Leptonic Decays of Hadrons

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The experimental data on leptonic decays of baryons have been re-examined in the light of a two-angle Cabibbo theory, where the two angles, θ_V and θ_A, are characteristic of the vector and axial-vector baryon currents, respectively. With certain assumptions about the energy dependence of the form factors in the vector (K_{e3}) and axial-vector ($K_{\mu 2},\pi_{\mu 2}$) decays of mesons, it can be shown that the angles θ_V and θ_A derived from baryon decays are compatible with the corresponding angles derived from meson decays, thus justifying our use of a two-angle theory. There is no discrepancy between the information from hyperon and meson decays and the information from the superallowed nuclear beta decays (O^{14}, Cl^{34}, etc.). Using the assumptions about the meson form factors, a fit of all data on leptonic decays of hadrons gives the values $\theta_V=0.212\pm0.004$, $\theta_A=0.268\pm0.001$, $\alpha=0.665\pm0.018$, where the parameter α defines the relative content of D coupling in the baryonic axial-vector current.

I. INTRODUCTION

THE extended notion of universality proposed by Cabibbo[1] made use of one angular parameter, θ, common to all leptonic interactions of hadrons. At the time of this proposal the available experimental information could be fitted roughly by choosing $\theta=0.26$, taking a suitable mixture of F and D coupling in the axial-vector baryon current.

However, it soon became evident[2] that the meson data, at least, required two angles differing by 10–20%. Such a difference might well be attributed to strong renormalization effects.[3] A detailed description of the leptonic baryon decays would then require two extra parameters (vector and axial-vector renormalization constants) for each pair of isomultiplets.[4] In view of the limited experimental information available, one should aim at a more economical parametrization.

We consider here a parametrization in terms of two angles, $\theta_V^{(B)}$ and $\theta_A^{(B)}$, for the vector and axial-vector baryon currents, respectively. In Sec. II we show that, to first order in the mass-splitting interaction, such a description may be possible. In Sec. III we show that the angles, $\theta_V^{(B)}$ and $\theta_A^{(B)}$, derived from the present experimental information on baryon decays are compatible with the corresponding angles, $\theta_V^{(M)}$ and $\theta_A^{(M)}$, derived from meson decays, when suitable assumptions about the energy dependence of the form factors in the meson currents are made. Section IV is devoted to various comments.

II. THE TWO-ANGLE THEORY

It is natural to assume that the leptonic interactions of hadrons (we consider only the usual octets of $\frac{1}{2}^+$ baryons and 0^- mesons) can be described in terms of one bare angle Θ in the absence of strong and electromagnetic interactions. The relevant effects of the inclusion of strong interactions[5] are changes in particle masses and in the form of the vertex functions, i.e., the occurrence of renormalization factors and new covariants.

Let the strong Hamiltonian be $H_s=H_0+H_8$, of which H_0 preserves the SU_3 symmetry, whereas H_8 is responsible for the splitting of masses according to the Gell-Mann–Okubo mass formula.[6] The part H_0 does not lead to a renormalization of the vector coupling constant but it may be partly responsible for induced terms in the meson and baryon currents, and for renormalization of the coupling strengths of the axial-vector currents. However, as the effects due to H_0 alone are common to all pairs of baryon or meson states (as members of a given SU_3 multiplet), it is evident that none of these effects requires a modification of the Cabibbo description. Hence, if such a modification really should be required it must be attributed to the term H_8.

An important theorem first proved by Ademollo and Gatto[7] states that for vanishing momentum transfer there are no first-order corrections (first-order in H_8) to the vector coupling constants, neither for $|\Delta S|=0$ nor for $|\Delta S|=1$ currents, although the $|\Delta S|=1$ vector

[1] N. Cabibbo, Phys. Rev. Letters 10, 531 (1963).
[2] N. Brene, B. Hellesen, and M. Roos, Phys. Letters 11, 344 (1964).
[3] J. Sakurai, Phys. Rev. Letters 12, 79 (1964).
[4] Actually we need further parameters for the induced terms.
[5] We shall completely disregard complications due to the electromagnetic interaction.
[6] M. Gell-Mann and Y. Ne'eman, *The Eightfold Way* (W. A. Benjamin, Inc., New York, 1964); S. Okubo, Progr. Theoret. Phys. (Kyoto) 27, 949 (1962).
[7] M. Ademollo and R. Gatto, Phys. Rev. Letters 13, 264 (1964).

current is not conserved in the presence of H_8. The theorem applies to meson-meson as well as to baryon-baryon vector currents.[8] Hence, the apparent angle θ_V, derived from observed $|\Delta S|=1$ vector currents, deviates from the bare angle Θ only to second or higher order in H_8. The usefulness of the Ademollo-Gatto theorem depends, of course, critically on the assumption that the second and higher order terms are small. We assume here that they are small indeed.

The proof of the Ademollo-Gatto theorem cannot be applied to the axial-vector current. Hence we may find first-order (in H_8) correction factors to the axial-vector coupling strengths. These correction factors might be different for all pairs of isospin multiplets. The success of Cabibbo's original fit[1] showed that the axial-vector currents obey SU_3 approximately: In all experimentally known baryon-baryon transitions the effective axial-vector coupling constant G_A, divided by the appropriate SU_3 Clebsch-Gordan coefficient and Cabibbo factor, has a value approximately equal to $(G_A)_N$, the value determined in the beta decay of the neutron. Thus, the correction factors due to H_8 are not very different from each other.

We shall here investigate the possibility that the set of all these correction factors can be accounted for essentially by the use of two correction factors, $\beta^{(0)}$ for $|\Delta S|=0$ transitions and $\beta^{(1)}$ for $|\Delta S|=1$ transitions. It is then natural to absorb the ratio $\beta^{(1)}/\beta^{(0)}$ in the definition of the observed axial-vector angle $\theta_A{}^{(B)}$ through the relation

$$\tan\theta_A{}^{(B)}=(\tan\Theta)\beta^{(1)}/\beta^{(0)}, \quad (1)$$

and use one common coupling constant, $G_A=(G_A)_N$, for all the leptonic interactions of baryons. The following considerations show that such a simplification of the situation is not inconceivable.[9]

Assuming that H_S is invariant under charge conjugation, the weak current can be expanded[10] to first order in H_8, as

$$\begin{aligned} a_0\,\mathrm{Tr}(\bar{B}B\lambda_j)&+b_0\,\mathrm{Tr}(\bar{B}\lambda_jB)+a\,\mathrm{Tr}(\bar{B}B\{\lambda_j,\lambda_8\}_+)\\ &+b\,\mathrm{Tr}(\bar{B}\{\lambda_j,\lambda_8\}_+B)+c\,\mathrm{Tr}(\bar{B}\lambda_jB\lambda_8)\\ &+d\,\mathrm{Tr}(\bar{B}\lambda_8B\lambda_j)+h[\mathrm{Tr}(\bar{B}\lambda_j)\,\mathrm{Tr}(B\lambda_8)\\ &+\mathrm{Tr}(\bar{B}\lambda_8)\,\mathrm{Tr}(B\lambda_j)]. \quad (2)\end{aligned}$$

In this expression all Dirac matrices are suppressed. Expressions of this kind are valid for first-class covariants: $\bar{u}\gamma_\lambda u$, $\bar{u}\sigma_{\lambda\kappa}q_\kappa u$, $\bar{u}\gamma_\lambda\gamma_5 u$, and $\bar{u}\gamma_5 q_\lambda u$.

Assume that the constants c, d, h are small compared with the largest of a_0, b_0, a, b:

$$\max[|c|,|d|,|h|]\ll\max[|a_0|,|b_0|,|a|,|b|]. \quad (3)$$

Then, the current (2) can be written as[11]

$$(a_0+ad_{j8j})\,\mathrm{Tr}(\bar{B}B\lambda_j)+(b_0+bd_{j8j})\,\mathrm{Tr}(\bar{B}\lambda_jB).$$

In the weak currents we use $j=1\pm i2$ ($\Delta S=0$) for which $d_{j8j}=2/\sqrt{3}$, and $j=4\pm i5$ ($|\Delta S|=1$) for which $d_{j8j}=-1/\sqrt{3}$. Hence, if the condition

$$a/b=a_0/b_0 \quad (4)$$

is fulfilled in addition to the condition (3), we can write

$$J_j=\beta^{(j)}(\alpha\,\mathrm{Tr}(\bar{B}\{\lambda_j,B\}_+)+(1-\alpha)\,\mathrm{Tr}(\bar{B}[\lambda_j,B]_-)),$$

where

$$\begin{aligned}\beta^{(j)}&=b_0+2b/\sqrt{3} \quad \text{for} \quad \Delta S=0,\\ &=b_0-b/\sqrt{3} \quad \text{for} \quad |\Delta S|=1, \quad (5)\end{aligned}$$

and

$$\alpha=\frac{a_0+b_0+2a/\sqrt{3}+2b/\sqrt{3}}{2(b_0+2b/\sqrt{3})}=\frac{a_0+b_0-a/\sqrt{3}-b/\sqrt{3}}{2(b_0-b/\sqrt{3})}.$$

Apart from the different $\beta^{(j)}$'s, which can be hidden in the experimental Cabibbo angle, this looks formally like a pure octet current.

In the next section we show that the present experimental data are consistent with a two-angle description of the baryon currents. This suggests that the conditions (3) and (4) are valid to a good approximation.

These considerations shed no light on the possibility of a relation between the ratio $\beta^{(1)}/\beta^{(0)}$ for baryon transitions and the corresponding ratio for meson decays ($K_{\mu2},\pi_{\mu2}$), i.e., between $\theta_A{}^{(B)}$ and $\theta_A{}^{(M)}$.

III. MATRIX ELEMENTS AND EXPERIMENTAL FITS

For the baryon-baryon matrix elements of the vector and axial-vector currents, we shall use expressions almost identical to those in Ref. 2:

$$\begin{aligned}\langle B|J_{V,\lambda}|A\rangle=T(\theta_V{}^{(B)},\Delta S)\bar{u}_B\Big[&f^{iBA}\mathfrak{F}_V(q^2)\gamma_\lambda\\ &+\{\alpha_M d^{iBA}+(1-\alpha_M)f^{iBA}\}\\ &\times\frac{\mu_n-\mu_p}{2M_N}\sigma_{\lambda\kappa}q_\kappa\Big]u_A, \quad (6)\end{aligned}$$

$$\begin{aligned}\langle B|J_{A,\lambda}|A\rangle=T(\theta_A{}^{(B)},\Delta S)&\\ \times\{\alpha d^{iBA}+(1-\alpha)f^{iBA}\}&\beta\mathfrak{F}_A(q^2)\\ \times\bar{u}_B\Big[\gamma_\lambda\gamma_5&-i\frac{M_A+M_B}{q^2+m^2}\gamma_5q_\lambda\Big]u_A, \quad (7)\end{aligned}$$

[8] The content of this theorem was somewhat clarified by C. Bouchiat and Ph. Meyer, Nuovo Cimento **34**, 1122 (1964), who showed that the first-order correction is accounted for by the use of the unrenormalized coupling constants and wave functions of the actual physical masses.

[9] Our point of view is contrary to that of M. Gourdin, Phys. Letters **18**, 82 (1965).

[10] We use the notation of Ref. 7 from which this expansion is borrowed.

[11] The coefficients d_{abc} are defined in M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).

TABLE I. Experimental data.

Decay	Branching ratio	(Total rate)$^{-1}$ (sec)	Comments
$K^+ \to \mu^+\nu$	$(63.5 \pm 0.7) \times 10^{-2}$	$(1.243 \pm 0.004) \times 10^{-8}$	a
$K^+ \to \pi^0 e^+\nu$	$(4.49 \pm 0.25) \times 10^{-2}$		a
$K_2^0 \to \pi^\pm e^\mp \nu$	$(37.4 \pm 2.1) \times 10^{-2}$	$(8.66 \pm 0.14) \times 10^{-11}$	a
$\pi^+ \to \mu^+\nu$	1	$(2.601 \pm 0.002) \times 10^{-8}$	b
$\Lambda \to p e^-\nu$	$(0.88 \pm 0.08) \times 10^{-3}$	$(2.61 \pm 0.02) \times 10^{-10}$	c
$\Lambda \to p\mu^-\nu$	$(1.35 \pm 0.6) \times 10^{-4}$		d
$\Sigma^- \to n e^-\nu$	$(1.28 \pm 0.16) \times 10^{-3}$		e
$\Sigma^- \to n\mu^-\nu$	$(0.62 \pm 0.12) \times 10^{-3}$	$(1.65 \pm 0.02) \times 10^{-10}$	f
$\Sigma^- \to \Lambda e^-\nu$	$(0.75 \pm 0.28) \times 10^{-4}$		g
$\Sigma^+ \to \Lambda e^+\nu$	$(0.13 \pm 0.13) \times 10^{-4}$	$(0.81 \pm 0.01) \times 10^{-10}$	h
$\Xi^- \to \Lambda e^-\nu$	$(1.2 + 0.8) \times 10^{-3}$	$(1.75 \pm 0.05) \times 10^{-10}$	i

[a] Reference 19. [b] Reference 20. [c] Reference 14.
[d] World average based on information in Ref. 22 because Ref. 14 is erroneous.
[e] The branching ratio is a weighted mean of the four experiments quoted in Ref. 14. The lifetime is a weighted mean of the values in Refs. 14 and 23.
[f] Weighted mean of the values in Refs. 24 and 21.
[g] Reference 21.
[h] The branching ratio is evaluated from Ref. 21 which gives $\Gamma(\Sigma^+ \to \Lambda e^+\nu)/\Gamma(\Sigma^+ \to n\pi^+) \approx 0.3 \times 10^{-4}$. The lifetime is a weighted mean of the values in Refs. 14, 23, and 25.
[i] Since three events have been seen (Ref. 26), of which one is questionable, we have taken 2.5 ± 0.5 events over a denominator of 1225 (Ref. 26) +900 (Ref. 27) nonleptonic events. The lifetime is from Ref. 14.

where

$$
\begin{aligned}
T(\theta,\Delta S) &= \cos\theta\,, \quad \text{for} \quad \Delta S = 0\,,\\
&= \sin\theta\,, \quad \text{for} \quad |\Delta S| = 1\,,\\
\mathfrak{F}_{V,A}(q^2) &= 1 - \tfrac{1}{6}\langle r_{V,A}^2\rangle q^2\,,\\
q_\lambda &= (P_B - P_A)_\lambda\,,\\
\alpha_M &= \tfrac{3}{2}[1/(1-\mu_p/\mu_n)]\,,\\
m &= m_\pi \quad \text{for} \quad \Delta S = 0\,,\\
&= m_K \quad \text{for} \quad |\Delta S| = 1\,,
\end{aligned}
$$

and μ_n and μ_p are the anomalous magnetic moments of the nucleons.

As the effects of the correction terms (momentum dependence, weak magnetism, and induced pseudoscalar term) are small, of the order of a few percent, we shall not discuss whether these terms are changed in exactly the same ratio as the corresponding large terms, i.e., whether it is correct to use the same factor $T(\theta_V,\Delta S)$ $[T(\theta_A,\Delta S)]$ for all vector (axial-vector) terms.

Second-class currents[12] (vector term proportional to q_λ, axial-vector term proportional to $\sigma_{\lambda\kappa}q_\kappa\gamma_5$) are omitted. They vanish in the symmetric limit[13] (assuming that H_s is invariant under charge conjugation), but their effects are not necessarily small compared with those of the well established magnetic and pseudoscalar terms. However, in view of the uncertainty of the fit of the data on hyperon decays (Fig. 1) we would not be able to decide whether reasonably small second-class terms are present or not. The situation here corresponds closely to that of the meson currents. From the data on K_{l3} decay $[\Gamma(K_{\mu3})/\Gamma(K_{e3})$ and muon polarization$]$ it can only be concluded that the second-class form factor, $f_-(q^2)$, is small compared with the first-class form factor $f_+(q^2)$.

[12] W. Drechsler, Nuovo Cimento **38**, 345 (1965).
[13] L. Wolfenstein, Phys. Rev. **135**, B1436 (1964).

In our fit of the experimental data we have used the following numerical values:

$$
\begin{aligned}
G^2/2 &= 0.6755 \times 10^{-22}\ \text{MeV}^{-4}\,,^{14,15}\\
\beta &= 1.18 \pm 0.025 \quad (\pm 1\%\ \text{unknown corrections}^{16})\,,\\
\sqrt{\langle r_V^2\rangle} &= (0.80 \pm 0.01)\ \text{F}\,,^{17}\\
\sqrt{\langle r_A^2\rangle} &= (0.88_{-0.15}^{+0.30})\ \text{F}\,,^{18}\\
\mu_p &= 1.7928\,,^{14}\\
\mu_n &= -1.9131\,.^{14}
\end{aligned}
\qquad (8)
$$

The experimental information on hyperon decays consists of the leptonic branching ratios and inverse total rates (lifetimes) given in Table I,[14,19–27] and in

[14] A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirz, and M. Roos, Rev. Mod. Phys. **37**, 633 (1965).
[15] Derived from the muon lifetime, Ref. 14, using the radiative correction of T. Kinoshita and A. Sirlin, Phys. Rev. **113**, 1652 (1959).
[16] C. P. Bhalla, Phys. Letters **19**, 691 (1966).
[17] R. J. Wilson and J. S. Levinger, Ann. Rev. Nucl. Sci. **14**, 135 (1964).
[18] M. Paty, CERN, Report 65-12, 1965 (unpublished).
[19] G. H. Trilling, University of California Technical Report No. UCRL 16473, 1965 (unpublished); and Proceedings of the International Conference on Weak Interactions, Argonne National Laboratory, Report No. ANL-7130, 1965 (unpublished).
[20] M. Eckhause, R. J. Harris, Jr., W. B. Shuler, R. T. Siegel, and R. E. Welsh, Phys. Letters **19**, 348 (1965).
[21] H. Courant, H. Filthuth, P. Franzini, A. Minguzzi-Ranzi, A. Segar, R. Engelmann, V. Hepp, E. Kluge, R. A. Burnstein, T. B. Day, R. G. Glasser, A. J. Herz, B. Kehoe, B. Sechi-Zorn, N. Seeman, G. A. Snow, and W. Willis, Phys. Rev. **136**, B1791 (1964).
[22] B. Ronne, C. Baglin, J. Six, W. L. Knight, F. R. Stannard, and A. Haatuft, Phys. Letters **11**, 357 (1964).
[23] Chung-Yan Chang, thesis, Columbia University, New York Nevis Report No. 145, 1965 (unpublished).
[24] M. Bazin, H. Blumenfeld, U. Nauenberg, L. Seidlitz, R. J. Plano, S. Marateck, and P. Schmidt, Phys. Rev. **140**, B1358 (1965).
[25] N. L. Carayannopoulos, G. W. Tautfest, and R. B. Willmann, Phys. Rev. **138**, B433 (1965).
[26] H. H. Bingham, Proc. Roy. Soc. (London) **A285**, 202 (1965).
[27] J. R. Hubbard, J. P. Berge, G. R. Kalbfleisch, J. B. Schafer, F. T. Schmitz, M. L. Stevenson, S. G. Wojcicki, and P. G. Wohlmut, Phys. Rev. **135**, B183 (1964).

addition the ratio

$$(G_A/G_V)_\Lambda = -1.14_{-0.33}^{+0.23}$$

deduced[28] from four experiments on the angular distributions of the decay products from polarized Λ hyperons.

Defining $\chi_\alpha^2(x,y)$ through

$$\chi_\alpha^2(x,y)=\sum_j \left[\frac{b_j-\bar{b}_j(x,y,\alpha)}{\sigma_j}\right]^2,$$

$$x=\sin^2\theta_V^{(B)},$$

$$y=\sin^2\theta_A^{(B)},$$

where b_j and σ_j represent the mean value and standard deviation (s.d.) for an experimental branching ratio and $\bar{b}_j(x,y,\alpha)$ the corresponding expression deduced from the currents (6) and (7), we determine the set of values (x_α,y_α) which minimize χ_α^2 for a given value of α.

The points (x_α,y_α) form a curve[29] shown in Fig. 1. The small value of χ_α^2 in the interval $0.65\leqslant\alpha\leqslant 0.70$, $\chi_\alpha^2 \lesssim 3$ for 5 degrees of freedom, gives support to the form (6), (7) of the baryon-baryon matrix elements.

The information contained in the curve of Fig. 1 can be compared to the information obtained from nuclear beta decay and from meson decays.

The vector coupling constant G_V for the superallowed $0^+\rightarrow 0^+$ beta decays must fulfil the relation

$$\frac{G^2-G_V^2}{G^2}=\sin^2\theta_V^{(B)}.$$

Using the values given by Freeman *et al.*,[30] we obtain

$$\sin^2\theta_V^{(\beta)}=0.0436\pm 0.0025. \qquad (9a)$$

From the $K_{\mu 2}$ and $\pi_{\mu 2}$ data, one derives immediately

$$\sin^2\theta_A^{(M)}=0.0699\pm 0.0009, \qquad (10)$$

neglecting the possibility that the form factor might take different values at $q^2=-m_\pi^2$ and $q^2=-m_K^2$, corresponding to π decay and K decay, respectively.

For the determination of the angle $\theta_V^{(M)}$ we have chosen not to use the information on $K_{\mu 3}$ decay as a reliable theoretical calculation of the ratio $\xi(q^2)=f_-(q^2)/f_+(q^2)$ is lacking.

[28] G. Conforto, Ecole Internationale de la Physique des Particules Elémentaires, Herzeg Novi, 1965 (unpublished lecture notes). We are grateful to Dr. G. Conforto, CERN, for informing us about this result prior to publication. The sign of $(G_A/G_V)_\Lambda$ is such as to favor equal sign of $\theta_V^{(B)}$ and $\theta_A^{(B)}$, corresponding to equal sign of $V_{\rm ch}$ and A of Table II.

[29] By definition, x and y lie in the interval (0,1). Hence, acceptable solutions are found only when α lies inside certain intervals. Of these only the interval $0.20\leqslant\alpha\leqslant 0.70$ gives reasonably low values of χ^2.

[30] The average of the ft-values for O^{14}, Cl^{34}, Sc^{42}, V^{46}, Mn^{50}, Co^{54} reported in J. M. Freeman, J. H. Montague, G. Murray, R. E. White, and W. E. Burcham, Phys. Letters **8**, 115 (1964); and J. M. Freeman, G. Murray, and W. E. Burcham, *ibid.* **17**, 317 (1965) agrees excellently with the average ft-value used by C. S. Wu [Rev. Mod. Phys. **36**, 618 (1964)].

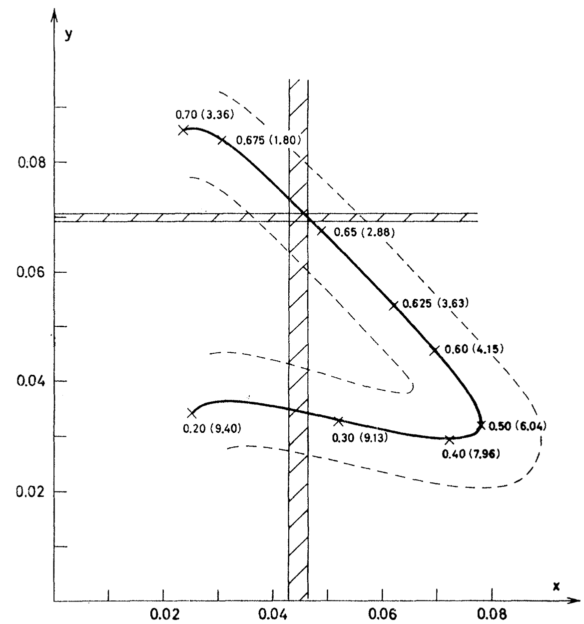


FIG. 1. The values of (x,y) which minimize χ^2 for a given value of α. Corresponding values of α and χ^2 (in parentheses) are indicated along the curve. The dashed curves represent 1 standard deviation. The vertical and horizontal bands represent the 1-standard-deviation regions around the values of x and y derived from (11) and (10), respectively.

From the K_{e3}^+ and K_{e3}^0 data we obtain (again neglecting a possible energy dependence of the form factor)

$$\sin^2\theta_V^{(M)}=0.0493\pm 0.0028,$$

and

$$\sin^2\theta_V^{(M)}=0.0501\pm 0.0038,$$

respectively. The mean value is

$$\sin^2\theta_V^{(M)}=0.0496\pm 0.0022. \qquad (9b)$$

If an energy dependence of the form factor is introduced,[31] corresponding to the mass $M=M(K^*)$ of an intermediate state, the value (9b) is reduced 10%. It is interesting that this correction leads to a value very near the value (9a), although the difference between (9a) and (9b) might be purely statistical (1.8 s.d.). From (9a) and (9b), using an intermediate mass $M=M(K^*)$ for K_{e3} decay, we obtain

$$\sin^2\theta_V=0.0442\pm 0.0015. \qquad (11)$$

In Fig. 1 we have drawn bands corresponding to the values (10) and (11). Note that these bands cross each other well inside the region favored by the hyperon decays. This fact seems to exclude a strong energy dependence of the form factor in the axial-vector-meson current. If we try to introduce an energy dependence corresponding to a mass of 1 BeV, say, in the axial-vector-meson form factor, the value (10) is reduced

[31] S. Oneda and J. Sucher, Phys. Rev. Letters **15**, 927 (1965); **15**, 1049 (1965).

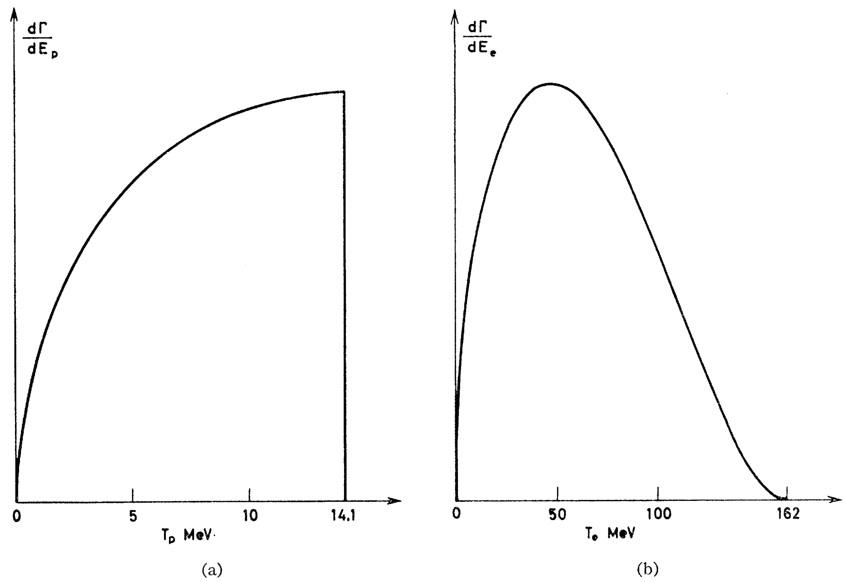


FIG. 2. Predicted proton (a) and electron (b) spectra for the decay mode $\Lambda \rightarrow p + e^- + \bar{\nu}$.

about 40%. Truly this brings the corrected axial-vector-meson band into the lower part of the region partially favored by hyperon decays but the larger value of χ_α^2 here, larger than 9 for 5 degrees of freedom, makes such a solution unlikely.

Thus, apparently we must face the following rather unsatisfactory situation: In the baryon current the vector part and the axial-vector part have roughly the same energy dependence. In the meson current the vector part has roughly the same energy dependence as the baryon current, whereas the axial-vector part is fairly independent of energy.

If we use the values (8) and make the following assumptions:

(a) The use of the Ademollo-Gatto theorem makes sense;
(b) The energy dependence of the form factor $f_+(q^2)$ in K_{e3} decay corresponds to the mass of K^*;
(c) $\theta_A^{(B)} = \theta_A^{(M)}$ at vanishing momentum transfer;
(d) The energy dependence of the form factor in the axial-vector mesonic current is negligible,

then we can use all the data in a determination of θ_V, θ_A, and α (7 degrees of freedom).
The result is

$$\begin{aligned} \sin^2\theta_V &= 0.0442 \pm 0.0015\,, & \sin^2\theta_A &= 0.0700 \pm 0.0007\,, \\ \theta_V &= 0.212 \pm 0.004\,, & \theta_A &= 0.268 \pm 0.001\,, \quad (12) \\ \alpha &= 0.665 \pm 0.018\,, & \chi^2 &= 2.53\,. \end{aligned}$$

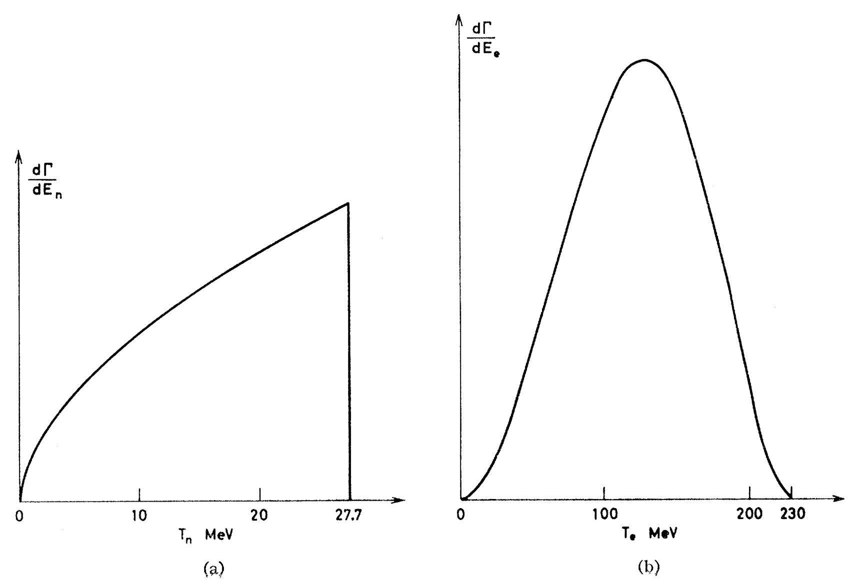
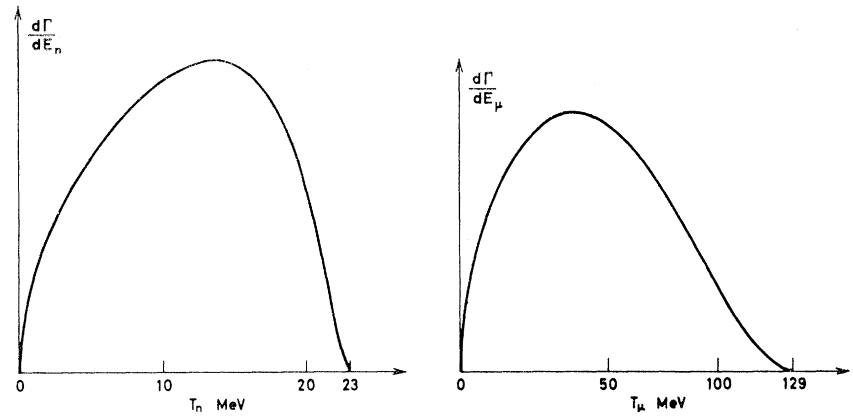


FIG. 3. Predicted neutron (a) and electron (b) spectra for the decay mode $\Sigma^- \rightarrow n + e^- + \bar{\nu}$.



(a) (b)

FIG. 4. Predicted neutron (a) and muon (b) spectra for the decay mode $\Sigma^- \rightarrow n + \mu^- + \bar{\nu}$.

We have used the solution (12) to predict the relative coupling constants in the baryon current, the leptonic branching ratios for baryons, the baryon and lepton spectra, Figs. 2–4, and the angular parameters, α_{AX}, for decays of polarized baryons (Table II). The relative coupling constants are defined through

$$\langle B|J_\lambda|A\rangle = \bar{u}_B\Bigg[V_{\rm ch}(q^2)\gamma_\lambda + V_{\rm magn}(q^2)\frac{\mu_n-\mu_p}{2M_N}\sigma_{\lambda\kappa}q_\kappa + A(q^2)\left(\gamma_\lambda\gamma_5 - i\frac{M_A+M_B}{m^2+q^2}q_\lambda\gamma_5\right)\Bigg]u_A\,, \quad (13)$$

and the parameter α_{AX} is defined through the angular distribution

$$W(\theta_{A,X}) = 1 + P_A\alpha_{AX}\cos\theta_{A,X}\,, \quad (14)$$

where $\theta_{A,X}$ is the angle between the spin of the decaying particle A and the direction of emission of the particle X, and P_A is the degree of polarization of the particle A.

IV. COMMENTS

In the determination of $\sin^2\theta_V^{(M)}$, we used data from K decays only. In principle we could also use the branching ratio

$$R = \frac{\Gamma(\pi^+ \rightarrow \pi^0 + e^+ + \nu)}{\Gamma(\pi^+ \rightarrow \mu^+ + \nu)}\,.$$

However, this information is still too inaccurate to be useful. The experimental value is[14,32]

$$R = (1.025 \pm 0.075) \times 10^{-8}\,,$$

and the theoretical value,[32] including radiative corrections and corrections for π^0 recoil and electron mass, is

$$R = 1.075 \times 10^{-8}\cos^2\theta_V^{(M)}\,.$$

This gives

$$\sin^2\theta_V^{(M)} = 0.047 \pm 0.070\,.$$

[32] K. Kleinknecht (private communication).

Our analysis suggests that the form factor in the axial-vector-meson current has the same value in $K_{\mu 2}$ and $\pi_{\mu 2}$ decay. Starting from the assumption that the form factors are all unity [apart from the factor $T(\theta,\Delta S)$ and the SU_3 coefficient] for vanishing momentum transfer, there are two possible interpretations (at least) of this result:

(a) The $K_{\mu 2}$ and $\pi_{\mu 2}$ form factors have an energy dependence corresponding to the masses M_1 and M_0, respectively, of the relevant intermediate system, such that $M_1/M_0 \approx m_K/m_\pi$. This is a rather strong requirement on the masses of the $(\mathrm{I},|Y|) = (\frac{1}{2},1)$ and (1,0) axial-vector mesons, if such mesons exist at all.

(b) The $K_{\mu 2}$ and $\pi_{\mu 2}$ form factors have an energy dependence corresponding to the same mass M, but the renormalization ratio $(\beta^{(1)}/\beta^{(0)})_B$, for baryons, and the corresponding ratio $(\beta^{(1)}/\beta^{(0)})_M$ for mesons, fulfill the relation

$$(\beta^{(1)}/\beta^{(0)})_B \approx (\beta^{(1)}/\beta^{(0)})_M \frac{M^2 - m_\pi^2}{M^2 - m_K^2}\,.$$

In a recent publication by Nieh[33] a model is considered which leads to relations between $\theta_A^{(M)}$, $\theta_A^{(B)}$, and θ_V $(=\theta_V^{(M)} = \theta_V^{(B)})$:

$$\tan\theta_A^{(B)} = (1+\alpha)^{-1/2}\tan\theta_A^{(M)}\,,$$

$$\tan\theta_A^{(B)} = (1+\alpha)^{-1}\tan\theta_V\,,$$

where α is a parameter[34] depending on the decay rates of the vector mesons ρ and K^*. The two relations can be combined into

$$\tan\theta_V \tan\theta_A^{(B)} = \tan^2\theta_A^{(M)}\,.$$

[33] H. T. Nieh, Phys. Rev. Letters **15**, 902 (1965).

[34] Nieh gives the value

$$(1+\alpha) = 0.76_{-0.11}^{+0.17}.$$

However, the errors are only statistical. When also systematic errors (for scale factors see Ref. 14) of the decay rates are included, we obtain

$$(1+\alpha) = 0.99_{-0.21}^{+0.56}.$$

TABLE II. Predicted relative coupling constants, branching ratios, and angular parameters.

Decay	Relative coupling constants $V_{\text{ch}}(0)$	$V_{\text{magn}}(0)$	$A(0)$	Branching ratio	Angular parameters α_{AB}	α_{Al}	$\alpha_{A\nu}$
$n\to pe^-\nu$	0.978	0.978	1.138 ± 0.026	1	-0.50	-0.06	0.99
$\Lambda\to pe^-\nu$	-0.258	-0.125	-0.213	$(0.87\pm0.03)\times10^{-3}$	-0.54	0.06	0.99
$\Lambda\to p\mu^-\nu$	±0.005	±0.003	±0.007	$(1.48\pm0.05)\times10^{-4}$	-0.41	-0.08	0.99
$\Sigma^-\to ne^-\nu$	-0.211	$+0.116$	$+0.103$	$(1.29\pm0.13)\times10^{-3}$	0.77	-0.81	-0.29
$\Sigma^-\to n\mu^-\nu$	±0.004	±0.002	±0.022	$(0.62\pm0.06)\times10^{-3}$	0.70	-0.63	-0.31
$\Sigma^-\to\Lambda e^-\nu$	0	$+0.618$	$+0.618$	$(0.70\pm0.04)\times10^{-4}$	0.09	-0.71	$+0.64$
$\Sigma^+\to\Lambda e^+\nu$		±0.001	±0.022	$(0.21\pm0.01)\times10^{-4}$	-0.08	0.70	-0.64
$\Xi^-\to\Lambda e^-\nu$	$+0.258$	-0.0084	$+0.043$	$(0.43\pm0.03)\times10^{-3}$	-0.37	0.19	0.40
$\Xi^-\to\Lambda\mu^-\nu$	±0.005	±0.0003	±0.005	$(0.12\pm0.01)\times10^{-3}$	-0.31	0.12	0.39
$\Xi^0\to\Sigma^+e^-\nu$	$+0.211$	$+0.211$	$+0.312$	$(0.31\pm0.04)\times10^{-3}$	-0.38	-0.28	0.96
$\Xi^0\to\Sigma^+\mu^-\nu$	±0.004	±0.004	±0.007	$(0.24\pm0.04)\times10^{-5}$	-0.18	-0.13	0.97

Using the values (10) and (11), this relation gives us the value

$$\sin^2\theta_A{}^{(B)}=0.108\pm0.005\,,$$

which is several standard deviations outside the region allowed by the experimental results.

The induced pseudoscalar term contributes 2–3% to the axial-vector part of the rate for decays involving the emission of a muon. However, with the data used here, the influence of the pseudoscalar term on the values (12) is negligible (a fraction of 1%). Hence, a possible break-down of the Goldberger-Treiman relation involving strangeness-changing currents[35] will not affect the values (12), although some of the results given in Table II and in Figs. 2–4 might be changed by a few percent.

[35] C. Kacser, P. Singer, and T. N. Truong, Phys. Rev. **137**, B1605 (1965); **139**, AB5(*E*) (1965).

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Errata

Sum Rules for the Axial-Vector Coupling Constant Renormalization in β Decay, STEPHEN L. ADLER [Phys. Rev. **140**, B736 (1965)]. In Eqs. (73) and (77), the coefficient of the isospin-2 cross section $\sigma_\pi{}^{t,2}$ should be $\frac{5}{3}$ rather than $\frac{2}{3}$. None of the conclusions of Sec. IV is changed. I wish to thank Dr. A. N. Kamal for pointing out this error.

General $SU(3)$ Crossing Matrices and the Projection Operators of 3×8, M. M. NIETO [Phys. Rev. **140**, B434 (1965)]. The following misprints should be corrected:

The second of the equations labeled (3.12) is (3.13).

The first $\frac{5}{4}$ in Eq. (4.9) should be $\frac{5}{2}$.

The first $\frac{5}{16}$ in Eq. (4.11) should be changed to $\frac{5}{8}$ so that it reads

$$(P_{15})_{\alpha\beta;ij}=\tfrac{5}{8}\delta_{\alpha\beta\gamma}\delta^i{}_j+\tfrac{3}{16}d_{\alpha\beta\gamma}\lambda^{(\gamma)i}{}_j-\tfrac{5}{16}if_{\alpha\beta\gamma}\lambda^{(\gamma)i}{}_j. \quad (4.11)$$

In (4.12) the subscript m should be j and the $-\frac{1}{8}$ should be $+\frac{1}{8}$ so that it reads

$$(P_{6^*})_{\alpha\beta;ij}=\tfrac{1}{4}\delta_{\alpha\beta\gamma}\delta^i{}_j-\tfrac{3}{8}d_{\alpha\beta\gamma}\lambda^{(\gamma)i}{}_j+\tfrac{1}{8}if_{\alpha\beta\gamma}\lambda^{(\gamma)i}{}_j. \quad (4.12)$$

Note that with these corrections the projection operators satisfy the relation

$$(P_3)_{\alpha\beta;ij}+(P_{6^*})_{\alpha\beta;ij}+(P_{15})_{\alpha\beta;ij}=\delta_{\alpha\beta}\delta^i{}_j,$$

as they should.