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CP Violation with $|\Delta T| > \frac{1}{2}$: A Re-examination of Truong's Model for $K_L{}^0 \to 2\pi$

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The possibility of the failure of CP invariance in an electromagnetic interaction provokes interest in the possibility that CP-violating amplitudes with $|\Delta T| > \frac{1}{2}$ play a significant role in the decays $K_L{}^0 \to \pi^+\pi^-$ and $K_L{}^0 \to 2\pi^0$. Such a model has been invented by Truong. We give a reformulation of this model which is characterized by an exceedingly simple derivation. The assumptions or approximations made at any point can be clearly stated and simply understood. Using a representative value of the pion-pion S-wave coupling parameter, and the measured branching ratio $|\eta^{+-}|^2$ for $K_L{}^0 \to \pi^+\pi^-$ to $K_S{}^0 \to \pi^+\pi^-$, we estimate on the basis of this model (I) the branching ratio for $K_L{}^0 \to 2\pi^0$ to $K_L{}^0 \to \pi^+\pi^-$ to be about 1.8; (II) the branching ratio for $K_L{}^0(K_S{}^0) \to l^- + \pi^+ + \bar{\nu}_l$ to $K_L{}^0(K_S{}^0) \to l^+ + \pi^- + \nu_l$ to be about 1 ± 0.0001 (with $\Delta S = \Delta Q$); (III) the magnitude of the mass difference between $K_S{}^0$ and $K_L{}^0$ to be $|\Delta m| = |m_S - m_L| \cong 0.42$, in units of the $K_S{}^0$ decay rate; (IV) the phase of η^{+-} to be η (or $\pi + \eta$) $\cong 1.2$ radians for $\Delta m > 0$, or $\cong 1.9$ radians for $\Delta m < 0$. The first two numbers are significantly different from those given earlier by Truong. The predictions (I) and (II) differ decidedly from the corresponding predictions of the theory with "superweak," CP-violating, $|\Delta S| = 2$ interactions. If the amplitude for $K_S{}^0 \to 2\pi$ is depressed by $SU(3)$ symmetry by a factor $\cong 3$, we find that the real and imaginary parts of the $|\Delta T| > \frac{1}{2}$ amplitude in $K^0(\bar{K}^0) \to 2\pi$ are comparable in magnitude, as might be expected if they arise from a CP-violating electromagnetic correction to a $|\Delta T| = \frac{1}{2}$ interaction, or, more generally, if CP violation is intrinsic to whatever causes $|\Delta T| > \frac{1}{2}$.

I. INTRODUCTION

THE question of the CP invariance of electromagnetic interactions has recently been raised.[1,2] It was found[2] that the experimental evidence is not inconsistent with a significant failure of CP and T invariance in an electromagnetic interaction. This question was raised[1,2] specifically in connection with the observation[3-7] of the decay $K_L{}^0 \to \pi^+\pi^-$. In Ref. 1 it was argued that a CP-noninvariant electromagnetic interaction could provide a radiative correction to a CP invariant weak interaction satisfying the leptonic and nonleptonic $|\Delta T| = \frac{1}{2}$ rules and that this could lead to $|\Delta T| > \frac{1}{2}$ nonleptonic decay amplitudes which would violate CP and perhaps play a significant role in $K_L{}^0 \to \pi^+\pi^-$. Of course, such CP-violating radiative corrections would also occur in the off-diagonal elements of the mass-matrix,[8] p^2 and q^2, which serve to define the states $K_S{}^0$ and $K_L{}^0$.

From a less specific point of view, the possibility of CP violation in $|\Delta T| > \frac{1}{2}$ amplitudes can be examined without prejudice as to the origin of these amplitudes. They might be intrinsic to the weak interaction itself. This is the viewpoint taken by Truong,[9] who proceeded to construct and parametrize such a model and to estimate its consequences.

In this paper Truong's model is reformulated. The derivation is exceedingly elementary, which befits the essential simplicity of the model. The approximations that are made are clearly stated. The two essential parameters of the model are estimated and from these, four quantities, presently being measured in several experiments, are calculated. The resulting estimates for these quantities on the basis of this model are: (I) the branching ratio for $K_L{}^0 \to 2\pi^0$ to $K_L{}^0 \to \pi^+\pi^-$ is about 1.8; (II) the branching ratio for $K_L{}^0(K_S{}^0) \to l^- + \pi^+ + \bar{\nu}_l$ to $K_L{}^0(K_S{}^0) \to l^+ + \pi^- + \nu_l$ is about 1 ± 0.0001 (with $\Delta S = \Delta Q$); (III) the magnitude of the mass difference between $K_S{}^0$ and $K_L{}^0$ is $|\Delta m| = |m_S - m_L| \cong 0.42$, in units of the $K_S{}^0$ decay rate; (IV) the phase η of the amplitude for $K_L{}^0 \to \pi^+\pi^-$ relative to that of the amplitude for $K_S{}^0 \to \pi^+\pi^-$ satisfies η (or $\pi + \eta$) $\cong 1.2$ radians for $\Delta m > 0$, or $\cong 1.9$ radians for $\Delta m < 0$. Results (I) and (II) are different from those initially obtained by Truong.[9] This is due, in part, to calculational errors in Ref. 9.[10]

In the next two sections we give the model and calculate estimates of its experimental consequences. In the last section, we discuss the possible value of the model and its manifest shortcomings.

II. MODEL FOR $K_L{}^0 \to 2\pi$

The basic assumption is to attribute a significant CP violation to a $|\Delta T| > \frac{1}{2}$ interaction assumed to be in-

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[8] C. N. Yang and T. T. Wu, Phys. Rev. Letters **13**, 380 (1964). We shall adhere, as closely as possible, to the phenomenological notations defined in this paper.
[9] T. N. Truong, Phys. Rev. Letters **13**, 358 (1964).
[10] Private communication from T. N. Truong. I thank Professor Truong for a helpful conversation. It is my understanding from Professor Truong that these points have been made independently by Professor L. Wolfenstein.

volved in the decays $K^0 \to 2\pi$, $\bar{K}^0 \to 2\pi$. Such an interaction will lead to the $T=2$, S-wave state of two pions, with amplitudes defined as follows[8]:

$$A_2 e^{i\delta_2} = a_2(\bar{p}/2m)^{1/2} \cos\delta_2 e^{i(\delta_2+\varphi)} = \langle(-)2\pi(T=2)|H_W|K^0\rangle, \quad (1a)$$

$$A_2^* e^{i\delta_2} = a_2(\bar{p}/2m)^{1/2} \cos\delta_2 e^{i(\delta_2-\varphi)} = \langle(-)2\pi(T=2)|H_W|\bar{K}^0\rangle. \quad (1b)$$

H_W is the weak interaction Hamiltonian; φ is the CP-violating phase; δ_2 is the pion-pion scattering phase shift in the $T=2$, S-wave state at total center-of-mass energy m, the K-meson mass; $\bar{p}$ is the pion momentum at this energy. The amplitude a_2 is real, and δ_2 is approximated as real. The real[8] amplitude $A_0(a_0)$ for decays into the $T=0$, S-wave state of two pions, with phase shift δ_0, is defined by

$$A_0 e^{i\delta_0} = a_0(\bar{p}/2m)^{1/2} \cos\delta_0 e^{i\delta_0} = \langle(-)2\pi(T=0)|H_W|K^0\rangle = \langle(-)2\pi(T=0)|H_W|\bar{K}^0\rangle. \quad (2)$$

The factors $e^{i\delta_j}\cos\delta_j (j=0,2)$ will be recognized as simply the Gell-Mann–Watson[11] final-state factors that were generalized by Takeda[12] and by Dalitz and Tuan[13] to the two-particle decay of a system at rest. Now it is assumed that φ is a constant. Further, for the moment, we treat a_0 and a_2 as constants. We will modify this assumption in a particular way in the paragraph following Eq. (16) of this section. The latter assumption is quite restrictive because the amplitude a_j is the off-diagonal element (without the CP-violating phase) of the 2×2 K matrix, K_j, formed from the K^0 (or $\bar{K}^0$) and the two-pion state.[13] All of the elements of K_j have a pole at a dynamical resonance in the two-pion system. We will, in fact, implicitly assume no resonant or near-resonant behavior for δ_0 and δ_2, at least at "low" energies, of order m. This assumption, and the model of pion-pion S-wave scattering discussed below which embodies it, do not blatantly contradict any experimentally *established* knowledge of this system.

The short- and long-lived states are given by

$$|K_S^0\rangle = (2)^{-1/2}(p|K^0\rangle + q|\bar{K}^0\rangle), \quad (3a)$$

$$|K_L^0\rangle = (2)^{-1/2}(p|K^0\rangle - q|\bar{K}^0\rangle), \quad (3b)$$

with[8]

$$q^2 = A_0^2 + A_2^2 + x_l + iy_l + x_{3\pi} + iy_{3\pi} + iM_r - M_i, \quad (4a)$$

$$p^2 = A_0^2 + (A_2^*)^2 + x_l - iy_l + x_{3\pi} - iy_{3\pi} + iM_r + M_i. \quad (4b)$$

The next assumption is to set $y_l = y_{3\pi} = 0$ ($y_l = 0$ would, of course, follow from a $\Delta S = \Delta Q$ rule). Thus, CP violation here resides in $\varphi \neq 0$, and/or $M_i \neq 0$. M_i will be nonzero, and it is in fact, the essence of the model to compute M_i, under the further *assumption* that dominant contributions to $(iM_r - M_i)$ come from the $T=0$ and $T=2$ two-pion states and can be expressed in terms of A_0, A_2, δ_0, and δ_2. From the viewpoint of CP-violating radiative corrections there will *surely* be other contributions to M_i; these are simply neglected (as is $y_{3\pi}$) in the hope that they are small.[14] With all these approximations, we have

$$p^2 = p_0^2 + p_2^2 e^{-2i\varphi}, \quad (5a)$$

$$q^2 = q_0^2 + q_2^2 e^{+2i\varphi}, \quad (5b)$$

$$p_j^2 = q_j^2 \quad j=0,2, \quad (5c)$$

$$2M_i = p^2 - q^2 + 2i|A_2|^2 \sin2\varphi, \quad (5d)$$

where the subscripts $j=0$ and $j=2$ denote the contributions from the $T=0$ and $T=2$ two-pion states, respectively.

Now we consider the following expression for the p_j^2, with μ the pion mass:

$$p_j^2 = +iI_j(m) = -\frac{ia_j^2}{4\pi}\int_{4\mu^2} \frac{|e^{i\delta_j}\cos\delta_j|^2[(s-4\mu^2)/s]^{1/2}ds}{(s-m^2-i\eta)}. \quad (6)$$

This is the elementary expression for the two-pion self-energy graph with the *assumption* that at vertices for emission (absorption) of the two pions by K^0 or $\bar{K}^0$ we have the factors $a_j e^{i\delta_j}\cos\delta_j (a_j e^{-i\delta_j}\cos\delta_j)$ for virtual pions, as well as for real pions. The problem is to express the quantity in Eq. (6) in terms of a model of pion-pion S-wave scattering.

Consider a model of this scattering defined by the Hamiltonian

$$H = H_0 + 4\pi\lambda \sum_{j,k=1}^{3} \phi_j^2 \phi_k^2, \quad (7)$$

with H_0 the free-field Hamiltonian and λ the dimensionless pion-pion S-wave coupling parameter. By well-known manipulations,[15] the equation satisfied by the T-matrix element in a given isotopic spin state j, for the scattering of pions of momenta $\mathbf{k}$ and $-\mathbf{k}$ into pions of momenta $\mathbf{p}$ and $-\mathbf{p}$, is found to be

$$\begin{aligned}T_k^j(p) &= 4\pi\langle(-)\mathbf{p},-\mathbf{p}|\lambda_j\phi^3|-\mathbf{k}\rangle\rho_k \\ &= 4\pi\langle-\mathbf{p}|\lambda_j\phi^2|-\mathbf{k}\rangle\rho_p\rho_k \\ &\quad + (4\pi)^2\langle-\mathbf{p}|\lambda_j\phi^3(2\omega_p - H + i\eta)^{-1}\lambda_j\phi^3|-\mathbf{k}\rangle\rho_p\rho_k \\ &\quad - (4\pi)^2\langle-\mathbf{p}|\lambda_j\phi^3[H + 2\omega_p - (\omega_p+\omega_k)]^{-1} \\ &\quad\quad \times\lambda_j\phi^3|-\mathbf{k}\rangle\rho_p\rho_k. \quad (8)\end{aligned}$$

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[11] M. Gell-Mann and K. M. Watson, Ann. Rev. Nucl. Sci. 4, 219 (1954).

[12] G. Takeda, Phys. Rev. **101**, 1547 (1956).

[13] R. H. Dalitz and S. F. Tuan, Ann. Phys. (N.Y.) **10**, 307 (1960).

[14] Recall that if one considers only the $T=0$ two-pion state with *constant CP* violating phase, then $K_L^0 \not\rightarrow 2\pi$. See S. Weinberg, Phys. Rev. **110**, 782 (1958). The failure of this argument for energy-dependent phases was first noted in R. G. Sachs and S. B. Treiman, Phys. Rev. Letters **8**, 137 (1962).

[15] See, for example, G. C. Wick, Rev. Mod. Phys. **27**, 339 (1955).

In Eq. (8), $\omega_k=(\mathbf{k}^2+\mu^2)^{1/2}$, $\omega_p=(\mathbf{p}^2+\mu^2)^{1/2}$, $\rho_k=(2\omega_k)^{-1/2}$, $\rho_p=(2\omega_p)^{-1/2}$, $\lambda_j/4=5\lambda$ for $j=0$ and $\lambda_j/4=2\lambda$ for $j=2$; ϕ^2 and ϕ^3 are merely symbolic for a product of two and three distinct pion field operators, respectively. The following approximations are now made on the right-hand side of Eq. (8): (a) expand in a complete set of eigenstates of H on one side of the energy denominators and retain only the "lowest" state—that with two pions (of momenta $\mathbf{q}$ and $-\mathbf{q}$, to be summed over); (b) in the energy denominator of the third term neglect quantities of order $(\omega_p+\omega_k)/(2\omega_p+2\omega_q)$; (c) treat the first term in the Born approximation. [Note that it is the four-pion intermediate state in the second term of Eq. (8) which contains the so-called "bootstrap" type of graph—this is therefore being neglected.] Then Eq. (8) reads

$$T_k{}^j(p)=4\pi\lambda_j(\rho_p\rho_k)^2-\int\frac{d\mathbf{q}(T_p{}^j(q))^*T_k{}^j(q)(4\omega_q)}{[(2\omega_q)^2-(2\omega_p)^2-i\eta](2\pi)^3}. \tag{9}$$

Define $h_j(\omega_p)$ by

$$T_k{}^j(p)=4\pi h_j(\omega_p)\lambda_j(\rho_p\rho_k)^2. \tag{10}$$

Then Eq. (9) reads

$$h_j(\omega_p)=1-\frac{2\lambda_j}{\pi}\int_\mu\frac{|h_j(\omega_q)|^2qd\omega_q}{(2\omega_q)^2-(2\omega_p)^2-i\eta}. \tag{11}$$

The well-known solution is

$$h_j(\omega_p)=\left[1+\frac{\lambda_j}{2\pi}\int_\mu\frac{qd\omega_q}{(\omega_q{}^2-\omega_p{}^2-i\eta)}\right]^{-1}. \tag{12}$$

We assume that cutoff functions implicit in the interaction Hamiltonian in Eq. (7) and in the weak vertices of the self-energy graph, as represented for *illustration* by a cutoff M on the integrals in Eqs. (6), (9), (11), and (12), serve to define a finite result. From Eqs. (10) and (12) and the following expression for the differential cross section

$$\frac{d\sigma_j}{d\Omega}=\left|\frac{e^{i\delta_j}\sin\delta_j}{p}\right|^2=\frac{1}{(2\pi)^2}\frac{\omega_k}{2k}|T_k{}^j(p)|^2p^2\frac{dp}{dE}\bigg|_{E=2\omega_p=2\omega_k}, \tag{13}$$

we have

$$h_j(\omega_p)=-\frac{4\omega_p}{\lambda_jp}e^{i\delta_j}\sin\delta_j, \tag{14a}$$

$$\tan\delta_j=-\frac{\lambda_jp}{4\omega_p}\left[1+\frac{\lambda_j}{2\pi}P\int_\mu^M\frac{qd\omega_q}{\omega_q{}^2-\omega_p{}^2}\right]^{-1}=-\frac{\lambda_j{}^r(\omega_p)p}{\omega_p},$$

with

$$\lambda_j/4=\left\{\lambda_j{}^r(\omega_p)\Big/\left[1-\frac{2\lambda_j{}^r(\omega_p)}{\pi}P\int_\mu^M\frac{qd\omega_q}{\omega_q{}^2-\omega_p{}^2}\right]\right\}=Z_j(\omega_p)\lambda_j{}^r(\omega_p). \tag{14b}$$

Assume $\lambda_j{}^r(m/2)\cong\lambda_j/4$, for simplicity. Substituting from Eq. (14a) for h_j into Eq. (11), and noting the approximation in the preceding sentence, we have for $2\omega_p=m$ and $2\omega_q=\sqrt{s}$

$$Z_j{}^{-1}(m/2)\cot\delta_j(e^{i\delta_j}\sin\delta_j-Z_j(m/2)\tan\delta_j)=-\frac{\lambda_j{}^r(m/2)Z_j(m/2)}{\pi}\int_{4\mu^2}^{4M^2}\frac{|(4\omega_q/q\lambda_j)e^{i\delta_j}\sin\delta_j|^2[(s-4\mu^2)/s]^{1/2}ds}{(s-m^2-i\eta)}, \tag{15}$$

or

$$-i(\bar{p}/2m)e^{i\delta_j}\cos\delta_j\cong-\frac{1}{4\pi}\int_{4\mu^2}^{4M^2}\frac{|Z_j{}^{-1}(\omega_q)e^{i\delta_j}\cos\delta_j|^2[(s-4\mu^2)/s]^{1/2}ds}{s-m^2-i\eta}. \tag{16}$$

Consider now the modification of the integrand on the right-hand side of Eq. (6) that would result from modifying the constant K-matrix elements a_j to include in *perturbation theory* the *sum* of any number of two-pion "bubbles" following, or preceding, one of the two weak vertices. Expanding the expression for $Z_j{}^{-1}(\omega)$ from Eq. (14a) in powers of $\lambda_j/4$ we immediately recognize this series as the desired sum. Thus, such a modification would result in inserting a factor of $Z_j{}^{-2}(\omega_q)$ into the integrand of Eq. (6). (Such a modification, in dispersion theory, results in insertion of the square of the Muskhelishvili-Omnes factor,

$$[\cos\delta_j(s)]^{-1}\exp\left(\frac{P}{\pi}\int_{4\mu^2}\frac{\delta_j(s')ds'}{s'-s}\right).$$

Upon expanding the exponential factor to lowest order in $\lambda_j{}^r$, using $\delta_j\approx\tan\delta_j$, one shows the essential equivalence of this to $Z_j{}^{-1}(s)\cos\delta_j(s)$, for *slowly varying*, small phase shifts.)

Comparing the right-hand of Eq. (16) with that of the modified Eq. (6) we obtain an approximate relationship between the self-energy part and the pion-pion scattering,

$$p_j^2=q_j^2=a_j^2(\bar{p}/2m)e^{+i\delta_j}\cos\delta_j. \tag{17}$$

Equation (17) represents the essence of this simple model derived in an elementary manner, with the many approximations rather manifest.

Since the quantity $|A_2|$ can be estimated from the rate for $K^+\to\pi^+\pi^0$, the parameters of the model are λ and φ. In the next section we estimate φ from experiment, and choosing a representative value of λ, we calculate from the model several quantities to be measured by further experiments.

III. CALCULATIONS AND RESULTS

We estimate $|A_2/A_0|$ from the experimental ratio of the rate for $K^+\to\pi^+\pi^0$ to that for $K_S{}^0\to 2\pi$,

$$\gamma=|A_2/A_0|^2\approx x/630, \tag{18}$$

where x is a factor <1 arising from the *possible* $SU(3)$ depression[16,17] of the amplitude for $K_S{}^0\to 2\pi$. From Eqs. (1), (2), (4), (5), and (17), we have[8]

$$\frac{q}{p}=\frac{1-\frac{1}{2}\epsilon}{1+\frac{1}{2}\epsilon}\cong 1-\epsilon\approx 1-ie^{+i\Delta}|A_2/A_0|^2(\cos\delta_0/\cos\delta_2)\sin 2\varphi, \tag{19a}$$

with

$$\Delta=(\delta_2-\delta_0),$$

$$\eta^{+-}=\frac{\epsilon}{2}+\frac{i}{\sqrt{2}}e^{i\Delta}\,\mathrm{Im}(A_2/A_0), \tag{19b}$$

$$M_i=+|A_2|^2\tan\delta_2\sin 2\varphi, \tag{19c}$$

$$M_r=m_S-m_L=(2i)^{-1}(p^2+q^2-2A_0{}^2-2|A_2|^2\cos 2\varphi-2x_l-2x_{3\pi})\approx -i(p_0{}^2-A_0{}^2)\approx +A_0{}^2\tan\delta_0. \tag{19d}$$

(A) Because of the smallness of ϵ given by Eq. (19a), we can obtain an estimate of φ to first order in γ from Eq. (19b), neglecting ϵ.

$$\frac{\mathrm{Rate}(K_L{}^0\to\pi^+\pi^-)}{\mathrm{Rate}(K_S{}^0\to\pi^+\pi^-)}=|\eta^{+-}|^2\approx\tfrac{1}{2}|\mathrm{Im}(A_2/A_0)|^2=\left(\frac{x}{1260}\right)\sin^2\varphi. \tag{20}$$

From the experimental result,[3] $|\eta^{+-}|^2\cong 3.6\times 10^{-6}$, we obtain, for $x=1$, $|\varphi|\cong 0.067$ rad; for $x=\frac{1}{10}$, $|\varphi|\cong 0.2$ rad. (B) We must now take a value for the pion-pion coupling parameter, λ, which determines $\lambda_0{}^r$ and $\lambda_2{}^r$. [It is clear from Eq. (19d) that λ fixes Δm, or can be fitted to Δm; we consider that Δm has not been experimentally established.] As a representative value, not inconsistent with what is known from strong-interaction experiments relating to the pion-pion S-wave interaction at low energies,[18] we take $\lambda=\mp 0.2$. Then $\lambda_0{}^r(m/2)\cong\mp 1$, $\lambda_2{}^r(m/2)\cong\mp 0.4$, and we expect $\Delta m\cong\pm 0.42\,\Gamma_S$, where the upper sign refers to an attractive pion-pion interaction, and the lower sign to a repulsive pion-pion interaction. (C) We estimate[19] the branching ratio, b_L, for $K_L{}^0\to 2\pi^0$ to $K_L{}^0\to\pi^+\pi^-$

$$\begin{aligned} b_L&=\frac{1}{2}\left|\frac{\eta^{00}}{\eta^{+-}}\right|^2=\frac{1}{2}\left|\frac{\gamma(\cos\delta_0/\cos\delta_2)\sin 2\varphi-2\sqrt{2}(\gamma)^{1/2}\sin\varphi}{\gamma(\cos\delta_0/\cos\delta_2)\sin 2\varphi+\sqrt{2}(\gamma)^{1/2}\sin\varphi}\right|^2,\\ &\cong\frac{1}{2}\left|\frac{(2\sqrt{\gamma})(\cos\delta_0/\cos\delta_2)-2\sqrt{2}}{(2\sqrt{\gamma})(\cos\delta_0/\cos\delta_2)+\sqrt{2}}\right|^2\cong 1.75\quad\text{for}\quad x=1,\end{aligned} \tag{21}$$

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[16] N. Cabbibo, Phys. Rev. Letters **12**, 62 (1964).
[17] M. Gell-Mann, Phys. Rev. Letters **12**, 155 (1964).
[18] See, in particular, H. J. Schnitzer, Phys. Rev. **125**, 1059 (1962).
[19] We assume here $a_2/a_0>0$ (with $-\pi/2\leq\varphi\leq\pi/2$). The assumption $a_2/a_0<0$ gives $b_L\cong 2.3$.

where[8]

$$\eta^{00}=\tfrac{1}{2}\epsilon-i\sqrt{2}e^{i\Delta}\,\mathrm{Im}(A_2/A_0).$$

(D) We estimate the phase η of η^{+-}. We have

$$|\eta^{+-}|e^{i\eta}\approx\tfrac{1}{2}e^{i(\pi/2+\Delta)}\{\gamma(\cos\delta_0/\cos\delta_2)\sin2\varphi+\sqrt{2}(\gamma)^{\frac{1}{2}}\sin\varphi\},$$

or

$$\eta \quad \text{or} \quad \pi+\eta\cong(\pi/2)+\Delta. \tag{22}$$

Taking into account the unknown sign of Δm, we calculate for $x=1$

$$\begin{aligned}\eta \quad \text{or} \quad \pi+\eta &\approx 1.20 \text{ radians for } \Delta m>0,\\ &\approx 1.95 \text{ radians for } \Delta m<0.\end{aligned} \tag{23}$$

(E) Finally, we estimate the magnitude of the charge asymmetry, $|r|$, defined as one minus the ratio of the rate for $K_L^0(K_S^0)\to l^-+\pi^++\bar{\nu}_l$ to that for $K_L^0(K_S^0)\to l^++\pi^-+\nu_l$.

$$\begin{aligned}|r|&=|1-|q/p|^2|\\ &\cong|2\gamma(\cos\delta_0/\cos\delta_2)\sin\Delta\sin2\phi|\cong0.013\%,\end{aligned}$$

for $x=1$. Thus, for all practical purposes, a zero charge asymmetry is expected from the particular mechanism of CP violation in the $|\Delta T|>\frac{1}{2}$ two-pion amplitude.

IV. DISCUSSION

We have given an elementary derivation of a model invented by Truong to illustrate the possible consequences of CP violation in the $|\Delta T|>\frac{1}{2}$ amplitudes for $K^0(\bar{K}^0)\to 2\pi$. The estimates for four quantities of experimental importance are summarized in (I)–(IV) of the Introduction—(I) and (II) would seem to be crucial tests of the model. Both of these predictions differ decidedly from the corresponding predictions of the rather appealing theory[20,21] with "super-weak," CP-violating, $|\Delta S|=2$ interactions. (See, however, the concluding paragraph below.)

Some remarks about the model: It is surely oversimplified, and many approximations are made. Yet, after the phase φ is fixed from the basic observation of $K_L^0\to\pi^+\pi^-$, the model makes semiquantitative estimates of some self-energy effects in terms of a single parameter, λ, which, in principle, is obtainable from and capable of correlating information on strong interactions in the pion-pion S-wave system. The rate for $K_L^0\to 2\pi^0$, the charge asymmetry, the mass difference, the phase of η^{+-}, are correlated through λ and φ. The self-energy effects are small and nearly the same results could be obtained from order-of-magnitude estimates using only the basic assumptions of $\varphi\neq0$, $\gamma\ll1$. Nevertheless, the model serves to illustrate what we feel is a useful point: if there is some $SU(3)$ suppression of the $K_S^0\to 2\pi$ rate ($x\approx10^{-1}$), then the $K^+\to\pi^+\pi^0$ rate *can* possibly be understood as arising from an electromagnetic correction. The CP violating phase, $|\varphi|\cong0.2$, is then such that the real and imaginary parts of the $|\Delta T|>\frac{1}{2}$ amplitude A_2 are at least comparable in magnitude.[22] This might not be unexpected if there were a significant failure of CP invariance in an electromagnetic interaction.[1,2] The $|\Delta T|>\frac{1}{2}$ interaction may well be intrinsic to the weak interactions.[8,9] In this case also, comparable real and imaginary parts of A_2 would imply that the CP violation is at least not a tiny part of a "weaker" interaction.

Finally, a somewhat evident word of caution: It is clear that the self-energy effects, as represented by M_i, could be much larger. For example, even within the framework of this model and using cutoff functions as representative of incomputable damping effects, the cutoff functions at the weak vertices of the two-pion self-energy graph may damp down the integral in Eq. (6) much more slowly than a corresponding function in the pion-pion interaction Hamiltonian does so for the integral in Eq. (16). The correspondence between the integrations over high energies would fail. Further, an energy-dependent phase may cause a contribution to M_i from the $T=0$, two-pion self-energy. Two consequences for the measurements of the increased significance of M_i (see e.g., Ref. 21) may well be (1) to lower b_L in the direction of the value of $\sim\frac{1}{2}$ for $|\Delta T|=\frac{1}{2}$; and (2) to give rise to a charge asymmetry 1 to 2 orders of magnitude larger than that estimated here.

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[20] L. Wolfenstein, Phys. Rev. Letters 13, 562 (1964).
[21] T. D. Lee and L. Wolfenstein, Phys. Rev. 138, 1490 (1965).

[22] Implicit in this argument is the assumption that an $SU(3)$ enhancement of the "true" ratio on the right-hand side of Eq. (18) by $1/x$ is accompanied by an $SU(3)$ *depression* of the "true" phase by about $\sqrt{x}$.