lambda^*\gamma\delta^*)B'^*B'\\&\qquad+(\lambda\beta\gamma^*+\lambda^*\alpha\delta^*)B^*B'+(\lambda\alpha^*\delta+\lambda^*\beta^*\gamma)B'^*B\,,\end{aligned} \tag{3.18}$$

where (of course) λ varies from term to term, and we have used the inverse of (3.1). Under time reversal, (3.18) transforms into

$$\begin{aligned}&(\lambda\alpha^*\beta+\lambda^*\alpha\beta^*)B^*B+(\lambda\gamma^*\delta+\lambda^*\gamma\delta^*)B'^*B'\\&\qquad+\zeta^*\zeta'(\lambda\alpha^*\delta+\lambda^*\beta^*\gamma)B^*B'\\&\qquad\qquad+\zeta\zeta'^*(\lambda\beta\gamma^*+\lambda^*\alpha\delta^*)B'^*B\,,\end{aligned} \tag{3.19}$$

where we have omitted the σ_2's since they have the same effect on all the terms here. We note that the coefficients of B^*B and B'^*B' do not change; so the same must be true of the remaining coefficients. This yields the equation

$$\lambda\beta\gamma^*+\lambda^*\alpha\delta^*=\zeta^*\zeta'(\lambda\alpha^*\delta+\lambda^*\beta^*\gamma) \tag{3.20}$$

and its complex conjugate. Returning to (2.4), we see that there are terms with $\lambda=-\sqrt{3}i$ (the Σ-n terms) and terms with $\lambda=\sqrt{2}x^2$ (the Ξ-A terms); substituting these values for λ into (3.20) and combining the two equations yields

$$\beta\gamma^*=(\zeta^*\zeta')\alpha^*\delta. \tag{3.21}$$

Combining this with (3.2) yields the restrictions

$$\begin{gathered}|\alpha|=|\beta|=|\gamma|=|\delta|=\sqrt{\tfrac{1}{2}}\,,\\\gamma=\pm i(\zeta\zeta'^*)^{1/2}\alpha\,,\quad\text{and}\quad\delta=\mp i(\zeta\zeta'^*\beta)^{1/2}.\end{gathered} \tag{3.22}$$

Substitution into (3.1) leads to

$$\begin{aligned}B&=\eta(\sqrt{\tfrac{1}{2}})(\mathbf{B}-\xi\mathbf{B}')\\B'&=\eta'(\sqrt{\tfrac{1}{2}})(\mathbf{B}+\xi\mathbf{B}')\,,\end{aligned} \tag{3.23}$$

where $\eta=\sqrt{2}\alpha$, $\eta'=\pm i(2\zeta\zeta'^*)^{1/2}\alpha$, and $\xi=-\beta/\alpha$ (note all have unit magnitude).

Thus, since $|B\rangle\sim B|0\rangle$, etc.,[4] we see from (3.7) and (3.23) that time-reversal invariance for the baryonic **55** yields the same result as assuming the symmetry-breaking interactions H_1, and neglecting H_3 [Eqs. (3.3) and (3.5)].

Substituting from (3.23) into (2.3), (2.4), and (2.5) now yields the following expressions in terms of the physical baryons, where we have set $\zeta'=\zeta$ for simplicity:

$$\begin{aligned}\mathbf{55}_{\text{top}}&=(\overline{\mathbf{10}},\mathbf{1})_{\text{top}}\\&=-(\sqrt{\tfrac{1}{32}})\{[\Xi_-^{-*}n_-+\Xi_-^{0*}p_-+\Xi_+^{-*}n_++\Xi_+^{0*}p_+]+[']\\&\qquad-\sqrt{3}[\Xi_-'^{-*}n_-+\Xi_-'^{0*}p_-+\Xi_+'^{-*}n_++\Xi_+'^{0*}p_+]+\sqrt{3}[']\}\,,\end{aligned} \tag{3.24}$$

$$\begin{aligned}(\mathbf{8},\mathbf{3})_{\text{top}}=(1/\sqrt{24})\{&[\sqrt{2}\,\mathrm{Re}(\xi)\Xi_-^{-*}\Sigma_+^0+\sqrt{2}\,\mathrm{Re}(x\xi)\Xi_-^{-*}A_++\sqrt{3}\,\mathrm{Im}(x\xi)\Xi_-^{0*}\Sigma_+^+-\sqrt{3}\,\mathrm{Im}(\xi)\Xi_-^{-*}n_+\\&+\sqrt{2}\,\mathrm{Re}(x\xi)\Sigma_-^{0*}p_++\sqrt{2}\,\mathrm{Re}(\xi)A_-^*p_+]-[']+[\sqrt{2}\,\mathrm{Im}(\xi)\Xi_-'^{-*}\Sigma_+^0+\sqrt{2}\,\mathrm{Im}(x\xi)\Xi_-'^{-*}A_+\\&-\sqrt{3}\,\mathrm{Re}(x\xi)\Xi_-'^{0*}\Sigma_+^++\sqrt{3}\,\mathrm{Re}(\xi)\Sigma_-'^{-*}n_++\sqrt{2}\,\mathrm{Im}(x\xi)\Sigma_-'^{0*}p_++\sqrt{2}\,\mathrm{Im}(\xi)A_-'^*p_+]+[']\}\,,\end{aligned} \tag{3.25}$$

[4] We adopt the convention that B contains the operators which create *particles*, in order that one-particle states and their corresponding fields shall have the same transformation properties.

$$(\mathbf{8,1})_{\text{top}}=(1/\sqrt{24})\{[\sqrt{2}\Xi_-^{-*}(\Sigma_-^0-A_-)+\sqrt{2}\Xi_+^{-*}(\Sigma_+^0-A_+)+\sqrt{2}(A_-^*-\Sigma_-^{0*})p_- +\sqrt{2}(A_+^*-\Sigma_+^{0*})p_+ +\Xi_-^{0*}\Sigma_-^+ +\Xi_+^{0*}\Sigma_+^+ -\Sigma_-^{-*}n_- -\Sigma_+^{-*}n_+]+[']\}. \quad (3.26)$$

The symbol $[']$ denotes an expression just like the preceding one except that unprimed baryons have been primed and vice versa. Note that all coefficients are real, owing in part to our choice of over-all phase factors in writing down (2.3), (2.4), and (2.5).

IV. COMPARISON WITH EXPERIMENT

It has become customary to express $SU(3)$ octet combinations in terms of the quantities F (antisymmetric) and D (symmetric) defined by Gell-Mann.[5] We define analogous expressions F_1 and D_1 for spin-1 combinations, and F_0 for spin-zero; the top vectors of these (corresponding to the $SU(3)$ weight [1 1]) are

$$F_{1\,\text{top}}=2[A_-^*p_+-\Xi_-^{-*}A_+-\Sigma_-^{0*}p_++\Xi_-^{-*}\Sigma_+^0]+\sqrt{2}[\Xi_-^{0*}\Sigma_+^+-\Sigma_-^{-*}n_+],$$

$$D_{1\,\text{top}}=(\tfrac{2}{3})[A_-^*p_++\Xi_-^{-*}A_++\Sigma_-^{0*}p_++\Xi_-^{-*}\Sigma_+^0]+\sqrt{2}[\Xi_-^{0*}\Sigma_+^++\Sigma_-^{-*}n_+], \quad (4.1)$$

and

$$F_{0\,\text{top}}=\sqrt{2}\{[A_-^*p_--\Xi_-^{-*}A_--\Sigma_-^{0*}p_-+\Xi_-^{-*}\Sigma_-^0]+[_{++}]\}+\{[\Xi_-^{0*}\Sigma_-^+-\Sigma_-^{-*}n_-]+[_{++}]\},$$

where $A\equiv(\sqrt{\tfrac{3}{4}})\Lambda+(\sqrt{\tfrac{1}{4}})\Sigma^0$, the lower subscripts + or − denote spin projections, and $[_{++}]$ indicates a repetition of the preceding backetted expression but with positive spin indices.[6] Comparing (4.1) with (3.25) and (3.26), we see that

$$(\mathbf{8,3})=f_1F_1+d_1D_1+\cdots$$

and

$$(\mathbf{8,1})=f_0F_0+\cdots, \quad (4.2)$$

where

$$f_1=-(\tfrac{1}{8})\,\text{Im}(x^2\xi),\quad d_1=-\tfrac{1}{8}\sqrt{3}\,\text{Re}(x^2\xi),\quad f_0=1/\sqrt{24}, \quad (4.3)$$

and the dots now denote terms involving primed physical baryons. The result that $(\mathbf{8,1})$ is purely F-type is, of course, what we must have if it is to contain the nonrelativistic limits of vector currents. Since it is customary to express the strengths of the axial-vector $(\mathbf{8,3})$ currents relatively to those of the vector $(\mathbf{8,1})$ currents, we define

$$f\equiv f_1/f_0=-(\sqrt{\tfrac{3}{8}})\,\text{Im}(x^2\xi),$$

and

$$d\equiv d_1/f_0=-\tfrac{3}{4}\sqrt{2}\,\text{Re}(x^2\xi). \quad (4.4)$$

Thus $R(11)$, together with time-reversal invariance, yields a relationship between f and d:

$$R(11):\ 3f^2+d^2=9/8. \quad (4.5)$$

This is, of course, a weaker prediction than the corresponding result from $SU(6)$

$$SU(6):\ f=\tfrac{2}{3},\ d=1. \quad (4.6)$$

Note that the values (4.6) do not satisfy (4.5); thus the prediction of $R(11)$ is different from, as well as weaker than, that of $SU(6)$.

The Cabibbo–Gell-Mann theory of weak interactions yields a number of predictions in terms of f and d; the one which has been most accurately tested experimentally is[7]

$$(G_A/G_V)_{n\to p}=f+d. \quad (4.7)$$

If we assume that the magnetic-moment operator also belongs to $(\mathbf{8,3})$, with the same $SU(3)$ identification as the charge operator, then various predictions for magnetic moments result; for example, for the nucleons,

$$\mu_p/\mu_n=-\tfrac{1}{2}(1+3f/d). \quad (4.8)$$

The predictions from (4.5)–(4.8) are compared with

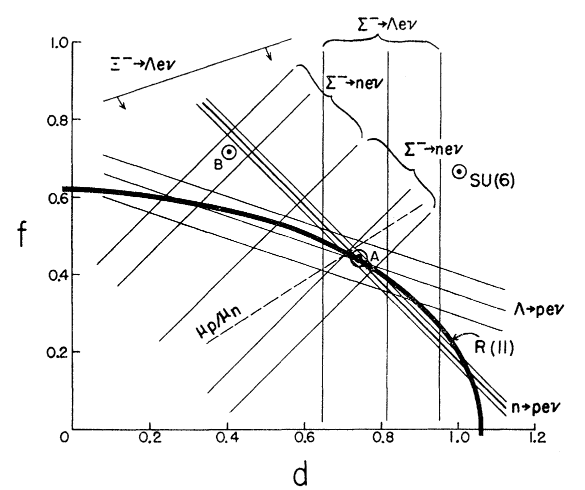

FIG. 1. Comparison of the $SU(6)$ and $R(11)$ predictions with experiment. The experimental limits indicated by solid lines are obtained from leptonic-decay rates as in Ref. 7, but by using the more recent data of Ref. 8; points A and B represent the consistent values found in Ref. 7 on the basis of the earlier data. The dashed line is obtained from the experimental value of μ_p/μ_n.

TABLE I. Comparison of predictions with experiment.

Ratio	$SU(6)$	$R(11)$	Experiment
μ_p/μ_n	−1.50	(Input)	−1.46
$(G_A/G_V)_{n\to p}$	1.67	1.17	1.18±0.02

[5] M. Gell-Mann and Y. Ne'eman, *The Eightfold Way* (W. A. Benjamin, Inc., New York, 1964), pp. 51, 52.

[6] We have defined Λ with the opposite sign from that used in Ref. 5. Note that A_-^* has the opposite spin projection from A_-, and so on.

[7] W. Willis *et al.*, Phys. Rev. Letters **13**, 291 (1964).

experimental values[8,9] in Table I. The comparison in the case of $SU(6)$ is already well-known: excellent for the magnetic-moment ratio, bad for $(G_A/G_V)_{n\to p}$. While $R(11)$ makes only one prediction, this one number is in excellent agreement with experiment.

Since the predictions of $SU(6)$ and $R(11)$ beyond those of $SU(3)$ are really just the numbers f and d, it is the latter which should be compared with experiment. In Fig. 1 we have plotted the limits of f and d resulting from the experimental limits on various leptonic-decay rates,[8] as was done in Ref. 7, along with the ratio f/d from measurements of nucleon magnetic moments.[8] It is seen that the leptonic-decay rates favor solution A of Ref. 7 even more strongly than before (and this solution need hardly be shifted at all), while solution B is now very unlikely. The interpretation of magnetic moments in terms of f and d is somewhat less direct since the magnetic coupling to the electromagnetic field differs from the electric coupling by an unknown dimensional parameter; however, present experimental data are reasonably consistent with the $SU(3)$ predictions $\mu_{\Sigma^+}=\mu_p$ and $\mu_\Lambda=\frac{1}{2}\mu_n$; and the f/d ratio determined from μ_p/μ_n is in excellent agreement with the leptonic-decay data, as Fig. 1 shows. Thus it is reasonable to suppose that the coordinates of a point close to A ($f=0.44$, $d=0.74$) are the correct values which should be reproduced by any higher symmetry such as $SU(6)$ or $R(11)$. As Fig. 1 shows, the point in the f-d plane predicted by $SU(6)$ is so far away as to indicate a rather large degree of symmetry breaking. On the other hand, the prediction based on $R(11)$, while it is weaker (being a curve rather than a point), is entirely consistent with the experimental data.

V. SUMMARY

When $R(11)$ symmetry is broken down to that of $SU(3)\otimes R(3)$, a degeneracy occurs since the baryon multiplet **32** decomposes into two identical $SU(3)\otimes R(3)$ spin-$\frac{1}{2}$ octets. This does not occur in the case of $SU(6)$, where the baryon multiplet **56** decomposes into a spin-$\frac{1}{2}$ octet and a spin-$\frac{3}{2}$ decuplet. Because of the resulting indeterminacy in identifying the physical baryons, $R(11)$ by itself yields almost no predictions concerning the parameters f and d describing the nonrelativistic limit of the axial-vector current octet; in fact, any point on *or within* the ellipse in Fig. 1 is allowed. However, f and d are restricted to values *on* the ellipse by either of two assumptions:

(i) that $R(11)$ symmetry is broken down to just that of $SU(3)\otimes R(3)$ in the simplest way, i.e., that H_3 [defined in (3.5), which respects also $R(8)\otimes R(3)$ symmetry] is much less important than H_1 [defined in (3.3)]; or

(ii) that the baryonic **55**-plet of currents must be of such a form as to allow a time-reversal invariant coupling to a mesonic **55**-plet of currents.

This restriction on f and d is entirely consistent with experimental observations of leptonic decay rates; by contrast, the values of f and d predicted by $SU(6)$ are not.

Earlier,[1] it appeared that the low-lying baryons strongly favored $SU(6)$ over $R(11)$, since the most appropriate $R(11)$ representation has the decomposition $\mathbf{32}\to(8,2)+(8,2)$, requiring a second spin-$\frac{1}{2}$ octet which had not been observed. Now, however, there is fairly good evidence for the zero-strangeness member of such a second octet,[2] split from the nucleons by about 470 MeV [compared to 300 MeV for the corresponding splitting in the $SU(6)$ multiplet **56**]. As yet quarks remain undiscovered[10]; if they really do not exist, then that is a point in favor of $R(11)$. On the other hand, $R(11)$ does predict 2 decuplets of spin-zero mesons which have not been observed; but yet, one cannot really say that their existence in appropriate mass ranges has been ruled out by experiment. Thus, in view of the favorable f-d relationship derived above, it seems that $R(11)$ is still in the running as an alternative to $SU(6)$.

APPENDIX A: MAPPING $[SU(3)/Z_3]\otimes R(3)$ INTO $R(11)$

Table II defines the mappings we have used. The labels on the Dynkin diagrams[11] refer to simple positive roots, which correspond to diagonal (commuting) generators of the Lie algebras. For $SU(3)$ there are two simple positive roots and hence two commuting generators, H_A and H_B; the corresponding raising and lowering operators are denoted by $E_{\pm A}$ and $E_{\pm B}$. Basis vectors are labeled by the eigenvalues A and B of H_A and H_B, often written as the pair $[A\ B]$ and called a "weight"; the operators $E_{\pm A}$ and $E_{\pm B}$ then take us from one basis vector to another. There is a unique highest weight in any irreducible representation; and in fact the highest weight can be used as a label for the representations. For example, the highest weight [1 1] corresponds uniquely to the octet representation of $SU(3)$. Figure 2(a) gives the basis vectors for this representation, and the normalized raising and lowering operators which connect the various states. This is the representation to which the generators belong (i.e., the "regular" representation) as Fig. 2(b) indicates; it is also the one to which the baryons belong, as shown in Fig. 2(c).[4]

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[8] A. H. Rosenfeld, Rev. Mod. Phys. **37**, 633 (1965).

[9] This value for G_A/G_V is quoted by S. L. Adler, Phys. Rev. Letters **14**, 1051 (1965).

[10] However, one might argue that the successes of the quark model (as presented, e.g., by R. H. Dalitz in Ref. 2) give an indirect indication of their existence.

[11] E. B. Dynkin, *American Mathematical Society Translations Series 2* (American Mathematical Society, Providence, Rhode Island, 1957), Vol. 6, pp. 319–362 [English transl. of Trudy Moskov. Mat. Obšč. **1**, 39 (1962)].

Table II. Labels and mappings.

Definition of labels				
Group	$R(3)$	$SU(3)$	$R(8)$	$R(11)$
Algebra	B_1	A_2	D_4	B_5
Dynkin diagram	○ m	○—○ A B	○—○—○ (with ○ attached to b) a b c d	○—○—○—○=● α β γ δ ϵ
Weight	$[m]$	$[A\ \ B]$	$[a\ \ b\ \ c\ \ d]$	$[\alpha\ \ \beta\ \ \gamma\ \ \delta\ \ \epsilon]$

Mappings: $SU(3)/Z_3$ into $R(8)$	$R(8)\otimes R(3)$ into $R(11)$	
$E_A=E_a+xE_c+x^2E_d$	$E_a=E_\alpha$	$H_a=H_\alpha$
$E_B=E_{a+b}+E_{c+b}+E_{d+b}$	$E_b=E_\beta$	$H_b=H_\beta$
$=[E_a+E_c+E_d,\ E_b]$	$E_c=E_\gamma$	$H_c=H_\gamma$
$H_A=H_a+H_c+H_d$	$E_d=\frac{1}{2}E_{\delta+\gamma+\epsilon+\delta+\epsilon}$ $=\frac{1}{2}[E_\delta,[E_\gamma,[E_\epsilon,[E_\delta,E_\epsilon]]]]$	$H_d=H_\gamma+2H_\delta+H_\epsilon$
$H_B=H_a+3H_b+H_c+H_d$	$E_m=E_\epsilon$	$H_m=H_\epsilon$

From Table II we see that $R(11)$ has five commuting generators, and hence its weights consist of the five eigenvalues of these operators. Figure 3 gives the weight diagram for the 32-dimensional "spinor" representation. Under restriction to the subgroup $R(8)\otimes R(3)$, this representation splits into two distinct octets with spin $\frac{1}{2}$; specifically, the weight $[0\,0\,0\,0\,1]$ maps into the $R(8)$ weight $[0\,0\,0\,1]$, the top of one of the $R(8)$ octets, while $[0\,0\,1\,0\,-1]$ maps into $[0\,0\,1\,0]$, the top of a second $R(8)$ octet. When $R(8)$ is restricted to $SU(3)/Z_3$, these both map into the greatest weight $[1\,1]$ of the $SU(3)$ octet. Figure 4 gives the assignment of the baryons to **32** which results from combining the $SU(3)$ identification of the baryons given in Figs. 2(a) and 2(c) with the mapping of the generators of $SU(3)\otimes R(3)$ into those of $R(11)$ given by Table II. [Note that while other equivalent mappings could have been chosen, this one exhibits the spin dependence most directly, since the third component of spin is just $\frac{1}{2}$ the eigenvalue of the $R(3)$ operator $H_m=H_\epsilon$.]

We now look for the **55** which occurs in the product $\mathbf{32}\otimes\mathbf{32}$ of the baryon multiplet with its conjugate. The weight diagram for **55** is shown in Fig. 5. Since weights are additive in direct products, we look for all pairs of weights in $\mathbf{32}\otimes\mathbf{32}$ which add up to the greatest weight $[0\,1\,0\,0\,0]$ in **55**. From Fig. 3 we find eight such pairs; the particular combination of these which cannot be raised with $R(11)$ operators (cf. Fig. 3), and hence must represent the top vector of **55**, is

$$\mathbf{55}_{\rm top}=\begin{bmatrix}0&0&0&0&1\\0&1&0&0&-1\end{bmatrix}-\begin{bmatrix}0&0&0&1&-1\\0&1&0&-1&1\end{bmatrix}+\begin{bmatrix}0&0&1&-1&1\\0&1&-1&1&-1\end{bmatrix}-\begin{bmatrix}0&1&-1&0&1\\0&0&1&0&-1\end{bmatrix}$$
$$+\begin{bmatrix}0&1&0&0&-1\\0&0&0&0&1\end{bmatrix}-\begin{bmatrix}0&1&0&-1&1\\0&0&0&1&-1\end{bmatrix}+\begin{bmatrix}0&1&-1&1&-1\\0&0&1&-1&1\end{bmatrix}-\begin{bmatrix}0&0&1&0&-1\\0&1&-1&0&1\end{bmatrix}. \quad \text{(A1)}$$

The upper weight in each pair of brackets is from the first representation in the product and the lower is from the second. Recalling that the product is of the form B^*B, using the particle assignments from Fig. 4 and its "conjugate," and normalizing, we obtain Eq. (2.3). The identification of the top of the **55** with the

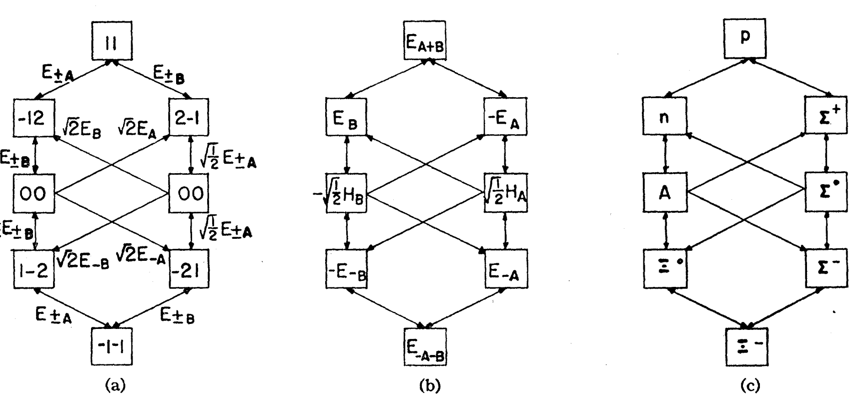

Fig. 2. The octet representation of $SU(3)$ and the assignments of the generators and baryons to this representation. Note that

$$\Lambda\equiv(\sqrt{\tfrac{3}{4}})\Lambda+(\sqrt{\tfrac{1}{4}})\Sigma^0,$$

in (c).

top of the $SU(3)\otimes R(3)$ representation $(\overline{\mathbf{10}},\mathbf{1})$ results from the mapping

$$H_A=H_\alpha+2H_\gamma+2H_\delta+H_\epsilon,$$
$$H_B=H_\alpha+3H_\beta+2H_\gamma+2H_\delta+H_\epsilon, \qquad \text{(A2)}$$

and

$$H_m=H_\epsilon$$

obtained from Table II. Thus the $R(11)$ weight $[0\,1\,0\,0\,0]$ corresponds to the $SU(3)$ weight $[0\,3]$ and $R(3)$ weight $[0]$; we write $[0\,3,0]$ for short. These latter are the greatest weights of the $SU(3)$ representation $\overline{\mathbf{10}}$ and the $R(3)$ representation $\mathbf{1}$ (spin-zero) [or $(\overline{\mathbf{10}},\mathbf{1})$ for short].

From Fig. 5 and Eqs. (A2) we see there is only one basis vector in **55** with the $SU(3)\otimes R(3)$ weight $[1\,1,2]$ corresponding to the top of **(8,3)** namely, the one with weight $[1\,0\,0\,-1\,2]$. To obtain this vector in terms of the baryons, we lower (A1) with the $R(11)$ operators $E_{-\beta}$, $E_{-\gamma}$, and $E_{-\delta}$ (in that order) and then use the baryon assignments given by Fig. 4 and its conjugate. The result, when normalized, is Eq. (2.4).

To find the **(8,1)**, with greatest weight $[1\,1,0]$, we note that there are four $R(11)$ weights in **55** with this

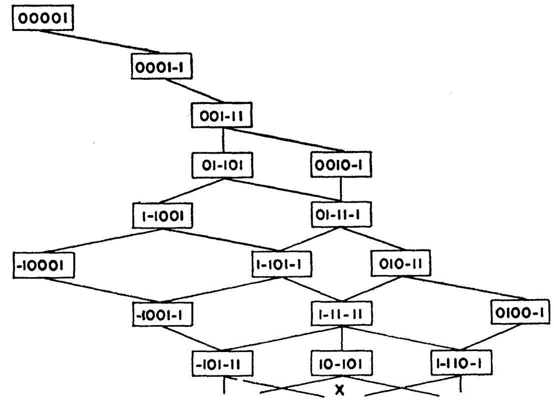

FIG. 3. The weight diagram for **32**. Only the upper half is shown; the lower half is obtained by a reflection about point X, with a reversal of signs. The action of $E_{\pm\alpha}$ is indicated by lines sloping down gently to the left; of $E_{\pm\beta}$, steeply to the left; of $E_{\pm\gamma}$, straight down; of $E_{\pm\delta}$, steeply to the right; and of $E_{\pm\epsilon}$, gently to the right. No normalizing factors are needed in this representation.

$SU(3)\otimes R(3)$ weight. Referring to Fig. 5, we see that the weight $[1\,0\,0\,0\,0]$ corresponds to the weight $[1\,1,0]$ of the **(8,3)**, since $E_{-\epsilon}$ just lowers the spin. The other three, $[-1\,0\,1\,0\,0]$, $[1\,0\,-1\,1\,0]$, and $[1\,0\,1\,-1\,0]$, can

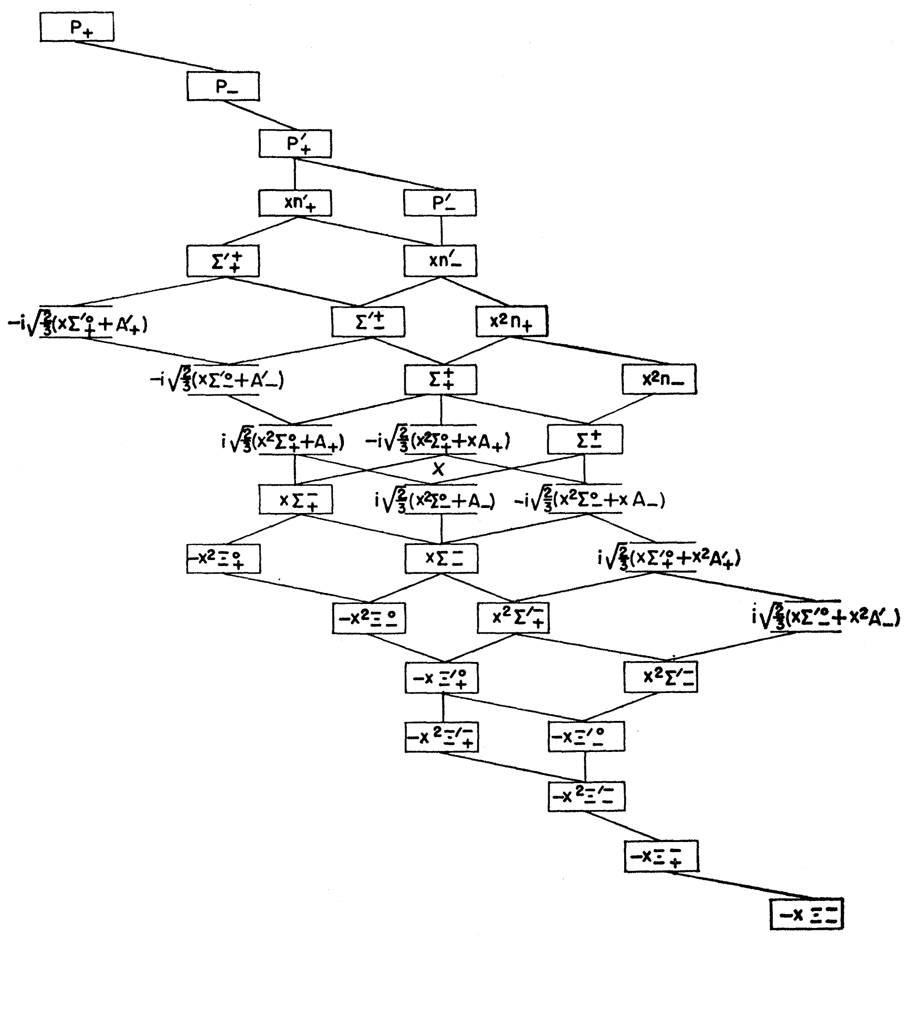

FIG. 4. Assignment of the B and B' baryons to the **32**. The assignments for the B^* and B'^* fields are obtained by starring the particle symbols and their coefficients, reflecting about the point X, and then changing the signs of the boxes in the even rows (i.e., the second, fourth, etc.). The lower $\pm$ signs denote third components of spin, $A\equiv(\sqrt{\tfrac{3}{4}})\Lambda+(\sqrt{\tfrac{1}{4}})\Sigma^0$, and $x\equiv\exp(2\pi i/3)$.

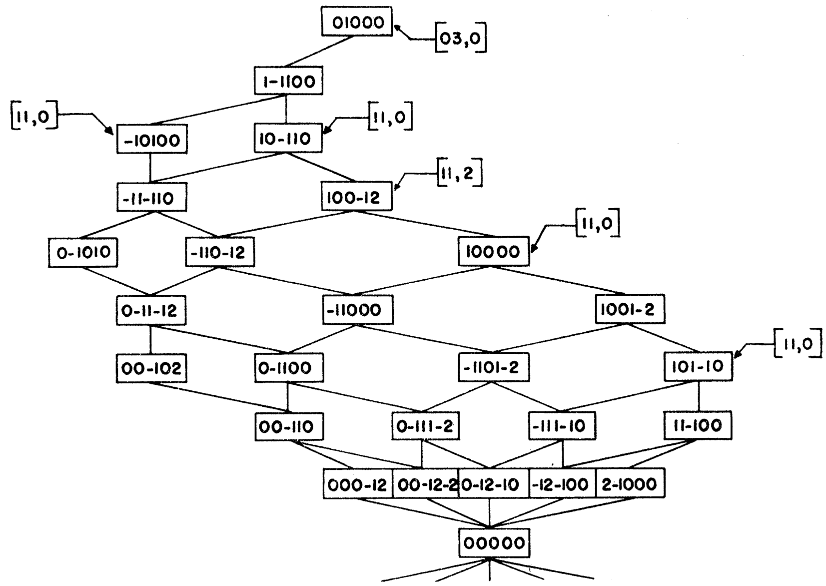

FIG. 5. Weight diagram for **55**, The lower half of the diagram is obtained by a reflection through the weight [0 0 0 0 0], with a reversal of all signs. Note that [0 0 0 0 0] is fivefold degenerate. Sloping lines indicate the actions of the raising and lowering operators, as in Fig. 3, except that in some cases here a normalizing factor $\sqrt{\frac{1}{2}}$ is needed.

be obtained from (A1) by use of appropriate $R(11)$ lowering operators; and the top of (**8**,**3**) is just that combination of these vectors which cannot be raised by the $SU(3)$ operators

$$E_A=E_\alpha+xE_\gamma+\tfrac{1}{2}x^2E_{\delta+\gamma+\epsilon+\delta+\epsilon}\,,$$
and
$$E_B=[E_\alpha+E_\gamma+\tfrac{1}{2}E_{\delta+\gamma+\epsilon+\delta+\epsilon},\,E_\beta]\,. \tag{A3}$$

(Cf. Table II.) That is, we set

$$\left.\begin{matrix}E_A\\E_B\end{matrix}\right\}\{a[-1\,0\,1\,0\,0]+b[1\,0\,-1\,1\,0]+c[1\,0\,1\,-1\,0]\}=0\,, \tag{A4}$$

which yields

$$b/a=x \quad\text{and}\quad c/a=x^2\,, \tag{A5}$$

where $x\equiv\exp(2\pi i/3)$. Normalizing and identifying weights with baryons yields (2.5).

APPENDIX B: ELECTROMAGNETIC AND WEAK-CURRENT OPERATORS

We now introduce a new notation to distinguish the different F and D baryonic currents; we shall, for example, write $F_J{}^M(\chi_a)$ for the antisymmetric combination with spin J and third component M which occupies the same position in an $SU(3)$ weight diagram as the generator χ_a [see Fig. 2(b)]. So the expressions defined in (4.1) will be denoted as follows:

$$F_{1\,\mathrm{top}}=F_1{}^1(E_{A+B})\,,$$
$$D_{1\,\mathrm{top}}=D_1{}^1(E_{A+B})\,, \tag{B1}$$
$$F_{0\,\mathrm{top}}=F_0(E_{A+B})\,;$$

and Eqs. (4.2) correspond to

$$(\mathbf{8},\mathbf{3})_{\mathrm{top}}=f_1F_1{}^1(E_{A+B})+d_1D_1{}^1(E_{A+B})+\cdots\,,$$
$$(\mathbf{8},\mathbf{1})_{\mathrm{top}}=f_0F_0(E_{A+B})+\cdots \tag{B2}$$

(where the dots represent terms involving the second octet of physical baryons).

The charge operator Q is easily expressed in terms of H_A and H_B by comparing the eigenvalues of those operators, given in Fig. 2(a), with the charges of the baryons, from Fig. 2(c); the result is

$$Q=\tfrac{2}{3}H_A+\tfrac{1}{3}H_B\,. \tag{B3}$$

Since the charge ($\mathcal{Q}$) and magnetic-moment ($\mathfrak{M}$) currents are to belong to the (**8**,**1**) and (**8**,**3**), respectively, we must have

$$\mathcal{Q}=\epsilon' f_0F_0(Q)+\cdots=\epsilon' f_0[\tfrac{2}{3}F_0(H_A)+\tfrac{1}{3}F_0(H_B)]+\cdots$$
and
$$\begin{aligned}\mathfrak{M}&=\epsilon''\{f_1F_1{}^0(Q)+d_1D_1{}^0(Q)\}+\cdots\\&=\epsilon''\{f_1[\tfrac{2}{3}F_1{}^0(H_A)+\tfrac{1}{3}F_1{}^0(H_B)]\\&\quad+d_1[\tfrac{2}{3}D_1{}^0(H_A)+\tfrac{1}{3}D_1{}^0(H_B)]+\cdots\,,\end{aligned} \tag{B4}$$

where ϵ' and ϵ'' are scale factors. The necessary baryon combinations can be obtained by noting (B1) and lowering expressions (4.1). The result is

$$\mathcal{Q}=(\epsilon' f_0/\sqrt{2})\{[\Sigma_-{}^{+*}\Sigma_-{}^{+}+p_-{}^*p_- -\Sigma_-{}^{-*}\Sigma_-{}^{-}-\Xi_-{}^{-*}\Xi_-{}^{-}]+[{}_{++}]+\cdots\} \tag{B5}$$

and

$$\begin{aligned}\mathfrak{M}=&(\epsilon''/9\sqrt{2})\\&\times\{[9f_1(\Sigma_-{}^{+*}\Sigma_-{}^{+}-\Sigma_-{}^{-*}\Sigma_-{}^{-}-\Xi_-{}^{-*}\Xi_-{}^{-}+p_-{}^*p_-)\\&+d_1(3p_-{}^*p_- -6n_-{}^*n_- -6\Xi_-{}^{0*}\Xi_-{}^{0}+3\Xi_-{}^{-*}\Xi_-{}^{-}\\&+3\Sigma_-{}^{+*}\Sigma_-{}^{+}+3\Sigma_-{}^{-*}\Sigma_-{}^{-}+8\Sigma_-{}^{0*}\Sigma_-{}^{0}-4A_-{}^*\Sigma_-{}^{0}\\&-4\Sigma_-{}^{0*}A_- -4A_-{}^*A_-)]+[{}_{++}]+\cdots\}\,,\end{aligned} \tag{B6}$$

where $[{}_{++}]$ denotes a repetition of the preceding bracketed expression but with positive spin indices. Clearly the expression for $\mathcal{Q}$ is consistent. From (B6) we can immediately read off the ratio

$$\mu_p/\mu_n = -(3f_1+d_1)/2d_1, \tag{B7}$$

which is equivalent to (4.8) since $f\equiv f_1/f_0$ and $d\equiv d_1/f_0$. For $R(11)$, we can substitute f and d from (4.4) to obtain

$$\mu_p/\mu_n = \mathrm{Re}(\xi)/\mathrm{Re}(x^2\xi); \tag{B8}$$

thus this magnetic-moment ratio depends only on the phase of ξ.

The non-strangeness-changing weak currents, with $SU(3)$ quantum numbers of the $\pi^\pm$ mesons, correspond to the operators $E_{\pm A}$, with weights $[\pm 2 \mp 1]$. Noting that the vector current $\mathcal{J}_V$ (in the nonrelativistic limit) is to belong to (**8**,**1**) and the axial-vector current $\mathcal{J}_A$ to (**8**,**3**), and lowering (4.1) [recall (B1)] to obtain the appropriate baryonic combinations, we find

$$\begin{aligned}\mathcal{J}_V &= \epsilon f_0 F_0(E_A)+\cdots\\ &= -\epsilon f_0\{[n_-{}^*p_- + \sqrt{2}\Sigma_-{}^{0*}\Sigma_-{}^+ + \sqrt{2}\Sigma_-{}^{-*}\Sigma_-{}^0\\ &\qquad + \Xi_-{}^{-*}\Xi_-{}^0] + [_{++}]\} + \cdots,\end{aligned} \tag{B9}$$

$$\begin{aligned}\mathcal{J}_A &= \epsilon f_1 F_1{}^0(E_A) + \epsilon d_1 D_1{}^0(E_A) + \cdots\\ &= -\epsilon f_1\{[n_-{}^*p_- + \sqrt{2}\Sigma_-{}^{0*}\Sigma_-{}^+ + \sqrt{2}\Sigma_-{}^{-*}\Sigma_-{}^0 + \Xi_-{}^{-*}\Xi_-{}^0]\\ &\qquad + [_{++}]\} - \epsilon d_1\{[n_-{}^*p_- - (\sqrt{2}/3)(\Sigma_-{}^{0*} - 2\Lambda_-{}^*)\Sigma_-{}^+\\ &\qquad + (\sqrt{2}/3)\Sigma_-{}^{-*}(\Sigma_-{}^0 - 2\Lambda_-) - \Xi_-{}^{-*}\Xi_-{}^0]\\ &\qquad + [_{++}]\} + \cdots.\end{aligned} \tag{B10}$$

(Note that $\mathcal{J}_A$ is defined to be the component of the axial current with zero spin projection. As usual we assume the same scale factor ϵ for $\mathcal{J}_V$ and $\mathcal{J}_A$.) To interpret these expressions in terms of the conventional coupling constants G_A and G_V, we note that vector and axial-vector currents have the following nonrelativistic limits:

$$\begin{aligned}&G_V\bar{\psi}\gamma_\mu\psi \to G_V\phi^\dagger\phi \text{ (space components vanish)}\\ &G_A\bar{\psi}i\gamma_5\gamma_\mu\psi \to G_A\phi^\dagger\sigma_m\phi\\ &\qquad\qquad \text{(time component vanishes)},\end{aligned} \tag{B11}$$

where ϕ denotes the "large" components in the representation $\gamma_0=\sigma_3\otimes I$, $\gamma_k = i\sigma_2\otimes\sigma_k$, and $i\gamma_5 = -\sigma_1\otimes I$. Thus, for example, the ratio of axial-vector to vector amplitudes in neutron decay is given by

$$\frac{G_A\langle p_+|\phi^\dagger\sigma_3\phi|n_+\rangle}{G_V\langle p_+|\phi^\dagger\phi|n_+\rangle} = \frac{\langle p_+|\mathcal{J}_A|n_+\rangle}{\langle p_+|\mathcal{J}_V|n_+\rangle}. \tag{B12}$$

Together with (B9) and (B10) this yields the relation

$$(G_A/G_V)_{n\to p} = (f_1+d_1)/f_0 = f+d. \tag{B13}$$

For $R(11)$, use of (4.4) leads to

$$(G_A/G_V)_{n\to p} = (\sqrt{\tfrac{3}{2}})\ \mathrm{Im}(x\xi). \tag{B14}$$

The strangeness-changing weak currents $\mathcal{K}_{V,A}$ transform under $SU(3)$ like $E_{\pm(A+B)}$, i.e., like $K^\pm$ mesons; thus, from (B1) and (4.1),

$$\begin{aligned}\mathcal{K}_V &= \epsilon f_0 F_0(E_{A+B}) + \cdots\\ &= \epsilon f_0\{[-\sqrt{2}(\Sigma_-{}^{0*} - \Lambda_-{}^*)p_- + \sqrt{2}\Xi_-{}^{-*}(\Sigma_-{}^0 - \Lambda_-)\\ &\qquad - \Sigma_-{}^{-*}n_- + \Xi_-{}^{0*}\Sigma_-{}^+] + [_{++}]\} + \cdots,\end{aligned} \tag{B15}$$

$$\begin{aligned}\mathcal{K}_A &= \epsilon f_1 F_1{}^0(E_{A+B}) + \epsilon d_1 D_1{}^0(E_{A+B}) + \cdots\\ &= \epsilon f_1\{[-\sqrt{2}(\Sigma_-{}^{0*} - \Lambda_-{}^*)p_- + \sqrt{2}\Xi_-{}^{-*}(\Sigma_-{}^0 - \Lambda_-)\\ &\qquad - \Sigma_-{}^{-*}n_- + \Xi_-{}^{0*}\Sigma_-{}^+] + [_{++}]\}\\ &\qquad + \epsilon d_1\{[(\sqrt{2}/3)(\Sigma_-{}^{0*} + \Lambda_-{}^*)p_-\\ &\qquad + (\sqrt{2}/3)\Xi_-{}^{-*}(\Sigma_-{}^0 + \Lambda_-) + \Sigma_-{}^{-*}n_- + \Xi_-{}^{0*}\Sigma_-{}^+]\\ &\qquad + [_{++}]\} + \cdots.\end{aligned} \tag{B16}$$