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# Parity-Conserving Hyperon Decay and the Pole Model

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B. W. Lee has shown that, in the limit $\delta=0$, where $\delta=(m_\Sigma-m_\Lambda)/(m_\Xi-m_N)$, a $\lambda_6$-type pole model for parity-conserving (p.c.) hyperon decay gives rise to an effective interaction which behaves like $\lambda_7$. Here we generalize this result to include a 27-plet baryon pole, and show that, under certain plausible conditions, the generalization holds to first order in $\delta$. As a result of this generalization, the p.c. decay amplitudes satisfy two sum rules which have previously been derived for parity-violating amplitudes from the $SU(3)$ properties of the Cabibbo current×current interaction.

## INTRODUCTION

An interesting theorem associated with the pole model for nonleptonic hyperon decay[1] has been proven by B. W. Lee.[2] He shows that if the parity-conserving (p.c.) pole term behaves like the $\lambda_6$ component of an octet, and if baryon masses are linear in hypercharge, then the *effective* p.c. decay Hamiltonian must behave like the $\lambda_7$ component of an octet. An immediate corollary is that the p.c. decay amplitudes satisfy the Lee-Sugawara (L-S) triangle[3] in the limit $\delta=0$, where

$$\delta=(m_\Sigma-m_\Lambda)/(m_\Xi-m_N). \quad (1)$$

Apart from its intrinsic value, this theorem is particularly relevant to recent work based upon the partially conserved axial-vector current (PCAC).[4] Using PCAC, several authors[5,6] have shown that in the limit of zero pion mass, the p.c. amplitudes for nonleptonic hyperon decay are determined by baryon poles. If the interaction responsible for nonleptonic decay is of the current×current form, then its octet component behaves like $\lambda_6$,[7] and Lee's theorem is applicable as long as the (27)-component of the interaction can be neglected. The question then arises as to what happens when the (27)-component is not neglected.

To consider this question, we find it convenient to introduce a change of terminology. Instead of referring to $\lambda_6$ and $\lambda_7$ types of behavior, we shall use $T$-$L(1)$ and $T$-$L(2)$ invariance.[8] $T$-$L(1)$ invariance implies symmetry under the exchange of the $SU(3)$ indices 2 and 3, and in the case of an octet, it is equivalent to $\lambda_6$ behavior. $T$-$L(2)$ invariance implies antisymmetry under $2\leftrightarrow 3$ and corresponds to $\lambda_7$ in an octet.

In this terminology, Lee's theorem states that a $T$-$L(1)$-invariant-octet pole model gives rise to a $T$-$L(2)$-invariant effective interaction. Now, the essential feature of a $CP$-conserving current×current interaction is that both its octet component and its (27)-component are $T$-$L(1)$-invariant.[9] Therefore, a natural extension of Lee's theorem would be: A $T$-$L(1)$-invariant pole model with octet and (27)-components gives rise to a $T$-$L(2)$-invariant *effective* interaction which also transforms according to the octet and (27)-plet representations of $SU(3)$. In this note, we shall discuss the validity of this extension and its consequences for p.c. decay amplitudes.

For baryon poles alone, it is easy to show that the extension of Lee's theorem is valid in the limit $\delta=0$. With certain plausible assumptions, it remains valid when first-order terms in $\delta$ are taken into account. [This result is reminiscent of the Ademollo-Gatto theorem for conserved currents.[10]] The presence of an octet meson pole does not invalidate our conclusions, but a (27)-plet meson pole does. In view of the low-energy theorem mentioned above,[5,6] this may not be a serious drawback in practice.

When the extension is valid, the p.c. amplitudes for nonleptonic hyperon decay will satisfy two sum rules. Moreover, these sum rules are identical with the ones predicted by the Cabibbo current×current interaction for parity-violating (p.v.) amplitudes.[11] One, due to

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[1] G. Feldman, P. T. Matthews, and A. Salam, Phys. Rev. 121, 302 (1961).
[2] B. W. Lee, Phys. Rev. 140, B152 (1965).
[3] B. W. Lee, Phys. Rev. Letters 12, 83 (1964); H. Sugawara, Progr. Theoret. Phys. (Kyoto) 31, 212 (1964).
[4] Y. Nambu, Phys. Rev. Letters 4, 380 (1960); M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 705 (1960).
[5] Y. Hara, Y. Nambu, and J. Schechter, Phys. Rev. Letters 16, 380 (1966).
[6] S. Badier and C. Bouchiat, Phys. Letters 20, 529 (1966); see also L. S. Brown and C. M. Sommerfield, Phys. Rev. Letters 16, 751 (1966).
[7] M. Gell-Mann, Phys. Rev. Letters 12, 155 (1964).
[8] S. P. Rosen, Phys. Rev. 137, B431 (1965).
[9] The $T$-$L(1)$ invariance of the p.c. part of the current×current interaction does not require that vector and axial-vector currents have the same Cabibbo angle [N. Cabibbo, Phys. Rev. Letters 10, 531 (1963)]. This follows immediately from the assumption that octet currents conserve $CP$. The p.v. part, however, is $T$-$L(1)$-invariant only when $\theta_V=\theta_A$. See S. Coleman, S. L. Glashow, and B. W. Lee, Ann. Phys. (N.Y.) 30, 348 (1964), for a discussion of this point.
[10] M. Ademollo and R. Gatto, Phys. Rev. Letters 13, 264 (1964).
[11] S. P. Rosen, S. Pakvasa, and E. C. G. Sudarshan, Phys. Rev. 146, 1118 (1966).

Suzuki,[12] relates the $\Delta T=\frac{3}{2}$ amplitude in $\Lambda$ decay to the corresponding amplitude in $\Xi$ decay, and the other[11] is a generalization of the Lee-Sugawara triangle.[3] Thus the parallel between p.c. and p.v. decays which Lee and Swift[13] discovered in the octet pole model can be extended to a (27)-plet pole.

In the following section, we prove the extension of Lee's theorem, and in the last, we discuss the sum rules which follow from it.

## ANALYSIS

Following Lee,[2] we write the effective Hamiltonian for the pole model as

$$H=H^{(0)}+H^{(1)}, \tag{2}$$

where $H^{(0)}$ is the $SU(3)$-invariant free Hamiltonian for baryons and mesons, and $H^{(1)}$ contains the pole and the strong baryon-meson interaction terms. If we neglect meson poles, $H^{(1)}$ takes a form

$$H^{(1)}=(\Delta m)\bar{B}(F_8+\alpha D_8)B+f\bar{B}(F_6+\beta D_6+\gamma\{F_6,F_Q\})B+g\bar{B}(F_k+\lambda D_k)\gamma_5 BM_k \tag{3}$$

in which the anticommutator $\{F_6,F_Q\}$ gives rise to the (27)-plet pole.[14] To establish the $T$-$L(1)$ invariance of the pole terms in Eq. (3), we express the $F$ and $D$ matrices in terms of the $A_\nu{}^\mu$ generators of $SU(3)$ as

$$F_6=\tfrac{1}{2}(A_2{}^3+A_3{}^2),\quad D_6=\tfrac{1}{4}\{A_\nu{}^3,A_2{}^\nu\}+\tfrac{1}{4}\{A_\nu{}^2A_3{}^\nu\},\quad F_Q=F_3+(1/\sqrt{3})F_8=-A_1{}^1. \tag{4}$$

In order to remove the pole terms, we introduce the $SU(8)$ transformation

$$B\to B'=[1+i(2f/\sqrt{3}(\Delta m))\times(aF_7+bD_7+c\{F_7,F_Q\})]B. \tag{5}$$

Lee[2] has already shown that $F_7$ and $D_7$ conserve charge and the charge-conjugation property. The third term of Eq. (5) conserves charge because

$$[Q,\{F_7,F_Q\}]=[F_3+(1/\sqrt{3})F_8,\{F_7,F_3+(1/\sqrt{3})F_8\}]=0 \tag{6}$$

and under charge conjugation, it yields

$$\begin{aligned}C:\quad B_i'\to B_i'^{(c)}&=\textstyle\sum_j[1+i\theta\{F_7,F_Q\}]_{ij}\epsilon_j\bar{B}_j\\&=\epsilon_i\textstyle\sum_j[1+i\theta\epsilon_7\epsilon_Q\{F_7,F_Q\}]_{ij}\bar{B}_j.\end{aligned} \tag{7}$$

The $F$ matrices are antisymmetric, and so $\{F_7,F_Q\}$ is symmetric; in addition, the $\epsilon$ factor is given by[7,15]

$$\epsilon_7\epsilon_Q=\epsilon_7\epsilon_3=\epsilon_7\epsilon_8=-1. \tag{8}$$

Equation (7) now becomes

$$B_i'^{(c)}=\epsilon_i\textstyle\sum_j\bar{B}_j[1-i\theta\{F_7,F_Q\}]_{ji}=\epsilon_i\bar{B}_i' \tag{9}$$

and, therefore, all the terms of Eq. (5) commute with charge conjugation.

We assume that $f\ll\Delta m$ and consider the effects of Eq. (5) to lowest order in $f$. The first two terms of Eq. (3) are transformed into

$$(\Delta m)\bar{B}'(F_8+\alpha D_8)B'+f\bar{B}'(F_6+\beta D_6+\gamma\{F_6,F_Q\})B'+i(2f/\sqrt{3})\bar{B}'[(aF_7+bD_7+c\{F_7,F_Q\}),(F_8+\alpha D_8)]B', \tag{10}$$

where the commutator bracket is equal to

$$i(\sqrt{3}/2)(aF_6+bD_6+c\{F_6,F_Q\})+i\alpha(\sqrt{3}/2)(5bF_6/9+aD_6+c\{D_6,F_Q\})+\alpha b([D_7,D_8]-i(5\sqrt{3}/18)F_6). \tag{11}$$

In the limit $\alpha=0$, which corresponds to $\delta=0$ (see Eq. 1), we choose

$$a=1,\quad b=\beta,\quad c=\gamma \tag{12}$$

so that the second and third terms of Eq. (10) cancel one another. The effective Hamiltonian for nonleptonic decay is then generated by Eq. (5) from the strong baryon-meson interaction term of Eq. (3), and it takes the form

$$H_{\rm NL}=i[2fg/\sqrt{3}(\Delta m)]\bar{B}'[F_7+\beta D_7+\gamma\{F_7,F_Q\},F_k+\lambda D_k]\gamma_5 B'M_k. \tag{13}$$

It is not difficult to show that $H_{\rm NL}$ is an admixture of the octet and (27)-fold representations of $SU(3)$, because $F_7$ is related to the $A_\nu{}^\mu$ by

$$F_7=(1/2i)(A_2{}^3-A_3{}^2). \tag{14}$$

$H_{\rm NL}$ is also $T$-$L(2)$-invariant.

Next we consider the case in which $\alpha$ is not zero. This parameter is proportional to the $\Sigma$-$\Lambda$ mass difference $\delta$ [see Eq. (1)], and experimentally its value is about 0.3. Recent attempts[5,13,16] to fit the data on nonleptonic decay to a pole model indicate that, in the case of octet dominance, the parameter $\beta$ is of the same order of magnitude as $\alpha$. We, therefore, propose to deal with the case $\alpha\neq0$ by assuming that $\alpha$, $\beta$, and $\gamma$ [see Eq. (10)], are all of order $\delta$, and by neglecting second and higher order terms in $\delta$.

The weak-interaction pole terms of Eq. (10) can be transformed away by choosing

$$a+(5/9)\alpha b=1,\quad b+\alpha a=\beta,\quad c=\gamma. \tag{15}$$

To first order in $\delta$, Eq. (15) implies that

$$a\approx1,\quad b\approx\beta-\alpha,\quad c\approx\gamma. \tag{16}$$

The terms $\alpha b([D_7,D_8]-i(5\sqrt{3}/18)F_6)$ and $\alpha c\{D_6,F_Q\}$ of Eq. (11) belong to the ten-fold representation of $SU(3)$, but in view of Eq. (16), they are second-order in $\delta$. Therefore, we see that, to first order in $\delta$, the effective nonleptonic decay Hamiltonian is again generated from the baryon-meson interaction term of Eq. (3) by means

[12] M. Suzuki, Phys. Rev. **137**, B1602 (1965).

[13] B. W. Lee and A. R. Swift, Phys. Rev. **136**, B228 (1964).

[14] Terms of the form $\{F_i,F_j\}$ transform as an admixture of the $D$-type octet and the (27)-plet. To extract the (27)-component, we should subtract a trace term; we have not done so because this would merely redefine the arbitrary constant $\beta$ in Eq. (3).

[15] N. Cabibbo, Phys. Rev. Letters **12**, 62 (1964).

[16] R. H. Graham and S. Pakvasa, Phys. Rev. **140**, B1144 (1965).

of Eqs. (5) and (16). As before, it is an admixture of the octet and (27)-plet, and it is $T$-$L(2)$-invariant.

So far we have omitted the meson terms

$$(\Delta\mu^2)MD_8M+f'MD_6M+f''M\{F_6,F_Q\}M. \quad (17)$$

As pointed out by Lee,[2] the $D_6$ pole can be removed by an $SU(3)$ transformation. Thus when the (27)-plet term is absent from Eq. (17), all of the above conclusions are unaffected by the meson pole. To remove the octet and (27)-plet poles simultaneously, it is necessary to use an $SU(8)$ transformation that breaks the $T$-$L(2)$ invariance of the effective interaction.

## DISCUSSION

The practical consequences of our extension of Lee's theorem are most easily determined by constructing an effective p.c. Hamiltonian with the required properties. Its structure in $SU(3)$ space turns out to be exactly the same as that of the effective p.v. Hamiltonian derived by Rosen, Pakvasa, and Sudarshan,[11] from the symmetry properties of the Cabibbo current$\times$current interaction. Consequently, the p.c. amplitudes satisfy the same sum rules as the p.v. amplitudes. They are:

$$\begin{aligned}\Delta(\Lambda)+\Delta(\Xi)&=0\\ (\sqrt{\tfrac{3}{2}})\Delta(\Sigma)+\Delta(\Lambda)&=2\Delta(\text{L-S}),\end{aligned} \quad (18)$$

where

$$\begin{aligned}\Delta(\Lambda)&=B(\Lambda_-^0)+\sqrt{2}B(\Lambda_0^0),\\ \Delta(\Xi)&=B(\Xi_-^-)-\sqrt{2}B(\Xi_0^0),\\ \Delta(\Sigma)&=B(\Sigma_-^-)-B(\Sigma_+^+)-\sqrt{2}B(\Sigma_0^+),\\ \Delta(\text{L-S})&=-\sqrt{3}B(\Sigma_0^+)+B(\Lambda_-^0)+2B(\Xi_-^-).\end{aligned} \quad (19)$$

An explicit calculation of the p.c. amplitudes arising from the baryon pole model verifies that these sum rules are satisfied to first order in $\delta$ when the coefficients $\alpha, \beta, \gamma$ [see Eq. (10)] are of order $\delta$. When the (27)-plet meson pole is present, no sum rules can be obtained.

To conclude this discussion, we wish to emphasize the parallel between p.v. and p.c. decays. In the former case, we obtain certain predictions from the symmetry of the Cabibbo current alone, and in the latter, we find the same predictions arising from a baryon pole model. It is interesting to note that this model is a consequence of PCAC.[5,6]

*Note added in proof.* Dr. M. Suzuki has pointed out that if the coefficients $\alpha$ and $\beta$ of Eq. (3) are exactly equal and the coefficient $\gamma$ is negligible, then the p.c. amplitude $B(\Sigma_+^+)$ for $\Sigma^+\to n+\pi^+$ must vanish. To see why this happens, we note that Eq. (15) can be solved exactly for $a$ and $b$:

$$\begin{aligned}a&=\beta/\alpha-9(\alpha-\beta)/(5\alpha^2-9),\\ b&=9(\alpha-\beta)(5\alpha^2-9).\end{aligned}$$

When $\alpha=\beta$, the coefficient $b$ vanishes and $a=1$. Thus the transformation of Eq. (5) involves only the $SU(3)$ generator $F_7$ ($\gamma=0$ implies $c=0$) and, when applied to the strong baryon-meson term of Eq. (3), it yields an effective $H_{\text{NL}}$ in which the baryon-antibaryon coupling is pure octet. As is well known,[17] this coupling forbids $\Sigma^+\to n+\pi^+$.

It should be noted that this result is independent of the presence of meson poles because they also lead to octet $\bar{B}B$ coupling in $H_{\text{NL}}$. Thus the vanishing of $B(\Sigma_+^+)$ is a fundamental difficulty, not only for tadpole models,[18] but for all pole models in which $\alpha=\beta$.[19]

Finally, we note that if the coefficients $\lambda$ and $c$ of Eqs. (3) and (5) are zero, $B(\Sigma_+^+)$ will again vanish for the same reason as above.

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[17] S. P. Rosen, Phys. Rev. **143**, 138 (1966).
[18] S. Coleman and S. L. Glashow, Phys. Rev. **134**, B671 (1964) and Ref. (2). In pure tadpole models all p.c. amplitudes vanish.
[19] See for example, Y. Hara, Y. Nambu, and J. Schechter, Phys. Rev. Letters **16**, 380 (1966) and **16**, 875 (1966).