Diffraction Scattering and Form Factors*

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The asymptotic feature of diffraction scattering of hadrons is tentatively assumed to mean asymptotically pure-imaginary partial waves. An essential requirement is then inferred to be the existence of an infinity of reaction channels and asymptotically infinite inelasticity (assuming asymptotic vanishing of partial-wave amplitudes). This view is supported by construction of a physically sensible inelastic model which has all the above features. Finally, these assumptions are shown to imply asymptotic vanishing of form factors; this resolves a puzzling feature of conventional dispersion-theory solutions for form factors.

INTRODUCTION

OUR first aim in this paper will be to briefly explore the conjecture[1] that elastic partial-wave scattering amplitudes become pure imaginary asymptotically (as cm energy $\sqrt{s}\to\infty$); we refer to this behavior as diffraction scattering, henceforth DS. There are at least two motivations for considering this situation:

(a) Present high-energy elastic-scattering data[2] seem to imply that elastic forward total-scattering amplitudes become predominantly imaginary at high energy.

(b) In the Regge description of high-energy scattering, elastic scattering is dominated by the Pomeranchuk trajectory ("vacuon") exchange, which was constructed so as to give a pure imaginary forward elastic-scattering amplitude. (This provides a means for satisfying the Pomeranchuk theorems, as discussed in Ref. 3.) As a consequence of the above conditions and the general structure of Regge exchange amplitudes, it turns out that Pomeranchuk exchange gives partial-wave amplitudes exhibiting DS.

It should be noted that DS is neither necessary nor sufficient for the existence of strong forward peaking of scattering, or for the Pomeranchuk theorems, since both these aspects of high-energy scattering depend on the sum of all partial waves. For this reason, we do not intend to relate s- and t-channel behavior in the present discussion. We prefer to present heuristic arguments that an infinity of inelastic channels (and associated inelasticity in a one-channel N/D formalism) are essential features for DS from which DS could be inferred, via s-channel unitarity, as a possible result. These arguments are the contents of Sec. I. In Sec. II we exhibit an N/D model which simulates an infinity of inelastic channels, and results in asymptotic DS.

Another question we shall discuss is the asymptotic behavior of a form factor, such as the pion electromagnetic form factor. In this example, it has been noted[4] that if only elastic $\pi\pi$ scattering were possible, and the P-wave amplitude is expressible as N/D as usual, then $F=D(0)/D(s)$ satisfies the analyticity and unitarity requirements of the pion form factor. However, if we accept the convention, motivated by potential theory, that it is possible to normalize $D(\infty)\to$ constant [to within a $\ln(s)$ factor, say], then the above form factor F does not vanish[5] asymptotically [again, to within a $\ln(s)$ factor]. This conclusion also applies to a finite number of strong interaction channels coupled to $\pi\pi$. If we accept a possible $\ln(s)$ behavior of a typical elastic D function (as in Sec. II), then for N channels we might find that $F^{-1}\approx(\ln s)^N$. It is thus plausible that for an infinity of channels, F^{-1} might have the form $\sum a_n(\ln s)^n$ which is a possible $\ln(s)$ series expansion of s^r, so that F might indeed vanish asymptotically as a power of s. In Sec. III we shall develop another argument to show that an infinity of inelastic channels makes it possible for the D^{-1} type of solution for F to vanish asymptotically at least as fast as a power of s.

I. UNITARITY AND DS

We start by considering inelastic unitarity for an elastic two-body partial-wave amplitude, omitting irrelevant indices throughout:

$$\mathrm{Im}t\equiv t_I=R|t|^2, \tag{1}$$

where

$$R=1+\sum\frac{|t_{\text{inelastic}}|^2}{|t|^2}\times(\text{inelastic-channel phase space}).$$

The assumption that $t\to t_I$ (DS) therefore implies that for $s\to\infty$,

$$t_I\approx R^{-1}. \tag{2}$$

Our usual idea that partial-wave amplitudes satisfy unsubtracted dispersion relations implies that

$$t(s\to\infty)\to 0. \tag{3}$$

Notice that if the total amplitude exhibits an energy-independent forward peak ("nonshrinking"), then simple

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[1] P. Olesen and E. J. Squires, Nuovo Cimento 39, 956 (1956).

[2] See L. Van Hove, CERN lecture notes (1965), CERN Report No. 65-22 (unpublished).

[3] E. J. Squires, *Complex Angular Momentum and Particle Physics* (W. A. Benjamin, Inc., New York, 1963).

[4] S. D. Drell and F. Zachariasen, *Electromagnetic Structure of Nucleons* (Oxford University Press, London, 1961).

[5] G. F. Chew, Lawrence Radiation Laboratory (private communication).

partial-wave projection shows that partial-wave amplitudes tend to nonvanishing constants asymptotically. We therefore infer from Eq. (3) that we are dealing with an energy-dependent peak. The nature of the energy dependence can be elucidated by using the constraint that total cross sections tend to constants asymptotically; via the optical theorem this implies that elastic forward amplitudes grow as s^1. Examining the partial-wave projection equations then reveals that this latter condition implies a shrinking forward peak when Eq. (3) holds.

It follows from Eqs. (2) and (3) that DS requires that $R\rightarrow\infty$, which suggests that an infinity of reaction channels is relevant.

We now develop a second heuristic argument in support of the above assertion. We shall assume that all channels for which Pomeranchuk or "vacuum" exchange is possible (we henceforth refer to such channels as VE channels) are a power of s larger than non-VE channels, in each partial wave (as is the case for Regge two-body scattering). We may therefore consider only VE channels in the unitarity relations; if the contribution of non-VE channels is to become relevant, then in view of the above assumption an infinite number of non-VE channels would be necessary, proving our contention.

For a set of VE channels,

$$t_I^{ii}=\sum_n (t_R^{in})^2+(t_I^{in})^2. \tag{4}$$

We now heuristically assume that all VE amplitudes exhibit DS. It may be objected that there exist successful models of inelastic channels which do not exhibit DS; since these models deal with non-VE channels, no contradiction is involved in our assumption. With the further approximation that all t's are the same order of magnitude we finally obtain

$$t_I\approx 1/n(s)\,, \tag{5}$$

where $n(s)$ is the number of open VE channels at energy s. Therefore, because of Eq. (3),

$$n(s)\rightarrow\infty\,. \tag{6}$$

At this point one might question whether it was crucial to our argument to assume all t's are of similar magnitude; that this assumption can be modified without changing our conclusion can be seen as follows: We shall subdivide the set of open VE channels into two subsets: group (a), the set of channels with thresholds "near" the energy of interest, and group (b), comprising "lower energy threshold" channels.

For group (a) channels, the smallest allowed momentum transfer is likely to be large. For example in the two-particle process (mass μ+mass $\mu\rightarrow$ mass M+mass M) where $M\gg\mu$, we find that near the inelastic threshold $s=4M^2$, we have $|t|\geqslant M^2$, whereas for energies large compared to the rest masses involved, the minimum momentum transfer in the physical region approaches zero. Empirically, large-momentum-transfer processes seem to be very strongly suppressed, so that probably group (a) channels are less important than group (b) channels in our considerations of unitarity. This effectively means that in Eq. (5), $n(s)$ really counts the group (b) channels; our conclusion that $n(s)_{s\rightarrow\infty}\rightarrow\infty$ is unaffected, however.

II. SUFFICIENT CONDITIONS FOR DS: A MODEL

Thus far we have given only heuristic arguments about conditions which are necessarily implied by DS. In this section we shall employ N/D two-body partial-wave equations to examine possible situations which might suffice to give DS.

We shall therefore examine the ratio

$$X=t_I/t_R=-\mathrm{Im}D/\mathrm{Re}D$$
$$=\rho RN\bigg/\left[1-\frac{1}{\pi}\mathrm{P}\int_{s_1}^{\infty}\frac{\rho RN(s-a)}{(s'-s)(s'-a)}ds'\right]. \tag{7}$$

Here we subtracted D at $s=a$; R as usual is the inelasticity. We choose a pole-model force, with the left-hand cut of t being a δ function at $s=a$; we then find that independent of R,

$$N=G/(s-a)\,, \tag{8}$$

where G is a constant. The simplest model we might consider is that of a constant R, leading to

$$X=-RN(s)/[1-RI(s)]. \tag{9}$$

For large s (assuming that the integral does not tend to zero as $s\rightarrow\infty$) we thus find that X does not increase with increasing, large R.

The next complication we can study is contained in a system of n degenerate channels, with

$$N=f(s)G. \tag{10}$$

Here N and G are matrices, with G independent of s. For simplicity we can look at an "average" ratio X; we now find that

$$\bar{X}=\sum \mathrm{Im}t^{ii}/\mathrm{Re}t^{ii}$$

turns out to have the same value as for a single channel. Thus, this model also fails to guarantee that $X_{ii}=\mathrm{Im}t_{ii}/\mathrm{Re}t_{ii}$ increases with the number of channels, i.e., with the inelasticity.

We feel that these preliminary models lack an essential feature, namely, the existence of an *infinite* number of channels with thresholds *above* any given energy. We shall therefore construct a model having these features; this exercise is amusing in that it incorporates all physically reasonable features and predicts DS asymptotically.

The main assumption of our model is the form of R:

$$R-1=\sum_{n=1}^{\infty} A_n\theta(s-c_n). \tag{11}$$

The step functions admittedly introduce logarithmic singularities at the thresholds, but we tolerate this because the relevant features for our purposes are retained in this approximation, which has the virtue of giving simple expressions. We will disperse the amplitude[6] $t/p^2=N/D$, and take a one-pole model force so that we have

$$N(s)=G/(s-a). \tag{12}$$

We also use the approximation

$$\rho=\text{phase-space factor}=2p/\sqrt{s}\approx 1.$$

This is both convenient and accurate except when $p\to 0$.

From these specifications we now obtain, for $s \in (c_N,c_{N+1})$,

$$X(s)=(G\sum_1^N A_n)/Y, \tag{13a}$$

$$Y(s)=1-\frac{G}{\pi}\left[1+\frac{s-s_1}{s-a}\left\{\ln\left(\frac{s_1-a}{s-s_1}\right)-\sum_{n=1}^{\infty}A_n\ln\left|\frac{c_n-s}{c_n-a}\right|\right]\right.$$
$$\left.+\sum_1^{\infty}\frac{s_1-a}{c_n-a}A_n\right\}, \tag{13b}$$

where s_1 is the first channel threshold and Y is just ReD, with the above approximations used to simplify the integral. Notice that for any *finite* number of channels N_{total}, we find

$$X(s\to\infty)\approx N_{\text{total}}/N_{\text{total}}\times\ln(s)\to 0.$$

However, a radically new feature can emerge if we let $N_{\text{total}}\to\infty$. To illustrate this, we choose the following parameters for simplicity:

$$a=0;\quad c_n=n^2s_0;\quad A_n=A \quad \text{for all } n. \tag{14}$$

Consequently,

$$\sum_1^N A_n=AN_{\text{total open channels}}(s), \tag{15a}$$

$$\sum_1^{\infty}A_n\ln\left|\frac{c_n-s}{c_n}\right|=A\ln\left|\prod_{n=1}^{\infty}\left(1-\left(\frac{\pi(s/s_0)^{1/2}}{n\pi}\right)^2\right)\right|, \tag{15b}$$

and

$$s_1\sum_1^{\infty}\frac{A_n}{c_n}=A\frac{s_1}{s_0}\sum_1^{\infty}n^{-2}. \tag{15c}$$

[6] We are taking a P-wave amplitude as an example; this case typifies most extant calculations, and typically exhibits $D(s\to\infty)\approx\ln s$. This asymptotic behavior is in no way crucial to the model, however. For S-wave pole models, $D(s\to\infty)\to$ constant, and we find that the same model again gives DS.

With these parameters, each new channel provides an equal increment to the inelasticity (which is consonant with classical ideas of energy equipartition). The channel spacing would be typical of multi-pion channels, for instance, with $s_0=m_\pi^2$. For $s\to\infty$, we note that $R\simeq\sqrt{s}$, and also $Rp^2\times$numerator "N function"$\approx\sqrt{s}$. According to Olesen and Squires,[1] this behavior can result in DS. Furthermore, in a once-subtracted dispersion relation for D, the high-energy integrand in our model is proportional to $s^{-3/2}$. Thus, in spite of $R(s\to\infty)\to\infty$, the low-energy phenomena in this partial wave can still be dominated by long-range forces (i.e., the lower energy range of the integral for D).

The expressions of Eqs. (15) can be easily evaluated[7] with the result

$$Y=1-\frac{G}{\pi}\left[1-\frac{s-s_1}{s}\left\{\ln\frac{s-s_1}{s_1}+A\ln\left|\frac{\sin(\pi(s/s_0)^{1/2})}{\pi(s/s_0)^{1/2}}\right|\right\}\right.$$
$$\left.+A\frac{s_1}{s_0}\tfrac{1}{6}\pi^2\right]. \tag{16}$$

For $s\to\infty$ it follows that

$$X\approx\frac{\sqrt{s}}{\ln s}\to\infty, \tag{17}$$

as promised.

Again, we wish to emphasize the "cancellation" arising within the infinite series of channel contributions to Y. [Incidentally, if we took $\sqrt{c_n}=n$th root of a Bessel function, then in Eq. (16) the asymptotic behavior would turn out to be unchanged.]

Apparently the existence of channels opening up above any arbitrary energy gives essential features which cannot be obtained with any finite number of channels. This need not be disconcerting, since it is impossible to have one production channel without an infinity of many-body inelastic channels.

For the interested reader, we sketch in the Appendix the consequences of a choice of R with more appropriate threshold properties.

III. FORM FACTORS

In this section we shall adopt a "truncated" inelastic unitarity relation to examine the asymptotic behavior of a form factor F. This relation is

$$\text{Im}F\equiv F_I=rt^*F, \tag{18}$$

where

$$r=(\sum_{\text{all } n}t_{1n}^*F_{n1})/t_{11}^*F_{11};$$

[7] P. M. Morse and H. Feshbach, *Methods of Theoretical Physics* (McGraw-Hill Book Company, Inc., New York, 1953), p. 385.

here r can generally be a complex number. If we define

$$r=\rho e^{i\theta},$$
$$F=\mathfrak{F}e^{i\chi}, \tag{19}$$
$$t=\tau e^{i\omega},\ \theta,\ \chi,\ \omega \text{ functions of } s,$$

then Eq. (18) can be written

$$\mathfrak{F}\sin\chi=\tau\rho\mathfrak{F}e^{i(\theta+\chi-\omega)}. \tag{18$'$}$$

We emphasize the important consequence

$$rt^*=e^{-i\chi}\sin\chi. \tag{18$''$}$$

As long as the "inelasticity" r is finite, our usual assumptions about $t(s\to\infty)$ lead to the conclusion that $\chi\to 0$ as $s\to\infty$. However, if (and only if) r is asymptotically infinite, the possibility arises that rt^* does not vanish asymptotically, and therefore $\chi\to$ a nonzero constant as $s\to\infty$. To see the relevance of this, we additionally assume that F satisfies a dispersion relation (possibly once subtracted). The solution to Eq. (18) is then given, e.g., in Goldberger and Watson,[8] by

$$F(s)=F(0)\exp\left[\frac{s}{\pi}\int_s^\infty \frac{ds'\Phi(s')}{(s'-s)s'}\right], \tag{20a}$$

$$\tan\Phi=\frac{\mathrm{Re}(r^*t)}{1-\mathrm{Im}(r^*t)}=\tan\chi, \tag{20b}$$

that is,

$$\Phi=\chi.$$

Equation (20a) has the asymptotic behavior

$$F(|s|\to\infty)=F(0)|s|^{-\Phi(\infty)/\pi}e^{i\Phi(\infty)}[1+O(\ln s)\cdots]. \tag{21}$$

If r remains finite, then our above discussion implies that $F(\infty)\neq 0$. However, if $r(s\to\infty)\to\infty$, then it is possible to have $\chi(\infty)\neq 0$, which implies an asymptotically vanishing form factor [barring the possibility of $\chi(\infty)<0$, not reasonable physically].

An example, which is not necessarily realistic, is furnished by assuming that asymptotically $r\approx R$. In this case, DS implies that $\chi\to\pi/2$, so that asymptotically $F\approx 1/\sqrt{s}$. Asymptotic Regge behavior with Pomeranchuk trajectory exchange provides an example for such DS, giving rise to partial waves[9] of the form

$$\frac{1}{\ln s}\left[i-\frac{\pi}{2}\ln s\right].$$

Note that once we concede the possibility of $rt^*\not\to 0$, we also encounter the possibility of $rt^*=e^{i\chi}\sin\chi$ oscillating, with $\chi(s)$ increasing steadily as $s\to\infty$. Again, this is a feature not expected with a finite number of channels, but unfortunately we are now unlikely to be able to infer such behavior solely from our sparse knowledge of "t_{ni}."

[8] M. L. Goldberger and K. M. Watson, *Collision Theory* (John Wiley & Sons, Inc., New York, 1964).
[9] E. J. Squires, Nuovo Cimento **34**, 1277 (1964).

In particular, for $\chi(s)\approx\sqrt{s}$ asymptotically, the following behavior of F is possible:

(a) $F(s)$ does not vanish asymptotically but oscillates for $s\to+\infty$;
(b) $F(s)$ vanishes as $\exp[-\sqrt{|s|}]$ for $s\to-\infty$.

[The reader may convince himself that this is possible by noting the following identity,[10] relevant for evaluating Eq. (20a):

$$\mathrm{P}\int_0^\infty \frac{\sqrt{s'}\,ds'}{s'(s'-s)}\approx|s|^{-1/2},\quad s<0,\quad =0,\quad s>0,$$

and $F(s)\approx\exp(i\sqrt{s})$ everywhere.]

Currently, we do not have a good model for the form-factor inelasticity r (which probably will not be asymptotically the same as R). Nevertheless, the presence of asymptotically infinite "inelasticity" enables us to see how a form factor vanishes asymptotically, in principle, when calculated via present dispersion techniques.[11]

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APPENDIX: AN INELASTIC N/D MODEL

We briefly sketch the consequences of choosing an R which more appropriately preserves inelastic-threshold analyticity. For all P-wave channels an appropriate choice would be

$$R=\sum_n A_n\left(\frac{p_n}{p_1}\right)^3\theta(s-c_n). \tag{A1}$$

To provide easily integrable expressions we could approximate

$$\left(\frac{2p_1}{W}\right)\frac{p_1^2}{s'-a}\left(\frac{p_n}{p_1}\right)^3\equiv\left(\frac{2p_n}{W}\right)\frac{p_n^2}{s'-a}\quad\text{by}\quad\frac{p_n^2}{s'-a}. \tag{A2}$$

[10] Bateman Manuscript Project, *Tables of Integral Transforms* (McGraw-Hill Book Company, Inc., New York, 1954), Vol. II, p. 249, Entry 28.
[11] After this work was done, the author received an unpublished report by D. H. Lythe, University of Birmingham (1965) which covers much of the relevant material from a more mathematical approach. (He does not discuss form factors, however.) He notes that the features of DS as presently known experimentally are also consistent with the following assumptions:

(a) $t\to ia$, $a\neq 0$, but $=\frac{1}{2}$ for "complete absorption";
(b) R constant, $=1/a$ for one-channel case.

Allowing $t(\infty)\neq 0$ deviates from conventional assumptions in the literature of dynamical calculations; this results in the possibility

$$rt^*|_{s\to\infty}\neq 0.$$

Thus, it is again possible for $\chi(\infty)\neq 0$, with the consequences

$$F(s\to\infty)\to 0.$$

This at least prevents the occurrence of singularities at inelastic thresholds as in Sec. II. As a result we now have

$$\mathrm{Re}D-1 =\frac{-1}{\pi}\mathrm{Re}\left[\sum_n A_n\left\{1-\left(1-\frac{c_n}{s}\right)\ln\left(1-\frac{s}{c_n}\right)\right\}\right]. \quad \text{(A3)}$$

For $c_n\to\infty$ the individual terms at fixed s are $O(s/c_n)$ so that our previous choice of parameters still gives a convergent result. Choosing $A_n=A$, we sum the series using the identity

$$\int^s ds\,\ln\left(1-\frac{s}{c_n}\right)=(s-c_n)\ln\left(1-\frac{s}{c_n}\right)-s \quad \text{(A4)}$$

to obtain

$$\mathrm{Re}D-1=-\frac{A}{\pi}\left[\frac{1}{s}\int^s ds\,\ln\left|\frac{\sin(\pi(s/s_0)^{1/2})}{\pi(s/s_0)^{1/2}}\right|\right]. \quad \text{(A5)}$$

For $s\to\infty$, $\ln(\ )\sim\ln s$ and therefore (A5) is of order $\ln s$ also. Our conclusions of Sec. II are therefore unaffected.

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Coupled-Equations Method for the Scattering of Identical Particles. II

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The set of coupled, inhomogeneous equations describing the scattering of a fermion by a system of identical fermions, previously derived from the integral equation for scattering, is shown to follow directly from the Schrödinger equation and the symmetry properties of the exact scattering wave function.

INTRODUCTION

In a previous paper,[1] we derived a set of coupled, inhomogeneous differential equations that described the nonrelativistic scattering of a fermion (boson) by a system composed of identical fermions (bosons). The set of equations was obtained from the integral equation satisfied by the total scattering wave function. Our purpose in the present work is to give an alternative and simpler derivation of the main results of I. We use the Schrödinger equation rather than the integral equation to do this. This treatment enables us to avoid completely the use of certain factors arising from the re-arrangement collision nature of the scattering due to the Pauli principle which were encountered in I and which seem to have caused some confusion concerning the validity of the results of I. The present derivation is intended to clarify the situation, since these factors do not enter the discussion. Only elastic and inelastic scattering of single fermions is considered in the present work.

GENERAL COMMENTS

We follow the notation introduced in I, to which the reader is referred for details not given here. Let $\{\varphi_\alpha\}$ be a complete set of antisymmetrized states for the target, and let Ψ^A be the exact, antisymmetrized scattering wave function. Our goal in I was to expand Ψ^A via the complete set $\{\varphi_\alpha\}$ and then determine a set of coupled equations for the coefficients $\psi_\alpha\equiv\langle\varphi_\alpha|\Psi^A\rangle$, with a specific labeling for the φ_α's. We used the expansion

$$\Psi^A=\textstyle\sum_\alpha \varphi_\alpha[N]\psi_\alpha(N), \quad (1)$$

where $[i]=(1\cdots i-1,\ i+1,\ \cdots N)$ and (i) denotes a function of the coordinates of particle i only. By definition, ψ_α yields the proper scattering amplitude f_α, which is obtained from Ψ^A by first projecting Ψ^A onto the state φ_α and then requiring that $\langle\varphi_\alpha|\Psi^A\rangle$ have the asymptotic behavior $\langle\varphi_\alpha|\Psi^A\rangle\sim e^{i\mathbf{k}\cdot\mathbf{r}}\delta_{\alpha0}+f_\alpha e^{ik_\alpha r}/r$, where $\alpha=0$ denotes the ground state and k_α is the wave number corresponding to excitation of the state φ_α. We have not specified a labeling in φ_α since this is unnecessary due to the identity of the particles: the coordinate of any one of them may be in ψ_α, with the remainder in φ_α; we obtain the *same* scattering amplitude from each $\psi_\alpha(i)$. By choosing the labeling $[N]$, we have $\psi_\alpha(N)=\langle\varphi_\alpha[N]|\Psi^A\rangle$, and thus the expansion (1).

Equation (1) is a valid representation for Ψ^A because $\{\varphi_\alpha\}$ is complete, even though termwise (1) is not antisymmetric. This latter point is unimportant as long as the proper boundary conditions are imposed on the ψ_α in order to secure the correct amplitudes. Since Ψ^A is antisymmetric, we must have

$$P_{MN}\sum \varphi_\alpha[N]\psi_\alpha(N)=-\textstyle\sum_\alpha \varphi_\alpha[N]\psi_\alpha(N),$$

where P_{MN} is the two-body transposition operator.

[1] F. S. Levin, Phys. Rev. **140**, B1099 (1965). We refer to this work as I, and equations from it as (I-1), (I-2), etc. In both I and the present work, we assume that the target is infinitely heavy. This assumption is easily relaxed to include the case of recoil.