Second Series *May, 1920* *Vol. XV., No. 5*

THE

PHYSICAL REVIEW.

MAGNETIZATION AND HYSTERESIS IN HEMATITE CRYSTALS.

By T. Townsend Smith.

Synopsis.

Magnetization and Hysteresis in Hematite Crystals; Experimental Procedure.—In a previous paper, the author has given the results of an investigation of the magnetic properties of crystallized hematite, the measurements being of the force exerted upon a sphere when it is placed in a non-uniform magnetic field. The present paper contains some results obtained by measuring the couple exerted when a hematite sphere is suspended in a uniform magnetic field. The couple in a uniform field measures the component of magnetization perpendicular to the field; the force measures the component parallel to the field.

Experimental Data.—The paper contains (1) *magnetization curves* (Fig. 4) for two specimens of hematite; (2) *hysteresis loops* at fields from 9.5 to 1020 gausses, some obtained with a rotating field (Figs. 6, 7, 8, 9, 11), the others with an alternating field (Figs. 10, 11); (3) a graph representing *components of magnetization* parallel and perpendicular to the field (Fig. 12).

Deduction, The normal magnetization curve of hematite consists of a sharp rise followed by a slow increase, which is proportional to the field strength. The *hysteresis energy loss* increases with an increase in the maximum field applied up to a certain field, and thereafter the area of the hysteresis loop remains constant. For the specimen on which most of the present measurements were made, the hysteresis loss is constant for field strengths above about 150 gausses.

In a previous paper[1] the author has published the results of an investigation on the magnetic properties of hematite crystals. In that work it was shown that certain hematite crystals possessed a dual magnetic character, exhibiting hysteresis in the principal plane of the crystal, but being apparently free from hysteresis in a direction at right angles to this plane. It was further shown that the intensity does not even approach saturation at the highest field investigated, just under 4,000 gausses.

The crystals investigated consisted of a number from the Isle of Elba,

[1] Phys. Rev. (2), 8: 721, 1916.

and specimens from Ouropreto, Brazil, from Schabry in the Ural Mountains, and from Dognacska, Hungary (a twin).

The results presented in the previous paper were all based upon measurements of the pull exerted upon the spheres in a non-uniform magnetic field, and the magnetic quantity obtained was the component of the intensity parallel to the field. In the present paper some results are given which were obtained by measuring the couple exerted upon the spheres when they were suspended in a uniform field, which could be rotated at will around the axis of suspension. From this couple is obtained the component of magnetic intensity perpendicular to the exciting field.

The results of the present work verify the previous work with reference to the paramagnetic behavior along the axis for three of the specimens previously reported, but the increased sensitiveness of the torsion measurements has demonstrated that the twin crystal from Dognacska and all of the Elba crystals exhibit at least a slight amount of hysteresis along the axis.

In the case of the Brazil and Ural Mountain specimens further interesting properties have been discovered. These are: (1) the specimens behave as if the magnetization curve were represented by the equation

$$I = \pm (I_0 + kH), \tag{1}$$

inasmuch as a comparatively large value is quickly reached and the intensity then increases slowly in proportion to the field strength. This is the combination of permanent magnetism and paramagnetism which is sometimes used as a mathematical supposition.

There is (2) the further property of a maximum area to hysteresis in the principal plane. This means that both the energy loss and the coercive force reach maxima at comparatively low fields.

1. *Couple Exerted on a Suspended Crystal.*—If a magnetically non-isotropic sphere be suspended in a uniform field, the direction of magnetization will coincide with that of the field only when the direction of maximum susceptibility or of minimum susceptibility coincides with the direction of the field. For all intermediate positions the direction of the magnetization will make an angle with the exciting field, the direction of magnetization being closer than that of the field to the direction of maximum susceptibility. The sense of the component of magnetization perpendicular to the field will, therefore, be such as will tend to rotate the crystal in that direction which will make the direction of maximum susceptibility coincide with that of the exciting field. There will consequently be exerted upon the specimen a couple, which will act against

the torsional moment of the suspending wire in an effort to set the direction of maximum susceptibility parallel to the field.

Such at least is the simple phenomenon which is exhibited by crystals in which hysteresis is lacking, but in general with hysteresis present the position of no torque might be oblique to the direction of maximum permeability, on account of the component of the residual magnetization which is perpendicular to the field. There is one special case where this will not be true, and fortunately that is the one which is here dealt with—the case in which there is a direction of minimum susceptibility along which there is no hysteresis.

It does not seem to me to follow necessarily from this that the component of the exciting field in the direction of the principal plane will have the same effect as if the other component did not exist. The experimental results, however, seem to indicate that this is at least approximately true.

In Fig. 1 the hematite sphere is supposed to be suspended about an

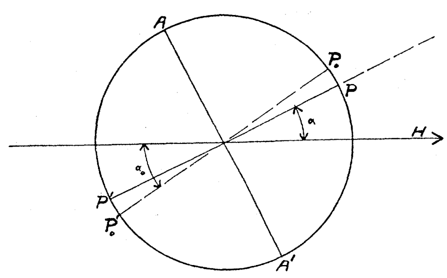

Fig. 1.

axis, which lies in the principal plane of the crystal, and which is, therefore, perpendicular to the plane of the sketch. PP' is the trace of the magnetic principal plane, and AA' is the direction of the magnetic axis of symmetry. The angle between H and PP' is α, and this angle will, of course, tend to diminish as the field increases. The value of α for a zero field strength is indicated by α_0, so that when the field is applied the sphere rotates through the angle $(\alpha_0 - \alpha)$.

If the torsion constant of the suspending wire be τ, then the couple exerted by the spring is $\tau(\alpha_0 - \alpha)$ dynes $\times$ cm. and the couple exerted by the field on a sphere, the volume of which is "V" c.c., is equal to $H \cdot I \cdot \sin(H, I) \cdot V$, or more simply, for a unit volume, the product of the field intensity into that component of the intensity of magnetization which is perpendicular to the field. This component can be readily written down and is $I_P \sin\alpha - I_A \cos\alpha$ which gives as the condition for equilibrium

$$(2) \qquad \tau(\alpha_0 - \alpha) = H[I_P \sin \alpha - I_A \cos \alpha]v.$$

It is known that the material here investigated is paramagnetic in the direction of AA' and the susceptibility in this direction has been determined to be $k = .000117$ (loc. cit., page 731), so that the component of the intensity in this direction may be calculated and the equation (1) may then be solved for I_P. This is the procedure which has been followed.

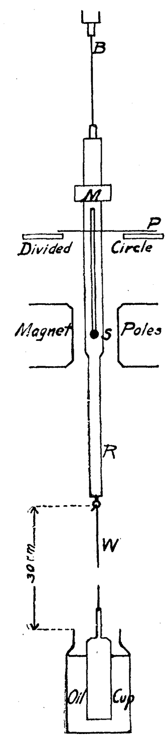

Fig. 2.

Suspension of Specimen.

B, Spring brass wire; *M*, Small mirror for telescope and scale; *P*, Glass capillary used for alidade; *S*, Sphere of hematite; *R*, Rod of maple, half cut away and grooved along its axis; *W*, Light brass rod, used to support the steadying and damping weight, which dipped into the oil cup beneath the table.

2. *The Suspension of the Crystal.*—The arrangement of the apparatus for suspending a sphere in a magnetic field is shown in Fig. 2. In the previous work the sphere was attached by means of a non-magnetic cement called cæmentium to a light wooden rod, which was about 3 or 4 millimeters in diameter, was about 7 or 8 centimeters long, and carried a light pointer of glass. In the present work the spheres are mounted on these wooden rods as before, and the rods are clipped with a little spring of brush copper to a heavier rod of maple. This maple rod was supported by a suspending wire of spring brass and was held plumb by a brass weight below. This weight dipped into a wide-mouthed bottle, which contained machine oil, and hence the weight served the additional purpose of damping the vibrations. The maple rod (R) could be unhooked at W and thus be lifted bodily out of the apparatus for the purpose of changing the specimen or making any other necessary adjustments.

The rod (R) was first turned to the desired diameter, half of it cut away along its whole length, with the exception of short distances for suspending purposes at the two ends, and then grooved along the axis. The rod was nicked opposite P to make a rest for the glass pointer, and a hollow was scooped out at S to make room for the spheres. The sphere to be investigated was thus situated in the axis of rotation of the suspending system, and the position of the sphere with reference to the field could be read by the pointer P on the divided circle, which was rigidly fastened to the electromagnet and rotated with the field. The magnet was mounted on a turn table, and was rotated by a bicycle-chain drive from outside of the box containing the apparatus.

The little mirror M was fastened to the rod R with beeswax and was used with a telescope and scale for measuring the deflections produced when the magnetic field was applied. In some of the work a second small mirror was placed just alongside of M, but attached to the divided circle, so that small angles of rotation of the magnet could be read accurately, using the same telescope and scale as was used for the deflections.

The suspension was altogether of non-magnetic materials, wood, brass, and copper, and gave a barely appreciable deflection with the sphere removed even at the strongest fields which were used. When first cut out the rod R was shellacked, but the shellac was sufficiently magnetic to give trouble so that it had to be carefully sandpapered off. The maximum deflection was then not greater than one or two tenths of a millimeter for a field of about 1,000 gausses, using a suspension whose torsion constant was 1.90 dynes $\times$ centimeters per centimeter deflection. The error here can be safely neglected, being in no case greater than about one part in 300.

3. *Calibrations.*—In order to make a determination of the magnetic intensity in the principal plane it was, according to equation (2), necessary to determine four quantities experimentally. These were

the torsion constant τ,
the angular deflection $(\alpha_0 - \alpha)$,
the field strength H,
the angle between principal plane and field, α.

The *torsion constants* were determined in two ways—first by means of a torsional pendulum, and secondly, by means of the torque exerted upon a small, flat coil of wire, which was suspended in the field and through which a small electric current was passed. This coil was mounted on a rod similar to R and this rod could be put in place of R when desired. The coil was designed for use in calibrating the magnetic field of the magnet, but after this work had been done and the magnetic field strength was known, the known field strength and the calibrated coil could be used for determining the torsion constant.

Two or three suspensions were broken during the course of the work, and it is unfortunately true that the torsion constant in several cases was not known with the accuracy which was thought to be the case at the time.

The *magnetic field strength* determinations in almost every case were made with the flat coil just referred to. Current leads for this coil were provided by the spring brass wire B and a fine coil of rolled galvanometer suspension, which was clipped to W. A current of about one thirtieth of an ampere was passed through this coil which was placed with its plane parallel to the direction of the magnetic field. The deflections produced were thus proportional to the field strength. The constant of proportionality was determined by means of an exploring coil and a standard of mutual inductance, both of which were used in the previous work.

In this way the magnetic field was determined under a variety of conditions. The magnetization curve for the electromagnet was determined, starting with the magnet carefully demagnetized by reversals, and then hysteresis loops for maximum fields of about 92, 190, 238, 335, 450, and 620 gausses were obtained in order that the specimens could be taken through such ranges with alternating fields.

I. Magnetization Values.

4. *The Magnetization Curves—Direction of the Axis.*—The torsional measurements afford a very delicate test of the paramagnetic character of the spheres in the direction of the axis, because a very small component of the magnetization in this direction can be detected by the couple which it exerts when the specimen is placed with the principal plane parallel to the field. If there is no hysteresis along the axis, then it will be possible to find the principal plane by setting for a position of no deflection or no torque. This will be the case when the axis of the crystal is perpendicular to the field, in which position (axial hysteresis assumed absent) there will be no rotation of the specimen caused by the field, even when the field is reversed in direction. If there is hysteresis along the axis, it will always be possible to find a direction for the field for which there will be no rotation when the field is applied, but in this case a reversal of the field causes a rotation of the specimen and it will be impossible to find a position for which the sphere will remain in its zero position when the direction of the field is thus changed.

The difference between the two cases may be seen by an inspection of Fig. 3, in which deflections in centimeters (proportional to the torque) are plotted against angular readings on the divided circle (the angle α of Fig. 1). The points for sphere G, axially paramagnetic, are for field strengths of 76 and 1,020 gausses, as indicated on the figure. The large circles represent a rotation with increasing positive angles, the smaller circles an opposite rotation. Concentric circles mean that the readings taken with opposite rotations are identical. At 1,020 gausses the barred circles represent angles in the neighborhood of 180° and the open circles points near 0°. These two sets do not quite coincide, though in each set the reversal of the field obviously does not change the deflection. No precise setting of the specimen at the center of the divided circle was attempted, and the discrepancy between the two sets is doubtless due to the axis of suspension being slightly to one side of the axis of rotation of the magnet. The points on the two curves

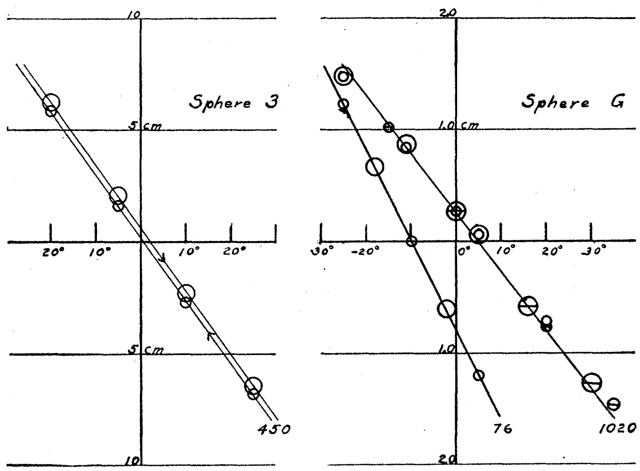

Fig. 3.

Axial Hysteresis.

At left —dependence of torque upon direction of rotation for sphere 3, Elba hematite.
At right—lack of axial hysteresis in sphere *G*.

for field strengths of 1,020 and 76 were taken with different set-ups, which accounts for the fact that the angle for no torque is different in the two cases. The readings for sphere 3, which were taken with the express purpose of investigating this question, show clearly the presence of hysteresis.

The net result of such tests is that there are three of the spheres which have shown themselves to be without hysteresis along the axis—*G* from Brazil, and J_1 and J_2 from the Ural Mountains. Hysteresis along the axis is exhibited by all specimens[1] of Elba Hematite so far tested, sphere 3 being an Elba specimen for which the hysteresis is slight. Sphere *F*, cut from a twin crystal from Dognacska, showed a slight trace of axial hysteresis.

5. *Magnetization Curves—Plane of Symmetry.*—Values for the intensity of magnetization for spheres *G* and *J* are shown in Fig. 4, all of the determinations being obtained from the torsion exerted in a uniform

[1] Loc. cit., p. 733.

field. The magnetic plane was set at a convenient, small angle (α) with the field, and the values of I_P were then determined with the aid of equation (2) above. The results are in fair agreement with the previous determinations made with a non-uniform field. (Loc. cit.)

The values for sphere G were determined in several runs with different set-ups:

(*a*) Field strengths up to about 100 gausses were obtained with two short solenoids, without the use of iron. These coils were mounted on

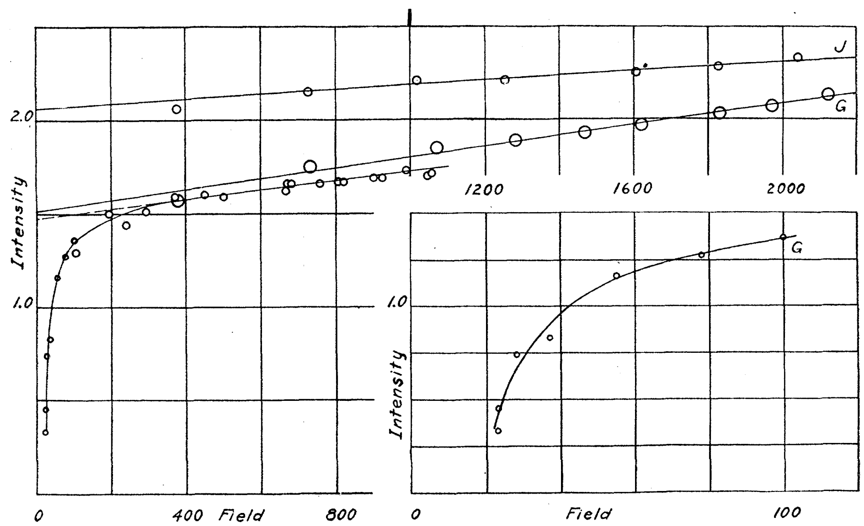

Fig. 4.

Magnetization Curves.

Lower curve for G—angle α small. The low field values, represented by the smallest circles are shown to a larger scale in the lower right-hand corner. These points are corrected for the demagnetizing field ($4\pi I/3$). The other points are not so corrected.
Upper curve for G—angle α is approximately 45°.
Top curve, J—angle α is approximately 45°.

the turntable in place of the electromagnet, coaxially and with just enough space between the two to make room for the rod R of Fig. 2. The points so determined are marked with the smallest circles in Fig. 4. The torsion constant of the suspending wire, number 35 brass about 9 centimeters long, was about 1.24 dynes × centimeters per centimeter deflection (scale distance about 72.5 centimeters) and the value of α_0 was about 8 degrees, the deflection being about half a degree for the largest field values.

(*b*) For the fields up to about a thousand gausses, the electro-magnet used in the previous work was fitted with a pair of flat-faced polepieces. The torsion constant was a little greater, 1.90 dynes × centimeters, and the value of α_0 was varied from four to eight degrees without making any appreciable difference in the results obtained.

(c) To obtain fields above 1,000 gausses a second pair of pole pieces with a smaller air gap was used and the values shown by the large circles for sphere G and the values shown above for sphere J were so obtained. In this case, to eliminate the inaccuracy in the small initial values of α_0, the principal plane was set at an angle of 45 degrees to the field, readings being taken in all four quadrants to avoid chance errors in the reading of the angles. The maximum field strength reached was about 3,000, and the torsion constant of the suspending wire[1] was about 63.0 dynes × centimeters.

The interpretation of these measurements rests, of course, on the assumption that the components of the field along and perpendicular to the axis of symmetry of the crystal may be treated independently. The concordance between these results and the results of the author's previous work[2] makes this assumption very probable.

Making this assumption, then, the values of the intensity of magnetization were calculated for various field strengths as is shown in Table I.

Table I.

Magnetization Values for Sphere G.

Initial Angle (α_0) 45°.

I. Applied Field, H.	II. Deflection, Cm.	III. Reduced Torque, Dyne × Cm.	IV. Angle, a.	V. Apparent Intensity, $T/H \sin a$.	VI. H_P, $H \cos a$.	VII. I_P.
530....	0.98 (1)	570	44° 37′	1.53	377	1.57
1,020....	2.03 (3)	1,180	44° 12′	1.66	733	1.75
1,480....	3.06 (1)	1,775	43° 48′	1.73	1,070	1.85
1,760....	3.64 (4)	2,110	43° 32′	1.74	1,278	1.89
2,020....	4.21 (1)	2,445	43° 20′	1.76	1,468	1.93
2,220....	4.65 (4)	2,700	43° 10′	1.78	1,620	1.97
2,500....	5.33 (4)	3,100	42° 53′	1.82	1,830	2.03
2,680....	5.79 (2)	3,350	42° 43′	1.84	1,970	2.07
2,880....	6.31 (4)	3,670	42° 30′	1.88	2,120	2.13

The table is self explanatory, save perhaps for columns III. and VII. The values in column III. are obtained from those in column II. by multiplying by the torsion constant 63.0, and dividing by the volume of the sphere, 0.1092 cubic centimeters. The numbers in column VII. are obtained by adding $.000117^2 \times H_P$ to the numbers of column V. to

[1] Number 26 brass about 9 centimeters long.

[2] Loc. cit.

[3] Loc. cit., p. 731.

allow for the paramagnetism along the axis of the crystal. The numbers in parentheses in column II. are the number of readings on which the deflection is based. A least squares calculation gives

TABLE II.

Magnetization Values for Sphere J_1.

Initial Angle (α_0) 45°.

Field (H_P).	376.	724.	1,018.	1,252.	1,605.	1,827.	2,040.	Mean.
Apparent Intensity	2.01	2.06	2.09	2.06	2.06	2.07	2.09	2.07
Intensity (I_P)	2.06	2.15	2.21	2.21	2.25	2.28	2.33	

$$I = 1.518 + .000291 \times H \tag{3}$$

as the magnetization curve, assuming the form (1), with a probable error of .001 in I_0 and of .0000002 in K.

An attempt was made to fit the data to the form

$$I = I_0 + K_1 H + K_3 H^3 \tag{4}$$

for the term in H^2 is obviously lacking, but the value of K_3 so obtained was so small that at a field of 3,000 the deflection due to this factor would be .003 centimeters, which is beyond the range of the experimental accuracy. It may be regarded as established, then, that so far as these observations go the form of equation (1) fits the facts.

This is even more strikingly shown in the J specimen, Table 2, where the apparent intensity of magnetization remains constant within the range from 700 to 2,000 gausses, and which exhibits, therefore, the curious phenomenon of a substance having a uniform paramagnetism superposed upon a ferromagnetism, the ferromagnetism conforming to the crystal structure.[1]

The rather striking agreement between (1) and the observed values induced me to collect all the data on hematite which bears upon this question, and this data is shown in Table III. The "force" values (column V.) are taken from the author's previous paper on hematite and Fig. 4 shows three out of the four "torsion" values.

This coexistence of ferromagnetism and paramagnetism in the plane of symmetry in hematite has been previously observed in other materials. Frivold[2] gives some susceptibility values for silver, which, when reduced

[1] The statement made in the earlier article that the magnetization values for spheres 3, 5, J_1, and J_2 would lie in good agreement with those for G is not contradictory to the present work, for the values referred to were in the range from 2,000 to 4,000 gausses, in which range the two spheres (G and J_1) have nearly the same values of intensity.

[2] Ann. d. Physik, 57: 471, 1918.

TABLE III.

Magnetization Values for Hematite.

Plane of Symmetry of Crystal.

I.	II.	III.	IV.	V.
Specimen.	Values of Constants.		Range of Field Strength Gausses.	Method of Experimenting.
	I_0.	k.		
G.	1.48	.00025	500–4,000	Force
G.	1.48	.00025	400–1,100	Torsion
G.	1.52	.00029	500–2,100	Torsion
J.	1.97(?)	.00012	2,000–4,000*	Force
J.	2.05	.00012	350– 620	Torsion
J.	2.07	.00012	530–2,040*	Torsion
F.	1.96	.00020	1,000–4,000	Force
9 *A*	1.13	.00024	600–4,000	Force

In column 4 for the series marked *, no measurements were made at field strengths either above or below the limits indicated.

to intensity, are analyzed in Fig. 5. The observed values are plotted in curve *A*. The straight line *B* is drawn through the downward slope of curve *A*, and is assumed to represent a diamagnetic component of

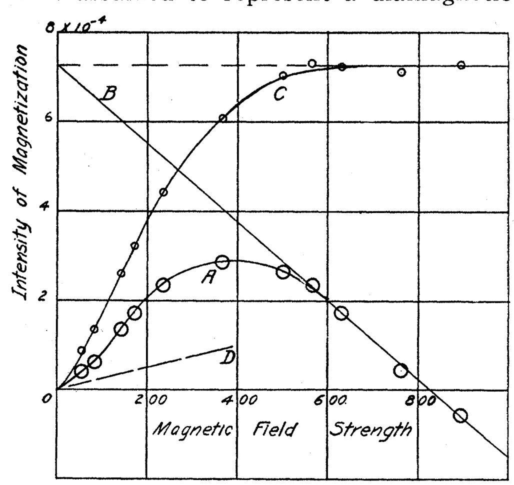

Fig. 5.

Magnetization of Silver.

Data for curves *A*, *B*, and *C* taken from a paper by Frivold.

Curve *A*: observed values of intensity of magnetization, for unit volume.

Curves *B* and *C*: the two assumed components of *A*, *B* diamagnetic, and *C* ferromagnetic.

Curve *D*: initial susceptibility of silver as observed by Shaw and Hayes, for fields below 12 gausses.

magnetization, which is added to the assumed ferromagnetic component C to give the observed values of curve A. The line OD represents an average value of the initial susceptibility of silver as observed by Shaw and Hayes[1] for fields below 12 gausses.

In this case, as also for a number given by Honda,[2] there is enough iron present as an impurity to account for the phenomenon. The behavior of this iron is, however, somewhat capricious. A specimen of silver containing but seven parts in a million of iron was found by Honda to have a diamagnetic susceptibility which increased with the field strength, whereas a specimen of cadmium containing forty-three parts of iron in a million had a constant susceptibility. In the case of hematite it seems improbable that the phenomenon can be ascribed to the presence of an iron impurity in the ordinary sense of the word, because the ferromagnetism conforms to the crystalline structure.

It is a possible inference that the normal curve for magnetization would be of this character. Such a superposed paramagnetism in iron, cobalt, nickel, or magnetite would be masked by the intense ferromagnetism of these substances. I have hunted through the accessible literature on the Heusler alloys without finding evidence one way or the other. It may be that, with relatively low intensities of magnetization of the order of magnitude met with in hematite, further evidence could be obtained. There are one or two minerals which offer promise in this direction, and I hope in the not distant future to be able to investigate this material.

II. Hysteresis.

The hysteresis losses accompanying a reversal in the direction of magnetization may be determined from torsional measurements in two ways: (1) by rotating the sphere in a uniform field, and measuring the torque at different angles, and (2) by suspending the sphere with the magnetic plane nearly parallel to the field, *i.e.*, with α small (Fig. 1) and then causing the field to increase from zero to a maximum value and then reversing, measuring again the torque as the field varies. Both methods were used, and the results from the two agree very closely.

6. *Rotational Hysteresis. Low Fields.*—In taking these values the solenoidal coils referred to in the preceding section were mounted on the rotating table and rotated about the specimen, angles being read from a divided circle carried by the coils and deflections being read by means of a telescope and scale. The set up was the same as that used in determining the component of the intensity of magnetization in the magnetic

[1] Electrician, 77: 708, 1916.

[2] Ann. d. Physik, 32: 1027, 1910.

plane, as described above. The results are shown plotted in Fig. 6, the scale deflection, which is proportional to the torque, being plotted against the angle of rotation. In Fig. 6 are also indicated the couples, reduced to absolute units of force and unit volume of specimen, these

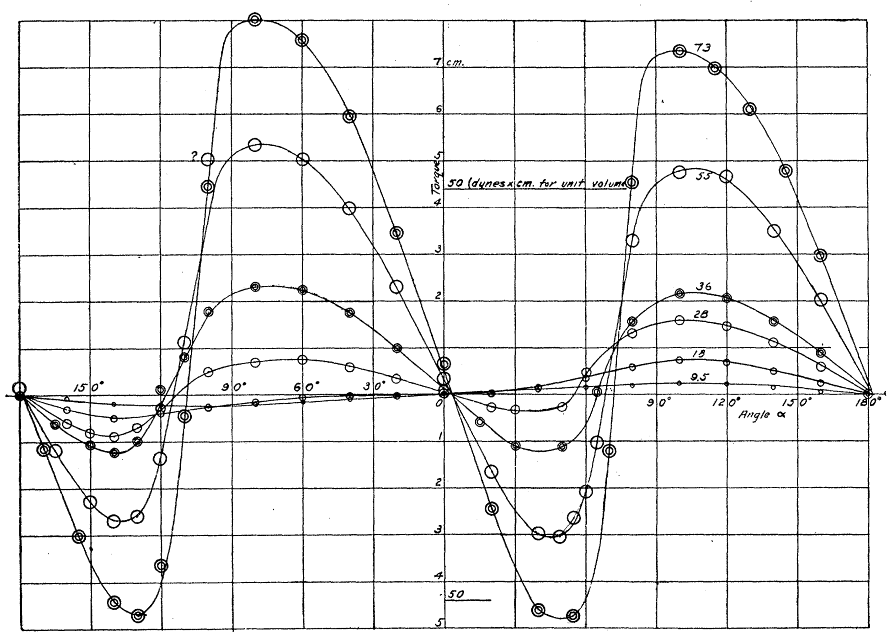

Fig. 6.

Torques at Low Fields.

Sphere *G*, Ouropreto, Brazil. Torque as dependent upon α (Fig. 1) for fields of 9.5, 18, 28, 36, 55, and 73 gausses.

values being obtained by multiplying the observed deflections by 1.90 (the torsion constant), and dividing by .1092 (the volume of the sphere). Fields above 73 gausses were not used because of the heating of the coils due to the exciting current.

Using the curves of Fig. 6, the values of the intensity of magnetization were calculated, and the values obtained are plotted in Fig. 7. The initial demagnetization was obviously imperfect, and strong hysteresis is apparent in the curves. They are likely not to be symmetrical at these low fields, though at higher fields this asymmetry disappears.

Intermediate Fields.—Rotational hysteresis curves were obtained with the same set up as was used in the magnetization curve work for fields of 76 gausses and 143 gausses (Section 5, *b*). The results of these meas-

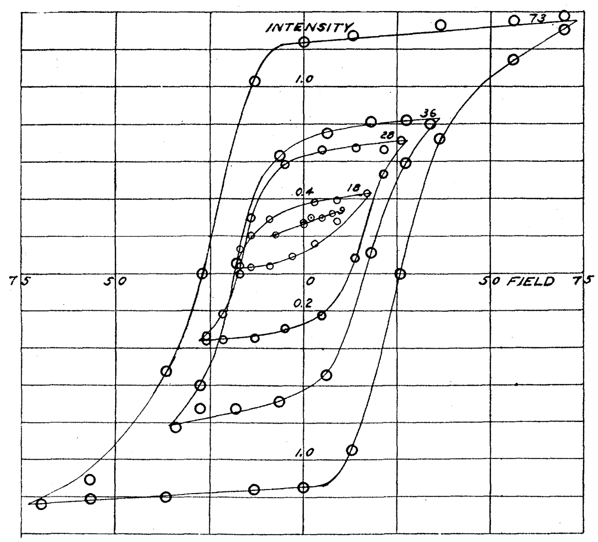

Fig. 7.

Hysteresis Loops for Sphere G.

Calculated from Fig. 6, on the assumption of the independence of the components of the magnetization along the axis and in the plane of symmetry.

urements are shown in Fig. 8, where the torques are plotted against the angle of rotation. Here the hysteresis is clearly in evidence, and the curves are nearly symmetrical. The results show the difference in the

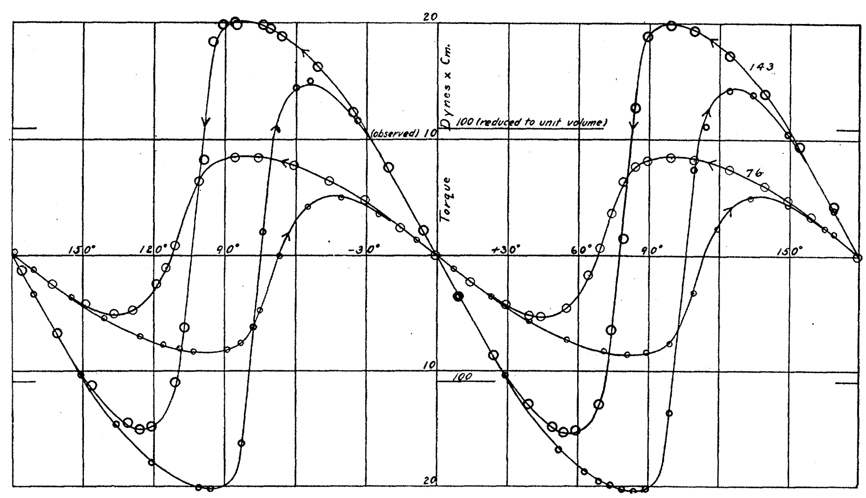

Fig. 8.

Torques at Intermediate Fields for Sphere G.

Curves taken with rotating electromagnet. Hysteresis is shown by the separation of the curve into two branches, corresponding to different directions of rotation.

torques obtained when the magnet is rotated first in the one and then in the other direction, the direction of rotation being indicated by arrow heads.

The hysteresis energy losses for the various loops of Figs. 6 and 8 are tabulated in Table IV., the cycles made with the electromagnet being designated with an *(e)*. To these have been added the losses calculated from several curves with alternating hysteresis, the method of obtaining which is described in the following section. The values obtained with the solenoids are a little greater than those obtained with the electromagnet, but if this difference be allowed for, it will be seen that the hysteresis energy losses for alternating and for rotating fields are the same.

Table IV.

Hysteresis Energy Losses for Sphere G.

Maximum Field.	Energy Loss in Ergs per Cycle.	Type of Hysteresis.
9	Not measurable	Rotational
18	7	"
28	30	"
36	52	"
55	106	"
73	144	"
76(*e*)	129	"
143(*e*)	211	"
39	60	Alternating
80	150	"
92(*e*)	152	"
188(*e*), 238(*e*), 340(*e*), 450(*e*)	211	"

The intensity of magnetization for these various fields was obtained from Fig. 4, and the logarithms of the intensity so obtained plotted against logarithms of the energy losses. The five points for fields of 36, 55, 73, 39, and 80 gausses are found to lie on a straight line. This is Steinmetz's law of energy loss[1] in hysteresis cycles, which for this specimen, is represented by

$$\text{Energy Loss} = 0.036 \times (4\pi I)^{3.0}.$$

These constants should be regarded as rough approximations.

The hysteresis energy loss seems not to increase appreciably as the field is made larger than 143 gausses, and the loop at this field strength

[1] Trans. American Inst. Elec. Eng., Jan. 19, 1892. Ewing, Magnetic Induction in Iron, etc., p. 111.

seems to have reached its final shape, which is represented in Fig. 9. The points calculated from the data represented in Fig. 8 are marked with the larger circles.

For fields above 143 gausses the spheres were unstable when the plane of symmetry, *i.e.*, the magnetic plane, was nearly perpendicular to the direction of the field. The direction of magnetization reversed and consequently the sign of the couple due to the magnetic field also reversed, changing rapidly, and the torsion constant of the suspending wire was not great enough to hold the sphere in place. It was thus impossible with the suspension used to get readings within a range of about 25 degrees when α was nearly 90° for fields only a little stronger than 143 gausses.

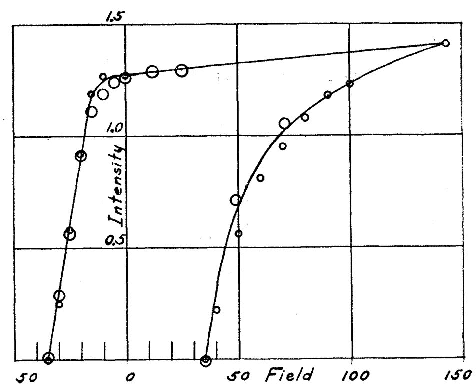

Fig. 9.

Limiting Hysteresis Loop for Sphere G.

The larger circles indicate points obtained from the torques in a rotating field of 143 gausses, Fig. 8. The smaller circles represent alternating hysteresis and the points are calculated from the curves of Fig. 10.

7. *Alternating Hysteresis.*—For this work the only change made in the experimental set-up was to attach to the rotating magnet a small mirror for the measurement of small angles with telescope and scale. The plane of symmetry of the specimen was set at a small angle to the magnetic field and the torques were measured as the field was increased to a maximum, decreased to zero, and then reversed. The maximum fields used were 92, 188, 238, 340, and 450 gausses, and the torques were measured in each case for a number of intermediate values. These torques for the four sets with maximum fields of 188, 238, 340, and 450 gausses are plotted in Fig. 10, the different sets being distinguished from one another in the plot by marking them with successively larger circles. In addition, the torques from which the magnetization curve of Fig. 4 was obtained are indicated by crosses in the branch *OB* of Fig. 10.

With decreasing field strengths (Branch *OA* of Fig. 10) and with increasing fields (branch *OB*), it appears that the magnetization follows the same path for all five sets of readings—the four hysteresis loops and the magnetization curve for which the specimen was carefully demagnetized.

On *OB* no torques are indicated for field strengths less than about 100 gausses. These torques are shown on *OCD*, to a scale ten times as great,

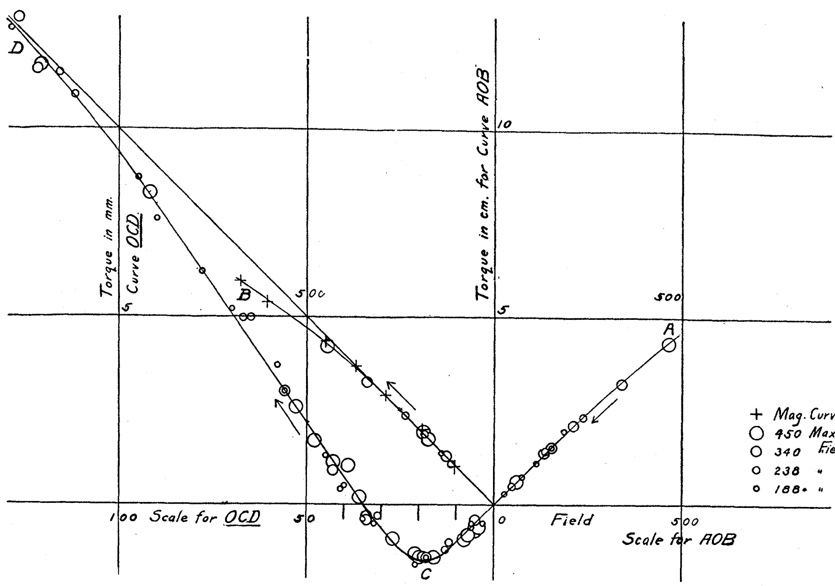

Fig. 10.

Alternating Hysteresis for Sphere G.

The angle between field and magnetic plane is here small.

The curve *AOB* represents the torques exerted as the field is changed from a maximum value in one direction to a maximum in the opposite direction, magnet stationary. Torques taken from the magnetization curve readings, Fig. 4, are shown with crosses in *OB*. The range from zero to 130 gausses is shown in the curve *OCD* to ten times the scale of *OB*.

The hysteresis loop of Fig. 9 is calculated from the curve *OCD*.

both for the deflection and field strength. Here again it is evident that the four sets of readings follow the same course, and that, within the limits of field strength here used, the hysteresis curve is independent of the maximum value of the magnetizing field. There is, then, a final hysteresis loop for hematite—at least for this particular specimen—which can be obtained from the readings plotted in Fig. 10. The smaller circles of Fig. 9 represent values calculated from Fig. 10.

It is seen that the agreement between these values and those representing the rotational hysteresis with a maximum field of 143 gausses is good. This agreement is perhaps the most direct evidence of the independence within these limits of field strength of the effects produced by the two components of the magnetic field, the one along the axis and the other in the plane of symmetry.

It was considered desirable to test this curve at higher fields and consequently with a stronger suspension a rotational hysteresis loop was taken with a field of 1,020 gausses, and the results are plotted in Fig. 11. It is, I think, obvious that any accurate determination of the

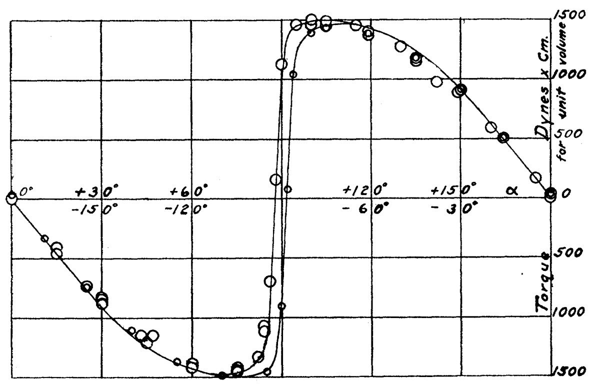

Fig. 11.

Rotational Hysteresis for Sphere G. Field of 1020 gausses.

area is impracticable, but such checks as can be applied agree with the assumption of a maximum hysteresis loop.

The areas check approximately and the coercive force checks very well. With a coercive force of 35, as shown in Fig. 9 and a field of 1,020, the width of the hysteresis loop should be just under four degrees, and the width of the loop in Fig. 11 is very nearly that.

There is no such maximum hysteresis loop within the range of field strengths which have been used for experiments with sphere 3, Elba hematite, in which the energy losses have the values of 79, 154, 195, 266, and 377 ergs for fields of respectively 76, 143, 208, 308, and 450 gausses. The question has not been tested with the *J* spheres, for which the hysteresis is certainly much less. In any case the significance of the observation in the case of sphere *G* consists in the existence of the maximum loop, and not in the area, which seems to depend upon the particular specimen used.

A hysteresis loop of maximum area has been observed by Weiss[1] in the case of pyrrhotite above 2,000 gausses, the energy loss being about 2,900 ergs per cycle, with a maximum intensity of magnetization of

[1] Journal de Physique (4), 4: 829, 1905.

about 47. This "normal" hysteresis loop he supposes to be rectangular, while in the case of hematite the normal loop would be a rhomboid, with an energy loss of about 211 ergs and having the curve cross the axis at about $I = 1.50$.

Baily's[1] calorimetric study of the hysteresis energy loss at high excitation in soft iron indicates that for alternating fields the hysteresis energy loss approaches a maximum as the iron approaches saturation. The limiting energy loss with the specimen of iron used by Baily was about 30,000 ergs per cycle.

Attention should be called, to the fact that the shape of the curve in Fig. 11 does not entirely bear out the assumption of the independence of the effects produced by the two components of the energizing field. The experimental curve changes less abruptly in the neighborhood of 90° than would be required by the calculated values.

According to Fig. 9, the residual intensity of magnetization should be 1.27, and with a field of 1,020 the torque at 90° should be, therefore, about 1,300 dynes × cm., when reduced to unit volume. The two observed points at 90° are 900 and 1,125 (Fig. 11), which are appreciably lower than the independence of the two field components would lead one to expect.

These differences are puzzling, in view of the excellent agreement exhibited in Figs. 4, 9 and 10. A possible explanation is that the assumption of the independence of the two components of the field is not strictly justifiable—at least not in the immediate neighborhood of the axis of the crystal. It is, also, possible that the differences are caused by the oscillation of the sphere in the field, arising from the elastic suspension and the step by step rotation of the magnet.

At a little distance from the axis, the agreement is better. In Fig. 11 the maximum torque is about 1,500, which would correspond to a component of magnetization perpendicular to the field of about 1.47, when the angle is about 75 degrees. Such a component would indicate an apparent intensity in the magnetic plane of about 1.515 for a field of about 264 gausses. Adding now .000117 × 264, to allow for the axial paramagnetism, the result is 1.55, which is close to the magnetization curve of Fig. 4.

III. Magnetization Oblique to the Principal Plane.

8. The product of the magnetic field strength multiplied by the component of the intensity of magnetization perpendicular to the field gives the torque exerted on a magnet of unit volume set obliquely to the

[1] Phil. Trans., A, 187: 715, 1896. Electrician, Nov. 22, 1895.

magnetic field. Consequently if the torques represented in Fig. 11 be divided by 1,020 the result will be the component of the intensity of magnetization perpendicular to the field. If the mean of the two branches of this curve be taken, the effect of hysteresis will be at least partially eliminated. This was done.

The perpendicular component so obtained is shown in curve B of Fig. 12, for values of α from zero to 90 degrees, the zero direction being

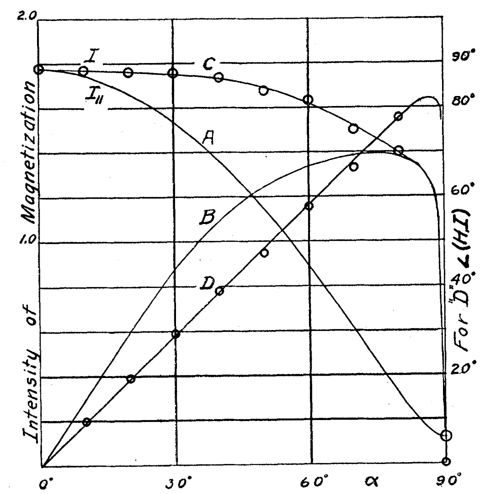

Fig. 12.

Magnetization Oblique to the Axis.

Sphere G in a field of 1020 gausses.
Curve A: component of intensity parallel to the field.
Curve B: component of intensity perpendicular to the field.
Curve C: resultant intensity of magnetization.
Curve D: angle between directions of field and of magnetization.
Scale for D shown to right of figure.

the direction of the principal plane. From Fig. 3 of the author's previous paper (loc. cit.) the component of magnetization parallel to the field may be determined by interpolation, and this is shown in curve A of Fig. 12. From these two sets of values the resultant magnetization (curve C) and the angle between H and I (curve D) were determined in steps of 10 degrees, giving the points indicated. Neither B, C, nor D are accurately determined near 90 degrees and the curves in that region indicate only that the values change rapidly with α.

The author is indebted to Professor E. F. Stimpson for some very timely assistance in the experimental work.

BLAKE PHYSICAL LABORATORY,
UNIVERSITY OF KANSAS,
August 31, 1919.