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Recombination Kinetics and Electroluminescence from Deep Levels in the Carrier Diffusion Region of a *p-n* Junction

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Approximate steady-state solutions of the continuity equation containing effects of diffusion, drift, and recombination are obtained for the carrier diffusion region of a forward-biased *p-n* junction over a wide range of injection. These are used to predict the voltage (V) dependence and the spatial distribution of electroluminescence (EL) originating from a deep level. Regions where ln(EL) versus eV/kT has slope 1, $\frac{1}{2}$, $(m+1)/m$, and $\frac{1}{2}$ are found (m is defined by the current-voltage characteristic: $I=I_0 \exp eV/mkT$). These predictions hold for either Shockley-Read-Hall or donor-acceptor-pair recombination. The spatial distribution of electroluminescence is a simple exponential in the first region. For the next two regions for each recombination mechanism, it contains a saturated layer of constant brightness adjacent to the junction which expands rapidly with voltage. The latter behavior persists in the fourth region for donor-acceptor-pair recombination, but for Shockley-Read-Hall recombination a further increase in brightness near the junction occurs. Comparison of these predictions with experimental results on gallium phosphide electroluminescent diodes is made in the following paper.

I. INTRODUCTION

Recently Morgan[1] has used the Sah-Noyce-Shockley theory[2] of deep-level recombination in *p-n* junctions to predict the voltage dependence of electroluminescence emitted from the space-charge layer of junctions. Mayburg and Black[3] have studied the kinetics of a competition between monomolecular and bimolecular light emission mechanisms in the space-charge-layer and diffusion regions of junctions under the restriction that these recombination mechanisms also determine the current-voltage characteristic.

A recent experimental study[4] of the kinetics of light emission from a deep recombination level in gallium phosphide *p-n* junctions has found three regions where the voltage dependence of the electroluminescence has a simple form. Observation of the spatial distribution of the emission has shown it to originate from the carrier diffusion region beyond the space-charge layer. The spatial distribution of emission was also measured and found to change with voltage. These results cannot be compared with Morgan's work[1] which applies only to space-charge-layer recombination, nor with the work of Mayburg and Black[2] which (1) assumes radiative recombination is dominant, (2) does not involve a saturable recombination mechanism (which is necessary to explain the data), and (3) considers only the case of conductivity modulation.

It is the purpose of this paper to construct a theoretical framework for the interpretation of these observations. The results obtained, however, can be applied to any semiconductor. We will obtain approximate solutions to the steady-state continuity equation containing effects of diffusion, drift, and recombination for the carrier diffusion region of a forward-biased *p-n* junction in four different regions of junction bias. The current-voltage characteristic, which depends on space-charge-layer as well as diffusion-region recombination, will be assumed experimentally known. Recombination via a Shockley-Read-Hall mechanism and a donor-acceptor-pair mechanism will be considered. From the solutions, predictions of the voltage dependence of the total light emission and its spatial distribution will be made for each voltage region.

II. RECOMBINATION STATISTICS

A. Shockley-Read-Hall Recombination

Shockley and Read[5] and Hall[6] showed that the steady-state recombination rate of nondegenerate distributions of holes and electrons through a single level in the forbidden gap is given by

$$U=(pn-p_1n_1)/[(n+n_1)\tau_{p0}+(p+p_1)\tau_{n0}], \qquad (1)$$

where p and n are the densities of free holes and electrons, p_1 and n_1 the densities when the Fermi level coincides with the recombination level ($p_1n_1=n_i^2$, where n_i is the intrinsic carrier concentration), τ_{p0} the lifetime for holes injected into highly n-type crystals, and τ_{n0} the lifetime for electrons injected into highly p-type crystals. We will assume that at least one of the transitions involved in Eq. (1) is radiative. If we assume that charge neutrality holds and that the majority carrier density under equilibrium conditions is large compared with the recombination level density,[5] then

$$n=n_0+\delta n, \qquad (2)$$

and

$$p=p_0+\delta n, \qquad (3)$$

where n_0 and p_0 are the equilibrium electron and hole

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[1] T. N. Morgan, Phys. Rev. **139**, A294 (1965).

[2] C. T. Sah, R. N. Noyce, and W. Shockley, Proc. IRE **45**, 1228 (1957).

[3] S. Mayburg and J. Black, J. Appl. Phys. **34**, 1521 (1963).

[4] M. Gershenzon, R. A. Logan, and D. F. Nelson, following paper, Phys. Rev. **149**, 580 (1966). See Sec. V and Figs. 10, 11, 16, and 17.

[5] W. Shockley and W. T. Read, Jr., Phys. Rev. **87**, 835 (1952).

[6] R. N. Hall, Phys. Rev. **83**, 228 (1951); **87**, 387 (1952).

densities. The recombination rate becomes

$$U=\delta n(1+c\delta n)/\tau_0(1+a\delta n)\,, \tag{4}$$

where

$$a\equiv(\tau_{p0}+\tau_{n0})/[(n_0+n_1)\tau_{p0}+(p_0+p_1)\tau_{n0}]\,,$$

$$c\equiv(n_0+p_0)^{-1}\,,$$

and

$$\tau_0\equiv(\tau_{p0}+\tau_{n0})c/a$$

which is the low excitation lifetime.

B. Donor-Acceptor Pair Recombination

Radiative recombination of electrons trapped on donors with holes trapped on spatially separated acceptors ("pair recombination") has been shown to be present in junction electroluminescence.[4] The calculation of the recombination rate of this process at low temperature is greatly complicated by the necessary integrations over the pair-separation distribution.[7] However, at higher temperatures redistribution of charges on the centers causes pair recombination to become a bimolecular process[7] between donor-acceptor pairs whose separation is such that their recombination rate is greater than or comparable to the ionization rate from the shallowest of the two levels. All other donors and acceptors act merely as traps. In this temperature regime, the spatial distribution of donor-acceptor pairs can be ignored and the pair recombination represented as merely a transition between two levels in the forbidden gap. The net transition rate between an electron on a donor and a hole on a nearby acceptor is therefore

$$U_{da}=f_{pa}N_ac_nf_dN_d-f_aN_ae_nf_{pd}N_d\,, \tag{5}$$

where f is the probability of occupancy for electrons, $f_p\equiv 1-f$ is the probability of occupancy for holes (d and a are subscripts denoting donor and acceptor, respectively), c_n is a capture coefficient and e_n an emission coefficient (units of cm^3/sec), and N_d and N_a are the densities of donors and acceptors which satisfy the closeness requirement stated above. At thermal equilibrium, Eq. (5) equals zero. Thus, $e_n/c_n=\exp(E_a-E_d)/kT\equiv K$, where E_a and E_d are the energies of the acceptor and donor levels. For wide-band-gap semiconductors at normal temperatures $K\approx 0$. Thus,

$$U_{da}=(f_{pa}f_d-Kf_af_{pd})c_nN_aN_d\,. \tag{6}$$

The net capture rate of free electrons by donors for a nondegenerate free-electron distribution is[5]

$$U_{cd}=(f_{pd}n-f_dn_1)/\tau_{n0}\,, \tag{7}$$

where $n_1\equiv N_c\exp(E_d-E_c)/kT$, N_c being the density of states and E_c the energy of the edge of the conduction band. The net capture rate of free holes by acceptors for a nondegenerate free hole distribution is[5]

$$U_{va}=(f_ap-f_{pa}p_1)/\tau_{p0}\,, \tag{8}$$

where $p_1\equiv N_v\exp(E_v-E_a)/kT$, N_v being the density of states and E_v the energy of the edge of the valence band. Valence-band → donor and conduction-band → acceptor transitions will be assumed negligible. At steady state $U=U_{cd}=U_{da}=U_{va}$. This yields

$$\begin{aligned}U=\{&Q[(p+Kp_1)/\tau_{p0}+(n+Kn_1)/\tau_{n0}]\\&+(p+p_1)(n+n_1)/\tau_{p0}\tau_{n0}\\&-[\{Q[(p+Kp_1)/\tau_{p0}+(n+Kn_1)/\tau_{n0}]\\&+(p+p_1)(n+n_1)/\tau_{p0}\tau_{n0}\}^2\\&-4Q^2(1-K)(pn-Kp_1n_1)/\tau_{p0}\tau_{n0}]^{1/2}\}/2Q(1-K)\,,\end{aligned} \tag{9}$$

where $Q\equiv c_nN_aN_d$. With the definitions of n_1 and p_1 used for the donor-acceptor-pair recombination $Kn_1p_1=n_i^2$.

The low recombination rate lifetime τ_0, defined by $U=\delta n/\tau_0$, can be found from Eq. (9) with the use of Eqs. (2) and (3) to be

$$\tau_0=\frac{Q[p_0\tau_{n0}+n_0\tau_{p0}+K(p_1\tau_{n0}+n_1\tau_{p0})]+(p_0+p_1)(n_0+n_1)}{Q(p_0+n_0)}\,. \tag{10}$$

The high excitation limit of Eq. (9) can be obtained by expanding the radical to first order in $\tau_{p0}^2\tau_{n0}^2/(p+p_1)^2(n+n_1)^2$ to be

$$U=\frac{Qpn}{(p+p_1)(n+n_1)}\times\left\{1-\frac{K}{pn}\left[p_1n_1+\frac{(p+n)(p_1+n_1)}{2(1-K)}\right]\right\}\,. \tag{11}$$

When $p\gg p_1$ and $n\gg n_1$, $U=Q$, which corresponds to the complete filling of both the donor and acceptor levels.

III. CONTINUITY EQUATION FORMULATION

We now derive the steady-state equation governing the diffusion, drift, and recombination of excess carriers in the diffusion region of a p-n junction. The steady-state continuity equations for free holes and electrons are

$$U=-\nabla\cdot\mathbf{J}_h/e\,, \tag{12}$$

$$U=\nabla\cdot\mathbf{J}_e/e\,, \tag{13}$$

where $\mathbf{J}_h$ and $\mathbf{J}_e$ are the hole and electron current

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[7] D. G. Thomas, J. J. Hopfield, and W. M. Augustyniak, Phys. Rev. **140**, A202 (1965).

densities, e the electron charge, and the recombination rate U is given by either Eq. (1) or (9) or a sum of such rates. The current densities are given in terms of the electric field $\mathbf{E}$, the hole and electron mobilities μ_h and μ_e, and the hole and electron diffusivities D_h and D_e ($D_i=\mu_i kT/e$, where $i=h,e$) by

$$\mathbf{J}_h=e\mu_h p\mathbf{E}-eD_h\nabla p\,, \tag{14}$$

$$\mathbf{J}_e=e\mu_e n\mathbf{E}+eD_e\nabla n\,, \tag{15}$$

and the total current density $\mathbf{J}$ is

$$\mathbf{J}=\mathbf{J}_h+\mathbf{J}_e. \tag{16}$$

By substituting Eqs. (14) and (15) into (12) and (13) and eliminating the $\nabla\cdot\mathbf{E}$ term, we obtain

$$\begin{aligned} U(\mu_e n+\mu_h p)=\mu_e D_h n\nabla^2 p+\mu_h D_e p\nabla^2 n \\ -\mu_e\mu_h\mathbf{E}\cdot(n\nabla p-p\nabla n). \qquad 17) \end{aligned}$$

Our interest here is in studying the electroluminescence emitted from the carrier-diffusion region on one side of the junction. The current-voltage characteristic of the junction is determined by recombination on both sides of the junction and in the space-charge layer. Thus, we can regard the total junction current density given by Eq. (16) as an experimentally known parameter and express the electric field $\mathbf{E}$ in terms of it. From Eqs. (14), (15), and (16) we have

$$\mathbf{E}=(\mathbf{J}/e-D_e\nabla n+D_h\nabla p)/(\mu_h p+\mu_e n). \tag{18}$$

Equation (18) can be used to eliminate $\mathbf{E}$ from Eq. (17).

We now use Eqs. (2) and (3) to obtain from Eqs. (17) and (18) an equation governing the excess carrier density δn. We will consider the p side of the junction ($n_0\approx 0$). Define the dimensionless quantities: $\Delta\equiv\delta n/p_0$, $l\equiv x[\tau_0 D_e]^{-1/2}$, $b\equiv\mu_e/\mu_h=D_e/D_h$, and $\Gamma\equiv b|J|\tau_0^{1/2}/ep_0D_e^{1/2}$. The coordinate system we use places the edge of the space-charge region at $x=0$ and the positive contact of the diode at $x=+\infty$. Thus,

$$\frac{1+2\Delta}{1+(b+1)\Delta}\frac{d^2\Delta}{dl^2}-\left\{\frac{\Gamma+(b-1)(d\Delta/dl)}{[1+(b+1)\Delta]^2}\right\}\frac{d\Delta}{dl}=\frac{\tau_0}{p_0}U. \tag{19}$$

The second derivative term represents the effect of diffusion, the first derivative terms the effects of drift (which act to transport excess carriers in the same direction as diffusion does), and the right-hand side represents recombination.

The Shockley-Read-Hall recombination rate becomes, in dimensionless form,

$$\tau_0 U/p_0=\Delta(1+\Delta)/(1+R\Delta)\,, \tag{20}$$

where $R\equiv ap_0$. The donor-acceptor pair recombination rate at low excitation becomes

$$\tau_0 U/p_0=\Delta\,, \tag{21}$$

where τ_0 is given by Eq. (10) for this case. At high excitation for a wide-band-gap semiconductor ($K\approx 0$), Eq. (11) yields

$$\tau_0 U/p_0=M\Delta(1+\Delta)/(1+A\Delta)(1+B\Delta)\,, \tag{22}$$

where $A\equiv p_0/n_1$, $B\equiv p_0/(p_0+p_1)$, and $M\equiv\tau_0 p_0 Q/n_1(p_0+p_1)$. For strongly p-type material B is of order unity and an adequate approximation is to put $(1+\Delta)/(1+B\Delta)\approx 1$. Saturation of U is then seen to occur when $A\Delta=\delta n/n_1\gg 1$. For this case

$$\tau_0 U/p_0=M/A\,. \tag{23}$$

Our problem is now to solve Eq. (19) with Eq. (20) or Eqs. (21) and (23). Equation (19) is badly nonlinear making a general solution impractical. However, Eq. (19) can be solved in several ranges of Δ and from these solutions the main physical predictions can be extracted. The boundary condition to be used at the space-charge layer edge for low to moderate forward biases corresponds to the quasi-Fermi levels being flat through the space-charge region.[2] Therefore

$$\Delta(x=0)\equiv\Delta_0=(n_i/p_0)^2\exp(eV/kT)\,, \tag{24}$$

where n_i is the intrinsic carrier concentration and V the applied voltage. When conductivity modulation ($\delta n\gtrsim p_0$) is reached the boundary condition becomes[2]

$$\Delta(x=0)\equiv\Delta_0=(n_i/p_0)\exp(eV/2kT)\,, \tag{25}$$

where V is then the applied potential minus the ohmic drop in the bulk crystal.

IV. SPACE AND VOLTAGE DEPENDENCES OF ELECTROLUMINESCENCE

A. Shockley-Read-Hall Recombination

We will now obtain approximate solutions of Eq. (19) in different ranges of Δ for the Shockley-Read-Hall recombination rate of Eq. (20). Three parameters appear: b, Γ, and R. In most semiconductors, b is larger than, but of order, unity. Among other parameters Γ contains the current density, which depends on voltage, as does the boundary condition parameter Δ_0. By expressing $J=J_0\exp eV/mkT$ where $1\leq m\leq 2$ and by using Eq. (24)

$$\begin{aligned} \Gamma=bJ_0\tau_0^{1/2}(\Delta_0 p_0^2/n_i^2)^{1/m}/ep_0D_e^{1/2} \\ \equiv\Gamma_0\exp(eV/mkT). \end{aligned} \tag{26}$$

If we consider a deep donor-like recombination center in the p-type material, R will be very large compared to unity.

Region 1. Linear Recombination, Diffusion Controlled

$\Delta_0\ll 1$ and $R\Delta_0\ll 1$. Equation (19) becomes simply

$$d^2\Delta/dl^2=\Delta \tag{27}$$

with a solution

$$\Delta=\Delta_0\exp(-l). \tag{28}$$

This gives the spatial distribution of the electroluminescence since in this limit U is proportional to Δ. Figure 1 illustrates the spatial distribution of electroluminescence in this and other regions. The total light emission from this carrier-diffusion region will be proportional to L where

$$L=\int_0^\infty U dl. \tag{29}$$

The p-side injection current will have the same dependence on V as L does. Hence, for this case

$$L=p_0\Delta_0/\tau_0=(n_i^2/p_0\tau_0)\exp eV/kT. \tag{30}$$

The total light emission is plotted versus voltage in Fig. 2.

The transition from region 1 to 2 will occur around a voltage V_{12} determined by $R\Delta_0=1$. This leads to

$$eV_{12}=E_g-kT\ln(N_vN_cR/p_0^2), \tag{31}$$

where E_g is the energy gap of the semiconductor.

Region 2. Saturated Recombination, Diffusion Controlled

$\Delta_0\ll 1$, $R\Delta_0\gg 1$, and $\Gamma(2R\Delta_0)^{1/2}\ll 1$. The last condition allows the neglect of the drift term containing Γ. The term in $(d\Delta/dl)^2$ is also negligible. Equation (19) becomes

$$d^2\Delta/dl^2=1/R, \tag{32}$$

with a solution

$$\Delta=l^2/2R+k_1l+\Delta_0, \tag{33}$$

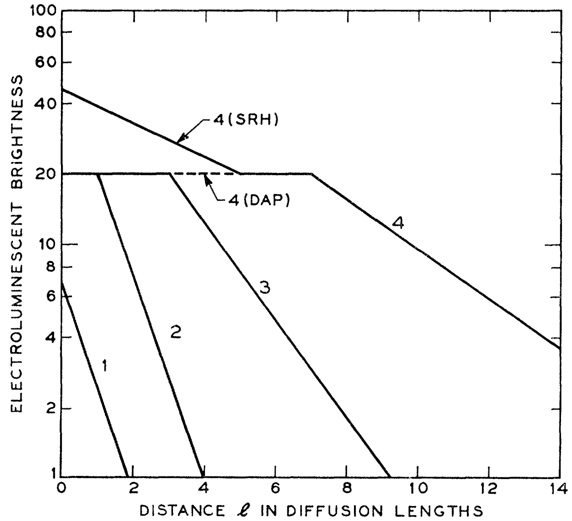

FIG. 1. Typical spatial distributions of electroluminescence from a deep level in regions 1 to 4 (see text) are plotted on a semilogarithmic scale. Shockley-Read-Hall (SRH) and donor-acceptor pair (DAP) recombination mechanisms give the same distributions in regions 1 to 3 but differ, as shown, in region 4. The sharp corners result from the approximate nature of the solutions.

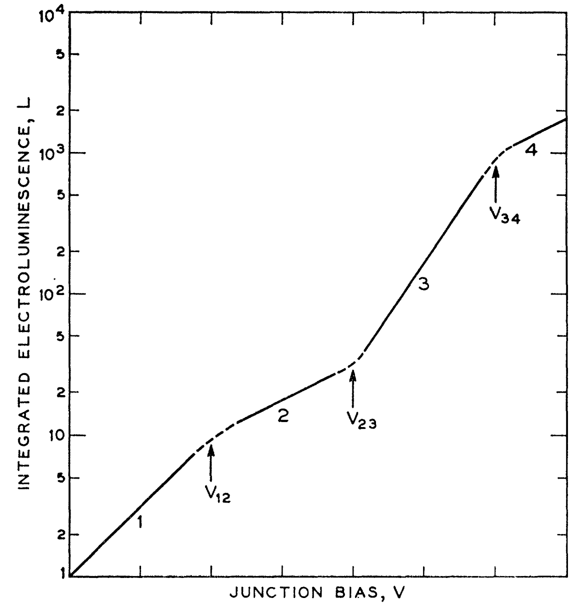

FIG. 2. The total electroluminescence (integrated over space) from a deep level is plotted versus junction bias V on a semilogarithmic scale for regions 1 to 4 (see text) for either Shockley-Read-Hall or donor-acceptor-pair recombination. The slope of $\ln L$ versus eV/kT is 1, $\frac{1}{2}$, $(m+1)/m$ and $\frac{1}{2}$ successively in regions 1 to 4 (m is defined by the current-voltage characteristic: $I=I_0\exp eV/mkT$).

k_1 being an arbitrary constant. This solution fails when $R\Delta\ll 1$, that is, for large l. For this region a solution of Eq. (27)

$$\Delta=\Delta_c\exp(l_c-l) \tag{34}$$

must be used. We can construct an approximate solution by extrapolating both Eqs. (33) and (34) to $l=l_c$, where $R\Delta=1$ [Δ_c of Eq. (34) becomes R^{-1}] and joining the functions and their derivatives continuously. This yields

$$l_c=(2R\Delta_0-1)^{1/2}-1\approx(2R\Delta_0)^{1/2}. \tag{35}$$

The spatial distribution of the electroluminescence in this approximation is constant (saturated) for $0<l<l_c$ and then falls exponentially for $l>l_c$. This distribution is plotted in Fig. 1. From Eqs. (35) and (24) we see that the width of this saturated region $l_c=[(2R)^{1/2}n_i/p_0]\times\exp eV/2kT$ expands rapidly with voltage. The total light output is

$$L=\frac{p_0}{\tau_0}\left\{\int_0^{l_c}\frac{dl}{R}+\int_{l_c}^{\infty}\Delta dl\right\}. \tag{36}$$

Therefore,

$$L=p_0(l_c+1)/R\tau_0\approx(2^{1/2}n_i/\tau_0R^{1/2})\exp eV/2kT, \tag{37}$$

which increases more slowly with voltage than L for region 1 does. This is shown in Fig. 2.

The transition from region 2 to 3 will occur around a voltage V_{23} determined by $\Gamma(2R\Delta_0)^{1/2}=1$. This leads to

$$eV_{23}=m/(m+2)[E_g+kT\ln(p_0^2/2\Gamma_0^2RN_cN_v)]. \quad (38)$$

Region 3. Saturated Recombination, Drift and Diffusion Controlled

$\Delta_0\ll1$, $R\Delta_0\gg1$ and $\Gamma(2R\Delta_0)^{1/2}\gg1$. For this case Eq. (19) becomes

$$\frac{d^2\Delta}{dl^2}-\Gamma\frac{d\Delta}{dl}=\frac{1}{R}, \quad (39)$$

with a solution of the form

$$\Delta=\Delta_0-l/R+k_2[\exp\Gamma l-1], \quad (40)$$

where k_2 is an arbitrary constant. Again we will extrapolate this solution to $l=l_c$ where $R\Delta=1$ and join it smoothly to a solution of

$$\frac{d^2\Delta}{dl^2}-\Gamma\frac{d\Delta}{dl}=\Delta. \quad (41)$$

The solution of this equation is

$$\Delta=R^{-1}\exp[P(l_c-l)], \quad (42)$$

where $P\equiv(\Gamma^2/4+1)^{1/2}-\Gamma/2$. Joining of the solutions leads to a transcendental equation for l_c of the form

$$\alpha-\Gamma l_c=\beta\exp(-\Gamma l_c), \quad (43)$$

where

$$\alpha\equiv\Gamma^2(R\Delta_0-1)+1-P\Gamma\approx\Gamma^2R\Delta_0\gg1$$

and

$$\beta\equiv1-P\Gamma\lesssim1.$$

An approximate solution of Eq. (43) is thus $l_c\approx\alpha/\Gamma$ or

$$l_c\approx R\Gamma\Delta_0=(R\Gamma_0n_i^2/p_0^2)\exp[eV(1+m^{-1})/kT]. \quad (44)$$

The spatial distribution of the electroluminescence for this case is again a constant for $0\leq l\leq l_c$ and an exponential fall-off for $l\geq l_c$ (see Fig. 1). Since $P<1$ this exponential fall is slower than for regions 1 and 2. Since $1\leq m\leq2$, the width of the saturated layer [Eq. (44)] increases faster with voltage than for region 2. The integrated light output is

$$L=p_0(l_c+P^{-1})/R\tau_0$$
$$\approx(\Gamma_0n_i^2/p_0\tau_0)\exp[eV(1+m^{-1})/kT]. \quad (45)$$

This also increases faster with voltage than in region 2 (see Fig. 2).

The transition from region 3 to 4 will occur around a voltage V_{34} determined by $\Delta_0=1$. This leads to

$$eV_{34}=E_g-kT\ln(N_cN_v/p_0^2). \quad (46)$$

Region 4. Linear Recombination, Conductivity Modulation

$\Delta_0\gg1$, $R\Delta_0\gg1$. We will assume the opposite side of the junction is more heavily doped than the p side considered here. Otherwise, conductivity modulation ($\delta n\gtrsim p_0$) on this side would not be reached. The drift terms in Eq. (19) are negligible because of the factors in the denominator. Equation (19) becomes

$$\frac{2}{b+1}\frac{d^2\Delta}{dl^2}=\frac{\Delta}{R}. \quad (47)$$

Its solution is of the form

$$\Delta=\Delta_0\exp(-Sl)+k_3[\exp(Sl)-\exp(-Sl)], \quad (48)$$

where $S\equiv[(b+1)/2R]^{1/2}\ll1$ and k_3 is an arbitrary constant. By the procedure used in the other regions we will join this solution smoothly to

$$\Delta=k_4-l/\Gamma R+k_5\exp\Gamma l, \quad (49)$$

which is the solution of Eq. (39), at $l=l_b$ where $\Delta=1$. In turn this solution must be joined smoothly to Eq. (42), which is the solution of Eq. (41), at $l=l_c$ where $R\Delta=1$. Two transcendental equations for l_c and l_b result from the continuity conditions. One is Eq. (43) with the approximate solution Eq. (44) except that l_c must be replaced by l_c-l_b and Δ_0 by 1. Hence, $l_c-l_b\approx\Gamma R$. The other equation is

$$\Gamma RS\{[1-\Delta_0\exp(-Sl_b)]\cosh Sl_b-\Delta_0\exp(-Sl_b)\}+1$$
$$=[l_c-l_b-\Gamma R+\Gamma]\{\exp[\Gamma(l_c-l_b)]-1\}^{-1}. \quad (50)$$

Since $\Gamma^2R\gg1$ and $\Gamma R\gg S^{-1}$ in this region, an approximate solution is $\cosh Sl_b\approx2^{-1}\exp Sl_b\approx\Delta_0$. With the use of Eqs. (25 and (26) (with $m=2$ as appropriate for conductivity modulation[2])

$$l_b=(\ln2n_i/p_0+eV/2kT)/S, \quad (51)$$

and

$$l_c=l_b+\Gamma_0R\exp^{eV/2kT}. \quad (52)$$

The spatial distribution of the electroluminescence in this excitation region decreases nearly exponentially [following Eq. (48)] from the space-charge-layer edge to $l=l_b$ (which increases linearly with voltage), is constant from $l=l_b$ to $l=l_c$ (which increases exponentially with voltage) and decreases exponentially for $l>l_c$. This is illustrated in Fig. 1. The integrated light output is found to be

$$L=(p_0/R\tau_0)[\Delta_0(1-\exp-Sl_b)/S$$
$$+2k_3(\cosh Sl_b-1)/S+l_c-l_b+P^{-1}]. \quad (53)$$

The terms containing $\exp-Sl_b$, k_3, and P^{-1} are negligible and so

$$L\approx(RS\tau_0)^{-1}(n_i+\Gamma_0Rp_0S)\exp eV/2kT, \quad (54)$$

which is shown in Fig. 2. In the high bias region the junction voltage V will be equal to the applied bias minus the ohmic drop in the crystal.

B. Donor-Acceptor Pair Recombination

Consider next donor-acceptor-pair recombination. At low excitation the use of Eq. (21) leads to a result identical to that of region 1 found above. Saturation of donor-acceptor-pair recombination, as expressed in Eq. (23), yields results analogous to those of regions 2 and 3 above but with R replaced by A/M. The predictions for region 4 differ somewhat, however, since the donor-acceptor-pair recombination remains saturated under conductivity modulation rather than again becoming linear in Δ as for Shockley-Read-Hall recombination.

Region 4. Alternate Saturated Recombination, Conductivity Modulation

$\Delta_0 \gg 1$, $A\Delta_0 \gg 1$. Equation (19) becomes

$$\frac{2}{b+1}\frac{d^2\Delta}{dl^2}=\frac{M}{A}, \tag{55}$$

with a solution

$$\Delta=(b+1)Ml^2/4A+k_6 l+\Delta_0. \tag{56}$$

With the procedure used before, this must be joined smoothly to Eq. (49) (with $R \rightarrow A/M$) at $l=l_b$ where $\Delta=1$. In turn, this solution must be joined smoothly to Eq. (42) (with $R \rightarrow A$) at $l=l_c$ where $A\Delta=1$. The joining conditions lead to an equation similar to Eq. (43) for l_c-l_b with the approximate solution $l_c-l_b \approx \Gamma A/M$ and to an equation for l_b which was an approximate solution $l_b \approx [4A\Delta_0/(b+1)M]^{1/2}$. The integrated light output with the use of Eqs. (25) and (26) becomes

$$L=p_0(Ml_c+1/P)/\tau_0 A \approx (\Gamma_0 p_0/\tau_0) \exp eV/2kT. \tag{57}$$

Thus, the dominant voltage dependence of the integrated light output is the same for donor-acceptor-pair recombination as for Shockley-Read-Hall recombination in the conductivity modulation region as well as in the other regions (see Fig. 2).

The spatial distribution, however, is not the same. The electroluminescent brightness is constant from the space-charge-layer edge out to l_c beyond which it diminishes exponentially with a rather slow fall-off constant. As before, the saturated region width $l_c \sim \exp^{eV/2kT}$. This is shown in Fig. 1.

C. Discussion

Since electroluminescence is often inefficient, it is of interest to consider two competing recombination mechanisms in Eq. (19), a dominant donor-like nonradiative Shockley-Read-Hall one and a donor-like radiative one like either of those considered above. This can be easily done by the approximate method used above and the same predictions result. The only difference is that the transition between regions 1 and 2 will usually be more extended since both recombination mechanisms must saturate in order for region 2 to be reached.

It should be noted that the existence of four separate regions depends upon the numerical values of the parameters in Eq. (19). For instance, if R were close to one, as for a very shallow recombination level, the condition necessary for the existence of region 2 would not be reached before that giving region 3 was reached. The results obtained above apply to a deep recombination center ($R \gg 1$), though the formalism could be modified to apply to shallow centers. Comparison of the predictions to experimental results obtained with gallium phosphide electroluminescent junctions will be made in the accompanying paper.[4]

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