Collective Nuclear "Breathing Mode" Model with Application to ^{4}He Monopole State

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A new collective model of the nuclear breathing mode is presented which avoids the usual assumption of a constant nuclear ground-state density. The 20.4-MeV 0^+ state in ^{4}He is treated as an example of such a breathing-mode state.

I. INTRODUCTION

WE have developed a simple collective model of the nuclear breathing mode, assuming a vibration of the ground-state-matter density distribution according to a coordinate scale factor. A quantization of this oscillation leads to an expression for the vibrational amplitude in terms of the (empirical) energy of the "breathing" excited state and gives the electromagnetic form factor for the transition from the ground state as a derivative with respect to momentum transfer of the elastic form factor. Using this model, we treat the 20.4-MeV, 0^+, $T=0$ excited state in ^{4}He as an example of the breathing-mode state; reasonable agreement with the experimental[1] form factor is obtained (from inelastic electron scattering) both as to magnitude and to q dependence. We also treat the second excited state as a dipole state and show that a 2^-, $T=0$ assignment is consistent with the electron scattering data.

Of the four familiar types of vibration of nuclear matter,[2] assumed to consist of interacting fluids of $p\uparrow$, $p\downarrow$, $n\uparrow$, and $n\downarrow$ (the arrows indicating spin direction), those with some of the fluids vibrating 180° out of phase with others have recently been discussed in connection with electromagnetic[3] and weak[4] interactions. The $T=1$ excitations have been observed[5] either directly, as in photo-excitation, or indirectly, as in total muon capture rates. The breathing mode (or compressional) vibration[6] where all four nuclear fluids move in phase and the $T=0$ spin waves have never been observed, and thus it seems to be most interesting that these states may occur in ^{4}He where the simple spectrum facilitates identification.

II. COLLECTIVE MODEL

Authors in the past have developed models of the breathing mode[7] in which the ground state density is taken to be constant out to a sharp nuclear "edge." Our model avoids this assumption by describing the oscillating excess density in the same way in which Ferrell and Visscher[8] and Griffin[9] originally generated the wave functions of the breathing mode state. That is, we describe the breathing mode vibration by introducing a time-dependent coordinate scale factor into the ground-state-matter distribution $\rho_0(r)$ and write for the vibrating density

$$\rho(\mathbf{r},t)=N(\eta)\rho_0[r(R-\eta)/R]=\rho_0(r)+\rho'(\mathbf{r},t). \quad (1)$$

To first order, $\eta(_j)$ represents the displacement amplitude at the rms radius R. The normalization

$$\int d^3r\ \rho_0(r)=\int d^3r\ \rho(\mathbf{r},t)=1 \quad (2)$$

determines $N(\eta)$ and we find to first order in η that the excess charge density (the classical analog of the quantal transition charge density) is

$$\rho'(\mathbf{r},t)\cong-\frac{\eta}{R}\frac{1}{r^2}\frac{d}{dr}[r^3\rho_0(r)]. \quad (3)$$

If now η is considered to be the position variable of a harmonic oscillator with Hamiltonian

$$H=\tfrac{1}{2}\mu\omega^2\eta^2+\tfrac{1}{2}\mu\dot{\eta}^2, \quad (4)$$

which we subsequently quantize, we obtain in terms of creation and annihilation operators

$$\eta=(2\mu\omega)^{-1/2}[a_{l,m}{}^\dagger+(-1)^m a_{l,-m}]. \quad (5)$$

The mass parameter μ is easily shown to be the total mass of the nucleus, mA. The equation of continuity

$$\dot{\rho}'(\mathbf{r},t)=-\nabla\cdot(\rho\mathbf{v}) \quad (6)$$

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[5] For 0^+, $T=0$ ground-state nuclei, the 1^-, $T=1$ isospin mode is the well-known photonuclear giant dipole resonance. The $(2^-,1^-,0^-)$ $T=1$ spin-isospin mode was studied experimentally by T. de Forest, Jr., J. D. Walecka, G. Vanpraet, and W. C. Barber, Phys. Letters **16**, 311 (1965). The $T=0$, $(2^-,1^-,0^-)$ spin-wave mode cannot be excited appreciably in these interactions.

[6] For 0^+, $T=0$ ground-state nuclei 0^+, $T=0$ excited states are generated.

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[8] R. A. Ferrell and W. M. Visscher, Phys. Rev. **102**, 450 (1956).

[9] J. J. Griffin, Phys. Rev. **108**, 328 (1957).

is satisfied to first order by $\mathbf{v}=\dot{\eta}\mathbf{r}/R$, $\rho=\rho_0$. Then the kinetic energy is

$$T=\tfrac{1}{2}mA\int d^3r\,\rho_0\mathbf{v}^2$$
$$=\tfrac{1}{2}mA\dot{\eta}^2/R^2\int d^3r\,\rho_0\mathbf{r}^2 \qquad (7)$$
$$=\tfrac{1}{2}mA\dot{\eta}^2.$$

The cross section for inelastic electron scattering may be written[3] to sufficient accuracy as

$$d\sigma/d\Omega=\sigma_M(\theta)\,|F_{\rm in}(\mathbf{q})|^2, \qquad (8)$$

with σ_M being the Mott scattering cross section. In terms of our model, the inelastic form factor is given by

$$F_{\rm in}(\mathbf{q})=Z\langle 0_{\rm exc}^+|\int d^3r\,e^{i\mathbf{q}\cdot\mathbf{r}}\rho'(\mathbf{r})\,|0_{\rm gnd}^+\rangle. \qquad (9)$$

Inserting $\rho'(r)$ from Eq. (3), the final expression for the inelastic form factor becomes

$$F_{\rm in}(\mathbf{q})=(Z/R)(2mA\omega)^{-1/2}q(dF(q)/dq), \qquad (10)$$

where $F(q)$ is the ground-state form factor.

III. APPLICATION TO ^{4}He

The state to which we apply our formalism is the 20.4-MeV, 0^+, $T=0$ state in ^{4}He. Its breathing mode character is established both by its single-particle description and by the fact that its energy is obtainable directly from the model. We discuss briefly these two points.

It has long been known[10] that the lowest shell-model excited state in ^{4}He which has the same quantum numbers as the ground state and the same spatial symmetry, [4], has energy $2\hbar\omega$. (Here $\hbar\omega$ is the shell-model level splitting.) The linear combination of single-particle states which eliminates the spurious center-of-mass motion is

$$\tfrac{1}{2}\sqrt{3}\,(1s)^{-1}(2s)+\tfrac{1}{2}(1s)^{-2}(1p)^2. \qquad (11)$$

However, the state is more simply described in terms of relative coordinates and in this description its collective character is more clearly exhibited.

We introduce the center-of-mass coordinate, $\mathbf{R}$, and three relative coordinates, $\boldsymbol{\varrho}$, $\mathbf{r}_{12}$, and $\mathbf{R}_4$ by the relations

$$\mathbf{R}=\tfrac{1}{4}(\mathbf{r}_1+\mathbf{r}_2+\mathbf{r}_3+\mathbf{r}_4),\quad \mathbf{r}_{12}=\mathbf{r}_1-\mathbf{r}_2, \qquad (12)$$
$$\boldsymbol{\varrho}=\tfrac{1}{2}(\mathbf{r}_1+\mathbf{r}_2)-\mathbf{r}_3,\quad \mathbf{R}_4=\tfrac{1}{3}(\mathbf{r}_1+\mathbf{r}_2+\mathbf{r}_3)-\mathbf{r}_4.$$

Then the symmetrized internal wave function of the excited state can be written

$$\Psi=\tfrac{1}{2}(1+P_{14}+P_{24}+P_{34})\Phi(R_4)\psi(\mathbf{r}_{12},\boldsymbol{\varrho}). \qquad (13)$$

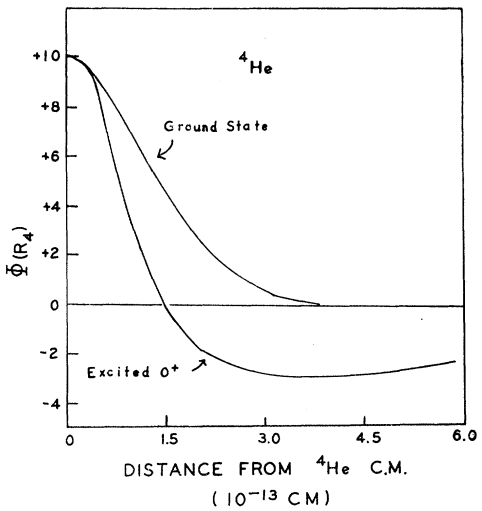

FIG. 1. The fourth-particle wave functions in the ground and first excited state of ^{4}He, taken from Ref. 11.

The P_{ij}'s are space-exchange operators, $\psi(\mathbf{r}_{12},\boldsymbol{\varrho})$ is the wave function for the three unexicted nucleons, and $\Phi(R_4)$ is the single-particle wave function for the fourth nucleon. In the relative coordinate system the excited state is simply $(1s)^{-1}(2s)^1$. In Fig. 1 we show the function $\Phi(R_4)$ for the ground state (a Gaussian consistent with the measured rms radius of ^{4}He) and for the 0^+. The latter function has been obtained in a resonating group calculation[11] in which Φ was treated as a continuum wave function. It is clear from the figure and from Eq. (13) that the transition density,

$$\rho'(\mathbf{r})=\langle\Psi|\sum_{i=1}^{4}\delta(\mathbf{r}-\mathbf{r}_i)\tfrac{1}{2}(1+\tau_{iz})\,|\Psi_{\rm gnd}\rangle \qquad (14)$$

has a radial node in it at about a distance from the origin equal to the rms radius. This is precisely the form of the excess density of a breathing-mode oscillation.

An even more striking demonstration of the collective nature of the 0^+ state is that the energy of the excitation can be derived from the simple model presented in the preceding section. If one calculates the internal energy of ^{4}He as a function of the rms radius, the following approximate Hamiltonian is obtained for the breathing mode.

$$H=\tfrac{1}{2}mA\dot{\eta}^2+\tfrac{1}{2}\partial^2E_{\rm int}/\partial\eta^2|_0\,\eta^2. \qquad (15)$$

Then the energy of this state is given by

$$E_{\rm exc}=\hbar\left(\frac{\partial^2E_{\rm int}}{\partial\eta^2}\Big/mA\right)^{1/2}. \qquad (16)$$

We take our estimate of the constant $\partial^2E_{\rm int}/\partial\eta^2$ from a calculation of the binding energies of the three- and four-nucleon systems by Mang and Wild.[12] From Fig. 13

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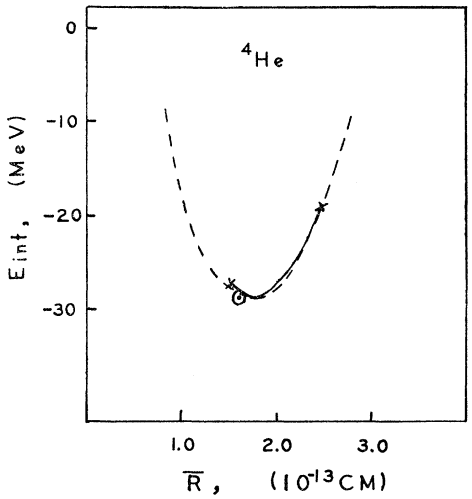

FIG. 2. The calculated internal energy (solid curve) as a function of the rms radius of ^{4}He, derived from Fig. 13 of Ref. 12. The dashed curve is a parabola fitted to the minimum and the maximum calculated point. The circled dot shows the experimental energy and radius.

of the paper by these authors we have obtained the internal energy of ^{4}He as a function of R, the rms radius, (their curve has the oscillator parameter as the abscissa) and have plotted this curve in Fig. 2. The dotted curve is a parabola which passes through the minimum of the energy curve and through the highest calculated value. The values of the spring constant and excitation energy which one obtains from the parabola are

$$\partial^2 E_{\rm int}/\partial\eta^2 = 40.1 \text{ MeV/F}^2, \qquad E_{\rm exc} = 20.5 \text{ MeV}. \tag{17}$$

The exact agreement with the experiment energy is, of course, fortuitous because the ratio of the single-particle frequencies to the collective frequency is only about 3, hardly large enough to justify our Born-Oppenheimer approximation in Eq. (15).

We now apply our breathing-mode model cross section to ^{4}He. In applying Eqs. (8) and (10) it must be remembered that the 0^+ state is actually in the continuum of the $t+p$ channel while the model implicitly assumes a bound excited state. What is measured is a differential cross section as a function of energy, $d\sigma/d\Omega dE$. The standard way (see the Appendix) to "spread out" the prediction based on a discrete state is to multiply the cross section by a Lorentz factor, i.e.

$$\frac{d\sigma}{d\Omega dE} = \frac{1}{\pi} \frac{\tfrac{1}{2}\Gamma_p}{(E_R+\Delta-E)^2+\tfrac{1}{4}\Gamma_p^2} \frac{d\sigma}{d\Omega}. \tag{18}$$

Since this state lies only $\sim$0.5 MeV above the $t+p$ threshold and $\sim$0.3 MeV below the ^{3}He$+n$ threshold, the width Γ_p and the level shift Δ are rapidly varying functions[13] of the electron energy E, or of the proton energy E_p in the $t+p$ c.m. system, and the shape of the spectrum is markedly asymmetric. In a previous analysis of $t+p$ scattering[14] these functions have been obtained in terms of reduced widths, for which we take $\gamma_n^2=\gamma_p^2=2.09$ MeV, and a channel radius of 4.2 F. Values of $d\sigma/d\Omega dE$ calculated with Eqs. 8 and 18 have the correct shape as compared to experiment,[1] but are too large by a factor of roughly 2. We note that the resonance extends into the energy region of the open ^{3}He$+n$ channel so that a term containing Γ_n should also be added to Eq. (13). However, the resonance is tailing off rapidly at the neutron threshold so we have ignored the neutron term. The effect of the closed neutron channel has been included in the level shift Δ.

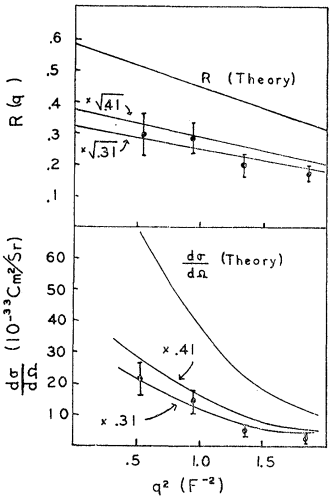

FIG. 3. Inelastic electron scattering cross scattering cross section of the 20.4-MeV state in ^{4}He and function $R(q)$; comparison of experimental points in Ref. 1 with theoretical curves from the breathing-mode model and with curves reduced by scale factors as indicated.

To give a convenient comparison with experiment, we show in Fig. 3 a comparison between the cross section from Eq. (18) integrated over energy between $E\hat{p}=0$ and $E\hat{p}=1$ MeV and the integrated experimental[1] cross section. Numerical integration of the Lorentz factor over the width of the experimental peak yields $0.76/\pi$ so that there is an effective reduction of the strength of the state because of its proximity to the thresholds. (With constant Γ_p and no level shift the integral over all $E\hat{p}$ would be 1.) With this factor and a Gaussian ground state form factor corresponding to $R=1.68$ F,[15] the theoretical expressions for $d\sigma/d\Omega$ and for the quantity

$$R(q)=F_{\rm in}(q)/1-F(q) \tag{19}$$

are plotted versus q^2 in Fig. 3 and compared with the experimental points.[1] Our breathing model thus yields matrix elements for the 0^+ excited state which are about 50% too large, which is a not unsatisfactory result.[16]

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[15] G. R. Burleson and H. W. Kendall, Nucl. Phys. **19**, 68 (1960); see also J. P. Repellin, P. Lehmann, J. Lefrancois, and D. B. Isabelle, Phys. Letters **16**, 169 (1965).

[16] See footnote 29 of H. Überall, Ref. 3.

It should be noted that Ferrell's monopole sum rule[17]

$$\sum_n \omega |\langle \eta | \sum_{i=1}^{A} \tfrac{1}{2} r_i^2 | 0 \rangle|^2 = R^2 A/2m \qquad (20)$$

is easily shown to be 100% exhausted by our breathing-mode model of Sec. I. If the sum over final states becomes an integral over continuum states the same Lorentz factor that appears in Eq. (18) appears inside the integral and our model exhausts about 25% of the monopole strength. This is still larger by a factor of 2 than the experimental strength, but is close enough to confirm the collective nature of the state.

The ^{4}He second excited state at 22.2 MeV has been assigned a $J^\pi = 2^-$ by Baz' and Smorodinskii[18]; an isospin $T=1$ is ruled out by a recent ^{3}He$+p$ phase-shift analysis.[19] The assignment $T=0$ is confirmed by a comparison of the experimental e,e' and p,p' cross sections[20] shown in Fig. 4. The identification of this level with a 2^- (or 1^-) $T=1$ spin—isospin collective mode would, using the theoretical electron-scattering cross section obtained by one of us,[3] lead to the dotted curve, in disagreement with the experimental e,e' data. However, the electro-excitation of a 2^- (or 1^-) $T=0$ spin-wave mode is reduced by a factor $(\mu_p+\mu_n)^2/(\mu_p-\mu_n)^2 = 0.035$ and such a state would not appear if it were a broad resonance as in the present case.

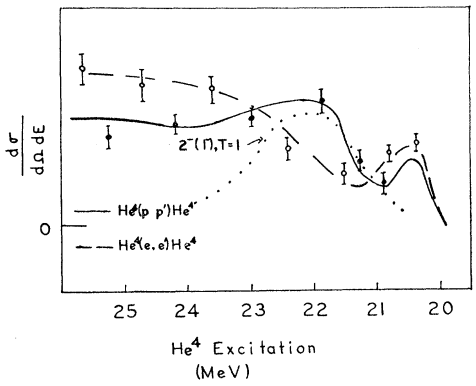

FIG. 4. Comparison of experimental excitation functions of ^{4}He states by proton (Ref. 20) (solid curve, $\theta_{\rm lab}=52°$) and electron[1] (dashed curve, $\theta_{\rm lab}=90°$) scattering. The dotted curve corresponds to the theoretical electron scattering cross section for a $T=1$ spin—isospin state at 22 MeV (with a width consistent with that in the proton scattering).

IV. GENERAL APPLICATIONS

It is an unsettled question whether or not breathing modes exist in nuclei other than ^{4}He. Glassgold, Heckrotte, and Watson[2] found that the compressional excitation in nuclear matter is unstable since the energy per unit volume turns out to be complex. They suggest that a physical basis for this result is a clustering of nucleons in the rarified regions. Such clustering could occur in nuclei heavier than ^{4}He and prevent the breathing mode state, one in which the ground-state wave function dilates adiabatically, from being an eigenfunction of the nuclear Hamiltonian.

Attempts have been made in the past to associate the lowest lying 0^+ states in ^{12}C and ^{16}O (the first excited state of ^{40}Ca is also 0^+) with breathing mode states. The 0^+ state in ^{16}O has received perhaps the most attention. Schiff[7] noted that the pair production rate from this state is an order of magnitude lower than the rate expected from a collective monopole state, and in electron scattering the 6-MeV 0^+ state exhausts only about 4% of the monopole sum rule.[21] In addition, the theoretical calculations of the energy of this state require large admixtures of two-particle–two-hole excitations[22–24] and/or four-particle–four-hole excitations[25] to lower the energy to the observed value. Since collective states are predominantly coherent admixtures of single-particle–single-hole states the first excited states of ^{4}He and ^{16}O seem to be of different natures.

Estimates of the excitation energy of the breathing mode in heavier nuclei can be obtained from the nuclear compressibility,

$$K = r^2(\partial^2 E_{\rm int}/\partial r^2) = bA\,. \qquad (21)$$

Taking $r^2 = R^2 = (r_0 A^{+1/3})^2$ we obtain from Eq. (16)

$$E_{\rm exc} = \hbar (b/m r_0^2)^{1/2} A^{-1/3}\,. \qquad (22)$$

For r_0 we choose 1.0 F (R is the rms radius, not the uniform radius) and a reasonable range[26] of values for b is $100 > b > 200$ (MeV). We obtain

$$65A^{-1/3} > E_{\rm exc} > 92A^{-1/3}, \qquad (23)$$

where the lower value seems more favored by experi-

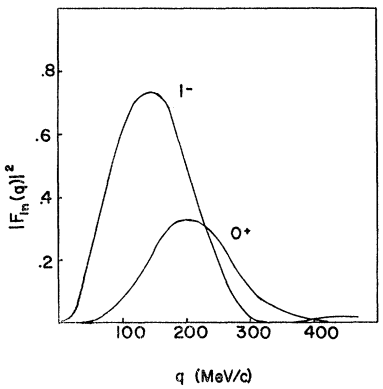

FIG. 5. Comparison of the squares of the inelastic form factors for collective dipole and monopole states. The excitation energy of both states is assumed to be 22 MeV and the Z and A are those of ^{16}O.

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[18] A. J. Baz and Ya. Smorodinskii, Zh. Eksperim. i Teor. Fiz. **27**, 382 (1954); W. E. Meyerhof, Bull. Am. Phys. Soc. **10**, 698 (1965).
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[23] Hartwig Schmidt, Z. Physik **181**, 532 (1964).
[24] J. M. Eisenberg, B. M. Spicer, and M. E. Rose, Nucl. Phys. **71**, 273 (1965).
[25] I. Talmi and I. Unna, Ann. Rev. Nucl. Sci. **10**, 353 (1960).
[26] K. A. Brueckner, Rev. Mod. Phys. **30**, 561 (1958).

mental estimates of K. This energy is somewhat lower than the giant dipole energy which is roughly given by $80A^{-1/3}$ and a bit higher than the monopole energy estimated by Walecka[7] $\sim 56A^{-1/3}$.

It is interesting to compare the momentum transfer values at which the breathing mode cross section is important to those where the giant dipole state is important in electroexcitation. One of us[3] has previously calculated the inelastic form factor for the giant dipole state and for convenience we write down the squares of the form factors together,

$$\begin{aligned} 1^-;\quad & |F_{\rm in}(q)|^2=\frac{Z^2q^2}{2mA\omega}|F(q)|^2, \\ 0^+;\quad & |F_{\rm in}(q)|^2=\frac{Z^2q^2}{2mA\omega}\frac{1}{R^2}\left|\frac{\partial F(q)}{\partial q}\right|^2. \end{aligned} \tag{24}$$

Since $F(q)$ is the ground-state form factor the monopole state becomes important near the first diffraction minimum of the elastic-scattering cross section. As a concrete example, we have plotted in Fig. 5 the squares of the inelastic form factors for a choice of $Z=8$, $A=16$, and $\omega=22$ MeV. The ground-state form factor $F(q)$ is taken to be that given by a normal-shell-model ground state[27] and assumes an rms radius for ^{16}O of 2.65 F.

APPENDIX

We give a derivation of Eq. (18) using the Wigner R-matrix description of a resonance.[13] In this formalism a discrete state X_R is introduced which is defined within a certain radius a to have the same shape as the continuum state at the resonance energy E_R. X_R is normalized in the same way as any bound state but differs from a bound state in that it is cut off abruptly at the nuclear "surface." One way to adopt calculations done with discrete states to physical systems in the continuum is to associate the model bound state with X_R and use the observed reduced widths to spread out the calculated cross sections in energy.

To accomplish this, we use first-order perturbation theory to write down an expression for the integrated (over energy) differential cross section for the scattering of an electron with initial energy E_i into a final state within solid angle $d\Omega$,

$$\frac{d\sigma}{d\Omega}=\frac{2\pi}{\hbar}\int\frac{d^3p_N}{(2\pi\hbar)^3}\int\frac{d^3p_e}{(2\pi\hbar)^3}|(\psi_f,V\psi_i)|^2 \times\delta(E_i-(p_N^2/2\mu)-E_{\rm th}-E_e)\delta(\Omega_e-\Omega). \tag{A1}$$

In the above equation, $E_{\rm th}$ is the energy relative to E_0, the energy of the initial state, at which the continuum begins; $\mathbf{p}_N$ and $\mathbf{p}_e$ are the final nuclear and electron momenta, and μ the reduced mass; $(\psi_f,V\psi_i)$ is the nuclear matrix element of the interaction with ψ_i the initial bound state and ψ_f the final continuum state.

We now choose X_R to be the same as ψ_f, aside from normalization, at the resonance energy. X_R is normalized to one over a sphere with channel radius a, and on the surface of the sphere is given by[13]

$$X_R=(y(a)/a)i^lY_l^m(\theta,\varphi)\psi_{\rm int}. \tag{A2}$$

Here $\psi_{\rm int}$ is the internal wave function of the separated fragments. The reduced width γ_R^2 is defined in terms of the channel wave function $y(a)$ by

$$\gamma_R^2=(\hbar^2/2\mu a)y^2(a). \tag{A3}$$

The ψ_f in Eq. (A1) is normalized such that at the channel radius

$$\psi_f\rightarrow(4\pi)^{1/2}(e^{i\delta_l}i^l/k_Na)\times[F_l\cos\delta_l+G_l\sin\delta_l]Y_l^m\psi_{\rm int}. \tag{A4}$$

For simplicity we assume a one-channel final state so that the phase shift δ_l is real.

We now make two assumptions: (1) The matrix element $(\psi_f,V\psi_i)$ receives no contributions from regions of configuration space such that the fragments are separated by more than a distance a. (2) Over the width of the resonance ψ_f has the same shape as X_R within the interaction region. With these assumptions Eq. (A1) can be rewritten

$$\frac{d\sigma}{d\Omega}\cong\frac{2\pi}{\hbar}|(X_R,V\psi_i)|^2\int\frac{d^3p_N}{(2\pi\hbar)^3}\int\frac{d^3p_e}{(2\pi\hbar)^3} \times\left[\frac{(4\pi)^{1/2}\hbar}{|\mathbf{p}_N|}\frac{F_0\cos\delta_0+G_0\sin\delta_0}{y(a)}\right]^2 \times\delta\left(E_i-\frac{p_N^2}{2\mu}-E_{\rm th}-E_e\right)\delta(\Omega_e-\Omega). \tag{A5}$$

We limit ourselves to S states so that the angular integration is trivial. After integrating over all variables except the nuclear energy, E_N, we obtain

$$\frac{d\sigma}{d\Omega}\cong\frac{2\pi}{\hbar}|(X_R,V\psi_i)|^2\frac{(E_i-E_R+E_0)^2}{(2\pi\hbar c)^3} \times\frac{1}{\pi}\int dE_N\frac{\frac{1}{2}\Gamma(E_N)}{(E_R+\Delta(E_N)-E_N)^2+\frac{1}{4}\Gamma^2(E_N)}. \tag{A6}$$

The quantities $\Gamma(E)$ and $\Delta(E)$ are related to the phase shift δ_0 and to γ_R^2 in the usual way, namely

$$\begin{aligned} &\Gamma=\frac{2k_Na}{(F_0^2+G_0^2)^{1/2}}\gamma_R^2, \qquad \delta_0=\delta_R+\alpha, \\ &\delta_R=\tan^{-1}\left(\frac{\frac{1}{2}\Gamma}{E_R+\Delta-E_N}\right), \quad \alpha=-\tan^{-1}(F_0/G_0). \end{aligned} \tag{A7}$$

The first factor in Eq. (A6) is the differential cross

[27] Paul Goldhammer, Rev. Mod. Phys. 35, 40 (1963).

section for electroexcitation of a discrete state. The second factor usually integrates to one so that the usual procedure of associating model differential cross section with experimental cross sections integrated over the resonance peak is in accord with our result. However, in the specific case of the ^{4}He 0^+ state the resonance falls between two thresholds and the width and level shift are rapidly varying functions of energy. The result is that the monopole strength inherent in the internal function X_R is effectively reduced. One can either divide the experimental cross section by the integral in Eq. (A6) or multiply the model differential cross section by the same integral. We have elected the latter possibility in our comparison with experiment.

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Accurate Excitation Energies of B^{11} States Below 7 MeV*

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A recent precision measurement of gamma-ray energy made with a lithium-drifted germanium detector disagrees with an earlier value of an excitation energy obtained by measurement of charged-particle energies. To investigate this discrepancy and to confirm the level assignment of the gamma-ray transition, excitation energies of B^{11} states were measured using both the $B^{10}(d,p)B^{11}$ and $Be^9(He^3,p)B^{11}$ reactions. Particle energies were measured with a broad-range spectrograph calibrated with a Po^{210} alpha source (5304.5 keV). Excitation energies determined with the data from the two reactions agree within 2.9 keV in all cases. The mean values are (in keV) 2124.4±0.7, 4444.4±1.4, 5018.9±1.7, 6742.9±1.8, and 6792.8±1.8. The uncertainties are combinations of an uncertainty in the shape of the calibration curve with the standard deviation of the mean or the assumed internal error, whichever is greater. These uncertainties may be considered as standard deviations. The energy difference between the last two levels was found to be 49.8±0.3 keV. The value 6792.8 agrees exactly with the gamma-ray energy, whereas this and the other excitation energies disagree by as much as 22 keV with earlier values from charged-particle measurement.

I. INTRODUCTION

THE advent of lithium-drifted germanium detectors has made possible the measurement of gamma-ray energies with a precision comparable to that of good charged-particle energy measurements. Thus, values of many of the nuclear excitation energies that have been determined with charged particles will now be subject to new and strict comparisons. It may be expected that some discrepancies will appear, particularly in cases where only one precise charged-particle measurement of excitation energies has been made. The B^{11} level at an excitation energy of about 6.8 MeV is a case in point, and is the object of the present work.

The first precise measurement of excitation energies around 6.8 MeV in B^{11} was made by Van Patter, Buechner, and Sperduto[1] using the $B^{10}(d,p)B^{11}$ reaction. A 180° annular magnet was used to measure particle energies and a series of magnetic-field settings was required to cover the range of particle momenta from that of the ground-state proton group, through that of the proton group leading to the 6.8-MeV level, to that of the elastically-scattered deuteron groups used to obtain the bombarding energy. The magnetic field was measured with a current balance, and it was necessary to assume that the ratio of the average field along the particle trajectory to the field at the fluxmeter probe was constant for all fields. When the value used by these authors for the energy of the calibrating Po^{210} alpha particles is changed to the currently accepted value, the excitation energy of the state in question becomes 6.815±0.013 MeV. These authors give a value of 50±2 keV for the difference in energy between this state and the next lower state. Other values for these excitation energies may be obtained from the work of Hinds and Middleton[2] on the $Be^9(He^3,p)B^{11}$ reaction. In this work the excitation energy of the B^{11} state near 4.44 MeV was used as a reference because the ground-state proton group was not recorded. If the value of Van Patter *et al.*[1] (corrected for the change in Po^{210} alpha energy) is used for the 4.44-MeV level, one obtains 6.810±0.010 MeV for the excitation of the upper state, whereas if the result of Jaidar *et al.*[3] is used for the 4.44-MeV level, one obtains 6.795±0.010 MeV. Hinds and Middleton give a value 52 keV lower for the excitation energy of the next-lower state.

Recently Alburger *et al.*[4] performed an experiment to determine the parity of Be^{11}. In the course of this work

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