exists such that

$$(2Q-N)\Psi=\begin{bmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & -1\end{bmatrix}\Psi.$$

The six permutation matrices and $2Q-N$ together with their products and linear combinations generate the nine matrices

$$(E_{ij})_{mn}=\delta_{im}\delta_{jn}.$$

Since the matrices E_{ij} are obtained from unitary matrices, they are the matrices that lead to $SU(3)$. In other words, it appears that $SU(3)$ has been built into the formalism.

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Necessity of Additional Unitary-Antisymmetric q-Number Terms in the Commutators of Spatial Current Components

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It is proved that to maintain consistency of the commutation relations among spatial current components with the Jacobi identity, a Schwinger term antisymmetric with respect to interchange of isotopic (or unitary) indices is needed. The proof is based on the use of the Jacobi identity for triple commutators and of the Lehman-Källen expression for the vacuum expectation value of a current commutator. Additional conditions to be satisfied by the new Schwinger term are derived from an analysis of the origin of a discrepancy between the Lee–Dashen–Gell-Mann and the Cabibbo–Radicati sum rules for magnetic moments, and a solution of the discrepancy is proposed.

I. INTRODUCTION AND SUMMARY OF RESULTS

THE quark model suggests the commutation relation (μ, $\nu\neq 4$)

$$[j_\mu{}^{(\alpha)}(x),\, j_\nu{}^{(\beta)}(y)]_{x_0=y_0}=i\delta^{(3)}(\mathbf{x}-\mathbf{y})\delta_{\mu\nu}f_{\alpha\beta\gamma}j_0{}^{(\gamma)}(x)+(\text{terms symmetric in }\alpha\beta)$$

between vector current components $j_\mu{}^{(\alpha)}(x)$ [$\alpha=1, 2, 3$ in $SU(2)$ or $\alpha=1, 2, \cdots, 8$ in $SU(3)$], or between chiral vector current components. We prove here the inconsistency of the above equation, essentially by inserting its expression into the vacuum expectation value of the Jacobi identity for a triple commutator of currents and performing a spectral analysis of the expression obtained. The validity of the Jacobi identity at equal times if postulated. We first develop the argument for $SU(2)$ currents, and then extend the proof to $SU(3)$. The inconsistency can be avoided if the antisymmetric part in α,β of the current commutator has a form different from that suggested by the quark model. The new antisymmetric Schwinger terms (R terms) must necessarily be q numbers, as directly shown by our proof. To derive further information on the new terms we perform a comparison of the Lee–Dashen–Gell-Mann sum rule[1] with the Cabibbo–Radicati sum rule[2] for the magnetic moments. A discrepancy of a factor of 2 between the two formulas[3] is analyzed for its possible origin and is shown to be directly imputable to the R terms. Furthermore, to eliminate the discrepancy, two alternative choices of supplementary conditions are proposed that

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[1] B. W. Lee, Phys. Rev. Letters **14**, 613 (1965); R. F. Dashen and M. Gell-Mann, Phys. Letters **17**, 142 (1965); these authors assume $SU(3)$ in deriving the sum rule. However it has been noted by C. Ryan, Phys. Rev. **140**, B480 (1965), that the $SU(2)$ subalgebra is sufficient for the derivation.

[2] N. Cabibbo and A. L. Radicati, Phys. Letters **19**, 697 (1966).

[3] F. Buccella, G. Veneziano, and R. Gatto, Nuovo Cimento **42**, 1019 (1966).

must be satisfied by the R terms. Further speculations presented include some remarks on higher moments and the resolution of a discrepancy between analogous sum rules derived for pseudoscalar mesons.

II. INCONSISTENCY OF COMMUTATION RELATIONS

To be definite let us assume the quark model and set

$$j_\mu^{(\alpha)}(x)=\tfrac{1}{2}i\bar{\varphi}(x)\gamma_\mu\lambda_\alpha\varphi(x)\,, \tag{1}$$

where $\varphi(x)$ is the quark field and α is a unitary index. With the definition

$$j_4^{(\alpha)}(x)\equiv ij_0^{(\alpha)}(x)\,,$$

the current $j_\mu^{(\alpha)}(x)$ is self-conjugate, i.e., for $\mu=1$, 2, 3, 0,

$$j_\mu^{(\alpha)}(x)^\dagger=j_\mu^{(\alpha)}(x)\,. \tag{2}$$

Formal application of the canonical commutation relations among $\varphi(x)$ and $\bar{\varphi}(x)$ gives at equal time, for μ, $\nu\neq4$,

$$[j_\mu^{(\alpha)}(x),\, j_\nu^{(\beta)}(y)]_{x_0=y_0}=i\delta^{(3)}(\mathbf{x}-\mathbf{y})\delta_{\mu\nu}f_{\alpha\beta\gamma}j_0^{(\gamma)}(x) +(\text{terms symmetric in }\alpha\beta)\,. \tag{3}$$

Possible Schwinger terms symmetric in $\alpha\beta$ are included in the right-hand side (rhs) of Eq. (3). Commutation relations analogous to Eq. (3) hold for chiral vector currents in the combinations $V\pm A$.

We shall prove first that the commutation relation of Eq. (3) leads to an inconsistency both for $SU(2)$ (α, β, $\gamma=1$, 2, 3) and for $SU(3)$ (α, β, $\gamma=1$, 2, $\cdots$, 8) when one requires the validity of the Jacobi identity. A conservation law $\partial_\mu j_\mu^{(\alpha)}(x)=0$ will *not* be assumed. The proof will thus be valid also for chiral $V\pm A$ currents and for the σ model [in the latter case $j_\mu^{(\alpha)}(x)$ stands for the nucleonic part of the isospin current]. The inconsistency implies the existence of a Schwinger term *antisymmetric* in α, β in the rhs of Eq. (3).

A similar inconsistency in the commutation relation ($\nu\neq4$) $[j_4^{(\alpha)}(x),\, j_\nu^{(\beta)}(y)]$ was proved by Schwinger,[4] assuming $\partial_\mu j_\mu^{(\alpha)}(x)=0$, and was also proved without this assumption by one of the authors.[5] Such arguments are not, however, directly applicable here to prove the inconsistency of Eq. (3), as we are considering a case μ, $\nu\neq4$ and including additional symmetric terms in the rhs of Eq. (3).

We suppose that Eq. (3) is valid with inclusion in its rhs of any Schwinger term symmetric in α,β. Let us set

$$R_{\alpha\beta\gamma}{}^{\mu\nu\lambda}(x,y,z) =[j_\mu^{(\alpha)}(x),\,[j_\nu^{(\beta)}(y),j_\lambda^{(\gamma)}(z)]]_{x_0=y_0=z_0}\,, \tag{4}$$

where μ, ν, $\lambda\neq4$. We write the Jacobi identity for the triple commutator (4):

$$R_{\alpha\beta\gamma}{}^{\mu\nu\lambda}(x,y,z)+R_{\beta\gamma\alpha}{}^{\nu\lambda\mu}(y,z,x)+R_{\gamma\alpha\beta}{}^{\lambda\mu\nu}(z,x,y)=0\,, \tag{5}$$

and we take its vacuum expectation value. Let us first restrict ourselves to $SU(2)$ (α, β, $\gamma=1$, 2, 3). Symmetric terms in α,β from the rhs of Eq. (3), will not contribute to the vacuum expectation value of Eq. (4) because of isospin invariance with invariant vacuum. One has

$$\langle R_{\alpha\beta\gamma}{}^{\mu\nu\lambda}(x,y,z)\rangle_0 =i\delta_{\nu\lambda}\epsilon_{\beta\gamma\delta}\delta^{(3)}(\mathbf{y}-\mathbf{z})\langle[j_\mu^{(\alpha)}(x),j_0^{(\delta)}(y)]_{x_0=y_0}\rangle_0\,, \tag{6}$$

and similarly for the other terms of Eq. (5). Let us now consider the Lehman-Källen representation

$$\langle[j_\mu^{(\alpha)}(x),j_\nu^{(\beta)}(y)]\rangle_0 =\delta_{\alpha\beta}\int_0^\infty d(m^2)[\delta_{\mu\nu}\rho_1(m)-\rho_2(m)\partial_\mu\partial_\nu]\Delta(x-y,m)\,, \tag{7}$$

where we have again assumed isospin invariance. The spectral weights $\rho_1(m)$ and $\rho_2(m)$ are defined by

$$\frac{1}{(2\pi)^3}\sum_n\delta^{(4)}(p-p_u)\langle0|j_\mu^{(\alpha)}(0)|n\rangle\langle n|j_\nu^{(\beta)}(0)|0\rangle =\delta_{\alpha\beta}[\delta_{u\nu}\rho_1(m)+p_up_\nu\rho_2(m)];\quad(p^2+m^2)=0\,. \tag{8}$$

As noted elsewhere[5] they must satisfy the inequalities

$$m^2\rho_2(m)\geqslant\rho_1(m)\geqslant0\,. \tag{9}$$

The conservation law $\partial_\mu j_\mu^{(\alpha)}(x)=0$—not assumed here, for the sake of generality—would actually imply $m^2\rho_2(m)=\rho_1(m)$. Equation (7) for $\mu=4$, $\nu\neq4$, and $x_0=y_0$ implies

$$\langle[j_4^{(\alpha)}(x),j_\nu^{(\beta)}(y)]_{x_0=y_0}\rangle_0 =\delta_{\alpha\beta}\frac{\partial\delta^{(3)}(\mathbf{x}-\mathbf{y})}{\partial x_\nu}\int_0^\infty d(m^2)\rho_2(m)\,. \tag{10}$$

We now insert Eqs. (6) and (10) into the vacuum expectation value of Eq. (5) and obtain for the expectation value

$$\epsilon_{\alpha\beta\gamma}\int_0^\infty d(m^2)\rho_2(m)\bigg[\delta_{\nu\lambda}\frac{\partial\delta^{(3)}(\mathbf{x}-\mathbf{y})}{\partial x_\mu}\delta^{(3)}(\mathbf{y}-\mathbf{z}) +\delta_{\lambda\mu}\frac{\partial\delta^{(3)}(\mathbf{y}-\mathbf{z})}{\partial y_\nu}\delta^{(3)}(\mathbf{z}-\mathbf{x}) +\delta_{\mu\nu}\frac{\partial\delta^{(3)}(\mathbf{z}-\mathbf{x})}{\partial z_\lambda}\delta^{(3)}(\mathbf{x}-\mathbf{y})\bigg]=0\,. \tag{11}$$

Choosing in particular $\nu=\lambda\neq\mu$ in Eq. (11), one finally

[4] J. Schwinger, Phys. Rev. Letters 3, 296 (1959); R. Johnson, Nucl. Phys. 25, 431 (1961).

[5] S. Okubo, Nuovo Cimento (to be published).

gets

$$\int_0^\infty d(m^2)\rho_2(m)=0. \tag{12}$$

Comparing this result with Eq. (9) we see that

$$\rho_1(m)=\rho_2(m)=0 \tag{13}$$

provided there are no zero-mass particles. Then Eq. (8) together with the Federbush-Johnson argument[6] shows that

$$j_\mu{}^{(\alpha)}(x)\equiv 0$$

unless Eq. (2) does not hold. This completes our argument for $SU(2)$.

To extend the argument to the $SU(3)$ currents ($\alpha=1, 2, \cdots, 8$), we write the Jacobi identity for the antisymmetrized operator

$$\tilde{R}_{\alpha\beta\gamma}{}^{\mu\nu\lambda}(x,y,z)=R_{\alpha\beta\gamma}{}^{\mu\nu\lambda}(x,y,z)-R_{\alpha\gamma\beta}{}^{\mu\nu\lambda}(x,y,z).$$

Putting

$$[j_\mu{}^{(\alpha)}(x),j_\nu{}^{(\beta)}(y)]_{x_0=y_0}=S_{\mu\nu}{}^{\alpha\beta}(x,y)+A_{\mu\nu}{}^{\alpha\beta}(x,y),$$

where we have written $S_{\mu\nu}{}^{\alpha\beta}$ for all the terms symmetric in $\alpha\beta$ and $A_{\mu\nu}{}^{\alpha\beta}$ for all the terms antisymmetric in $\alpha\beta$, we find for the contribution of the S terms to the vacuum expectation value of the Jacobi identity for $\tilde{R}$

$$\langle[j_\lambda{}^{(\gamma)}(z),S_{\mu\nu}{}^{\alpha\beta}(x,y)]\rangle_0+\langle[j_\nu{}^{(\beta)}(y),S_{\lambda\mu}{}^{\gamma\alpha}(z,x)]\rangle_0 -(\text{same with }\beta\leftrightarrow\gamma). \tag{14}$$

Assuming $SU(3)$ (with invariant vacuum) we see that the indices $\alpha\beta\gamma$ must be combined as in $d_{\alpha\beta\gamma}$, with $f_{\alpha\beta\gamma}$ excluded because it is completely antisymmetric. The expression (14) is, however, antisymmetric in $\beta\gamma$, so it can only be zero. The proof of the inconsistency now runs exactly as for the case of $SU(2)$, apart from the trivial extension $\epsilon_{\alpha\beta\gamma}\leftrightarrow 2f_{\alpha\beta\gamma}$.

We have thus proved that Eq. (3) cannot be valid, both in $SU(2)$ and in $SU(3)$. We note that our argument may not be used to directly disprove the once-integrated relation

$$\left[j_\mu{}^{(\alpha)}(x),\int_{x_0=y_0} d^3y\, j_\nu{}^{(\beta)}(y)\right]=i\delta_{\mu\nu}f_{\alpha\beta\gamma}j_0{}^{(\gamma)}(x) +(\text{symmetric terms in }\alpha\beta), \tag{15}$$

once Eq. (3) has been relaxed. Equation (15) might still be valid even though Eq. (3) is not. This fact may be relevant for many applications of the algebra of currents, as we shall see shortly.

At any rate we have found that, to avoid inconsistency, the antisymmetric term in the rhs of Eq. (3) must be different from its assumed simple form. Additional Schwinger terms proportional to $f_{\alpha\beta\gamma}$ must be present in Eq. (3). Familiar arguments demand that Eq. (3) for μ, $\nu\neq 4$ be modified as

$$[j_\mu{}^{(\alpha)}(x),j_\nu{}^{(\beta)}(y)]_{x_0=y_0}=i\delta^{(3)}(\mathbf{x}-\mathbf{y})\delta_{\mu\nu}f_{\alpha\beta\gamma}j_0{}^{(\gamma)}(x) +f_{\alpha\beta\gamma}\frac{\partial\delta^{(3)}(\mathbf{x}-\mathbf{y})}{\partial x_k}[R_{\mu\nu,k}{}^{(\gamma)}(x)-R_{\nu\mu,k}{}^{(\gamma)}(y)] +(\text{symmetric terms in }\alpha\beta), \tag{16}$$

where the repeated index k means automatic summation over $k=1, 2, 3$, and $R_{\mu\nu,k}{}^{(\gamma)}(x)$ is an unknown function of x. As we have remarked, the once-integrated commutator Eq. (15) could still be valid. If we conjecture that it is in fact valid, we must impose the condition

$$\frac{\partial}{\partial x_k}R_{\mu\nu,k}{}^{(\gamma)}(x)=0. \tag{17}$$

We shall see that such an additional constraint may reconcile the discrepancy between the Cabibbo-Radicati[2] (C-R) and the Lee-Dashen-Gell-Mann[1] (L-D-G) sum rules. It is worth while to emphasize that $R_{\mu\nu,k}{}^{(\gamma)}(x)$ must be a q number. If it is a c number, our proof of the inconsistency still applies. In fact any c-number term in the rhs of Eq. (3) does not contribute to the triple commutator of Eq. (4) and therefore cannot affect our proof of the inconsistency. We may also remark that for the commutation relation

$$[j_0{}^{(\alpha)}(x),j_0{}^{(\beta)}(y)]_{x_0=y_0}=if_{\alpha\beta\gamma}\delta^{(3)}(\mathbf{x}-\mathbf{y})j_0{}^{(\gamma)}(x) \tag{18}$$

we are unable to prove or disprove the existence of possible inconsistencies. Equation (18) was used to derive the C-R relation.

III. DISCREPANCY BETWEEN THE CABIBBO-RADICATI AND THE LEE-DASHEN-GELL-MANN SUM RULES

The L-D-G relation was obtained by considering the sum of commutators

$$[m_{12}{}^{(\alpha)},m_{12}{}^{(\beta)}]+[m_{21}{}^{(\alpha)},m_{21}{}^{(\beta)}] -[m_{12}{}^{(\alpha)},m_{21}{}^{(\beta)}]-[m_{21}{}^{(\alpha)},m_{12}{}^{(\beta)}], \tag{19}$$

where

$$m_{il}{}^{(\alpha)}=\int d^3x\, x_i j_l{}^{(\alpha)}(x), \tag{20}$$

taken between proton states at rest with N and N^* intermediate states. In this way one obtains

$$\left(\frac{\mu_p-\mu_n}{2m}\right)^2-2\left(\frac{\mu^*}{2m}\right)^2=\tfrac{1}{6}\langle r^2\rangle, \tag{21}$$

if one starts from Eq. (3), or equivalently from Eq. (16), neglecting the contributions from the $R_{\mu\nu,k}{}^{(\alpha)}(x)$ term. In Eq. (21) the notation is standard and μ^* represemts the N^*-N transition magnetic moment.

[6] P. Federbush and R. Johnson, Phys. Rev. **120**, 1926 (1960).

On the other hand, the C-R sum rule, under the same restriction as to the intermediate states, gives[3]

$$\left(\frac{\mu_p-\mu_n}{2m}\right)^2-2\left(\frac{\mu^*}{2m}\right)^2=\tfrac{1}{3}\langle r^2\rangle. \tag{22}$$

We note a discrepancy of a factor of 2 between the right-hand sides of the two relations, Eq. (21) and Eq. (22). Equation (21) may of course be incorrect simply because of the neglect of a contribution from the Schwinger terms $R_{\mu\nu,k}{}^{(\gamma)}(x)$. However, the difference of just a factor of 2 might not be accidental. In order to pinpoint the possible origin of this rather curious coincidence we note that Eq. (19) can actually be split into a sum of relations such as

$$[m_{12}{}^{(\alpha)},m_{12}{}^{(\beta)}]=if_{\alpha\beta\gamma}\int d^3x(x_1)^2j_0{}^{(\gamma)}(x)-f_{\alpha\beta\gamma}\int d^3x(x_1)^2\frac{\partial}{\partial x_k}R_{22,k}{}^{(\gamma)}(x), \tag{23}$$

$$[m_{12}{}^{(\alpha)},m_{21}{}^{(\beta)}]=-f_{\alpha\beta\gamma}\int d^3x\times\left\{x_2[R_{21,1}{}^{(\gamma)}(x)-R_{12,1}{}^{(\gamma)}(x)]+x_1x_2\frac{\partial}{\partial x_k}R_{21,k}{}^{(\gamma)}(x)\right\}, \tag{24}$$

where we have made use of Eq. (16). The additional terms in Eq. (16) symmetric in $\alpha\beta$ need not be considered in the above equations. This is obvious for Eq. (23). To show it for Eq. (24), we take the equation between states $<p,S_z|$ and $|p,S_z'>$, where S_z and S_z' are the spin components along z and $\mathbf{p}$ is also directed along z. The left-hand side of Eq. (24) gives

$$\begin{aligned}\langle p,S_z|[m_{12}{}^{(\alpha)},m_{21}{}^{(\beta)}]|p,S_z'\rangle&=\langle p,S_z|R(\pi/2)R^{-1}(\pi/2)\times[m_{12}{}^{(\alpha)},m_{21}{}^{(\beta)}]R(\pi/2)R(\pi/2)^{-1}|p,S_z'\rangle\\&=e^{i(\pi/2)(S_z-S_z')}\langle p,S_z|[m_{21}{}^{(\alpha)},m_{12}{}^{(\beta)}]|p,S'_z\rangle,\end{aligned}$$

where we have introduced a rotation of 90° around the z axis. However, S_z-S_z' can only assume values 0, ±2, ±4 because of the selection rules due to angular momentum. In particular for spin $\frac{1}{2}$, $S_z=S_z'$ and hence

$$\langle p,S_z|[m_{12}{}^{(\alpha)},m_{21}{}^{(\beta)}]|p,S_z'\rangle=\langle p,S_z|[m_{21}{}^{(\alpha)},m_{12}{}^{(\beta)}]|p,S_z'\rangle,$$

showing that the commutator behaves as an antisymmetric object in α,β. We have thus justified our neglect of symmetric terms in Eq. (24).

Let us now suppose, for a moment, that somehow the R terms in Eqs. (23) and (24) do not contribute at all to matrix elements between one-nucleon states. Then, of course,

$$\langle p|[m_{12}{}^{(\alpha)},m_{12}{}^{(\beta)}]|p\rangle=if_{\alpha\beta\gamma}\langle p|\int d^3x\,x_1{}^2j_0{}^{(\gamma)}(x)|p\rangle, \tag{25}$$

$$\langle p|[m_{12}{}^{(\alpha)},m_{21}{}^{(\beta)}]|p\rangle=0, \tag{26}$$

where for simplicity we have omitted the spin indices. These are the equations one obtains by starting from Eq. (3). We can show that Eq. (25) leads to the C-R relation, while Eqs. (25) and (26) together, inserted into the sum of Eq. (14), lead to the L-D-G relation, always in the N and N^* dominance model. Hence, the simultaneous validity of both Eqs. (25) and (26) is contradictory.

We note that intermediate contributions to Eq. (25) or (26), saturated in the rest system $\mathbf{p}=0$, arise only from states of positive parity and with total angular momentum $J\leqslant\frac{5}{2}$. Let us now compare

$$\langle n|m_{12}{}^{(\alpha)}|p\rangle \quad\text{and}\quad \langle n|m_{21}{}^{(\alpha)}|p\rangle,$$

n being a state of positive parity and with $J\leqslant\frac{5}{2}$. By a 90° rotation around z one finds

$$\langle n|m_{12}{}^{(\alpha)}|p\rangle=-e^{i(\pi/2)(S_z-S_z')}\langle n|m_{21}{}^{(\alpha)}|p\rangle$$

with the selection rule $S_z-S'_z=0$, ±2. Intermediate states with $J=\frac{1}{2}$ [such as the nucleon pole, or the suggested $N^*(1425)$, etc.] simply give

$$\langle J=\tfrac{1}{2}|m_{12}{}^{(\alpha)}|p\rangle=-\langle J=\tfrac{1}{2}|m_{21}{}^{(\alpha)}|p\rangle.$$

As for $N^*(1238)$ with $J=\frac{3}{2}^+$, one finds for the M_{1^+} transition, for which $S_z=S_z'$

$$\langle N^*|m_{12}{}^{(\alpha)}|p\rangle=-\langle N^*|m_{21}{}^{(\alpha)}|p\rangle.$$

The E_2 contribution (experimentally negligible) also vanishes for degenerate masses. Finally, $N^*(1518)$ and other negative parity states do not contribute. Contributions from $N^*(1688)$ with $J=\frac{5}{2}^+$, etc., will be reduced considerably because of centrifugal effects.

We have thus seen that the approximate saturation of Eqs. (25) and (26) seems to indicate that the left-hand sides of the two equations are roughly of equal magnitude, in contrast to the right-hand sides. At the same time one notes that the validity of Eq. (25) will lead to the C-R relation, Eq. (22). This fact explains why the insertion of Eqs. (25) and (26) into Eq. (19) gives the L-D-G formula, Eq. (21). The conclusion seems to be that the Schwinger terms $R_{\mu\nu,k}{}^{(\alpha)}(x)$ cannot be neglected in the lhs of Eq. (24), although they can be neglected in Eq. (23). Such a circumstance can easily be explained if $R_{\mu\nu,k}{}^{(\gamma)}(x)$ satisfies either one of the following alternative conditions:

(i) $$\frac{\partial}{\partial x_k}R_{\mu\nu,k}{}^{(\gamma)}(x)=0, \tag{28}$$

(ii) $$R_{\mu\nu,k}{}^{(\gamma)}(x)=-R_{\nu\mu,k}{}^{(\gamma)}(x). \tag{29}$$

It is easy to see from Eqs. (23) and (24) that for either of the two alternatives [Eqs. (28) and (29)], Eq. (25) remains valid, but not Eq. (26). The first alternative, Eq. (28), was suggested in the previous section [see Eq. (17)] as a sufficient condition to preserve the validity of the once-integrated commutation relation, Eq. (15). The second alternative, Eq. (29), appears very simple and may in fact be obtained by the ordinary limiting procedure to get the Schwinger term if we use a particular limiting assumption.

Another possibility, on an entirely different basis, is that the commutation relations have a different form from that in the quark model, namely

$$[j_\mu^{(\alpha)}(x),j_\nu^{(\beta)}(y)]_{x_0=y_0}=i\delta_{\mu\nu}\delta^{(3)}(\mathbf{x}-\mathbf{y})f_{\alpha\beta\gamma}J_0^{(\gamma)}(x) \quad \text{(terms symmetric in } \alpha\beta\text{)},$$

with $J_0^{(\gamma)}(x)$ satisfying, for some reason,

$$\langle p|\int d^3x\, x_1^2 J_0^{(\gamma)}(x)|p\rangle=0.$$

The inconsistency would be solved and, saturating with N and N^*, one would find

$$\mu^*=(5/4)(2\sqrt{2}/5)(\mu_p-\mu_n)=1.25(2\sqrt{2}/3)\mu_p,$$

in surprisingly perfect agreement with data (see, for instance, Dalitz's analysis of photoproduction[7]).

We would now like to study briefly what effects would follow from the addition of the antisymmetric Schwinger term to the commutation relations of higher momenta. Sum rules for higher momenta have been studied by Bietti.[8] One easily sees that only the commutation relations involving components of j along the same direction are free from contributions from the Schwinger terms introduced. On the basis of Eq. (16), one would obtain the interesting relation

$$\left[\int d^3x\, x_3x_1j_2^{(\alpha)}(x),\int d^3y\, y_3y_1j_2^{(\beta)}(y)\right]_{x_0=y_0}=if_{\alpha\beta\gamma}\int d^3x\, x_1^2x_3^2j_0^{(\gamma)}(x).$$

One would not obtain the relation given in Eq. (6) of the second paper by Bietti[8]. His conclusions would thus be invalidated. We may also comment about Schnitzer's argument on the mean-square radius of the pion.[9] Schnitzer evaluates Eq. (19) between psuedoscalar-meson states truncating the intermediate summation to vector-meson contributions. We can show, by the same procedure we have used above for the L-D-G relation, that the contributions from the Schwinger terms in Eqs. (22) and (24) cannot be neglected without contradiction. Under either of the two alternatives, Eq. (28) and Eq. (29), we can again neglect the contributions from the R terms in Eq. (23), and we find that Schnitzer's prediction for $\langle r_\pi^2\rangle$ has to be multiplied by a factor of $\frac{1}{2}$, in agreement with the C-R result for this quantity, as obtained from the commutation rule in Eq. (18).

When this work was completed, one of us (S. O.) was informed by Professor D. Amati that recently Bouchiat and Meter[10] have also been able to resolve the discrepancy between the L-D-G and C-R sum rules. They assume the existence of an antisymmetric Schwinger term by a plausibility argument instead of proving its existence, and suppose the validity of our Eq. (29).

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[10] C. Bouchiat and P. H. Meyer (to be published).