THE MEASUREMENT OF VERY SHORT TIME INTERVALS.

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SYNOPSIS.

Measurement of very short time intervals; a partial deflection method.—After briefly reviewing the usual method of determining time intervals of the order of microseconds, the author describes a method in which the potential difference of a condenser after it has been discharged for the interval of time to be measured, is accurately determined by connecting the condenser through a ballistic galvanometer to a known potential difference, which is made nearly the same as that to be measured, so that the small throw observed merely measures the small correction to be applied to the known potential difference to give the unknown. A steady current through a known resistance is used as the source of known potential difference. The arrangement of circuits is described and also a gang switch which simplifies the experimental manipulation and eliminates possible error from absorbed charge in the condenser. The working equation for computing the time interval from the observations shows that the attainable accuracy depends on the accuracy of a resistance, a resistance ratio and a capacity. The method was tested with the aid of a Helmholtz pendulum and it was shown that an interval of 250 microseconds may be measured with such *accuracy* that the probable error of each observation need not exceed 0.15 per cent. The *precision* was such as to enable the determination of an overall temperature coefficient of the Helmholtz pendulum and accessories amounting to about 1 microsecond per degree for the interval just mentioned.

Measurement of the rate of detonation of explosives; a source of error.—An attempt to apply the above method to measure the rate of detonation of dynamite and trinitrotoluol gave inconsistent results, probably because, on account of the intense ionization, the condenser continued to discharge through the gas after the wire was broken.

I. GENERAL.

NO particular novelty attaches to the determination of time intervals of the order of microseconds. A method for doing this, utilizing the partial discharge of a condenser through a resistance—both of known values—seems first to have been described by Sabine.[1] It was employed by Kennelly[2] in 1889, and later by various other investigators.[3] Briefly, the usual procedure is to measure the initial and remaining charges or potentials of the condenser, respectively, before and after the discharge

[1] Phil. Mag., 1 (5), 337, 1876.

[2] Reference in paper by Kennelly and Northrup, Jour. Frank. Inst., 172, 30, 1911.

[3] Radakovic, Wien. Akad. Ber., 109 (II.a), 276 and 941, 1900; Parker, PHYS. REV., 16, 243, 1903; Kozak, Mitteil. über Gegenst. d. Art.-u. Geniewes., 893, 1903 and 556, 1904; Edelmann, Phys. ZS., 5, 461, 1904; Devaux-Charbonnel, C. R., 142, 1080, 1906; Coulsen, PHYS. REV., 4, 40, 1914; Barnett, PHYS. REV., 12, 103, 1918. The method has also been used by A. G. Webster for a number of years.

has been allowed to proceed during the interval being measured. Then, from the usual relations

$$\tau = CR \log_\epsilon \frac{Q_0}{Q} \qquad (1a)$$

or

$$\tau = CR \log_\epsilon \frac{E_0}{E}, \qquad (1b)$$

the time is computed. To determine the charges represented in the first of these equations, a ballistic galvanometer is used. The potential differences represented in the second equation are obtained with an electrometer. The latter method is described in an unpublished paper by one of A. G. Webster's students. The writer is indebted to Dr. Webster for a copy of this paper.

A galvanometer is generally conceded to be a more workable instrument than an electrometer. In seeking to develop a method of this kind to give maximum accuracy with thorough convenience, one therefore turns to the former instrument as offering greater possibilities. From the standpoint of accuracy null methods or, if these are impracticable, partial deflection methods[1] of measurement are generally superior to total deflection methods. The application of eq. (1a) illustrates the latter method. Here we presuppose an unvarying source of potential difference charging the capacity C with a quantity Q_0. This total quantity, as well as the remaining quantity Q, are in turn discharged through the ballistic galvanometer, resulting in readings which are theoretically proportional to the respective quantities. Even with a calibrated scale, increasing the charging potential and therefore the throws, adds nothing to the probable percentage accuracy of the results, except with a very slow galvanometer, because of the increasing rapidity of reversal of the motion with increasing throws. In the total deflection method, also, one cannot assume strict proportionality of throws and quantities, because of non-uniformity of the galvanometer field;[2] and to maintain accuracy of results, frequent calibration is necessary.

In the measurement of capacities with a ballistic galvanometer, this difficulty was overcome by the method of mixtures, in which the unknown capacity and a known capacity neutralize each other's charges, more or less completely, and only the excess charge is passed through the galvanometer. Kennelly and Northrup, in the paper cited, applied this method to the determination of the remaining charge Q, after the fully charged condenser had discharged during the unknown interval.

[1] Of this principle the Brooks deflection potentiometer is an excellent illustration.

[2] Phys. Rev., 7, 637, 1916.

The work here described had been completed before the writer had learned of the work of Kennelly and Northrup. The application of the method of mixtures to the problem, as mentioned in the preceding paragraph, had been tried, but rejected, because the purpose of the work was to find a method which, if need should arise, might be used in the field as well as in the laboratory. The above method, because of the fact that two condensers of good quality and known capacity are needed, and because of the somewhat complicated circuits, did not seem to meet the requirements.

To secure the same advantage of small throws, and of making the accuracy of the measurement depend upon fixed quantities, preferably resistances and resistance ratios, another method of attack presented itself. This is to oppose, through a ballistic galvanometer, the remaining potential difference at the condenser terminals, after the partial discharge,

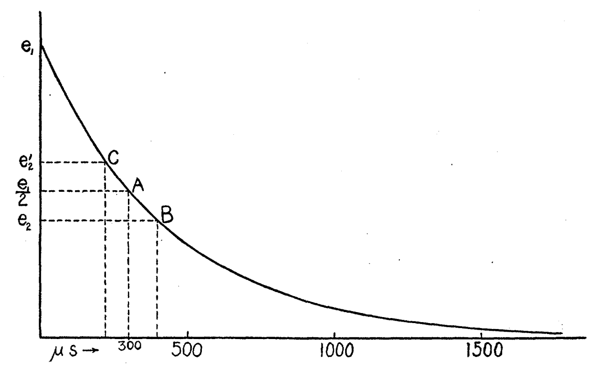

Fig. 1.

by an equal or almost equal *steady* potential difference. Then, if both the potential differences mentioned are equal, there will be zero galvanometer throw; but if the one or the other is greater, that across the condenser terminals will assume the value of the steady potential difference, the equalizing charge passing through the galvanometer, with a resulting small positive or negative throw. This idea forms the basis of the present method.

The discharge curve, Fig. 1, shows clearly what is involved. The curve represents clearly the relation between time and remaining potential difference across the condenser when the latter discharges through a pure resistance. Suppose the time interval during which the condenser discharges is of the order of 300 microseconds (300×10^{-6} sec.) and that the resistance through which the discharge takes place is of such value that the initial potential falls, in this interval, to exactly half value.

This is represented by point A on the curve. With the same capacity and resistance, but with a longer or shorter interval, the points B or C, respectively, might represent the final condition. In the first instance, no throw would result if the steady potential difference $e_1/2$ were thrown across the condenser terminals through the galvanometer; but in the second and third cases, a small throw would result, to the right or left, respectively. In the second case, the throw would correspond to a change of potential difference across the condenser from e_2 to $e_1/2$; and in the third case, the throw would represent the equalizing charge as the potential difference across the condenser changed from e_2' to $e_1/2$.

From what has been said, it is obvious that any partial deflection method of this kind requires approximate knowledge of the time interval to be measured. Otherwise it would be impossible to keep the galvanometer throws small, and the advantages of such a method would be lost. But in nearly any case which may arise one can, by computation or preliminary measurement, gain sufficient knowledge of the conditions of the experiment to enable the successful application of a partial deflection method.

2. Arrangement of Circuits.

Fig. 2 represents in diagram the apparatus for making determinations in the manner suggested. Let F_1 and F_2 represent two contacts which may be opened in such quick succession that the interval between breaks is of the desired magnitude. This may be accomplished with a Helmholtz pendulum,[1] or with a fall apparatus of a type as described by Hiecke[2] and Webster.[3] With the reversing switch R closed in position a, a fall of potential e_1 exists at the terminals of r_1, and e_2 at the terminals of r_2. Arbitrarily let $r_1 = 2r_2$, although any other ratio in which $r_1 > r_2$ would answer. These potential differences are proportional to the resistances, since the same steady current flows through both the latter. The condenser C is thus kept at a potential difference e_1 so long as F_1 remains closed. At the instant of opening F_1, however, the battery is cut off and the condenser discharges through r_1 until F_2 is opened.

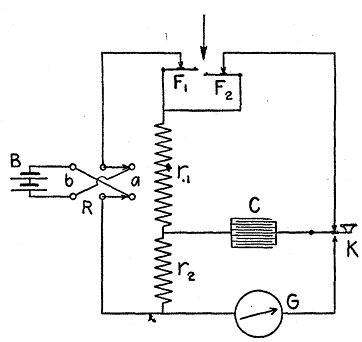

Fig. 2.

[1] Ann. d. Physik, 3, 274, 1900.

[2] Hiecke, Sitzber. d. k. Akad. d. Wiss. in Wien, Abt. IIa, 96, 114, 1887.

[3] Webster, Phys. Rev., O. S., 6, 297, 1898.

As stated before, if the resistance r_1, the capacity and the interval between breaks are properly related, the potential difference at the condenser terminals will now have become e_2, or half its original value. With resistances and capacity fixed, any deviation from the value e_2 is due to an actual interval different from the expected interval. The galvanometer throw is observed by first closing F_1—or, more conveniently, a switch in parallel with F_1—then reversing R, and finally depressing K. These operations have put a potential difference e_2 across r_2 in the proper direction to oppose the remaining potential difference across C, and have brought the latter to the value e_2 by means of the equalizing charge which has passed through the galvanometer.

3. Details of Apparatus.

In the apparatus as finally designed, all the operations necessary for observation of the galvanometer throw are accomplished with a single motion of a special gang switch. An additional feature of the switch is the provision which is made so that connection between condenser and galvanometer can persist for an instant only. In this manner only the "free charge"[1] or geometric capacity of the condenser is used, and errors which might otherwise arise from absorbed charge effects are eliminated.

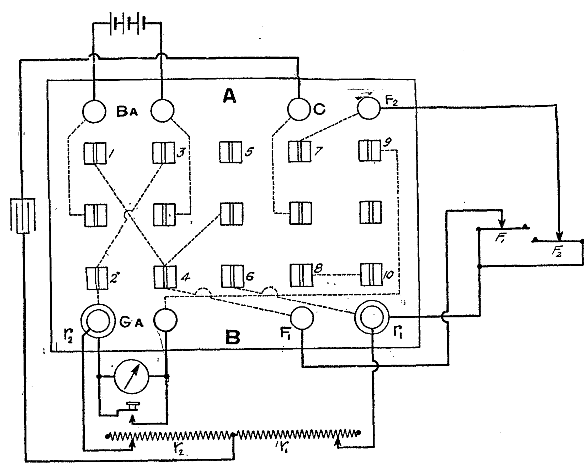

Fig. 3.

The arrangement of this gang switch is shown in Fig. 3, in which dotted lines represent internal, and solid lines external connections.

The switch is of the rocker or quick throw type, with the lengths of its blades so proportioned that the operations outlined in connection

[1] Zeleny, A., Phys. Rev., O. S., 22, 1906, 65.

with Fig. 2 take place in proper sequence as the switch is thrown from position "A" to position "B." Normally its position is "A," the blades engaging in the odd-numbered clips. With the switch in this position, the breaks at F_1 and F_2, which delimit the interval being measured, are made to occur. In throwing the switch, to obtain the galvanometer reading, the sequence of operations is as follows:

(1) Open at 7; disconnects condenser from charging source. (2) Close at 6; equivalent to closing F_1. (3) Open at 1 and 3, close at 2 and 4; reverses direction of fall of potential along r_2. (4) Close at 8 and 10; this opposes the fall of potential along r_2 to that across the condenser, through the galvanometer. During this operation 9 and 10 are bridged by the blade. (5) Open at 9. The combination of (4) and (5) keeps the condenser and galvanometer connected for a very brief interval only, as stated above. The switch is kept in position "B" until the reading has been taken.

4. Theoretical.

The reasoning which must be followed in order to obtain the equation upon which to base the measurements, and by which the observed values are reduced, is now evident. Upon the basis (1) of the known capacity, (2) of a predetermined ratio of initial to remaining potential difference at the condenser terminals, and (3) of the estimated interval, we must first determine the values of r_1 and r_2 so that, during the expected interval, the condenser shall discharge by an amount determined by the ratio. The equation giving the relations between these quantities will enable us to determine τ from the observations.

Let us suppose that the operations are carried out as previously described, and that a small galvanometer throw is observed, which may be positive or negative. The quantity producing this throw will have the value

$$\pm \Delta Q = \left(\frac{Q}{C} - e_2\right) C, \tag{2}$$

and the direction of its discharge depends upon the relative magnitudes of Q/C and e_2. In this equation Q, the remaining charge in the condenser has the value eC, where e is the remaining potential difference at the end of the interval τ. From (2) we obtain

$$Q = e_2 C \pm \Delta Q. \tag{3}$$

We know, according to equations (1) that

$$\tau = Cr_1 \log_\epsilon \frac{Q_0}{Q}, \tag{4}$$

and that the initial charge of the condenser

$$Q_0 = e_1 C. \tag{5}$$

If e_0 is the total fall of potential along r_1 and r_2 in series,

$$e_1 = e_0 \frac{r_1}{r_1 + r_2}, \tag{6}$$

and

$$e_2 = e_0 \frac{r_2}{r_1 + r_2}. \tag{7}$$

With proper substitutions of equations (3), (5), (6) and (7) in (4), we immediately have

$$\tau = Cr_1 \log_\epsilon \frac{1}{\frac{r_2}{r_1} \pm \frac{\Delta Q}{e_0 C}\left(\frac{r_1 + r_2}{r_1}\right)}. \tag{8}$$

This equation enables us to determine the values of r_1 and r_2 to be used in the measurement; for we know that in the ideal case, when the estimated and actual intervals are equal, ΔQ is zero. To illustrate, let us suppose the interval to be 10^{-4} second, the ratio $r_1 : r_2 \doteq 2 : 1$, and the capacity 1 microfarad. Then

$$r_1 = \frac{\tau}{C \log_\epsilon 2} = \frac{10^{-4}}{10^{-6} \cdot 0.6932},$$

or

$$r_1 = 144.3 \text{ ohms},$$

and

$$r_2 = 72.15 \text{ ohms}.$$

Equation (8), as it stands, cannot be used advantageously for obtaining τ from an observed throw corresponding to the discharge of the quantity ΔQ, for it would still involve obtaining the relatively very large throw resulting from the discharge of the quantity $e_0 C$ through the galvanometer. The requisite modification, which amounts to finding a ready means for determining the ballistic constant of the galvanometer, is readily made.

Suppose the potential difference across the condenser terminals to change by a small amount, from e_1' to e_2', and that the small quantity of electricity q, corresponding to this change, passes through the galvanometer. This condition is realized by choosing r_1' only slightly greater than r_2', their sum still remaining $r_1 + r_2$, as in the measurement, and throwing the gang switch from "A" to "B." We then have

and

$$\left.\begin{aligned} q &= (e_1' - e_2')C = i(r_1' - r_2')C, \\ e_0 C &= (e_1 + e_2)C = i(r_1 + r_2)C. \end{aligned}\right\} \tag{9}$$

Since i is the same in both cases, because of the equality of $r_1' + r_2'$

and $r_1 + r_2$, a steady battery being assumed,

$$\frac{r_1 + r_2}{e_0 C} = \frac{r_1' - r_2'}{q}. \qquad (10)$$

By substituting this value of the left-hand member of the equation in eq. (8), putting for the small quantities of electricity involved the corresponding proportional throws of the galvanometer, and changing to common logarithms, we arrive at the working equation,

$$\tau = 2.303\ Cr_1 \log \frac{1}{\frac{r_2}{r_1} \pm \frac{d_x}{d_0}\left(\frac{r_1' - r_2'}{r_1}\right)}. \qquad (11)$$

In this equation d_x represents the throw corresponding to the quantity ΔQ, and d_0 the throw corresponding to q, or the calibrating throw. The positive sign is used when the throws are in the same direction, and negative when opposite.

The equation shows that the accuracy of measurement attainable depends chiefly upon the accuracy of the resistance values and of their ratio, and of the capacity value of the condenser. When the actual and estimated intervals differ by a small amount, as they usually will, the second term in the denominator in eq. (11) may be regarded as a correction to be applied to the ratio of the resistances.

The sensitivity of the method may be made very great by using a sensitive ballistic galvanometer and a sufficiently high source of potential difference. In the ordinary or total deflection method one is limited by the fact that only small quantities of electricity can be measured, even at a full scale throw of the galvanometer. The present method is equivalent to suppressing the origin of readings quite out of the range of the scale, or to securing a much magnified angle with strict proportionality of scale readings and quantity, together with ease of reading. The experiments made have indicated that with a properly constructed "absolute" fall apparatus of the Hiecke[1] or Webster[2] type, condenser capacities may be determined by this method with considerable accuracy.

5. EXPERIMENTAL.

As a means for investigating the possibilities of this method, a Helmholtz pendulum was kindly placed at the writer's disposal by the Bureau of Standards.[3] This instrument does not so readily lend itself to com-

[1] Loc. cit. [2] Loc. cit.

[3] The experiments were conducted in the Electrical Division of the Bureau. To the chief and the members of this division the writer expresses his thanks for their helpful interest, and for the use of equipment. In this connection he desires especially to mention Drs. W. F. G. Swann and P. G. Agnew.

putation of the intervals as do the fall devices of Hiecke and Webster; but its design and construction are of such excellence that its performance in accurately reproducing any given interval within its range left nothing to be desired. The electrical apparatus, represented in Fig. 3, consisted of Wolff resistance boxes, a Leeds and Northrup precision condenser and Type R galvanometer. The special gang switch was also constructed in the shops of the Leeds and Northrup Company.

The first step in the experiment was to find the relative positions of F_1 and F_2 on the Helmholtz pendulum for simultaneous breaks. F_1 was fixed in position so that it should be opened at the instant the pendulum passed through its lowermost position. F_2, on a micrometer slide, was set so that it should open slightly later than F_1. In eq. (11) one con-

TABLE I.

C = 1.000 microfarad, $r_1' = r_1$, $r_2' = r_2$, $r_2/r_1 = .4$, d_0 and d are single readings. Second column indicates number of complete turns of the micrometer head which shifts position of F_2. Scale distance, 60 cm. $T = 21°$ C.

Obs. No.	No. Turns.	d_0, Cm.	d, Cm.	r_1.	r_2.	$t(\mu s)$.
a.......	1.000	12.24	−.04	175	70	161.2
b.......	2.000	12.27	−.02	350	140	321.5
c.......	2.000	12.27	+.21	360	144	321.6
d.......	3.000	12.29	−.04	525	210	483.7
e.......	3.000	12.32	+.13	535	214	482.1
f.......	3.000	12.33	+.26	545	218	482.5
g.......	3.000	12.31	−.31	505	202	482.6
h.......	4.000	12.33	−.05	700	280	646.0
i.......	5.000	12.32	−.03	875	350	805.1
j.......	5.000	12.34	+.58	950	380	805.8
k.......	5.000	12.32	−1.22	750	300	807.8
l[1].......	1.000	12.20	−.05	175	70	161.5
m[1]......	1.000	12.22	+.80	195	78	160.4

[1] Repetitions of obs. "*a*," turning micrometer back to original setting.

dition for $\tau = 0$, or simultaneous breaks, is that $r_1 = r_2$ simultaneously with $d_x = 0$. This was made the basis for the adjustment. With r_1 and r_2 equal, and connections as in Fig. 3, the pendulum was "dropped," and d_x observed. F_2 was then moved in small steps, so as to diminish the interval, until a zero throw was observed after dropping the pendulum and throwing the switch. The possibility was kept in mind that zero throw might be obtained in case F_2 should open before F_1. To check this point, F_1 and F_2 were interchanged by means of a double throw switch, and the adjustment continued until for either connection the throw remained zero. The position of F_2 so determined was taken as the origin of readings for its adjusting micrometer.

The set of observations and results recorded in Table I., representing a calibration of the pendulum, is typical of the results attainable by this method. Where observations of the interval for a given setting of the micrometer were repeated, with different electrical constants, excellent agreement invariably resulted. It should be noted that any error in a result includes those of mechanical imperfections in the pendulum as well as those of electrical quantities. The close agreement indicates that both of these must be small.

In another series of observations, the question of reproducibility of a short interval was particularly considered. From the calibration of Table I., the micrometer setting corresponding to an interval of 250 microseconds was obtained. Keeping the contacts fixed for this interval, and using various combinations of capacity and resistance values, sets of ten readings with each particular combination were taken. The results are given in Table II.

In connection with Table II., two interesting things may be pointed out.

It seemed desirable to know the effect upon the value of τ computed by eq. (11) of an error in the observed throw d_x. The relation was found by the usual process of partial differentiation, assuming constant values for all quantities except for the ratio d_x/d_0, d_x being the variable, and for τ. The probable error in the mean of the ten observations and in the single determination of d_x, were used in the resulting expression to determine the corresponding probable errors in τ. Observation series c may be taken as illustrative of the entire group. Here it is found that the probable error in the mean τ is .04 per cent., and in the individual observation, :13 per cent. The values in the last column of Table II., which are computed from the mean value of d_x, are therefore probably correct to 0.2 microsecond or better, as indicated.

The question at once arises as to the reason for the differences of several microseconds in the different values of the last column, since the settings of the pendulum contacts were fixed. Consideration of this question leads to the second interesting point.

It should be noticed first that the electrical constants were the same in series c, e and f only, so that the differences in these three cases cannot be ascribed to different probable errors in the values of the electrical constants. We should, upon first thought, expect closer agreement, because the values should be relatively correct, apart from the question

TABLE II.

Obs. No.	Temp.	Cap M.F.	r_1'.	r_2'.	d_0.	r_1.	r_2.	d_x.	t (μs).
a, 1–17–'18, a.m. . .	20°	1.0002	275	265	1.58	360	180	−.23 −.23 .23 .26 .25 .23 .25 .26 .23 .23	252.6±.1
b, 1–17–'18, p.m. . .	21°	1.0002	201	191	2.16	280	112	+.37 +.26 .37 .29 .40 .33 .29 .32 .30 .31	253.2±.2
c, 1–17–'18, p.m. . .	21°	0.5006	560	520	1.58	720	360	−.14 −.18 .15 .16 .17 .17 .16 .15 .15 .14	253.8±.1
d, 1–18–'18, a.m. . .	15°.6	0.5006	395	375	1.12	550	220	+.20 +.18 .19 .18 .20 .18 .21 .17 .18 .18	248.1±.1
e, 1–18–'18, a.m. . .	15°.8	0.5006	560	20	1.58	720	360	+.05 +.02 .05 .08 .03 .06 .09 .05 .05 .06	248.5±.2
f, 1–18–'18, p.m. . .	19°	0.5006	560	520	1.59	720	360	−.05 −.06 .09 .06 .06 .05 .06 .04 .08 .04	251.3±.2

of their correctness in absolute values. However, it is seen in the column of temperatures, observations "*d*" were made on a fuel-saving day, on which the temperature in the laboratory was 5.4° cooler than on the preceding day, when observations "*c*" were obtained. An obvious conclusion is that the apparatus had a temperature coefficient. This coefficient turns out to be + 1.05 microsecond per degree. Accordingly, the result at 19° should be 251.7 microseconds. The actual result found in series "*f*" was 251.3. Numerous other experiments, the results of which are not here tabulated, as well as those recorded in Table II., are in agreement with the conclusion stated.

Making proper allowance for the temperature coefficient, and taking into account the probable errors in the values of the capacities and resistances used, the discrepancies become very small, and we find that the method, with simple means, gives results considerably more accurate than can be expected from total deflection methods unless extreme precautions are used. We also find that the Helmholtz pendulum is capable of reproducing the very short time intervals in a manner which, when we consider the factors involved, must be called surprising. In intervals as small as 10 microseconds, an overall accuracy of two per cent. was easily attainable.

6. Applications.

The necessity for making measurements of intervals of the magnitude here dealt with arises in certain kinds of investigations, as is seen from the list of papers cited at the beginning of this article.

By the method here described, velocities of high power rifle bullets, as high as 900 meters per second, have been successfully determined between two points 20 cm. apart. It should be understood, however, that for routine field testing of ammunition, other methods are more adaptable since operators with little skill and training are able by them to make determinations more successfully than by a method involving the use of a galvanometer. In velocity measurements, the greatest errors are likely to arise from lack of sharpness or preciseness of the breaks produced by the projectile. They are mechanical errors, rather than errors in the electrical quantities. This question, because of other urgent work, could not be investigated in great detail. Preliminary work showed that glass-hard drill rod, or silvered glass strip, or mercury in capillary glass tubes should prove superior to ordinary wires for preciseness of the breaks.

The failure of the method in an attempted application to determining the detonation rates of dynamite and trinitrotoluol had its cause in an interesting phenomenon. The apparatus was set up as in Fig. 3, and wires were drawn through the stick of dynamite, at two points 100 cm. apart, to provide for the necessary breaks. All circuits were very carefully insulated, so that no leakage errors could occur from this source. With supposedly uniform explosives—the rate of detonation of which is of the order of 5,000 meters per second—the results varied by as much as five per cent. These variations were found to be probably due to the intense ionization produced by the explosion which causes leakage between the broken ends of the wires, and thus causes deviations from the theoretical value in the discharge rate of the condenser.