Theory of Nonleptonic Hyperon Decays*

Y. T. CHIU AND J. SCHECHTER
The Enrico Fermi Institute for Nuclear Studies, The University of Chicago, Chicago, Illinois
AND
Y. UEDA†
Physics Department, Indiana University, Bloomington, Indiana
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In the generalized Sugawara-Suzuki model of nonleptonic hyperon decays all amplitudes are given in terms of three parameters describing the current-current "spurion." We attempt to estimate the value of these parameters by approximately "saturating" the current-current product with $\frac{1}{2}^+$ octet and $\frac{3}{2}^+$ decuplet intermediate states. Fairly good agreement with experiment is found. This seems to show that weak-interaction universality holds for nonleptonic as well as leptonic decays and to indicate that the so-called "octet dominance" arises automatically.

I. INTRODUCTION

In this paper we attempt to calculate all the nonleptonic hyperon decay amplitudes without introducing *any arbitrary* parameters. We use the generalized Sugawara-Suzuki method[1–5] and the assumption that the weak Hamiltonian is of Cabibbo's current-current form. Our input information consists of the universal leptonic coupling constant G, the Cabibbo[6] angle $\sin\theta$, and nucleon and NN^* current form factors. The main conclusions are the following:

(i) A current-current Hamiltonian seems adequate to explain S-wave nonleptonic hyperon decays.
(ii) Octet dominance tends to emerge automatically.
(iii) The factor $\sin\theta$ appears to be necessary for nonleptonic as well as leptonic decays.

The basic idea behind this calculation is as follows. First, the generalized Sugawara-Suzuki method enables us to obtain all S- and P-wave amplitudes in terms of three parameters. These three parameters are the amount of $SU(3)$ {27}, {8s}, and {8a} representations contained in the matrix element of the current-current operator product evaluated between *single* baryon states. Then we estimate these three parameters by summing over a set of intermediate states inserted between the two currents. This approach was pointed out by Sugawara[1] in his original paper. Here we shall attempt to "saturate" the current-current product with only the octet $\frac{1}{2}^+$ baryons and the decuplet $\frac{3}{2}^+$ baryons. In a previous paper[7] we have found that the octet contribution alone gave qualitatively the three features mentioned above. Now we note that the inclusion of the decuplet tends to improve the agreement with experiment. It will be seen that this tendency comes from just $SU(3)$ factors rather than the details of integration over form factors.

The absolute magnitudes of the contributions, however, do depend somewhat on the details of the form factors as well as the choices of degenerate octet baryon mass M and degenerate decuplet mass M^*. Nevertheless, various reasonable fits can be made for different reasonable values of these parameters. In view of the present status of experimental information on form factors, this is probably all that we can hope for.

In Sec. II the general formalism is introduced, the amplitudes are expressed in terms of the weak spurion, and the relations among the amplitudes which follow from $SU(3)$ invariance are given. Section III contains a discussion of the relevant form factors and brings the theory to a stage where only numerical integrations need be done to obtain the final result. In Sec. IV we give a discussion of the results for a typical choice of form factors and draw some conclusions. Finally, Sec. V contains a detailed discussion of various fits.

II. GENERAL FORMALISM

Our initial assumption is that all weak interactions, nonleptonic as well as leptonic, are described by the universal current-current Hamiltonian,

$$H_w=\frac{G}{\sqrt{2}}\times\tfrac{1}{2}[J_\mu,J_\mu^\dagger]_+ , \qquad (1)$$

where $G\simeq10^{-5}/M_p^2$ and the current is taken to be of Cabibbo's form,[6]

$$J_\mu=J_\mu^{(\text{leptonic})}+\cos\theta[(V_2^1)_\mu+(P_2^1)_\mu]+\sin\theta[(V_3^1)_\mu+(P_3^1)_\mu]. \qquad (2)$$

In Eq. (2), $(V_b^a)_\mu$ and $(P_b^a)_\mu$ are, respectively, the

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[1] H. Sugawara, Phys. Rev. Letters 15, 870 (1965); 15, 997 (1965).
[2] M. Suzuki, Phys. Rev. Letters 15, 986 (1965).
[3] Y. Hara, Y. Nambu, and J. Schechter, Phys. Rev. Letters 16, 380 (1966).
[4] S. Badier and C. Bouchiat, Phys. Letters 20, 529 (1966).
[5] L. S. Brown and C. M. Sommerfield, Phys. Rev. Letters 16, 751 (1966).
[6] N. Cabibbo, Phys. Rev. Letters 10, 531 (1963).
[7] Y. T. Chiu and J. Schechter, Phys. Rev. Letters 16, 1022 (1966). See also S. Biswas, A. Kumar, and R. Saxema (to be published) whose results do not agree with ours. Similar work has also been done by Y. Hara (private communication to Y. Nambu). Work on K decays has been done by E. Ferrari, V. Mathur, and L. K. Pandit (to be published), and S. K. Bose and S. N. Biswas (to be published).

vector and pseudovector octet currents. By now it seems reasonably well established[8] that $\sin\theta \simeq 0.26$ does not arise as an $SU(3)$ symmetry-breaking renormalization but represents something more fundamental. The matrix elements of Eq. (2) between octet baryon states at zero momentum transfer were postulated by Cabibbo to be of the form

$$\cos\theta[(F^V)_2{}^1+d(D^A)_2{}^1+f(F^A)_2{}^1] + \sin\theta[(F^V)_3{}^1+d(D^A)_3{}^1+f(F^A)_3{}^1], \quad (3)$$

where we have omitted the Lorentz indices and

$$(F^V)_b{}^a=\bar{N}_c{}^a\gamma N_b{}^c-\bar{N}_b{}^c\gamma N_c{}^a, \quad (4a)$$

$$(F^A)_b{}^a=\bar{N}_c{}^a\gamma\gamma_5 N_b{}^c-\bar{N}_b{}^c\gamma\gamma_5 N_c{}^a, \quad (4b)$$

$$(D^A)_b{}^a=\bar{N}_c{}^a\gamma\gamma_5 N_b{}^c+\bar{N}_b{}^c\gamma\gamma_5 N_c{}^a - \tfrac{2}{3}\delta_b{}^a\bar{N}_n{}^m\gamma\gamma_5 N_m{}^n. \quad (4c)$$

Furthermore, the experimental data[9] indicate that

$$g_A=d+f\simeq 1.18, \quad (5a)$$

$$d/f\simeq 1.7. \quad (5b)$$

The justification for writing only octet terms in Eq. (3) is, perhaps, provided by the generalized[10] Ademollo-Gatto theorem.[11]

Now we note that the part of Eq. (1) which gives rise to nonleptonic decays is

$$H_w{}^{\mathrm{N.L.}}=\frac{G}{\sqrt{2}}\cos\theta\sin\theta(\tfrac{1}{2}) \times\{[(V_2{}^1+P_2{}^1),(V_1{}^3+P_1{}^3)]_+ +(2\leftrightarrow 3)\}. \quad (6)$$

Next we use the technique of Sugawara,[1] Suzuki,[2] and Nambu and Shrauner[12] to obtain the following formula[5] for the hyperon nonleptonic decay matrix elements:

$$\begin{aligned}\frac{2Mi}{g_{\pi NN}}&(2k_0)^{1/2}\langle N'(p')\pi_b{}^a(k)|H_w{}^{\mathrm{N.L.}}(0)|N(p)\rangle\\ &=A\bar{u}(p')u(p)-B\bar{u}(p')\gamma_5 u(p)\\ &=\frac{\sqrt{2}}{g_A}\{\langle N'(p')|[B_b{}^a(0),H_w{}^{\mathrm{N.L.}}(0)]|N(p)\rangle\\ &\quad-\sum_{n=\text{baryon octet}}(\langle N'(p')|B_b{}^a(0)|n\rangle \times\langle n|H_w{}^{\mathrm{N.L.}}(0)|N(p)\rangle\\ &\quad-\langle N'(p')|H_w{}^{\mathrm{N.L.}}(0)|n\rangle\langle n|B_b{}^a(0)|N(p)\rangle)\}, \quad (7)\end{aligned}$$

where A and B are defined to be the S- and P-wave amplitudes for each decay and $B_b{}^a$ is the axial-vector "charge" given by

$$B_b{}^a=i\int d^3x(P_b{}^a)_4.$$

The great simplification achieved in Eq. (7) is that all matrix elements are taken between one-particle states. The first term on the right-hand side contributes only to S waves[2] while the second term, which corresponds to baryon pole diagrams,[3] contributes only to P waves.

First let us consider the S-wave amplitudes. Using the assumed equal-time commutators,

$$[B_b{}^a(0),V_d{}^c(\mathbf{x},0)]=\delta_d{}^aP_b{}^c(\mathbf{x},0)-\delta_b{}^cP_d{}^a(\mathbf{x},0), \quad (8a)$$

$$[B_b{}^a(0),P_d{}^c(\mathbf{x},0)]=\delta_d{}^aV_b{}^c(\mathbf{x},0)-\delta_b{}^cV_d{}^a(\mathbf{x},0), \quad (8b)$$

and inserting Eq. (6) into Eq. (7), we find

$$\begin{aligned}A=\frac{1}{2g_A}G\sin\theta\cos\theta\{&\delta_1{}^a(V_{2b}{}^{13}+P_{2b}{}^{13}+(2\leftrightarrow 3))\\ &-\delta_b{}^1(V_{21}{}^{a3}+P_{21}{}^{a3}+(2\leftrightarrow 3))\\ &+\delta_2{}^a(V_{b1}{}^{13}+P_{b1}{}^{13})-\delta_b{}^2(V_{31}{}^{1a}+P_{31}{}^{1a})\}, \quad (9)\end{aligned}$$

where the vector and axial-vector "spurions" are defined by

$$V_{bd}{}^{ac}=\langle N'|[V_b{}^a,V_d{}^c]_+|N\rangle, \quad (10a)$$

$$P_{bd}{}^{ac}=\langle N'|[P_b{}^a,P_d{}^c]_+|N\rangle. \quad (10b)$$

It is convenient to introduce the total spurion also:

$$S_{bd}{}^{ac}=V_{bd}{}^{ac}+P_{bd}{}^{ac}. \quad (10c)$$

This has the following decomposition into irreducible $SU(3)$ tensors[13]

$$\begin{aligned}S_{bd}{}^{ac}=\tau T_{bd}{}^{ac}&+\delta\{(\delta_b{}^cD_d{}^a+\delta_d{}^aD_b{}^c)-\tfrac{2}{3}(\delta_b{}^aD_d{}^c+\delta_d{}^cD_b{}^a)\}\\ &+\phi\{(\delta_b{}^cF_d{}^a+\delta_d{}^aF_b{}^c)-\tfrac{2}{3}(\delta_b{}^aF_d{}^c+\delta_d{}^cF_b{}^a)\}\\ &+\sigma\{\delta_d{}^a\delta_b{}^c-\tfrac{1}{3}\delta_b{}^a\delta_d{}^c\}\langle\bar{N}N\rangle, \quad (11a)\end{aligned}$$

where τ, δ, ϕ, and σ are coefficients which, respectively, give the amount of spurion in the {27}, {8s}, {8a}, and {1} $SU(3)$ representations. We note that the $\{\overline{10}\}$ and {10} representations do not appear because of the symmetry of $S_{bd}{}^{ac}$. Furthermore, we have

$$\langle\bar{N}N\rangle=\bar{N}_n{}^mN_m{}^n, \quad (11b)$$

$$\begin{aligned}T_{bd}{}^{ac}=(&\bar{N}_b{}^aN_d{}^c+\bar{N}_d{}^cN_b{}^a+\bar{N}_d{}^aN_b{}^c+\bar{N}_b{}^cN_d{}^a)\\ &-\tfrac{1}{5}(\delta_b{}^aD_d{}^c+\delta_d{}^cD_b{}^a+\delta_d{}^aD_b{}^c+\delta_b{}^cD_d{}^a)\\ &-\tfrac{1}{6}(\delta_b{}^a\delta_d{}^c+\delta_d{}^a\delta_b{}^c)\langle\bar{N}N\rangle, \quad (11c)\end{aligned}$$

[8] L. K. Pandit and J. Schechter, Phys. Letters 19, 56 (1965); D. Amati, C. Bouchiat, and J. Nuyts, *ibid.* 19, 59 (1965); A. Sato and S. Sasaki (to be published); C. A. Levinson and I. J. Muzinich, Phys. Rev. Letters 15, 715 (1965); R. Oehme, *ibid.* 16, 215 (1966).

[9] W. Willis *et al.*, Phys. Rev. Letters 13, 291 (1964).

[10] J. Schechter and Y. Ueda, Phys. Rev. 144, 1338 (1966); G. Guralnik, V. Mathur, and L. K. Pandit, Phys. Letters 20, 64 (1966).

[11] M. Ademollo and R. Gatto, Phys. Rev. Letters 13, 264 (1964).

[12] Y. Nambu and E. Schrauner, Phys. Rev. 128, 862 (1962).

[13] S. Okubo, Progr. Theoret. Phys. (Kyoto) 27, 949 (1962).

TABLE I. P-wave amplitudes.

Decay	B'[Eq. (16)]
Λ_-^0	$(1/\sqrt{6})[2(f+d)(\phi+\delta)+(f-d)(\phi-\delta)-(6/5)f\tau+(2/5)d\tau]$
Λ_0^0	$(-1/\sqrt{12})[2(f+d)(\phi+\delta)+(f-d)(\phi-\delta)-(6/5)f\tau-(18/5)d\tau]$
Ξ_-^-	$(-1/\sqrt{6})[(f+d)(\phi+\delta)+2(f-d)(\phi-\delta)+(6/5)f\tau+(2/5)d\tau]$
Ξ_0^0	$(1/\sqrt{12})[(f+d)(\phi+\delta)+2(f-d)(\phi-\delta)+(6/5)f\tau-(18/5)d\tau]$
Σ_+^+	$-(4/3)d\delta-2f\tau-(2/5)d\tau$
Σ_0^+	$(-1/\sqrt{2})[(f-d)(\phi-\delta)-(4/5)f\tau+(4/5)d\tau]$
Σ_-^-	$(f-d)(\phi-\delta)-(4/3)d\delta+(6/5)f\tau+(2/5)d\tau$

where $D_b{}^a$ and $F_b{}^a$ are the symmetric and antisymmetric $SU(3)$ matrices [as in Eq. (4)].

In this scheme all the information about the nonleptonic hyperon decays is contained in the three parameters τ, δ, and ϕ. From Eqs. (9) and (11a) we obtain the S-wave, Λ and Ξ, $\Delta I=\frac{1}{2}$ relations

$$A(\Lambda_-^0)+\sqrt{2}A(\Lambda_0^0)=0\,, \tag{12a}$$

$$A(\Xi_-^-)+\sqrt{2}A(\Xi_0^0)=0\,, \tag{12b}$$

and the Sugawara-Suzuki[1,2] relations

$$A(\Sigma_-^-)+\sqrt{2}A(\Sigma_0^+)=-A(\Sigma_+^+)\,, \tag{13a}$$

$$A(\Lambda_-^0)+2A(\Xi_-^-)=-3/\sqrt{6}A(\Sigma_-^-)\,. \tag{13b}$$

The three independent S-wave amplitudes which we take to be $A(\Sigma_+^+)$, $A(\Lambda_-^0)$, and $A(\Xi_-^-)$ are given in terms of τ, δ, and ϕ as follows:

$$A(\Sigma_+^+)=\frac{G}{2g_A}\sin\theta\cos\theta(-2\tau)\,, \tag{14a}$$

$$A(\Lambda_-^0)=\frac{G}{2g_A}\sin\theta\cos\theta\left(\frac{1}{\sqrt{6}}\right)(-6/5\tau+\delta+3\phi)\,, \tag{14b}$$

$$A(\Xi_-^-)=\frac{G}{2g_A}\sin\theta\cos\theta\left(\frac{1}{\sqrt{6}}\right)(-6/5\tau+\delta-3\phi)\,. \tag{14c}$$

Furthermore all the P-wave amplitudes are given,[3] in this model, in terms of τ, δ, and ϕ and the quantities d and f of Eq. (3). We evaluate the P-wave term of Eq. (7) by using the method of Nambu and Shrauner[12] and note that

$$\langle N'|H_w{}^{\text{N.L.}}|N\rangle=\frac{G}{\sqrt{2}}\cos\theta\sin\theta \times(\tfrac{1}{2}(S_{31}{}^{12}+S_{21}{}^{13})\bar{u}(p')u(p)) \tag{15}$$

gives the B amplitudes as

$$B=\frac{2M}{\Delta M}\frac{G\sin\theta\cos\theta}{2g_A}B'\,, \tag{16}$$

where ΔM is the magnitude of the mass difference between initial and final baryons and B' is given for each decay in Table I. The (B')'s satisfy two relations for deviations from octet dominance given by Pakvasa, Graham, and Rosen[14]:

$$\Delta(\Lambda)+\Delta(\Xi)=0\,, \tag{17}$$

$$2\Delta(LS)=(\sqrt{\tfrac{3}{2}})\Delta(\Sigma)+\Delta(\Lambda)\,, \tag{18}$$

where

$$\Delta(\Lambda)=B'(\Lambda_-^0)+\sqrt{2}B'(\Lambda_0^0)\,, \tag{19a}$$

$$\Delta(\Xi)=B'(\Xi_-^-)+\sqrt{2}B'(\Xi_0^0)\,, \tag{19b}$$

$$\Delta(\Sigma)=B'(\Sigma_+^+)-B'(\Sigma_-^-)-\sqrt{2}B'(\Sigma_0^+)\,, \tag{19c}$$

$$\Delta(LS)=B'(\Lambda_-^0)+2B'(\Xi_-^-)-\sqrt{3}B'(\Sigma_0^+)\,. \tag{19d}$$

III. CALCULATION OF THE SPURION

From the last section we see that all the amplitudes for nonleptonic hyperon decays are given in terms of the current-current spurion $S_{bd}{}^{ac}$ defined in Eq. (10). We now estimate this spurion by summing over a set of intermediate $\frac{1}{2}^+$ baryon and $\frac{3}{2}^+$ baryon states inserted between the two currents. Here it is assumed that higher intermediate states may be neglected. We offer no justification for this assumption but note that the results we obtain are quite reasonable. In any event a more complete calculation would involve merely adding more terms to our results.

This "saturation" of the current-current spurion involves integration over the vector and pseudovector current form factors for N-N and N-N^* transitions. Since the former are far better known we discuss the two contributions separately. It is therefore convenient to write the spurion coefficients of Eq. (11) as sums of two terms:

$$\tau=\tau_8+\tau_{10}\,, \tag{20a}$$

$$\delta=\delta_8+\delta_{10}\,, \tag{20b}$$

$$\phi=\phi_8+\phi_{10}\,, \tag{20c}$$

$$\sigma=\sigma_8+\sigma_{10}\,, \tag{20d}$$

where τ_8, for example, is the contribution from octet intermediate states and τ_{10} is the contribution from decuplet intermediate states.

First let us consider the octet intermediate-state contribution. We need the vector and axial-vector form factors, defined by

$$\langle N'(p')|V_{b\mu}{}^a(0)|N(p)\rangle =\frac{-iM}{(p_0p_0')^{1/2}}\bar{u}(p')\left\{\gamma_\mu[(F_1{}^p(q^2)+\tfrac{1}{2}F_1{}^n(q^2))\times F_b{}^a-\tfrac{3}{2}F_1{}^n(q^2)D_b{}^a] +\frac{\sigma_{\mu\nu}q_\nu}{2M}[(F_2{}^p(q^2)+\tfrac{1}{2}F_2{}^n(q^2))\times F_b{}^a-\tfrac{3}{2}F_2{}^n(q^2)D_b{}^a]\right\}u(p)\,, \tag{21a}$$

[14] S. Pakvasa, R. H. Graham, and S. P. Rosen, Phys. Rev. 149, 1200 (1966).

$$\langle N'(p')|P_{b\mu}{}^a(0)|N(p)\rangle = \frac{-iM}{(p_0p_0')^{1/2}}\bar{u}(p')\Big\{\gamma_\mu\gamma_5[f(q^2)F_b{}^a+d(q^2)D_b{}^a] + \frac{iq_\mu}{2M}\gamma_5[f'(q^2)F_b{}^a+d'(q^2)D_b{}^a]\Big\}u(p), \quad (21b)$$

where $q=p-p'=(\mathbf{q},iq_0)$, $\sigma_{\mu\nu}=\frac{1}{2}i[\gamma_\nu,\gamma_\mu]$, and $D_b{}^a$ and $F_b{}^a$ are, respectively, the symmetric and antisymmetric $SU(3)$ matrices. We have assumed $SU(3)$ invariance so that all the vector form factors are given in terms of the known nucleon form factors. We use the following[15] empirical fit to the nucleon form factors:

$$F_1{}^{p,n}(q^2)=\left(\frac{1}{1+q^2/4M^2}\right)\times\left[G_E{}^{p,n}(q^2)+\frac{q^2}{4M^2}G_M{}^{p,n}(q^2)\right], \quad (22a)$$

$$F_2{}^{p,n}(q^2)=\left(\frac{1}{1+q^2/4M^2}\right)[G_M{}^{p,n}(q^2)-G_E{}^{p,n}(q^2)], \quad (22b)$$

with

$$G_E{}^p(q^2)=\frac{1}{\mu_p}G_M{}^p(q^2)=\frac{1}{\mu_n}G_M{}^n(q^2) =\left(\frac{1}{1+q^2/0.71}\right)^2,\ q \text{ in BeV}/c, \quad (23a)$$

$$G_E{}^n(q^2)=0, \quad (23b)$$

and $\mu_p\simeq 2.79$, $\mu_n\simeq -1.91$.

We may relate the two axial-vector form factors to each other by using Nambu's form[16] of the hypothesis of partially conserved axial-vector current (PCAC). This gives

$$d'(q^2)=\frac{4M^2}{q^2+M_\pi{}^2}d(q^2), \quad (24a)$$

$$f'(q^2)=\frac{4M^2}{q^2+M_\pi{}^2}f(q^2). \quad (24b)$$

Actually in Eqs. (24) we must replace $M_\pi{}^2$ by $M_K{}^2$ when the relevant current is strangeness-changing, but this turns out to have negligible effects for our purposes so that we may even neglect this mass.

Finally, we relate $d(q^2)$ and $f(q^2)$ to the nucleon electromagnetic form factors by using chiral $SU(3)\otimes SU(3)$ symmetry. Hara[17] has pointed out that the [(6,3), (3,6)] representation of this group gives the usual $SU(6)$ results. It also gives the form-factor predictions

$$d(q^2)=\tfrac{3}{5}g_A[F_1{}^p(q^2)+\tfrac{3}{2}F_1{}^n(q^2)], \quad (25a)$$

$$f(q^2)=\tfrac{3}{5}g_A[\tfrac{2}{3}F_1{}^p(q^2)-\tfrac{1}{2}F_1{}^n(q^2)], \quad (25b)$$

where we have normalized the result to obtain

$$d(0)+f(0)=g_A=1.18.$$

Substitution of the form factors of Eqs. (21)–(25) into the current-current spurion gives the following contribution from octet intermediate states

$$\tau_8=\frac{M^3}{2\pi^2}(K_1{}^V-1/9\mu_p{}^2K_2{}^V-4/9K_1{}^A -8/3\mu_pK_2{}^A-\mu_p{}^2K_3{}^A), \quad (26a)$$

$$\delta_8=\frac{M^3}{2\pi^2}(-9/5K_1{}^V+1/5\mu_p{}^2K_2{}^V+4/5K_1{}^A +24/5\mu_pK_2{}^A+9/5\mu_p{}^2K_3{}^A), \quad (26b)$$

$$\phi_8=\frac{M^3}{2\pi^2}(4/3\mu_p{}^2K_2{}^V+16/3K_1{}^A+4\mu_pK_2{}^A), \quad (26c)$$

$$\sigma_8=\frac{M^3}{2\pi^2}(-3/2K_1{}^V+3/2\mu_p{}^2K_2{}^V+6K_1{}^A +4\mu_pK_2{}^A+3/2\mu_p{}^2K_3{}^A), \quad (26d)$$

where we have set $\mu_n=-\frac{2}{3}\mu_p$, the $K_i{}^V$ are integrals from the vector-vector part of the spurion, and the $K_i{}^A$ are integrals from the pseudovector-pseudovector part of the spurion. All these are given in Appendix A. Details of the evaluation of these integrals will be discussed in Sec. V. At this point we only note that they converge rapidly, the dominant contribution coming from values of q^2 for which the empirical fit of Eqs. (23) is known to be good.[5] We also remark that the integrals are somewhat dependent on the degenerate mass of the baryon octet.

Next let us consider the decuplet intermediate state contribution to the current-current spurion. We define the decuplet-octet transition form factors in the $SU(3)$ limit as

$$\langle D(p')|V_{b\alpha}{}^a(0)|N(p)\rangle =\frac{1}{\sqrt{2}}\epsilon_{bmk}\left(\frac{MM^*}{p_0p_0'}\right)^{1/2}\bar{D}_\beta{}^{amn}(p') \times[\delta_{\beta\alpha}f_1+ip_\beta\gamma_\alpha f_2+p_\beta(p+p')_\alpha f_3 +(p-p')_\alpha p_\beta f_4]\gamma_5N_n{}^k(p) \quad (27a)$$

$$\langle D(p')|P_{b\alpha}{}^a(0)|N(p)\rangle =\frac{1}{\sqrt{2}}\epsilon_{bmk}\left(\frac{MM^*}{p_0p_0'}\right)^{1/2}\bar{D}_\beta{}^{amn}(p') \times[\delta_{\beta\alpha}g_1+ip_\beta\gamma_\alpha g_2+p_\beta(p+p')_\alpha g_3 +(p-p')\alpha p_\beta g_4]N_n{}^k(p), \quad (27b)$$

[15] L. Chan, K. Chen, J. Dunning, Jr., N. Ramsey, J. Walker, and R. Wilson, Phys. Rev. **141**, 1298 (1966).
[16] Y. Nambu, Phys. Rev. Letters **4**, 380 (1960).
[17] Y. Hara, Phys. Rev. **139**, B134 (1965).

where the f's and g's are related to the form factors of Albright and Liu[18] by

$$f_1=F_1{}^V(q^2)\,,\quad f_2=\frac{F_2{}^V(q^2)}{M+M^*}\,,$$
$$f_3=\frac{F_3{}^V(q^2)}{(M+M^*)^2}\,,\quad f_4=\frac{F_4{}^V(q^2)}{(M+M^*)^2}\,,$$
$$g_1=F_1{}^A(q^2)\,,\quad g_2=\frac{F_2{}^A(q^2)}{M+M^*}\,,$$
$$g_3=\frac{F_3{}^A(q^2)}{(M+M^*)^2}\,,\quad g_4=\frac{F_4{}^A(q^2)}{(M+M^*)^2}\,,$$

and our normalization of the decuplet $SU(3)$ wave function is such that, for example $\bar{\Omega}_-=(1/\sqrt{6})\bar{D}^{333}$. All the form factors may be taken to be real. M^* is the (degenerate) decuplet mass.

Using Eqs. (27) we may write the general result for the decuplet intermediate state contribution to the current-current spurion as

$$\tau_{10}=\frac{(M^*)^3}{2\pi^2}(-1/4)D\,, \tag{28a}$$

$$\delta_{10}=\frac{(M^*)^3}{2\pi^2}(-9/5)D\,, \tag{28b}$$

$$\phi_{10}=\frac{(M^*)^3}{2\pi^2}(3/2)D\,, \tag{28c}$$

$$\sigma_{10}=\frac{(M^*)^3}{2\pi^2}(15/8)D\,, \tag{28d}$$

where D is given in Appendix B.

Now obtaining D requires us to integrate over the N-N^* transition form factors which are very poorly known. In spite of this, we see from Eqs. (28) that the ratios of the decuplet contributions to the various decays are *uniquely fixed* independently of the details of integration. These contributions to

$$A(\Lambda_-{}^0),\ A(\Xi_-{}^-),\quad \text{and}\quad A(\Sigma_+{}^+)$$

are easily seen from Eqs. (28) and (14) to be in the ratio 1:–2: $1/\sqrt{6}$. Comparing this with our previous results[7] for the octet contribution, we see that this ratio may well improve the agreement with experiment for $A(\Lambda_-{}^0)$ and $A(\Xi_-{}^-)$ and only slightly worsen the agreement for $A(\Sigma_+{}^+)$.

To actually estimate D we need some idea of the form factors in Eqs. (27). For the vector form factors we used the results of Gourdin and Salin's pion photoproduction analysis[19] that

$$F_1{}^V(0)=-F_2{}^V(0)=5.6\,,$$
$$F_3{}^V(0)=F_4{}^V(0)\simeq 0\,.$$

To give this some momentum dependence we adopt the familiar empirical forms,

$$F_1{}^V(q^2)=\frac{F_1{}^V(0)}{(1+q^2/b)^2}\,, \tag{29a}$$

$$F_2{}^V(q^2)=\left(\frac{1}{1+q^2/4M^2}\right)\left(\frac{F_2{}^V(0)}{(1+q^2/b)^2}\right), \tag{29b}$$

$$F_3{}^V(q^2)=F_4{}^V(q^2)=0\,. \tag{29c}$$

The additional factor in Eq. (29b) was inserted to assure convergence of the relevant integral appearing in D. We regard the damping factor b as a parameter and, in Sec. V, give some arguments as to its correct value.

For the axial-vector form factors we rely on the analysis by Albright and Liu[18] of pion production by incident neutrinos. They find that a reasonable fit to the scanty experimental data is provided with

$$F_1{}^A(q^2)=\frac{-0.87}{(1+q^2/b)^2}\,, \tag{30a}$$

$$F_2{}^A(q^2)=F_3{}^A(q^2)=F_4{}^A(q^2)=0\,, \tag{30b}$$

for $b=0.71$ and $F_i{}^V(0)$ given as above.

As may be seen from Appendix B, the value of D also depends on the values of M^* and M we choose. We will show later that with reasonable values of b, M, and M^*, D also turns out to be of the proper value to give fairly good agreement with experiment for the decay amplitudes. In any event, we emphasize that D is only one number which could have been regarded as an arbitrary parameter if we had been less ambitious.

IV. GENERAL DISCUSSION

In this section we discuss the results for a typical choice of parameters in some detail and also compare with experiment. Other choices of parameters will be discussed in the next section, but the results are qualitatively the same.

For the choices,[20] $M=1.064$ BeV, $M^*=1.400$ BeV,

[18] C. H. Albright and L. S. Liu, Phys. Rev. **140**, B748 (1965); **140**, B1611 (1965). See this reference for further information on the experimental data.

[19] M. Gourdin and P. Salin, Nuovo Cimento **27**, 193 (1963). However, different results are quoted in B. V. Geshkenbein, Phys. Letters **16**, 323 (1965); H. Sugawara and F. Von Hippel, Phys. Rev. **145**, 1331 (1966).

[20] This choice has the disadvantage that it gives values somewhat too low for one set of total cross sections for neutrino-pion production. (See Ref. 18. We are grateful to Professor C. H. Albright for very helpful communications on this matter.) However, this choice gives the best ϕ/δ ratio for the weak spurion. The fact that b here is somewhat low compared with the corresponding quantity for nucleon electromagnetic form factors may reflect the fact that the NN^* transition form factor has a larger "radius." See the discussion in Sec. V.

TABLE II. Experimental and theoretical decay amplitudes (units of $10^{-7}\ M_\pi$).

	Process						
	Λ_-^0	Λ_0^0	Ξ_-^-	Ξ_0^0	Σ_+^+	Σ_0^+	Σ_-^-
A_{expt}	3.3	...	−4.4	3.32	−0.1	$\left\{\begin{matrix}-1.9\\-3.8\end{matrix}\right\}$	4.1
A_{theor}	3.0	−2.13	−3.84	2.72	0.6	−3.6	4.5
$(\Delta M/2M)B_{\text{expt}}$	2.42	...	1.46	−0.92	4.97	$\left\{\begin{matrix}2.22\\4.65\end{matrix}\right\}$	−0.48
$(\Delta M/2M)B_{\text{theor}}$	0.88	−0.88	0.43	0.06	1.7	0.88	−0.16

and $b=0.40$ (BeV)^2 we find for the coefficients in Eqs. (11)

$$\begin{aligned}\tau&=\tau_8+\tau_{10}=-0.38-0.91=-1.28M_\pi^3,\\ \phi&=\phi_8+\phi_{10}=6.4+5.4=11.8M_\pi^3,\\ \delta&=\delta_8+\delta_{10}=0.64-6.3=-5.8M_\pi^3.\end{aligned}\tag{31}$$

From Eq. (31) we notice first of all that octet dominance (small τ) emerges. Secondly, we note that the basic feature of the weak spurion, that of ϕ dominance, comes from the octet intermediate-state part of the contribution. Actually we see from Eqs. (28) that, irrespective of the details of the form factors, the decuplet intermediate-state contribution favors δ to ϕ. In spite of this, the decuplet contribution is sizable and improves the agreement with experiment from the earlier results with octet alone. (There we used a lower octet degenerate mass which increased slightly the size of the octet contribution.) Thirdly, we note that the ratio ϕ/δ is -2.

Substitution of the above results in Eqs. (14) and (16) gives all the decay amplitudes which are listed, side by side with the experimental values in Table II. We see that the S-wave results agree impressively with experiment. It might bear repeating that this was obtained by using the values of G and $\sin\theta$ from *leptonic* decays.

On the other hand, the P-wave decays come out roughly in the right ratios but less than one-half too small. This, however, is a consequence of our model[3,5] and is independent of the weak spurion. Presumably the generalized Sugawara-Suzuki formulation must be modified to adequately explain the P waves. We may remark that the P-wave amplitudes as given in Table I are rather sensitive to the fine details of the spurion. Reference to Table I also reveals the amusing point that if τ is nonzero but still small there may nevertheless be non-negligible violation of the $\Delta I=\frac{1}{2}$ rule for Λ and Ξ P waves. This is due to the large coefficient multiplying this term.

V. OTHER POSSIBLE FITS AND DISCUSSION OF NN^* FORM FACTORS

From Eqs. (14) and (20) and Appendices A and B we see that the amplitudes $A(\Lambda_-^0)$, $A(\Xi_-^-)$, and $A(\Sigma_+^+)$ (or alternatively τ, ϕ, and δ) can be expressed as complicated nonlinear functions of the degenerate octet mass M, the degenerate decuplet mass M^*, and the NN^* form-factor parameter b. Since the amplitudes depend somewhat on these variables (see Appendices) and only $b_8=0.71$ BeV^2 for the nucleon form factors is known accurately, it is desirable to investigate the effect of reasonable variations of the other parameters. Furthermore, the parameter b is obtained from the meager data on neutrino N^* production[18]; we will thus set up simple models by which b is related to b_8. To this end we consider two alternative hypotheses.

(A) Form factors fall off with universal slope, i.e., $b=b_8=0.71$ BeV^2.[15] Support for this hypothesis may possibly be found in the measurement of the charge radius of the π meson,[21] although the justification for generalization to N-N^* form factors is not entirely clear.

(B) It is eminently reasonable that the size of a particle increases with its mass (e.g., in nuclear physics). Now b is inversely proportional to $\langle r^2\rangle$; therefore, we shall consider the ansatz that $b/b_8\simeq(M/M^*)^2$. Other arguments in favor of considerable structure (larger size) in the vector N-N^* form factors can be found in the failure of the point-coupling absorption-model description of the process $\pi N\rightarrow\pi N^*$ via ρ exchange,[22] where anomalous threshold behavior is expected to occur at the ρNN^* vertex.

In the following we shall demonstrate that for reasonable choices of the degenerate masses, it is possible to obtain predictions for the nonleptonic decay amplitudes.

In order to illustrate the dependence of the amplitudes on the degenerate masses and the form factor parameters we refer to Tables III–VII.

We first note from Tables III, IV, and V that the amplitudes do vary with b, M, and M^*. To some extent

TABLE III. Octet intermediate-state contribution to decay amplitude versus M. $b_8=0.71$ BeV^2 and A is in units of $10^{-7}\ M_\pi$.

M (BeV)	$A(\Lambda_-^0)$	$A(\Xi_-^-)$	$A(\Sigma_+^+)$
0.94	2.9	−2.4	0.4
1.03	2.12	−1.83	0.248
1.064	1.95	−1.74	0.18
1.13	1.65	−1.54	0.145
1.218	0.95	−0.88	0.06

[21] C. W. Akerlof, W. W. Ash, K. Berkelman, and C. A. Lichtenstein, Phys. Rev. Letters **16**, 147 (1966); M. E. Nordberg and K. A. Kinsey (to be published).
[22] J. D. Jackson, Rev. Mod. Phys. **37**, 484 (1965).

TABLE IV. Decuplet intermediate-state contribution versus b for fixed $M=1.13$ BeV and $M^*=1.4$ BeV. A is in units of $10^{-7}\ M_\pi$.

b (BeV^2)	$A(\Lambda_-^0)$	$A(\Xi_-^-)$	$A(\Sigma_+^+)$
0.3	0.68	−1.36	0.277
0.4	1.09	−2.18	0.44
0.5	1.57	−3.4	0.64
0.71	5.4	−10.8	2.2
1.0	6.65	−13.3	2.71

TABLE V. Decuplet intermediate-state contribution versus M^* for fixed $M=1.13$ BeV and $b=0.71$ BeV^2. A is in units of $10^{-7}\ M_\pi$.

M^* (BeV)	$A(\Lambda_-^0)$	$A(\Xi_-^-)$	$A(\Sigma_+^+)$
1.13	2.46	−4.9	1.0
1.25	2.62	−5.23	1.04
1.4	5.4	−10.8	2.2
1.6	5.63	−11.3	2.3

this weakens our conclusions since it becomes impossible to obtain a unique choice of b, M, and M^* which fits the data. On the other hand, we recall that the choice of M and M^* is by no means completely arbitrary.

If we take the hypothesis A of universal form factors, it perhaps is slightly more consistent to calculate at a "universal" mass, although the experimental value of the form-factor parameters are *measured* for physical masses. Using the central octet mass, i.e., $M=M^*=1.13$ BeV, we read from Tables III and V the amplitudes $A(\Lambda_-^0)\simeq 4.11$, $A(\Xi_-^-)\simeq -6.44$, $A(\Sigma_+^+)\simeq 1.15$. These values are clearly too large, but have approximately the same ratios as the experimental amplitudes. With a slightly different treatment of the axial vector N-N form factors and a value of vector N-N^* form factor *smaller* than those of Gourdin and Salin,[19] Hara[7] has obtained values of the above amplitudes evaluated at the degenerate mass $M=1.13$ BeV to be in agreement with experiment. Although we have demonstrated that the values of the amplitudes change drastically if we vary the treatment of the form factors, we certainly cannot exclude hypothesis A because the large uncertainty inherent in the form-factor analyses obscures the calculation.

TABLE VI. Decuplet intermediate-state contribution versus M^* for fixed $M=1.13$ BeV and $b=0.4$ BeV^2. A is in units of $10^{-7}\ M_\pi$.

M^*(BeV)	$A(\Lambda_-^0)$	$A(\Xi_-^-)$	$A(\Sigma_+^+)$
1.13	0.664	−1.32	0.27
1.25	0.74	−1.44	0.303
1.4	1.09	−2.18	0.44
1.6	4.1	−7.9	2.36

TABLE VII. Decuplet intermediate-state contribution versus M^* for variable b (see hypothesis B) and fixed $M=1.13$ BeV. A is in units of $10^{-7}\ M_\pi$.

M^*(BeV)	b(BeV^2)	$A(\Lambda_-^0)$	$A(\Xi_-^-)$	$A(\Sigma_+^+)$
1.13	0.667	2.12	−4.22	0.86
1.25	0.506	1.23	−2.46	0.504
1.4	0.4	1.09	−2.18	0.43
1.6	0.306	6.15	−12.5	2.64

TABLE VIII. Most "reasonable" fits: A is in units of $10^{-7}\ M_\pi$.

Fit	M (BeV)	M^* (BeV)	b (BeV^2)	$A(\Lambda_-^0)$	$A(\Xi_-^-)$	$A(\Sigma_+^+)$
A	1.13	1.13	0.71	4.11	−6.44	1.15
B	1.064	1.4	0.4	3.04	−3.92	0.62
C	1.13	1.4	0.5	3.22	−4.94	0.79

On the other hand, if we assume that hypothesis B is correct, then we would have the value 0.4 BeV^2 for b. Since we do admit the variation of b for the octet and the decuplet, it would be more reasonable to include the corresponding variation of degenerate masses as well. Using $M=1.064$ BeV and $M^*=1.4$ BeV, we read from Tables III and VI the amplitudes

$$A(\Lambda_-^0)\simeq 3.04\,,\quad A(\Xi_-^-)\simeq -3.92\,,\quad \text{and}\quad A(\Sigma_+^+)\simeq 0.62\,.$$

This is in reasonable agreement with experiment.

The results for the above-mentioned choices of parameters are given in Table VIII as fits A and B. We are aware that there is no clear-cut *a priori* evidence in favor of our choices of fits A or B; however, it is remarkable that it is possible to obtain a fit consistent with our conclusion of weak-interaction universality and octet dominance for reasonable parametrization of the N-N^* form factors. With the present knowledge of N-N^* form factors, it is difficult to rule out any of our choices. However, our calculation seems to indicate a tendency to favor faster fall-off for these form factors. Further experimentation in this direction will be of great interest.

We note that our fit B is not completely consistent with *all* sets of the N^* neutrino production data, since a value of $b=0.4$ BeV^2 would predict too small a total cross section for one set of experimental data.[18] Assuming that the meager data available suggest the value of b to be higher, say $b=0.5$ BeV^2, it is then possible to obtain yet another fit with a central octet mass of $M=1.13$ BeV. This is shown as C in Table VIII. However, since fit B seems to predict the proper spurion ratio, we shall arbitrarily use B as our typical fit.

Finally, we point out that the integral D in Appendix B would be *linearly* divergent had we not introduced an extra damping factor of $(1+q^2/4M^2)^{-1}$ for $F_2^V(q^2)$ in Eq. (29b). However, it has been speculated by Wu and Yang[23] that for very large momentum transfers, the form factors are exponentially damped. Such a model would assure us of convergence for the integrals. On the other hand, we note that for $q^2\leq 2$–$3\,(\text{BeV}/c)^2$, i.e., the region from which D receives most of its contribution, the factor $(1+q^2/4M^2)^{-1}$ is approximately unity. Therefore, we believe that the introduction of our damping factor is quite reasonable.

[23] T. T. Wu and C. N. Yang, Phys. Rev. 137, B708 (1965).

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APPENDIX A

The integrals appearing in Eqs. (26) are

$$K_i^V=\int_1^\infty \frac{dx\,(x^2-1)^{1/2}Q_i}{[1+(2M^2/0.71\text{ BeV}^2)(x-1)]^4}, \tag{A1}$$

$$K_i^A=\left(\frac{g_A}{5/3}\right)^2\int_1^\infty \frac{dx(x^2-1)^{1/2}}{[1+(2M^2/0.71\text{ BeV}^2)(x-1)]^4}\frac{Q_i}{(x+1)^2}\left\{(x+2)-\frac{(x-1)(x-1+M_\pi^2/2M^2+M_K^2/2M^2)}{(x-1+M_\pi^2/2M^2)(x-1+M_K^2/2M^2)}\right\}, \tag{A2}$$

where $Q_1=1$, $Q_2=(x-1)$, and $Q_3=(x-1)^2$. In obtaining the above integrals we assumed that the initial and final nucleons were at rest. Setting $M_\pi=M_K=0$ in Eq. (A2) does not change the result essentially, as noted after Eqs. (24).

APPENDIX B

$$D=\int_1^\infty dx(x^2-1)^{1/2}\mathfrak{D}(x),$$

$$\begin{aligned}\mathfrak{D}(x)=(x-1)\{&(f_1)^2+M^2(x^2-1)[-\tfrac{1}{3}(M^2+M^{*2}+2xMM^*)(f_3)^2-\tfrac{1}{3}(M^2+M^{*2}-2xMM^*)(f_4)^2\\&+\tfrac{2}{3}(M^{*2}-M^2)f_3f_4-\tfrac{2}{3}(M^*-M)f_2f_3+\tfrac{2}{3}(M^*+M)f_2f_4+\tfrac{2}{3}f_1(f_3+f_4)]+M^2(x+1)\cdot[\tfrac{2}{3}(f_2)^2(x+2)\\&+\tfrac{4}{3}f_1f_2/M]\}+(x+1)\{(g_1)^2+M^2(x^2-1)[-\tfrac{1}{3}(M^2+M^{*2}+2xMM^*)(g_3)^2\\&-\tfrac{1}{3}(M^2+M^{*2}-2xMM^*)(g_4)^2+\tfrac{2}{3}(M^{*2}-M^2)g_3g_4-\tfrac{2}{3}(M^*+M)g_2g_3-\tfrac{2}{3}(M^*-M)g_2g_4\\&+\tfrac{2}{3}g_1(g_3+g_4)]+M^2(x-1)[\tfrac{2}{3}(g_2)^2(x-2)+\tfrac{4}{3}g_1g_2/M]\}.\end{aligned}$$

In this equation all form factors are functions of $q^2=2k_0M-M^2-M^{*2}$, and $x=k_0/M^*$.