Connection between *BBP* and *VPP* Vertices in the Quark Model

C. BECCHI AND G. MORPURGO
Istituto di Fisica dell'Università, Genova, Italy
and
Istituto Nazionale di Fisica Nucleare, Sezione di Genova, Genova, Italy
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It is shown that in the quark model it is possible to express the $N_{33}^* N\pi$ vertex and the $\rho\pi\pi$ vertex in terms of the axial-vector quark-pion coupling constant. By expressing then this constant in terms of the $NN\pi$ coupling constant f, results in fair agreement with the experimental data are obtained for the above vertices. A detailed comparison of this calculation with that of Gürsey, Pais, and Radicati is made, illustrating the different points of view and the different assumptions of the two procedures.

1. INTRODUCTION

IN previous papers we have applied the nonrelativistic quark model[1] to the calculation of some electromagnetic processes.[2,3] In particular, in Ref. 2 the rates of the radiative decays of the vector mesons have been calculated, and good agreement with experiment has been obtained.

The purpose of this paper, which we hope to expand in the future is to apply the same model and the same methods to the calculation of some strong-interaction vertices: more precisely, to connect the $NN\pi$, $N^*N\pi$, and $\rho\pi\pi$ vertices.

It should be remarked that this has already been discussed by Gürsey, Pais, and Radicati[4] in one of the first papers on SU_6, but, as will be apparent from the following, our point of view is substantially different from that of the authors mentioned above. Also, the results will be somewhat different, in spite of the apparent similarity.

2. STATEMENT OF THE PROBLEM

In this paper we shall confine ourselves to the three vertices ($NN\pi$, $N^*N\pi$, and $\rho\pi\pi$) listed above, purely for the sake of simplicity, although, of course, the same arguments can be used to connect all the *BBP* and *VPP* vertices, where B is a baryon, P is a pseudoscalar meson, and V is a vector meson. Briefly, our argument is as follows.

Consider the quark–quark–pseudoscalar-meson interaction. This has the form, in the static limit, $\bar{q}^\alpha \boldsymbol{\sigma} q_\beta \cdot \nabla P_\alpha{}^\beta$. The part of this interaction which refers to the pions is, more explicitly,

$$H_{q\pi}=\sum_i \sqrt{2}(f_q/\mu)(\tau_i^- \boldsymbol{\sigma}_i\cdot\nabla_i\Phi^\dagger(\mathbf{x}_i)+\text{H.c.})\,, \qquad (1)$$

where the sum refers to the quarks which are present, τ_i^- transforms a "p" quark into an "n" quark, $\boldsymbol{\sigma}_i$ is the spin of the ith quark, $\sqrt{2}f_q$ is the $qq\pi$ constant to be determined, and μ (inserted to make f_q dimensionless) is a mass which is chosen to have the numerical value of the pion mass. We have omitted, for simplicity, the neutral-pion part.

On the other hand, the conventional pion-nucleon interaction is

$$H_{N\pi}=\sqrt{2}(f/\mu)\tau^-\boldsymbol{\sigma}\cdot\nabla_{\mathbf{X}}\Phi^\dagger(\mathbf{X})+\text{H.c.}\,, \qquad (2)$$

where $f^2/4\pi=0.08$.

Now, if the nucleon is an object composed of three quarks with a wave function having the structure described in Ref. 1, the pion-nucleon vertex can be expressed in terms of the pion-quark vertex.

Neglecting the difference between the position $\mathbf{x}_i$ of the ith quark in the nucleon and the position $\mathbf{X}$ of the center of mass of the nucleon, one obtains in this way

$$f=f_q\langle W_p{\uparrow}|\sum_{i=1}^{3}\tau_i^+\sigma_{iz}|W_n{\uparrow}\rangle\,, \qquad (3)$$

where W_p and W_n are the proton- and neutron-spin–unitary-spin wave functions already given in Ref. 1. Because the matrix element in (3) is simply 5/3 we obtain

$$f_q=\tfrac{3}{5}f\,, \qquad (4)$$

a relation expressing the pseudovector quark-pion coupling constant in terms of the known pion-nucleon coupling constant.

We shall now use the pion interaction (1) with f_q determined in the way shown above to calculate the $N^*N\pi$ vertex and the $\rho\pi\pi$ vertex.

3. THE $N^*N\pi$ VERTEX

The calculation of the matrix element for the $N_{33}^*\to N+\pi$ transition is straightforward. It is of course convenient to calculate the matrix element for the decay $N_{33}^{*++}\to p+\pi^+$ with the N_{33}^* and p having values of the Z component of the spin, respectively $\frac{3}{2}$ and $\frac{1}{2}$.

This matrix element is

$$\mathfrak{M}_{\uparrow\uparrow}=\langle N^{*++}{\uparrow}|\sqrt{2}(f_q/\mu)\sum_i\boldsymbol{\sigma}_i\tau_i^+|p{\uparrow}\rangle\cdot\nabla\Phi(\mathbf{X})\,, \quad (5)$$

where we have again neglected the difference between the position $\mathbf{X}$ of the center of mass and the position $\mathbf{x}_i$ of the ith quark.

We obtain (noting that $\alpha_1\alpha_2\alpha_3p_1p_2p_3$ is the spin–

[1] G. Morpurgo, Physics 2, 95 (1965).
[2] C. Becchi and G. Morpurgo, Phys. Rev. 140, B687 (1965).
[3] C. Becchi and G. Morpurgo, Phys. Letters 17, 352 (1965).
[4] F. Gürsey, A. Pais, and L. A. Radicati, Phys. Rev. Letters 13, 415 (1964).

unitary-spin wave function of the $N^{*++}{}_{\uparrow}$ and using again the spin–unitary-spin wave function of the proton given in Ref. 1):

$$\mathfrak{M}_{\uparrow\uparrow}=-\frac{6\sqrt{2}}{\sqrt{18}}\frac{f_q}{\mu}\frac{p_x-ip_y}{\sqrt{(2\omega_p)}}, \tag{6}$$

where p and ω_p are the pion momentum and energy. From (6) the total decay rate of the N_{33}^* is calculated immediately as

$$\mathrm{Rate}(N_{33}^*\to N+\pi)=\frac{4}{3\pi}\frac{f_q^2}{\mu^2}p^3\frac{M_p}{M^*}=\frac{48}{25}\frac{f^2}{4\pi}\frac{p^3}{\mu^2}\frac{M_p}{M^*}, \tag{7}$$

where M_p is the proton mass and M^* is the N_{33}^* mass. We get for the width of the N_{33}^* resonance 80 MeV, to be compared with its experimental value of $\sim$100 MeV.[5] In Ref. 4 the same result is obtained apart from the substitution of M_p^{-2} with M_{00}^{-2}, where M_{00} is the central baryon mass.

4. THE $\rho\pi\pi$ VERTEX

Our calculation of the $\rho\pi\pi$ decay will be, admittedly, much more open to criticism than that of the $N_{33}^*\to N+\pi$ decay presented above. We are well aware of this; but to be as clear as possible we criticize the weak points of the calculation at the end, and proceed first ignoring these points.

The idea is the following. Equation (1) gives us the amplitude for a quark to flip its spin and emit a pion; now the transition $\rho\to\pi+\pi$ is precisely a transition between a triplet $q\bar{q}$ state (ρ) into a singlet $q\bar{q}$ state (π) plus a pion.

What we shall do will be to use the static amplitude (1) [where f_q is given by Eq. (4)] to calculate this transition. We shall fix our attention on the decay $\rho^0\to\pi^++\pi^-$.

It is appropriate to list the wave functions of the intervening particles, using the same (obvious) notation of Ref. 2. We have

$$\begin{aligned}\rho_{\uparrow}{}^0&=\tfrac{1}{2}\sqrt{2}(\alpha_1\alpha_{\bar{1}}-\alpha_2\alpha_{\bar{2}})f(r)\,,\\ \pi^+(p)&=\tfrac{1}{2}\sqrt{2}(\alpha_1\beta_{\bar{2}}-\beta_1\alpha_{\bar{2}})f(r)\ \exp i\mathbf{p}\cdot\mathbf{X},\\ \pi^-(p)&=\tfrac{1}{2}\sqrt{2}(\alpha_{\bar{1}}\beta_2-\alpha_2\beta_{\bar{1}})f(r)\ \exp i\mathbf{p}\cdot\mathbf{X}.\end{aligned} \tag{8}$$

The wave functions above describe, respectively, a ρ^0 with spin up (in the direction of the Z axis) and a π^+ and a π^- with momentum $\mathbf{p}$; $f(r)$ is the internal wave function, which we have assumed to be the same for the vector and pseudoscalar mesons; α_i, β_i and $\alpha_{\bar{i}}$, $\beta_{\bar{i}}$ are the spin wave functions of the quark and antiquark, respectively.

The total matrix element of the transition for decay of a ρ into $\pi^+(-\mathbf{p})+\pi^-(\mathbf{p})$ consists of two additive contributions: (a) The ρ^0 emits a π^- with momentum $\mathbf{p}$ and transforms into a π^+ with momentum $-\mathbf{p}$; (b) The ρ^0 emits a π^+ with momentum $-\mathbf{p}$ and transforms into a π^- with momentum $\mathbf{p}$. Because the final state is the same, the two matrix elements (a) and (b) must be added.

Noting also that the amplitude for emission of a pion by a quark or by an antiquark are the same, we obtain for the matrix element for the transition $\rho^0{}_{\uparrow}\to\pi^+(-\mathbf{p})+\pi^-(\mathbf{p})$ the expression

$$\mathfrak{M}=\frac{4f_q}{\mu}\frac{p_x-ip_y}{\sqrt{2}}\frac{1}{\sqrt{(2\omega_p)}}. \tag{9}$$

At this point we compare[6] this matrix element with the relativistic matrix element for the $\rho\to 2\pi$ decay taken in the nonrelativistic limit. The relativistic matrix element (written in the rest system of the ρ) is

$$\mathfrak{M}^{(\mathrm{rel})}=2g\boldsymbol{\varepsilon}\cdot\mathbf{p}/(8M_\rho\omega_p{}^2)^{1/2}, \tag{10}$$

where $\boldsymbol{\varepsilon}$ is the polarization unit vector of the ρ. It is known[7] that the width of the ρ is correctly obtained if g in (10) is chosen to be

$$g^2/4\pi=2.$$

Writing (10) for a ρ^0 with spin up, we have

$$\mathfrak{M}_{\uparrow}{}^{(\mathrm{rel})}=\frac{2g(p_x-ip_y)}{\sqrt{2}}\frac{1}{(8M_\rho\omega_p{}^2)^{1/2}}.$$

Putting both in (9) and in (10) $\omega_p=\frac{1}{2}M_\rho$, and equating expressions (9) and (10), we have

$$g=2\sqrt{2}f_qM_\rho/\mu. \tag{11}$$

Squaring and expressing the result in terms of the pion-nucleon constant f instead of f_q, we have

$$\frac{g^2}{4\pi}=\frac{9}{25}\times 8\left(\frac{M_\rho}{\mu}\right)^2\frac{f^2}{4\pi}. \tag{12}$$

Here a very serious ambiguity manifests itself. How must we take the nonrelativistic limit of Eq. (10)? Should we consider nonrelativistic a situation in which the mass of the ρ is taken to be 280 MeV, twice the pion mass, or instead, a situation in which the mass of the ρ has as its real value 750 MeV, and we imagine the mass of the pion increased to 375 MeV?

[5] In the tables by Rosenfeld *et al.* [Rev. Mod. Phys. **36**, 977 (1964)], a value of 125 MeV is reported for the width of the N_{33}^*. We are not entirely clear how such a large value is obtained. The fitting procedure briefly described in Ref. 10 of a paper by Becchi, Eberle, and Morpurgo [Phys. Rev. **136**, B808 (1964)] leads to $\sim$100 MeV.

[6] The procedure which we follow is the same as that used in Ref. 2; it amounts to neglecting the mass dependence of the vertex function. However, the ambiguities in the present case, as will be explained later in detail, are much larger than for the case of the $V\to P+\gamma$ decays treated in Ref. 2.

[7] J. Sakurai, *Proceedings of the Varenna School of Physics* (Academic Press Inc., New York, 1962). R. H. Dalitz, Ann. Rev. Nucl. Sci. **13**, 371 (1963). The $\rho\to 2\pi$ decay rate expressed in terms of the g appearing in Eq. (10) is $\frac{2}{3}(g^2/4\pi)(p^3/M_\rho{}^2)$, leading to a value $g^2/4\pi=2$. The g appearing in Ref. 4 is defined as our g divided by 2. Hence the value of $g^2/4\pi$ used in Ref. 4 is, correctly, $\frac{1}{2}$.

Note that, in spite of the fact that in Eq. (12) only the ratio M_ρ/μ appears, these two cases give different values for g because the value of μ in Eq. (11) is fixed; it is the value (140 MeV) which appears in the non-relativistic limit of the pion-nucleon interaction. Note also that we might even be willing to consider a third form of nonrelativistic limit, one in which for instance one "pion" has mass M_ρ and the other has mass zero. However, this last "nonrelativistic limit" has little to do with our process.

In short, the result is ambiguous. We shall comment later on how we plan in the future to improve the theory in order to avoid this kind of ambiguity. (Note, by the way, that for the $V\to P+\gamma$ decays considered in Ref. 2 the situation is much better because the matrix element is independent of M_V in the nonrelativistic limit.) But we emphasize, after these critical remarks, an important point: *The order of magnitude obtained for $g^2/4\pi$ is perfectly reasonable.*

In fact, if we insert in Eq. (12) $M_\rho=2\mu$, we get $g^2/4\pi\cong1$, while if we insert the real value of $M_\rho\cong5.5\mu$, we get $g^2/4\pi\cong7.5$. The correct value of $g^2/4\pi\cong2$ is in between the two values obtained.

5. COMPARISON WITH THE POINT OF VIEW OF GÜRSEY, PAIS, AND RADICATI

The procedure used in Ref. 4 to connect the $NN\pi$ and $\rho\pi\pi$ vertices is, as we have said, substantially different from the method used here.

Let us try (in order to show this difference) to summarize[8] the main arguments of Ref. 4. It is first of all stated that it is the meson SU_6 tensor defined for momentum zero of the mesons as

$$M_\alpha{}^\beta=\delta_i{}^J P_A{}^B+(\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon})_i{}^J V_A{}^B\,, \qquad (13)$$

(where P and V are, respectively, the pseudoscalar and vector mesons) that must intervene as a basic object in the baryon-baryon meson interaction. However, for odd-parity mesons, the tensor $M_\alpha{}^\beta$ cannot be coupled in the static approximation to the baryons, and it is argued in Ref. 8 that the relativistic completion of Eq. (13), which is the object which must be coupled to the baryons for odd-parity mesons, has the form

$$N_\alpha{}^\beta(\mathbf{q})=i[(\boldsymbol{\sigma}\cdot\mathbf{q})_i{}^J/\mu_{00}]P_A{}^B+\delta_i{}^J\epsilon_0 V_A{}^B\,. \qquad (14)$$

Here we use the notation of Ref. 8, where in particular μ_{00} is the central mass of the mesons. We do not reproduce the arguments of Ref. 8 but we only note, because this is important for the following discussion, that the fundamental assumption of Refs. 4 and 8 is that $N_\alpha{}^\beta$ must be coupled, globally, to the baryons. This assumption establishes the equality between, on the one hand, the axial-vector coupling constant f_A of the baryon tensor to the pseudoscalar part of the meson tensor and, on the other hand, the vector coupling constant of the baryon tensor f_V to the vector part of the meson tensor;

$$f_A=f_V\,. \qquad (15)$$

On writing explicitly the ρ^0 and π^0 interaction with protons in the static limit, one obtains in Refs. 4 and 8

$$f_V\bar{p}\gamma_4 p\rho_4{}^0+(5/3)(f_A/\mu_{00})\bar{p}\gamma_4\boldsymbol{\sigma}p\cdot\nabla\pi^0\,,$$

and, because of Eq. (15) defining f' as $(5/3)f_A$, one gets

$$f'=(5/3)f_V\,. \qquad (16)$$

Now the second important assumption of Ref. 4 is that the vector coupling of the ρ to the isospin current is universal. Writing the interaction

$$f_V\rho_\mu{}^0\bar{N}\gamma_\mu\tau_3N+g\rho_\mu{}^0\left(\pi_{\rm ch}\frac{\partial\pi_{\rm ch}{}^\dagger}{\partial x_\mu}-\pi_{\rm ch}{}^\dagger\frac{\partial\pi_{\rm ch}}{\partial x_\mu}\right), \qquad (17)$$

this universality implies $g=2f_V$, and therefore, on account of Eq. (16),

$$g=2\times\tfrac{3}{5}f'\,. \qquad (18)$$

At this point note that

$$f'/\mu_{00}=f/\mu\,,$$

where μ is the pion mass and $f^2/4\pi=0.08$. Therefore,

$$g=2\times\tfrac{3}{5}(\mu_{00}/\mu)f\cong6f \qquad (19)$$

if we take $\mu_{00}\cong5\mu$. It follows that

$$g^2/4\pi=36f^2/4\pi\cong2.9\,, \qquad (20)$$

in reasonable agreement with the experimental data. Note that in the last step we have deviated from Ref. 4, where one transforms the pseudovector constant f' into the pseudoscalar constant g_{PS}. Note also that, in spite of the agreement with experiment, there are some points which are not entirely clear in the above calculation connected with the value of central mass μ_{00}. But it is not on these which we want to comment.

The point which we want to make is instead the following: It is obvious from the reproduction of the results of Gürsey, Pais, and Radicati given above that there are two assumptions underlying such a calculation:

(a) The V and P mesons are put together in a tensor which interacts with the baryons globally; that is, the indices of $N_\alpha{}^\beta$ are saturated with those of the baryons.

(b) The coupling of vector mesons to the isotopic-spin current is "universal."

Unfortunately neither (a) nor (b) is an assumption having a well-defined and clear meaning. As far as (a) is concerned, the prescription used to construct the interaction is based on an invariance requirement which cannot be shared by the free Lagrangian or by the total S matrix. The meaning of requiring this invariance is obscure. As far as (b) is concerned, the difficulties in

[8] We shall use also the arguments and the notation of M. A. B. Bég and A. Pais, Phys. Rev. **133**, B1514 (1965); **138**, B692 (1965).

defining a universal coupling of vector particles with nonvanishing mass are well known.

On the other hand, in the quark model we do not have to make assumptions of the above kind. (This is the point we want to emphasize; this short reproduction of Ref. 4 has been inserted in order to be able to clarify this point as much as possible.) No universality of the coupling of vector mesons is postulated; instead, it is the quark–quark–pseudoscalar-meson axial-vector interaction which is, as we have seen, responsible in our scheme for the $\rho \rightarrow 2\pi$ decay. Also, we do not need special prescriptions for constructing the interactions. Once the static quark-quark pion interaction is written down in the only possible way, everything becomes automatic.

6. FINAL REMARKS AND A LIST OF PROBLEMS

The calculation which we have performed should be considerably improved, particularly for the $\rho \rightarrow 2\pi$ decay.

There are clearly three aspects of our calculation which are in need of improvement:

(a) We have dealt with the two pions in the $\rho \rightarrow 2\pi$ decay in a rather asymmetric way, treating one pion as a quantum of a field, the other as composite particle (q-$\bar{q}$ in a singlet state).

(b) Although the pions from the ρ decay move relativistically, we have used wave functions for them which can be correct only for pions moving slowly. (Note: we are not speaking of the internal dynamics; we are speaking of the motion of the pion as a whole.)

(c) Also the interaction should be extended relativistically, and it should in addition be investigated whether this extension is univocal.

The points (b) and (c) are related. It is in fact also possible to write down a wave function describing a bound state of two particles with total momentum P and total energy E in the case in which P/E is not substantially smaller than 1, that is, in the case in which the motion of this bound object is relativistic. Indeed, to obtain this wave function we need essentially to perform a kinematical transformation on the rest spinors α and β appearing in Eq. (8); that is, we must replace these rest spinors with relativistic spinors each satisfying a Dirac equation with momentum $P/2$, energy $E/2$, and mass $m/2$, where m is the mass of the bound object and not of the quark. One is therefore naturally led to what is usually called a Bargmann-Wigner description.

What is not entirely clear is how to transform the interaction (originally written in terms of quark operators satisfying a Dirac equation with mass M) in such a way as to exhibit how it has to operate on these bound spinors satisfying a Dirac equation with mass m.[9]

Also, an additional independent problem is [as stated under (c)] whether the relativistic extension of the interaction is univocal.[10]

Let us finally discuss briefly the point (a) mentioned above. Since we introduce in our Hamiltonian both the pion and the quark variables explicitly, and since we describe the pion aggregate both as a field and as a q-$\bar{q}$ aggregate, a consistency condition is necessary. This condition must express the fact that if one describes the pion as an independent particle or as a q-$\bar{q}$ composite particle one must obtain the same results. We conjecture that this condition is a set of restrictions on the internal wave function of the pion; more precisely, it relates the coupling constant to the internal wave function of the pion. Indeed, the requirement that the amplitude for a bare pion to become a composite pion must be 1 just leads, in the first order in the interaction, to a relation of the above kind, and our conjecture is that this result is more general.

If this is so, our argument is then that, insofar as our calculation does not depend on this internal wave function (this means: insofar as we may neglect the mass dependence of the vertex coupling constants) our results should not be affected by ambiguities of this kind.

It should be clear that, as its title indicates, the discussion of this last section is very tentative: Essentially we have only posed problems[11] and indicated a program.

We hope to be able to come back to these problems. The purpose of this paper has been only that of showing that even with a nonrelativistic calculation it is possible and straightforward in the quark model, by expressing the $N^*N\pi$ and $\rho\pi\pi$ vertices exclusively in terms of the axial-vector quark-pion coupling constant and determining this constant through the $NN\pi$ vertex, to obtain results in reasonable agreement with experiment.

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[9] What is required here is a kind of Furry transformation, or more precisely, a mixture of a Furry transformation and a transformation to the center-of-mass and relative coordinates. An attempt in a related, but somewhat different direction, has been made by Tavkhelidze in *Proceedings of the Seminar on Elementary Particles and High Energy Physics, Trieste, 1965* (International Atomic Energy Agency, Vienna, 1965).

[10] What we mean by this can be made clear by considering the $\Phi \rightarrow K+\bar{K}$ decay. Here the kinematics is clearly nonrelativistic. Now we could easily repeat for this case, as stated at the beginning, our nonrelativistic calculation for this process, assuming that we know the coupling constant of the interaction $\bar{q}_i \boldsymbol{\sigma} q_3 \cdot \nabla K$ $(i=1,2)$. The relativistic extension of this interaction is $\bar{q}_i\gamma_5\gamma_\mu q_3\partial_\mu K$. But the question is now: Will, in a correct relativistic calculation of the $\Phi \rightarrow K+\bar{K}$ decay (which is, as we have said, apparently a nonrelativistic process), only the coupling constant of this $\gamma_5\gamma_\mu$ interaction intervene, or can there be an independent contribution from the $\bar{q}_i\gamma_5 q_3 K$ interaction, a contribution having the same order of magnitude?

[11] Another problem which we do not consider in this paper is the following: What is the importance, in the binding of quarks, of the forces among quarks due to pion exchange? From the small value of the coupling constant f_q and from the fact that the pion force is strongly spin-dependent, it would superficially seem that the forces due to pions are insignificant for binding quarks together; but this is at the moment an entirely open question, also in view of the fact that at distances of the order m_V^{-1} exchange of more than one pion can be important.