Two-Pion-Exchange Effects in NN Scattering and Consistency with πN Scattering*

JOHN W. DURSO
Department of Physics, Michigan State University, East Lansing, Michigan

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The two-pion-exchange contribution to nucleon-nucleon scattering has been recalculated using the method of Amati, Leader, and Vitale: previous errors have been eliminated. The new results show that low-energy pion-nucleon scattering phenomena and two-pion exchange effects in nucleon-nucleon scattering can be explained in a consistent way. The results also show that the two-pion parts of the NN phase shifts are dominated by the cross-channel s-wave contribution, in contrast to the results of Amati *et al.* The cutoff sensitivity of both the total and high-wave cross-channel contributions is examined, and arguments are presented to explain the strong sensitivity of the high-wave part.

I. INTRODUCTION

SEVERAL years ago, Amati, Leader, and Vitale[1] (ALV) developed a method, based on the Mandelstam representation, for calculating two-pion exchange effects in nucleon-nucleon scattering. This method had the advantage that, if successful, it would relate low-energy pion-nucleon phenomenology directly to the nucleon-nucleon problem. ALV's subsequent numerical calculation gave results which were rather good in fitting high-wave $T=1$ phase shifts but were exceedingly poor for the $T=0$ phases. In addition, the original computations showed d and higher waves in the $N\bar{N}$ (t) channel, which made a sizeable contribution to the NN amplitudes. This was perhaps the most important conclusion to be drawn from the ALV calculation, since it contradicted the assumptions of the one-boson-exchange models which have been used by a number of authors[2–4] with some success in matching the NN phase shifts. It has been shown,[5,6] however, that some parts of the original ALV calculation contained numerical errors.

The purpose of this paper is threefold. First, we wish to check the entire ALV calculation; second, we would like to examine how well the results corroborate the connection between πN and NN scattering; and third, we want to re-examine the applicability of the method used.

The model used for these calculations is exactly that of ALV. Briefly, the theory consists of applying the Cini-Fubini[7] approximation to the NN problem in order to make use of our knowledge of πN scattering. The model used to describe πN scattering is basically that of Chew, Goldberger, Low, and Nambu[8] (CGLN) in which the πN elastic-scattering process is assumed to be dominated by the nucleon pole and the 3, 3 resonance (N^*) which is treated in the zero-width approximation. The amplitudes predicted by this model for πN are analytically continued to the $N\bar{N}$-$\pi\pi$ channel, then by unitarity and crossing, the latter are used to compute NN amplitudes. In terms of diagrams, we wish to include the contribution to NN amplitudes of fourth-order diagrams containing intermediate N^* nucleon lines as shown in Fig. 1. For a full exposition of the theory, the reader is referred to the ALV papers. A convenient summary is found in ALV IV. In all the following we use the same notation as ALV.

II. NUMERICAL RESULTS

In this section we present the results of the numerical calculations. Discussion of the results will be reserved for the following section.

A. The Complete 2π Continuum

As a reference point for possible future calculations, the results for the "complete" two-pion matrix elements $T_{ij}(l)$ are presented in Table I. By "complete" we mean that all $N\bar{N}$ partial waves are included and that the $N\bar{N}$-$\pi\pi$ amplitudes used in computing these $N\bar{N}$ amplitudes are derived from the *unmodified* CGLN model. It is well known that the CGLN amplitudes do not yield good agreement with low-energy s-wave πN scattering. Indeed, the two s-wave scattering lengths predicted by the unmodified CGLN model are far too large. Bowcock, Cottingham, and Lurié[9] (BCL) added a phenomenological constant to the amplitude $A^+(s,t)$ to account for distant cuts in the $\pi\pi$ channel. This constant brought the πN s-wave scattering lengths into better agreement with experiment. This correction of course would alter the $N\bar{N}$-$\pi\pi$ s-wave contribution as well and, as we shall see, in a desirable way. By "complete" 2π

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TABLE I. "Complete" CGLN contribution to the singlet-triplet nucleon-nucleon scattering matrix elements.

$E_{\rm lab}$ (MeV)	T_{11}	T_{1-1}	T_{10}	T_{01}	T_{00}	T_{ss}
			Isotopic spin = 1			
			$l=3$			$l=2$
20	0.023	-7.8×10^{-5}	1.9×10^{-4}	-4.7×10^{-5}	0.022	0.30
50	0.17	-0.0024	0.0048	-0.0068	0.16	1.17
100	0.57	-0.0094	0.018	-0.029	0.57	2.63
150	1.01	-0.017	0.034	-0.058	1.02	3.79
200	1.44	-0.024	0.050	-0.083	1.47	4.68
250	1.82	-0.031	0.065	-0.11	1.87	5.37
300	2.17	-0.037	0.079	-0.13	2.22	5.89
400	2.73	-0.048	0.10	-0.16	2.79	6.63
			$l=5$			$l=4$
20	1.6×10^{-4}	-6.5×10^{-8}	-3.6×10^{-7}	1.0×10^{-6}	1.5×10^{-4}	0.0018
50	0.0045	-1.4×10^{-5}	6.6×10^{-5}	-7.8×10^{-5}	0.0043	0.029
100	0.036	-1.4×10^{-4}	6.5×10^{-4}	-9.6×10^{-4}	0.035	0.16
150	0.097	-4.2×10^{-4}	0.0020	-0.0030	0.097	0.35
200	0.18	-7.8×10^{-4}	0.0038	-0.0058	0.18	0.58
250	0.28	-0.0012	0.0060	-0.0091	0.28	0.82
300	0.39	-0.0017	0.0085	-0.013	0.39	1.05
400	0.60	-0.0028	0.014	-0.020	0.62	1.49
			$l=7$			$l=2$
20	1.2×10^{-6}	4.9×10^{-10}	-5.4×10^{-9}	7.0×10^{-9}	1.2×10^{-6}	1.3×10^{-5}
50	1.4×10^{-4}	-1.4×10^{-7}	1.1×10^{-6}	-1.2×10^{-6}	1.3×10^{-4}	8.2×10^{-4}
100	0.0025	-3.9×10^{-6}	3.1×10^{-5}	-4.1×10^{-5}	0.0025	0.010
150	0.010	-1.8×10^{-5}	1.5×10^{-4}	-2.1×10^{-4}	0.010	0.034
200	0.024	-4.3×10^{-5}	3.7×10^{-4}	-5.2×10^{-4}	0.024	0.074
250	0.045	-8.1×10^{-5}	6.9×10^{-4}	-9.8×10^{-4}	0.045	0.13
300	0.071	-1.3×10^{-4}	0.0011	-0.0016	0.072	0.19
400	0.14	-2.6×10^{-4}	0.0022	-0.0031	0.14	0.32
			Isotopic spin = 0			
			$l=2$			$l=3$
20	0.97	0.23	0.37	0.36	-0.65	0.025
50	1.97	0.14	0.89	0.23	-1.00	0.16
100	3.07	-0.065	1.32	-0.20	-0.42	0.51
150	3.64	-0.21	1.57	-0.59	0.22	0.83
200	4.11	-0.28	1.65	-0.84	0.98	1.18
250	4.43	-0.32	1.69	-1.01	1.61	1.48
300	4.64	-0.33	1.70	-1.13	2.12	1.75
400	4.88	-0.32	1.68	-1.27	2.86	2.16
			$l=4$			$l=5$
20	0.0056	1.9×10^{-4}	6.1×10^{-4}	9.4×10^{-4}	-9.8×10^{-4}	1.9×10^{-4}
50	0.049	4.5×10^{-4}	0.0084	0.0024	-0.0087	0.0048
100	0.19	-8.7×10^{-4}	0.034	-0.0068	0.0085	0.034
150	0.34	-0.0036	0.068	-0.030	0.051	0.082
200	0.51	-0.0060	0.097	-0.054	0.16	0.15
250	0.68	-0.0080	0.12	-0.078	0.29	0.23
300	0.84	-0.0095	0.14	-0.10	0.43	0.32
400	1.12	-0.012	0.18	-0.14	0.70	0.49
			$l=6$			$l=7$
20	3.8×10^{-5}	4.8×10^{-7}	1.4×10^{-6}	3.9×10^{-6}	6.3×10^{-6}	1.6×10^{-6}
50	0.0014	4.9×10^{-6}	1.2×10^{-6}	4.7×10^{-5}	1.4×10^{-4}	1.6×10^{-4}
100	0.013	-2.3×10^{-5}	0.0013	-2.8×10^{-4}	0.0027	0.0026
150	0.033	-1.5×10^{-4}	0.0042	-0.0021	0.0075	0.0086
200	0.066	-3.1×10^{-4}	0.0078	-0.0047	0.025	0.021
250	0.11	-4.9×10^{-4}	0.012	-0.0080	0.050	0.038
300	0.15	-6.6×10^{-4}	0.016	-0.012	0.083	0.059
400	0.25	-9.8×10^{-4}	0.025	-0.020	0.16	0.11

continuum we do not mean that the whole of the 2π contribution is taken into account, since the integrations over contributing amplitudes in the $N\bar{N}$ channel are cut off at $t=16\mu^2$. Here t is the usual Mandelstam variable which, in the $N\bar{N}$ channel, is equal to the square of the total energy; μ is the pion mass. The constants used in this calculation differ from those quoted by ALV only in the value of the pion mass, which we have taken to be 135 MeV. It should be pointed out that these results are almost identical with those of ALV for their complete CGLN 2π contribution. However, the results presented in their Paper IV for the $T_{ij}(l)$ were with the $N\bar{N}$-$\pi\pi$ s

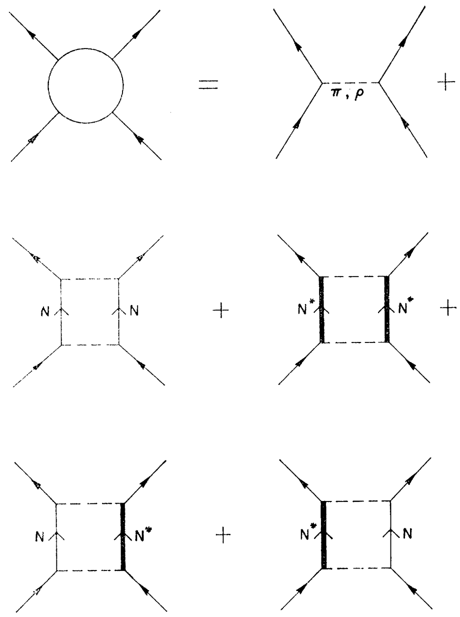
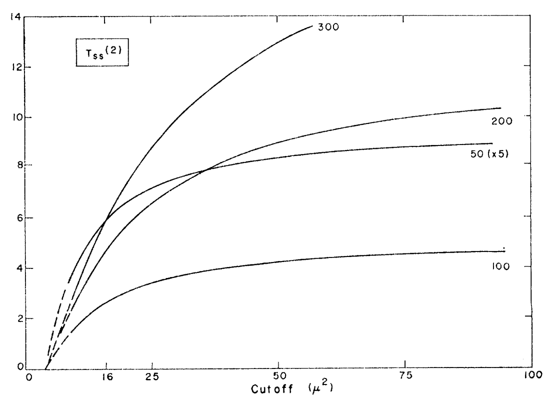


FIG. 1. Diagrammatic representation of contributions to NN scattering amplitudes computed in the model. Numbers labeling curves indicate lab energy in MeV, and scale.

and p waves subtracted out. It is at this point that the first discrepancy between the ALV results and our calculations occurs.

B. The "2π Basic" Contribution

In the original ALV calculation, as previously stated, the $N\bar{N}$ $d+$higher wave continuum contribution to the $T_{ij}(l)$ was extracted from the complete 2π continuum by subtracting the $N\bar{N}$-$\pi\pi$ s and p waves. The remainder was called "2π basic," suggesting that the results should be independent of the small details of the model, such as neglect of all πN amplitudes but the s and p waves, the use of the zero-width approximation, and the precise value of the cutoff. In their calculation, this $d+$higher wave remainder turned out to be a significant part of the complete two-pion contribution. However, in the present calculation and in an independent calculation by Furuichi and Watari,[6] this remainder was found to be negligible. In Table II, the complete 2π contribution (CGLN) is broken down into s, p, and $d+$higher wave contributions. It is seen that the s wave is by far the dominant one while the p wave is small, and the $d+$higher wave contribution is even smaller. Note that



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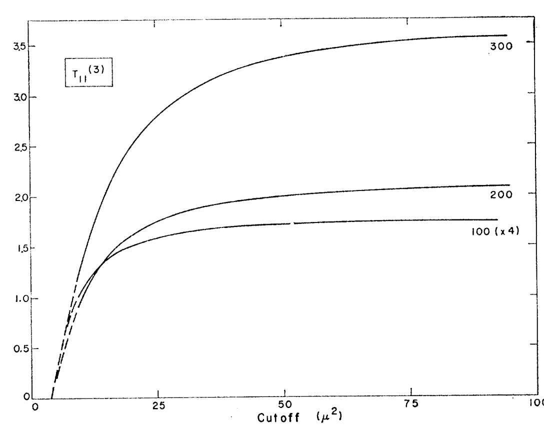

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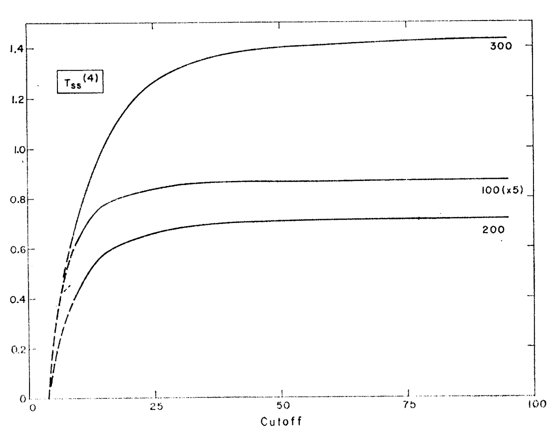

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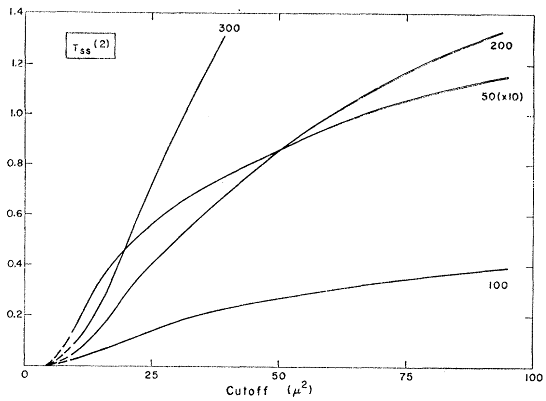
FIG. 2. Cutoff dependence of "complete" CGLN contribution to several NN matrix elements. Numbers labeling curves indicate lab energy in MeV *and scale.*



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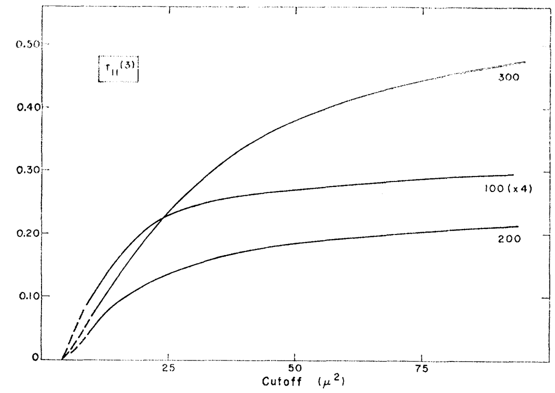

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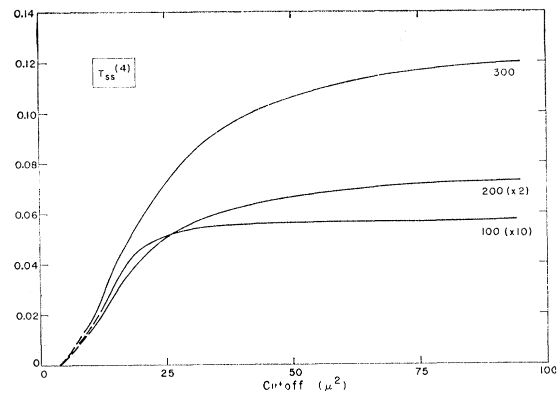

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FIG. 3. Cutoff dependence of d+higher wave $N\bar{N}$ contribution to several NN matrix elements.

TABLE II. Contribution of various $N\bar{N}$ partial waves to NN scattering matrix elements.

E_{lab} (MeV)	"Complete" CGLN	s wave	p wave	d+higher waves	ALV "2π-basic"
			$T_{ss}(2)$		
50	1.17	1.09	0.045	0.033	−1.68
100	2.63	2.44	0.11	0.076	−2.53
200	4.68	4.28	0.22	0.18	−2.36
300	5.89	5.29	0.29	0.31	−1.31
			$T_{11}(3)$		
50	0.17	0.18	−0.028	0.022	−0.31
100	0.57	0.61	−0.080	0.045	−0.76
200	1.44	1.54	−0.18	0.085	−1.09
300	2.17	2.29	−0.26	0.14	−0.98
			$T_{01}(3)$		
50	−0.0068	−0.0021	−0.0051	3.5×10^{-4}	−0.15
100	−0.030	−0.009	−0.022	0.001	−0.64
200	−0.082	−0.029	−0.050	−0.003	−1.06
300	−0.136	−0.056	−0.081	7.3×10^{-4}	−1.18
			$T_{1-1}(3)$		
50	−0.0024	0.0	−0.0031	7.1×10^{-4}	−0.0042
100	−0.0094	1.0×10^{-4}	−0.0088	-6.7×10^{-4}	−0.0086
200	−0.024	0.001	−0.022	−0.0037	−0.018
300	−0.037	0.001	−0.33	−0.0055	−0.019
			$T_{ss}(4)$		
50	0.029	0.028	0.0005	8.6×10^{-4}	−0.045
100	0.16	0.15	0.0041	0.0038	−0.175
200	0.58	0.54	0.020	0.016	−0.36
300	1.05	0.97	0.040	0.044	−0.39

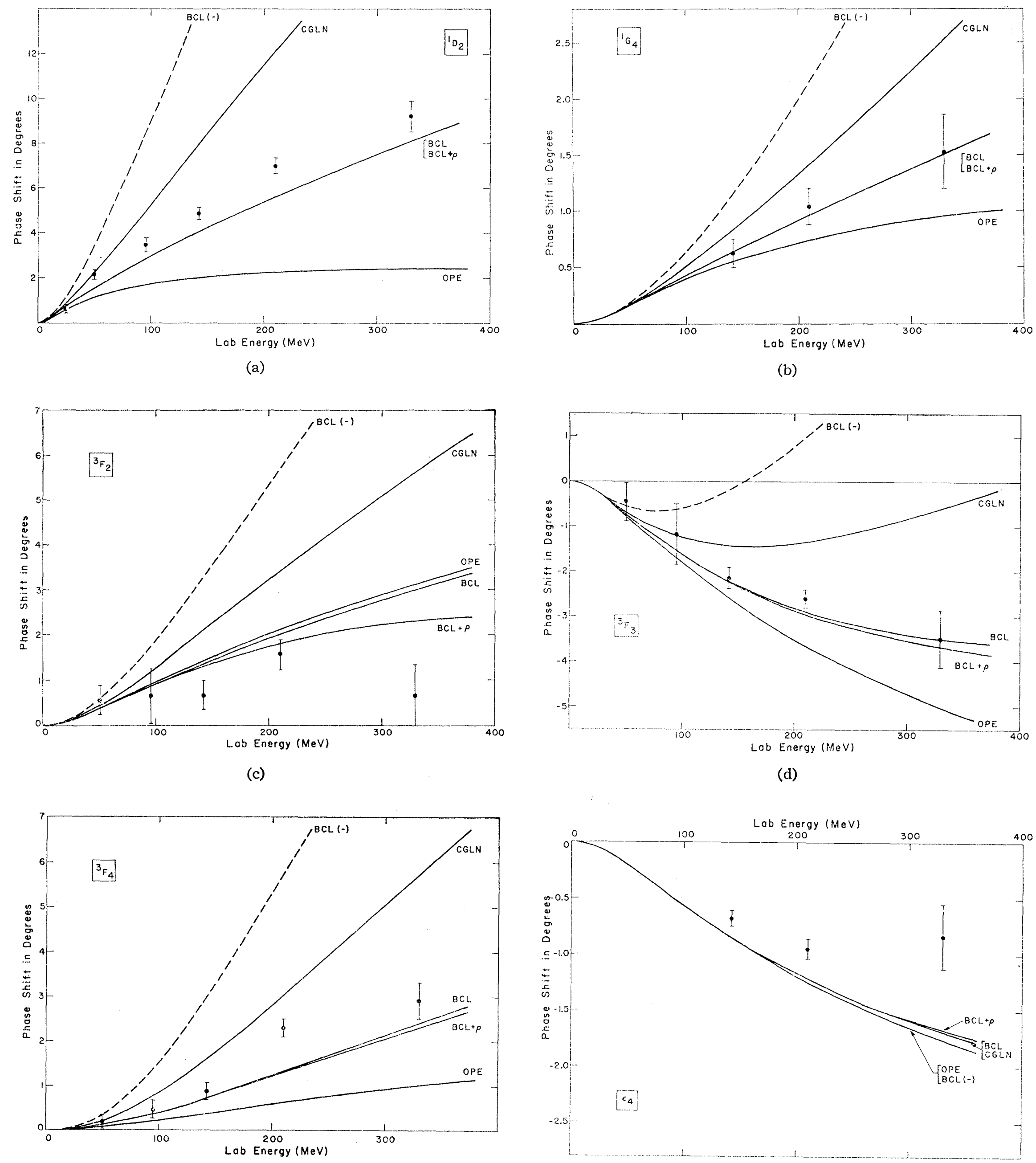
in the "small" matrix elements, T_{01} and T_{1-1}, the p wave is a significant part. However, one would not expect the s wave to contribute strongly to these matrix elements, and since they are so small, they have little effect on the phases. The (incorrect) ALV "2π basic" values are included for comparison.

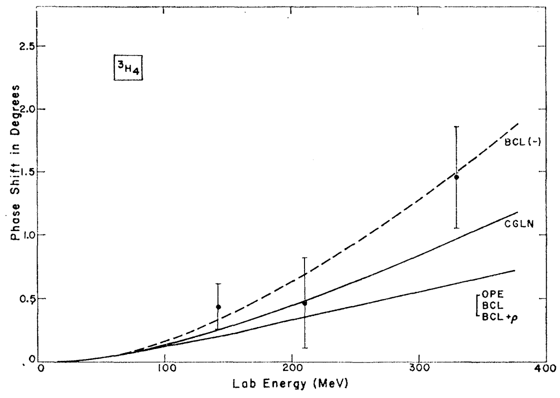
C. Sensitivity of the Results to the Cutoff

The question of whether and where to cut off the t-channel integration is not one which can be answered simply. There is the problem of the convergence of the Legendre expansion when continuing the πN amplitudes to the $N\bar{N}$-$\pi\pi$ channel. Since we cut off the expansion in the πN channel at $l=1$, there is no difficulty numerically in extending the series outside the region of convergence. However, one is reluctant to go too far since the higher waves might then become quite important. In addition, there is the problem of the onset of the three- and higher-pion cuts. There is no apparent reason why these should not be as large as the two-pion cut as we go to higher values of t, so clearly the calculation of the two-pion contributions from very high energy intermediate states does not appear to be meaningful. If we argue that the three-pion cut should be small for some region near threshold and combine that with the preceding arguments, we arrive at a value for the cutoff in the range $12–16\mu^2$.[10]

The hope is that for low energies and high angular momenta in the NN channel, the results will be relatively independent of the precise value of the cutoff. For the case we are considering, it turns out that this is true for NN D waves up to lab energies of about 100 MeV and for G waves up to about 300 MeV. However, this is mainly due to the dominance of the $N\bar{N}$ s waves. If one considers only the d+higher waves, the range of cutoff independence of the results shrinks markedly. This is illustrated in Figs. 2 and 3. One can see then that the "2π basic" amplitudes are not very cutoff-independent.

[10] The limit of convergence of the Legendre expansion from the πN to the NN-$\pi\pi$ channel is not precisely fixed, being dependent upon the value of the s-channel (πN) integration variable in the double integrals of the Mandelstam representation. For the s variable in the range of the mass squared of the N^*, the value $16\mu^2$ is a rough-average upper limit.





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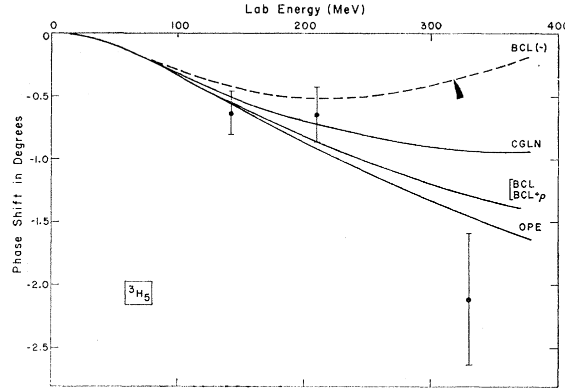

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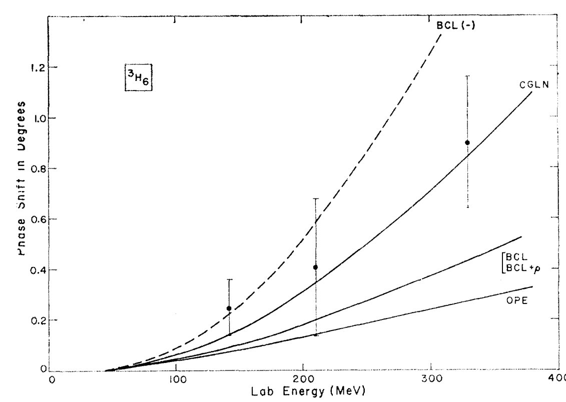

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FIG. 4. $T=1$ phase shifts predicted by modifications of the $N\bar{N}$-$\pi\pi$ s-wave amplitude in the model. Experimental phases are those of Arndt and MacGregor (Ref. 11).

D. Phase-Shift Predictions

In Figs. 4 and 5 (see Ref. 11) are shown the phase shifts for the higher partial waves computed on the basis of several models. The curves labeled CGLN are those predictions derived from the unmodified complete 2π contribution presented in Table I, plus one-pion exchange. The curves labeled BCL are obtained by modifying the πN and $N\bar{N}$-$\pi\pi$ s waves via the introduction of the BCL constant C_A^+. The curves labeled BCL$+\rho$ illustrate the effect of introducing the ρ as a discrete contribution using the coupling constants given in the analysis of Ball, Scotti, and Wong.[4] If one takes into account the experimental width of the ρ, then except for the sign of the amplitude one may calculate the effect of the ρ resonance on πN scattering in the manner of BCL.[9] It can easily be shown that the above choice of ρN coupling constants with the physical mass and width of the ρ has the same effect on the πN s-wave scattering lengths as the lower mass ρ resonance used by BCL. The small effect of the ρ on the NN phases is due to the strong interference between the vector and tensor parts of the ρN interaction.

The curve labeled BCL$(-)$ is obtained by (incorrectly) reversing the sign of C_A^+ in order to obtain agreement with the results of the original ALV calculation. That the BCL modification should reduce the 2π s-wave contribution is reasonable because that is just what it does for the s waves in πN scattering. That is, if the πN s waves yield a strong contribution to the $N\bar{N}$-$\pi\pi$ amplitudes, then a reduction of the πN s-wave amplitude will reduce the $N\bar{N}$-$\pi\pi$ s-wave amplitude. The dominant influence of the πN s wave on the $N\bar{N}$-$\pi\pi$ amplitudes is graphically demonstrated in Figs. 4 and 5 since by simply changing the sign of C_A^+ we can halve or double the CGLN contribution.

We have not subtracted out the 2π p-wave continuum as ALV did for two reasons. First, the p-wave continuum is not now so large that its inclusion will significantly alter any agreement or disagreement with experiment, and second, recent work on the nucleon isovector form factor[12] indicates that it is desirable to retain the p-wave contribution. This is because it has the effect of restoring agreement between the physical ρ mass and the mass estimated from the one-pole approximation of Frazer and Fulco.[13] That is, the p-wave continuum added to a resonant ρ peak at its physical mass has the same effect as shifting the peak to a lower, unphysical mass, while disregarding the p-wave continuum.

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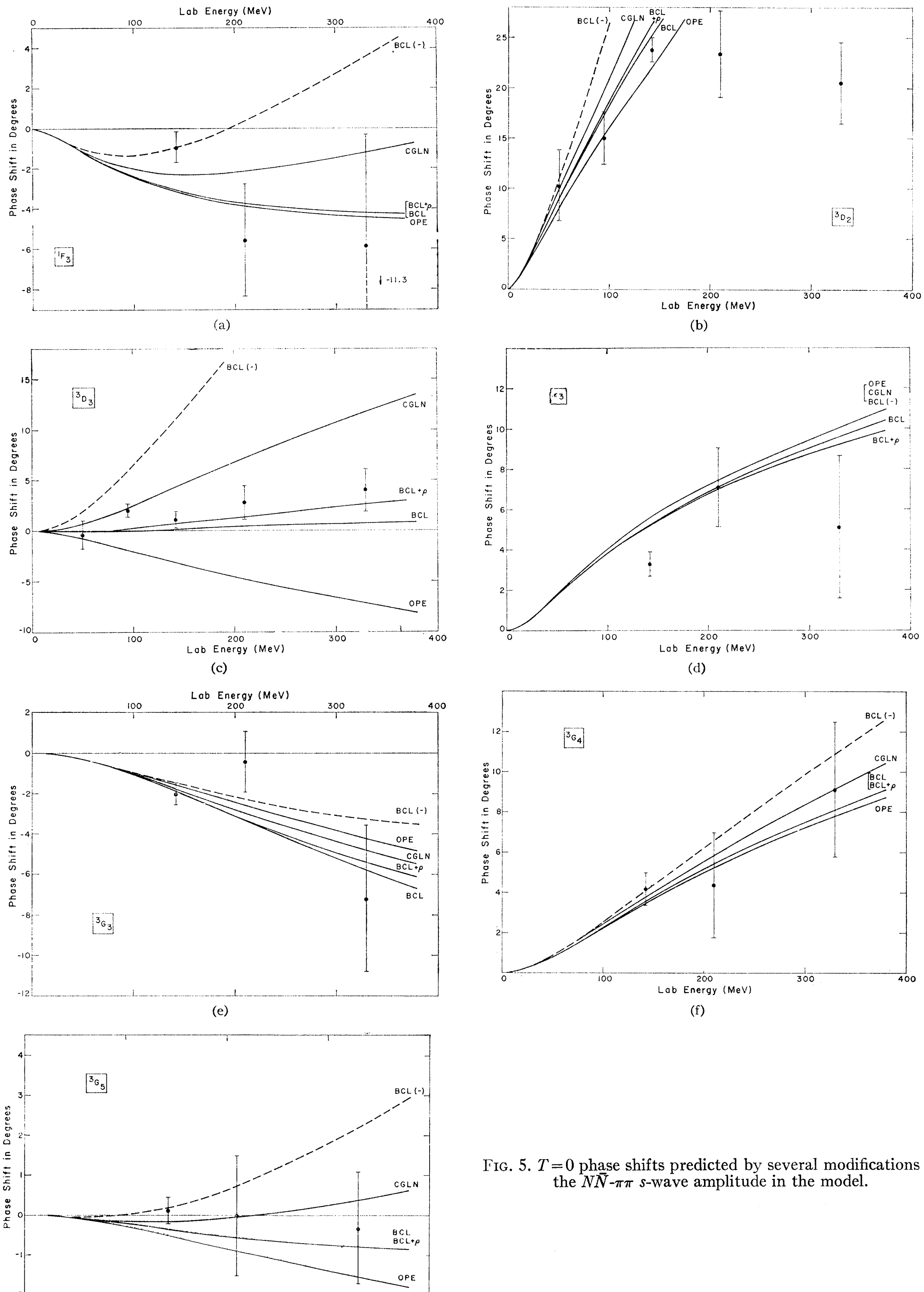

FIG. 5. $T=0$ phase shifts predicted by several modifications of the $N\bar{N}$-$\pi\pi$ s-wave amplitude in the model.

III. DISCUSSION

Concerning the "complete" 2π contribution as presented in Table I, there is little more to be said. The results are there, they agree with the ALV results at the corresponding stage of the calculation, and they agree with the results of Furuichi and Watari.[6] We feel confident that there are no errors in this part of the calculation.

As shown in Table II, there is a wide discrepancy between the 2π continuum minus the s and p waves, and the "2π basic" of ALV. This has been traced to the very large p-wave subtraction calculated by ALV which, incidentally, led them to replace their p-wave continuum contribution by the discrete ρ contribution in their final calculations. Here we have shown that both the p and $d+$higher wave contributions are small compared to the s wave and in fact could be ignored. The subtraction of an erroneously large p-wave contribution would leave a sizeable high-wave contribution of the opposite sign: this is what ALV found. ALV's error in the sign of the BCL constant then had the effect of restoring agreement with experiment for $T=1$ phases. Again, the results of our calculation are corroborated by those of Furuichi *et al.*

The reason that the $d+$ higher wave continuum makes so little contribution to the NN amplitudes appears to lie in the cutoff value. For a cutoff of $16\mu^2$ in the $N\bar{N}$-$\pi\pi$ channel, the momentum of the pions is still so low that the d and higher wave, and even the p-wave phase shifts are very small. (Note that the peak in the $\pi\pi$ wave amplitude, the ρ, occurs at approximately $t=30\mu^2$.)

If the cutoff is increased, all of the contributions—s, p, and $d+$ higher wave—increase. However, the high-wave contributions increase far more rapidly than the s-wave contribution. This is simple to understand. The energy dependence of the higher partial waves is stronger at low to moderate energies than that for low waves. If we assume an interaction radius of $0.5\mu^{-1}$, a simple impact-parameter estimate shows that a cutoff in the neighborhood of $16\mu^2$ lies right in the region where the p- and d-wave phase shifts are becoming significant and where the amplitudes would be growing most rapidly. However, to increase the value of the cutoff does not appear to be a satisfactory procedure for the reasons previously stated and because it contradicts the fundamental assumption and restriction of the Cini-Fubini approximation: i.e., that the amplitudes for low energies be dominated by nearby singularities.

From the predictions presented in Figs. 4 and 5 it can be seen that the model which fits low-energy πN scattering best also seems to fit low-energy, higher partial-wave NN scattering best. Let us emphasize several points here. We have only two parameters at our disposal, the constant $C_A{}^+$ and the ρN vector coupling constant, and these have been adjusted to fit the πN s-wave scattering lengths. There has been no adjustment of any parameter for fitting NN data. Even the choice of the value of the cutoff is suggested by the model. Also, using the same model, the behavior of the nucleon isovector form factors can be explained.

Furthermore, the dominant feature of fitting the πN scattering lengths is the cancellation of two large numbers to produce a small number; hence the adjustable constants, and therefore the two-pion contribution, will be very insensitive to variation of the πN scattering lengths. Thus, readjusting the constants of the model to give agreement with the latest values of the scattering lengths[14] produces an estimated maximum change in the two-pion contribution of 4%, although the scattering lengths change by almost 50%. Therefore any further refinement in our knowledge of the πN scattering lengths would have little bearing on the results of this calculation.

All this adds up to a rather encouraging situation, but whether the agreement obtained is truly of a fundamental nature, or is a fortuitous conjunction of parameters is open to question.

IV. CONCLUSIONS

One may state with some degree of confidence that the calculation of the 2π continuum contribution based on the ALV model is now finished. Our results on the high-wave t-channel continuum contribution to the NN amplitudes support the *ad hoc* assumption of the one-boson-exchange model; namely, that these high waves can safely be ignored, at least for low-energy, high-wave nucleon-nucleon interactions up to 400 MeV in the laboratory.

However, we believe that the model is on shaky ground as a basis for theoretically predicting nucleon-nucleon phase shifts. In spite of the apparently reasonable agreement with experiment of the BCL variation of the model, one is justified in raising the following theoretical objection: The agreement is due solely to the dominance of the t-channel s waves, about which we have the least information and for which the model is least valid.[15] Qualitative agreement with the higher wave NN phases could be obtained from a variety of models since any $\pi\pi$-$N\bar{N}$ s-wave interaction at all will produce attraction in the NN channel, which, in almost all cases, is the desired effect. However, these models would be expected to have very different πN p-wave scattering lengths.

The validity of the model for calculating the high-

[14] V. K. Samaranayake and W. S. Woolcock, Phys. Rev. Letters **15**, 936 (1965).

[15] The BCL analysis contains no information about low-energy $\pi\pi$-$N\bar{N}$ scattering except that from the πN channel. The BCL constant is *assumed* to be a contribution to the πN amplitudes from distant cuts in the $\pi\pi$-channel s wave, while the influence of nearby $\pi\pi$ singularities has been neglected.

wave $N\bar{N}$-$\pi\pi$ contribution is also suspect due to the strong cutoff dependence.

Now that there are other known πN resonances, these could be incorporated into the model. This would require a reparametrization of the πN model which could lead to significant effects in the nucleon-nucleon channel. This could be one way in which to test whether the agreement with experiment is largely a matter of chance or whether the model's success demonstrates the consistency of all low-energy phenomena involving pions and nucleons.

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Weak Axial-Vector Currents and the Baryon Field*

Austin M. Gleeson
Physics Department, Syracuse University, Syracuse, New York
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We study a postulated commutation relation between the chiral charge and the nucleon field. This relationship is used to generate a sum rule between $g_{\rho\pi\pi}$, $g_{\pi NN^*}$, and g_r. We find that the ρ meson and N^* baryon are insufficient to saturate the sum rule. The addition of strong S-wave dipion scattering is found to be adequate to explain the discrepancies of the above sum rule.

During the past year, there has been a great deal of research activity centered upon the algebras generated by equal-time commutators of currents.[1] At the beginning of these studies, it was pointed out that the proposed commutation relations between the currents or their associated charges was the most natural extension of the concept of universality[2] between the vector and axial-vector parts of the weak interaction. In this paper we propose to extend this universality to include the commutators of chiral charges and unobservables such as the baryon field.

The weak Lagrangian is assumed to be of the form of a current times a current. The current responsible for the weak decays is considered to be the sum of a vector and an axial-vector part. The charges associated with these currents are

$$Q^+(t)\equiv -i\int d^3x V_4(\mathbf{x},t) \tag{1}$$

and

$$X^+(t)\equiv -i\int d^3x A_4(\mathbf{x},t)\,. \tag{2}$$

Imposing the conserved-vector-current hypothesis,[3] we can identify $Q^+(t)$ as one of the generators of isospin rotations. This identification allows us to write down the equal-time commutator of $Q^+(t)$ with the baryon field as

$$[Q^+(t),\psi_n(\mathbf{x},t)]=0\,, \tag{3}$$

$$[Q^+(t),\psi_p(\mathbf{x},t)]=-\psi_n(\mathbf{x},t)\,, \tag{4}$$

where $\psi_{p(n)}(\mathbf{x},t)$ is the renormalized proton (neutron) field. We restrict ourselves to equal times in the presence of possible symmetry breaking. Invoking universality, we assume that

$$[X^+(t),\psi_n(\mathbf{x},t)]=0\,, \tag{5}$$

and

$$[X^+(t),\psi_p(\mathbf{x},t)]=-\gamma_5\psi_n(\mathbf{x},t)\,. \tag{6}$$

This appears as the simplest form consistent with space-time symmetries. We also point out that this form is implied by the free-field form of these operators.

For the future evaluation of the matrix elements which arise, we require the partially conserved axial-vector-current hypothesis (PCAC). We use PCAC in the form

$$\frac{d}{dt}X^+(t)=\mu^3 f_\pi\int d^3x\phi^+(\mathbf{x},t)\,, \tag{7}$$

where f_π is determined from the pion decay rate to be $f_\pi=0.935$.

Taking the matrix elements of (6) between a π^+ and

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[1] M. Gell-Mann and Y. Ne'eman, Ann. Phys. (N. Y.) **30**, 360 (1964); M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).

[2] N. Cabibbo, Phys. Rev. Letters **10**, 351 (1963).

[3] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).