section for electroexcitation of a discrete state. The second factor usually integrates to one so that the usual procedure of associating model differential cross section with experimental cross sections integrated over the resonance peak is in accord with our result. However, in the specific case of the ^{4}He 0^+ state the resonance falls between two thresholds and the width and level shift are rapidly varying functions of energy. The result is that the monopole strength inherent in the internal function X_R is effectively reduced. One can either divide the experimental cross section by the integral in Eq. (A6) or multiply the model differential cross section by the same integral. We have elected the latter possibility in our comparison with experiment.

PHYSICAL REVIEW VOLUME 149, NUMBER 3 23 SEPTEMBER 1966

Accurate Excitation Energies of B^{11} States Below 7 MeV*

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(Received 20 April 1966)

A recent precision measurement of gamma-ray energy made with a lithium-drifted germanium detector disagrees with an earlier value of an excitation energy obtained by measurement of charged-particle energies. To investigate this discrepancy and to confirm the level assignment of the gamma-ray transition, excitation energies of B^{11} states were measured using both the $B^{10}(d,p)B^{11}$ and $Be^9(He^3,p)B^{11}$ reactions. Particle energies were measured with a broad-range spectrograph calibrated with a Po^{210} alpha source (5304.5 keV). Excitation energies determined with the data from the two reactions agree within 2.9 keV in all cases. The mean values are (in keV) 2124.4±0.7, 4444.4±1.4, 5018.9±1.7, 6742.9±1.8, and 6792.8±1.8. The uncertainties are combinations of an uncertainty in the shape of the calibration curve with the standard deviation of the mean or the assumed internal error, whichever is greater. These uncertainties may be considered as standard deviations. The energy difference between the last two levels was found to be 49.8±0.3 keV. The value 6792.8 agrees exactly with the gamma-ray energy, whereas this and the other excitation energies disagree by as much as 22 keV with earlier values from charged-particle measurement.

I. INTRODUCTION

THE advent of lithium-drifted germanium detectors has made possible the measurement of gamma-ray energies with a precision comparable to that of good charged-particle energy measurements. Thus, values of many of the nuclear excitation energies that have been determined with charged particles will now be subject to new and strict comparisons. It may be expected that some discrepancies will appear, particularly in cases where only one precise charged-particle measurement of excitation energies has been made. The B^{11} level at an excitation energy of about 6.8 MeV is a case in point, and is the object of the present work.

The first precise measurement of excitation energies around 6.8 MeV in B^{11} was made by Van Patter, Buechner, and Sperduto[1] using the $B^{10}(d,p)B^{11}$ reaction. A 180° annular magnet was used to measure particle energies and a series of magnetic-field settings was required to cover the range of particle momenta from that of the ground-state proton group, through that of the proton group leading to the 6.8-MeV level, to that of the elastically-scattered deuteron groups used to obtain the bombarding energy. The magnetic field was measured with a current balance, and it was necessary to assume that the ratio of the average field along the particle trajectory to the field at the fluxmeter probe was constant for all fields. When the value used by these authors for the energy of the calibrating Po^{210} alpha particles is changed to the currently accepted value, the excitation energy of the state in question becomes 6.815±0.013 MeV. These authors give a value of 50±2 keV for the difference in energy between this state and the next lower state. Other values for these excitation energies may be obtained from the work of Hinds and Middleton[2] on the $Be^9(He^3,p)B^{11}$ reaction. In this work the excitation energy of the B^{11} state near 4.44 MeV was used as a reference because the ground-state proton group was not recorded. If the value of Van Patter *et al.*[1] (corrected for the change in Po^{210} alpha energy) is used for the 4.44-MeV level, one obtains 6.810±0.010 MeV for the excitation of the upper state, whereas if the result of Jaidar *et al.*[3] is used for the 4.44-MeV level, one obtains 6.795±0.010 MeV. Hinds and Middleton give a value 52 keV lower for the excitation energy of the next-lower state.

Recently Alburger *et al.*[4] performed an experiment to determine the parity of Be^{11}. In the course of this work

* This work was supported in part by the U. S. Office of Naval Research under Contract Nonr-1623(05).

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[4] D. E. Alburger, C. Chasman, K. W. Jones, J. W. Olness, and R. A. Ristinen, Phys. Rev. **136**, B916 (1964).

TABLE I. Results of Q-value measurements of the $B^{10}(d,p)B^{11}$ reaction leading to the fourth and fifth excited states of B^{11}. Units are keV. All values are based on a Po^{210} alpha energy of 5304.5 keV.

Run	Input energy	Angle of observation (°)	Q_4	Q_5	Q_4-Q_5
1	3996.0	90	2486.4	a	
2	3858.8	70	2489.6	a	
3	4200.3	60	2487.7	a	
4	4197.0	90	2486.4	a	
5	3503.1	90	2489.5	2438.7	50.8
6	3502.5	90	2488.4	2438.3	50.1
7	3501.5	70	2490.2	2440.7	49.5
8	3501.5	70	2488.7	a	
9	2731.3	130	b	b	
10	4003.5	130	2495.1	2446.2	48.9
Weighted mean			2489.0		49.9
Standard deviation of mean, σ_m			0.8		0.4
Internal error,[c] e_{int}			0.8		0.3

[a] Group not resolved because of target thickness.
[b] Group not recorded.
[c] An error of 0.1% of the Q value is assumed for each run.

they measured the energy of a gamma ray emitted by B^{11} in a transition to its ground state. A lithium-drifted germanium detector was used to compare the energy of this gamma ray with those from the decay of states in O^{16} at 6.132 MeV and 7.116 MeV. The B^{11} and O^{16} excited states were produced by beta decay of Be^{11} and N^{16}, produced in the $B^{11}(n,p)Be^{11}$ and $O^{16}(n,p)N^{16}$ reactions, respectively. Weighted means of the O^{16} excitation energies determined by Browne and Michael[5] (6.131±0.004 MeV and 7.115±0.003 MeV) and those listed in the tabulation of Ajzenberg-Selove and Lauritsen[6] (6.134±0.006 MeV and 7.118±0.006 MeV) were taken to obtain the energies of the reference gamma rays. An uncertainty of ±0.003 MeV is assigned to each of these reference energies.

The value found for the B^{11} gamma ray is 6.792 ±0.006 MeV. This is almost midway between the energies of the two states reported by Van Patter *et al.*[1] and thus a question is raised as to which level is involved. The lower excitation value, derived from the work of Hinds and Middleton suggests that it is the upper of the two levels whose decay was observed. When this situation was brought to our attention,[7] we rescanned for proton tracks the plates exposed in a broad-range spectrograph, in connection with measurements on the $B^{10}(d,\alpha)Be^8$ and $Be^9(He^3,\alpha)Be^8$ reactions. Measurement of proton groups from the $B^{10}(d,p)B^{11}$ and $Be^9(He^3,p)B^{11}$ reactions gave values for the excitation energy of the 6.74-MeV state considerably lower than that of Van Patter *et al.*, and hence in better agreement with the gamma-ray energy, under the assumption that the gamma ray comes from the upper state, and using the previously measured spacing of 50 keV.

The present work was undertaken to reduce the uncertainties in the charged-particle measurements in order to confirm the assignment of the gamma ray and to provide a test of agreement with a precision gamma-ray energy measurement made with a lithium-drifted germanium detector. To accomplish this, the uncertainty in the result should be no more than the 6 keV uncertainty in the gamma-ray measurement and, preferably, considerably less.

Standard techniques with broad-range spectrographs usually give uncertainties of no more than 10 keV in excitation energies of narrow nuclear states.[8] By paying attention to differential hysteresis in the magnetic field and averaging a large number of runs, uncertainties have been reduced to ±3 keV in at least one case.[5] The Notre Dame broad-range spectrograph and beam-analyzing magnets are now equipped with superstable current-regulated power supplies[9] which hold the current constant to 1 part in 10^5. The magnetic fields are set and monitored, as usual, with proton-resonance fluxmeters. Thus field drift during the course of a measurement has been virtually eliminated. It is found that when the electrostatic accelerator and ion source are allowed to come to equilibrium before beginning an exposure, the energy stabilizer holds the beam energy constant to a considerably higher degree than predicted by the calculated resolution of the beam-analyzing magnet. When the spectrograph field can be held constant throughout a reaction-energy measurement, the major uncertainty, aside from the ever-present target problems, comes from uncertainties and changes in the shape of the spectrograph calibration curve. This curve is a plot of trajectory radius versus position on the nuclear-track plate. The uncertainties arise mainly from differential hysteresis in the magnetic field. They are minimized when the spectrograph field is kept constant throughout a reaction-energy measurement and groups of known energy are placed near the unknown group on the plate. In the present work the proton groups corresponding to excitation energies up to 6.8 MeV could be recorded simultaneously, so all these B^{11} excitation energies were measured.

II. PROCEDURE AND RESULTS

Standard procedures for measuring Q values with broad-range spectrographs and electrostatic accelerators have been described many times, so only one typical measurement is referred to.[8] For the present work, targets were made by vacuum evaporation of elemental

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[9] Model AL105R made by Alpha Scientific Laboratories, Inc., Berkeley, California.

Be^9 or B^{10} onto Formvar backings. Electron bombardment was used to evaporate the boron.

The $B^{10}(d,p)B^{11}$ Reaction

At the bombarding energies and observation angles used, the proton groups leading to the two levels near 6.8-MeV excitation energy lay close to the group of deuterons elastically scattered from B^{10}. As the latter group was used to give the bombarding energy, a single spectrograph field setting served for the Q-value measurements. The energy range of the spectrograph is sufficient to allow the ground-state group to lie near the top end of the plate when the group corresponding to the 6.8-MeV level and the elastically-scattered deuteron group are placed near the lower end. In seven of the nine runs used to measure the Q value for the 6.74-MeV state, the groups were placed in this way. In the other two runs lower fields were used, so the excited-state groups lay near the middle of the plate and the ground-state group was not recorded. For these two Q values, uncertainties in the shape of the calibration curve introduce very small errors. Furthermore, errors in the absolute values of input and output energies nearly cancel. An assumption of an error of 0.1% in any one energy measurement with the spectrograph leads to an uncertainty of 0.1% in these Q values, which amounts to about ± 2.5 keV. For the ground-state Q value, however, the same percent error amounts to ± 9.2 keV. Thus the excitation energy, which is the difference between the ground-state and excited-state Q values, has a considerably larger uncertainty than the excited-state Q value itself.

The spacing between these two close-lying groups could be measured with higher precision than the Q values themselves, so the Q value for the higher-lying state was obtained by subtracting the mean of the measured spacings from the mean Q value for the lower state.

In Table I the result from each run is listed for the Q value for the fourth excited state of B^{11}, the Q value for the fifth excited state (when measured), and the difference between them. The bombarding energy and angle of observation used for each run are given in columns 2 and 3, respectively. Weighted means are shown for the Q value for the fourth state and for the difference of the Q values for the fourth and fifth states. The standard deviations of the means, σ_m, and the internal errors e_{int} are given. These were calculated from the expressions

$$\sigma_m^2=[\sum w_i\delta_i^2]/[(n-1)\sum w_i]$$

and

$$1/e_{\text{int}}^2=\sum[1/(\Delta E_i)^2],$$

where the w_i are weighting factors, the δ_i are the deviations of each measurement from the mean, n is the number of measurements, and the ΔE_i are the uncertainties assumed to exist in any one given measurement. For the fourth and the fifth states, this error is assumed to be 0.1% of the measured value; for the difference between these states, the uncertainty in the measurement of the group's separation on the plate is used to obtain the ΔE_i.

TABLE II. Excitation energies of the first three excited states of B^{11} and the ground-state Q value measured with the $B^{10}(d,p)B^{11}$ reaction. Units are keV.

Run[a]	Q_0	Excitation energy First state	Second state	Third state
1	9233.3 ±9.0	[b]	4444.5	5023.3
2	9238.4 ±9.0	2126.8	4451.9	[c]
3	9236.3 ±9.0	[b]	4450.5	5022.4
4	9232.8 ±9.0	[b]	4448.8	5017.3
5	[b]	[b]	[b]	[b]
6	9230.8 ±9.0	2125.3	4442.9	5019.8
7	[b]	[b]	[b]	[b]
8	9231.2 ±9.0	2124.4	[b]	5019.4
9	9228.5 ±4.0	2124.3	4443.4	5018.1
10	9235.3 ±3.0	2122.8	4440.9	5016.2
Weighted mean	9232.9	2124.6	4445.8	5019.2
Standard deviation of mean	1.2	0.6	1.7	0.9
Internal error	2.0	0.9	1.7	1.9

[a] Run numbers correspond with Table I.
[b] Group not recorded.
[c] Group obscured.

Excitation energies, which are listed below for these two states, were found by subtracting the mean Q values from the mean of the ground-state Q values. Excitation energies for the three lower states were found in the usual way, by averaging the excitation energies calculated from each run. These results are shown in Table II. Here the labeling of the runs corresponds to that given in Table I. The ground-state Q value for runs in which the group was recorded are listed in the second column of Table II, with an assigned error for each. In most cases this is 0.1% of the Q value, but for the last two, smaller uncertainties are given because a calibration check with a Po^{210} alpha source was made at the time of these runs. The standard deviation of the mean and the internal error are given for each mean listed in the table

The $Be^9(He^3,p)B^{11}$ Reaction

With this reaction it was again possible to place the ground-state group near the top of the plate and have the 6.8-MeV group near the bottom of the plate with a single magnetic-field setting. The input energy was found by measuring the position of the He^3 ions, which emerge with a single positive charge after elastic scattering from a gold target. This He^3 group usually lay about 10 cm from the ground-state proton group with the same magnetic field setting. It is felt that the excitation energies calculated for each run are more precise than the Q values, so these alone are given in Table III. Calibration groups of alpha particles from Po^{210} were place d on the plate at the time of, and with the same field setting as, many of the runs. The positions of these groups indicated a slight shift in the calibration

TABLE III. Excitation energies of B^{11} states found from the $Be^9(He^3,p)B^{11}$ reaction. Units are keV.

Run	Input energy	Angle of observation (°)	First state	Second state	Third state	Fourth state	Fifth state	E_4-E_5
1	3989.5	90	2122.0	4436.0	5013.7	6736.3	6787.6	51.3
2	4003.0	90	2125.4	4448.6	5020.7	6744.6	6792.5	47.9
3	4003.6	130	2122.5	4444.4	5017.8	6740.3	6791.7	50.4
4	2003.0	135	[a]	4444.1	5020.2	[a]	[a]	
5	2001.3	135	[a]	4442.7	5019.3	[a]	[a]	
6	3003.4	90	2124.2	4443.8	5019.7	6742.3	6792.4	50.1
7	3501.5	135	2125.0	4442.1	[b]	6741.5	6790.3	48.8
8	3999.7	110	2123.4	4445.1	5018.4	6739.7	6789.7	50.0
9	3999.7	110	2122.6	4442.5	5016.8	6739.4	6789.8	50.4
10	3003.8	90	2126.9	4444.9	5021.8	6744.8	6792.9	48.1
11	3500.0	135	2126.3	4443.6	[b]	6741.4	6792.0	50.6
Mean			2124.3	4443.4	5018.7	6741.1	6791.0	49.7
Standard deviation of mean			0.6	0.9	0.8	0.9	0.6	0.4
Internal error[c]			0.7	1.3	1.7	2.2	2.3	0.2

[a] Group not recorded. [b] Group obscured. [c] An error of 0.1% of the excitation energy is assumed for each run.

curve, necessitating a correction in the Q values, but making only a small change in excitation energies. A precision measurement of the ground-state Q value for the reaction will be reported on later. Standard deviations of the means and internal errors for the excitation energies are shown in the table.

III. ERRORS

Random errors include errors in setting the spectrograph magnetic field and observation angle, counting and plotting errors, part of the differential hysteresis effect, and, for the $Be^9(He^3,p)B^{11}$ reaction, field drift between recording the elastically scattered He^{3+} ions and recording the reaction protons. That these errors have been largely averaged out for the excitation energies by taking many runs is shown by the small values of the deviations of the means.

The important systematic errors are surface layers on the targets, and hysteresis and saturation effects in the magnetic field which cause uncertainties in the shape of the calibration curve. With the procedure used, a surface layer on the B^{10} targets would have a larger effect on the elastically-scattered deuterons than on the reaction protons, and hence too high a Q value would be measured for each state. There would, however, be very little effect on the excitation energies. For the $Be^9(He^3,p)B^{11}$ case, input energies were obtained from short exposures on clean gold targets. A surface layer on the beryllium would make all the Q values too low, but, again, would have a much smaller effect on the excitation energies. The good agreement between runs with different targets and the excellent agreement between the excitation energies found with the two reactions suggest that surface layers had little effect.

Uncertainties in the shape of the calibration curve are certainly important and an uncertainty of 0.03% of the excitation energy has been adopted for this error. To arrive at the final errors, the statistical uncertainty, which is taken as the larger of the standard deviation of the mean and the internal error, is combined with the calibration uncertainty. Many recalibrations of the spectrograph have been made over a period of years, and several check runs were taken during the present measurements. At the conclusion of these measurements a complete recalibration was done. The curve shape has remained constant within the errors of the measurements, although some displacement in trajectory radius for a given plate position frequently occurs when the magnetic field is cycled. In the various runs, groups occurred at somewhat different positions, and in two of the (d,p) runs the 6.8-MeV groups were placed at very different positions on the plate.

The final errors, which we give below, are to be considered standard deviations. They do not include the uncertainty (0.5 keV) in the energy of the Po^{210} alpha particles which were used for calibration.

IV. COMPARISONS OF RESULTS

The results of the present measurements using each reaction are displayed in columns 4 and 5 of Table IV. The excitation energy listed for the fifth state under the $Be^9(He^3,p)B^{11}$ reaction is 0.1 keV lower than the value in Table III, so that the spacing between the fourth and fifth states is the mean of that directly measured with the two reactions. The error listed for the fifth state is the square root of the sum of the squares of the internal error from Table III (0.03% of the excitation energy) and of the error in energy difference between the fourth and fifth states. A weighted mean and final error for each level is listed in column 6. Previous precise results of excitation-energy measurements are given in the first three columns. Values in the first column have been adjusted for the change in calibration energy from that used by these authors.[1] Values in column 2 were ob-

TABLE IV. Summary of excitation energies found for B^{11} and comparison with previous results. Based on a Po^{210} alpha-particle energy of 5304.5 keV.

Van Patter *et al.*[a]	Hinds and Middleton[b]	Jaider *et al.*[c]	Present work $B^{10}(d,p)B^{11}$	Present work $Be^{9}(He^{3},p)B^{11}$	Weighted mean
2140±14	2129±10	2128±10	2124.6±1.1	2124.3±0.9	2124.4±0.7
4464±14		4449± 8	4445.8±2.1	4443.4±1.8	4444.4±1.4
5039±14	5022±10	5023± 8	5019.2±2.4	5018.7±2.3	5018.9±1.7
6765±13	6738±10		6743.9±2.2	6741.1±3.0	6742.9±1.8
6815±13	6790±10		6793.8±2.2	6790.9[d]±3.1	6792.8±1.8

[a] See text Ref. 1. Values were adjusted for change in calibration energy.
[b] See text Ref. 2. These authors used the 4444-keV level as the energy standard. Tabulated values are our mean value for the level plus their value of level separation.
[c] See text Ref. 3.
[d] This value adjusted by 0.1 keV from value given in Table III to correspond to the directly-measured energy difference between the fourth and fifth excited states.

tained by adding the difference in excitation energy between the given state and the 4.44-MeV state, as quoted by the authors, to our mean value for the latter state.

Excellent agreement is achieved between the present results from the two reactions. The largest discrepancy is 2.9 keV and the average discrepancy is 1.8 keV. In each case the (d,p) reaction gave the higher value. Our results agree well with those of Jaidar *et al.*[3] for the three excitations they measured, and also agree well with the results of Hinds and Middleton.[2] The uncertainties in the spacing between levels found in the latter work would appear to be considerably smaller than the authors' stated uncertainties on the excitation energies. The values given by Van Patter *et al.*[1] disagree seriously with those from the other measurements. In each case the discrepancy is about 20 keV, whereas the stated uncertainties are 13 or 14 keV. The $B^{10}(d,p)B^{11}$ ground-state Q value found by Van Patter *et al.* (corrected for change in calibration energy) is 9245±11 keV, compared with our value of 9232.9±3.4 keV and with the value 9234±10 keV found by Jaidar *et al.* Our value of 49.8±0.3 keV for the spacing between the 6.74- and 6.79-MeV levels is in excellent agreement with the value 50.0±2.0 keV found by Van Patter *et al.*

Beautiful agreement occurs between the gamma-ray energy of 6792±6 keV given by Alburger *et al.*[4] and the mean value of 6792.8±1.8 keV which we find for the excitation energy, using the two reactions. The assignment of the gamma ray to the upper of the two closely-spaced levels is thus confirmed, and a satisfying agreement is achieved between gamma-ray energies measured with lithium-drifted germanium detectors and excitation energies measured with magnetic analysis of charged particles

ACKNOWLEDGMENT

We wish to thank Dr. D. E. Alburger for bringing this problem to our attention.