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η-Nucleon Interaction and η Production in π-Nucleon Scattering*

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A dynamical model of the η-nucleon interaction is constructed which satisfies the requirements of analyticity and unitarity as well as including the most important single-particle exchange terms as an interaction. The resulting model depends on two parameters: the η-nucleon coupling constant g_η and a cutoff. It is found that a reasonably good fit to the η-production data and the π-nucleon phase-shift analysis can be obtained. Finally it is shown that the strong rise in η production just above the threshold implies the existence of a $J=\frac{1}{2}$ odd-parity nonstrange baryon of mass 1420–1460 MeV.

I. INTRODUCTION

THE strong rise just above threshold observed in the η production from the π-nucleon initial state may be an indication of a nearby pole in the scattering amplitude. Such a pole corresponds to a baryon of $s=0$ and $J=\frac{1}{2}^-$, a hitherto unobserved set of quantum numbers, and might be the least massive of a new octet of baryons. Several attempts have been made to study questions of the existence and location of this pole by "effective range methods."[1–3] To study this possibility in more detail, a dynamical model of η-N scattering and production is constructed which satisfies the requirements of analyticity and unitarity as well as includes the most important single-particle exchange terms as an interaction. This model can then be used to fit the experimental data and thus determine the position of this pole as well as the η-N coupling constant which enters into the model. There are several reasons to expect that such a model may be a rather good description of η production from π-nucleon collisions.

First, the narrow decay width of the η meson indicates that in scattering processes involving strongly interacting particles, the η may be treated as a single-particle state to a good approximation, i.e., that the composite structure in terms of 4π and higher mass states plays a small role in scattering processes.

Secondly, the kinematical similarity between the η-N channel and the π-N channel, differing only by the isospin of the η and the η's higher mass, leads one to speculate that treatments of η-N scattering by means of dispersion theory might well reproduce the moderate successes of π-N dispersion relations.

Thirdly, the relatively weak coupling of the π-N to the Λ-K channel indicates that employing a scattering matrix for only two coupled channels (η-N and π-N) will be a good approximation.

Finally, the important interaction terms that couple the π-N and η-N channels are particularly simple consisting only of the single nucleon exchange, making the calculation rather clean in that it depends only on the η-N coupling constant.

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[1] F. Uchiyama-Campbell, Phys. Letters 18, 189 (1965).
[2] A. W. Hendry and R. G. Moorhouse, Phys. Letters 18, 171 (1965).
[3] P. N. Dobson, Phys. Rev. 146, 1022 (1966).

In Sec. II the kinematical preliminaries are discussed, following closely the treatment of π-N scattering. In the following section the partial-wave dispersion relations are obtained and the ND^{-1} method is formulated for these amplitudes. In Sec. IV the interaction terms, arising from single-particle exchange are discussed. In Sec. V the $S_{1/2}$ and $P_{1/2}$ scattering amplitudes are determined and compared with experimental data. In Sec. VI the position of the pole in the scattering amplitude is determined.

II. KINEMATICAL CONSIDERATION

Because of the similarity of η-nucleon scattering to π-nucleon scattering as well as the fact that these scattering channels are coupled, the usual notation of pion-nucleon scattering[4] will be suitably generalized and used throughout this paper. The pion-nucleon channel will be labeled channel 1, while η-nucleon is channel 2. The various 4-momenta are then $(P_1{}^i,P_2{}^j)$ the (initial, final) nucleon momenta and $(q_1{}^i,q_2{}^j)$ the (initial, final) meson momentum in the scattering of channel i to channel j. Since unitarity couples channels at the same total energy, each scattering amplitude will be evaluated at the same value of W (total energy in c.m. system). The invariant momentum transfer in the various channels is

$$t_{ij}=(q_1{}^i-q_2{}^j)^2=\mu_i{}^2+\mu_j{}^2-2\omega_i\omega_j+2q_iq_j\cos\theta_{ij}\,, \quad (2.1)$$

where ω_i, q_i is the meson energy, 3-momentum in the c.m. system;

$$q_i{}^2=(\omega_i{}^2-\mu_i{}^2)^{1/2}\,,\quad \omega_i=(W^2-M^2+\mu_i{}^2)/2W\,,$$

μ_1 is the pion mass, and μ_2 is the η-meson mass. The c.m. scattering angle is denoted $\chi_{ij}=\cos\theta_{ij}$ for the $i\to j$ scattering process.

The most general form of the scattering amplitude T_{ij} is

$$T_{ij}=-A_{ij}+i\gamma\cdot Q_{ij}B_{ij}\,, \quad (2.2)$$

where $Q_{ij}=\frac{1}{2}(q_1{}^i+q_2{}^j)$ and the operator T_{ij} is to be evaluated between the spinors $\bar{u}(p_2{}^j)$ and $u(P_1{}^i)$. The A_{11} and B_{11} are just the isotopic spin-$\frac{1}{2}$ amplitudes $A^{1/2}$ and $B^{1/2}$ for pion-nucleon scattering.

[4] W. R. Frazer and J. R. Fulco, Phys. Rev. 119, 1420 (1960).

The next step in obtaining the partial-wave amplitudes is to reduce the scattering amplitude to Pauli matrix form.

$$f_1{}^{ij}=\frac{[(M+E_i)(M+E_j)]^{1/2}}{8\pi W}[A_{ij}+(W-M)B_{ij}],\quad (2.3a)$$

$$f_2{}^{ij}=-\frac{[(E_i-M)(E_j-M)]^{1/2}}{8\pi W}\times[A_{ij}-(W+M)B_{ij}],\quad (2.3b)$$

where $E_i=(q_i^2+M^2)^{1/2}$. In terms of these amplitudes the differential cross section can be expressed

$$\frac{d\sigma_{ij}}{d\Omega}=\frac{q_j}{q_i}\left|\chi_f\left[f_1{}^{ij}+\frac{\sigma\cdot\mathbf{q}_2{}^j\sigma\cdot\mathbf{q}_1{}^i}{q^iq^j}f_2{}^{ij}\right]\chi_i\right|^2,\quad (2.4)$$

where χ_f and χ_i are the Pauli spinors for the final and initial nucleons.

Eigenamplitudes of definite parity and angular momentum are then given by

$$f_{l\pm}{}^{ij}=\frac{1}{16\pi W}\{[(E_i+M)(E_j+M)]^{1/2}\times[A_l{}^{ij}+(W-M)B_l{}^{ij}]+[(E_i-M)(E_j-M)]^{1/2}\times[-A_{l\pm1}{}^{ij}+(W-M)B_{l\pm1}{}^{ij}]\},\quad (2.5)$$

corresponding to $J=l\pm\frac{1}{2}$ where $A_l{}^{ij}$ and $B_l{}^{ij}$ are defined as follows:

$$A_l{}^{ij}=\int_{-1}^{1}d(\cos\theta_{ij})A_{ij}(s,t_{ij})P_l(\cos\theta_{ij}),$$
$$B_l{}^{ij}=\int_{-1}^{1}d(\cos\theta_{ij})B_{ij}(s,t_{ij})P_l(\cos\theta_{ij}).\quad (2.6)$$

As can esaily be seen from Eq. (2.5), the f's satisfy the MacDowell relation

$$f_l{}^{ij}(-W)=-f_{(l+1)-}{}^{ij}(W)$$

for reflections in the W plane.

III. PARTIAL-WAVE DISPERSION RELATIONS

We now introduce partial-wave amplitudes $h_J{}^{ij}$ from which all kinematical singularities in the W plane have been factored.

$$h_J{}^{ij}(W)=\frac{s^Jf_{l+}{}^{ij}(W)}{16\pi(q_iq_j)^{J-1/2}[(E_i+M)(E_j+M)]^{1/2}}.\quad (3.1)$$

These amplitudes are finite at the four thresholds in the W plane, $W=\pm(M+\mu_1)$ and $W=\pm(M+\mu_2)$. The only uncertainty in the analytic properties of h is the behavior at $s=0$ as h may have a $(2J-1)$-order zero at that point. This does not, however, influence the $J=\frac{1}{2}$ partial waves which will be our primary interest in this paper.

The unitarity condition satisfied by h written in matrix form is

$$\mathrm{Im}(h_J{}^{-1})=-\rho_J,\quad (3.2)$$

where

$$\rho=\begin{pmatrix}\rho_1 & 0\\ 0 & \rho_2\end{pmatrix}\quad\text{and}\quad \rho_i(W)=\frac{(E_i+M)q_i{}^{2J}}{s^J}.\quad (3.2)$$

The unitarity condition Eq. (3.2) gives the discontinuity of h_J across the cuts $W<-(M+\mu_1)$ and $W>(M+\mu_1)$. The $q_i{}^{2J}$ appearing in ρ_i is taken to be the positive branch of the square root for $W<-(M+\mu_1)$.

The matrix ND^{-1} equations are now employed to produce a unitary h_J given interaction singularities of h_J. Then Eq. (3.2) implies

$$D_{ij}=\delta_{ij}-\frac{W}{\pi}\left[\int_{-\infty}^{-W_i}+\int_{W_i}^{\infty}\right]dW'\frac{\rho_i(W')N_{ij}(W')}{W'(W'-W)},\quad (3.3)$$

where $W_i=M+\mu_i$.

If we define the integral over the interaction singularities of h_{ij} to be b_{ij}, then N_{ij} satisfies the following integral equation:

$$N_{ij}=b_{ij}+\sum_k\frac{1}{\pi}\left[\int_{W_k}^{\infty}+\int_{-\infty}^{-W_k}\right]dW'\frac{[W'b_{ik}(W')-Wb_{ik}(W)]}{W'(W'-W)}\rho_kN_{kj}.\quad (3.4)$$

The asymptotic behavior for large W of the b employed in the pion-nucleon scattering generally will not allow solutions to this equation. We therefore introduce a cutoff, replacing ∞ by W_c in all integrals for Eqs. (3.3) and (3.4). In this case, Eq. (3.4) becomes a Fredholm equation which can be solved numerically to obtain the scattering matrix.

IV. INTERACTIONS IN THE π-N AND η-N SCATTERING AND PRODUCTION

For the π-N elastic channel, the interaction used by Ball and Wong (BW)[5] will be employed. This interaction arises from the N, N^*, and ρ exchange and all the parameters controlling the strength of these terms will be fixed at the best values of BW, which are the following:

$$\gamma_{33}=0.06,\quad \gamma_1=-1.0,$$
$$m_\rho=5.4\mu_1,\quad g^2=14.$$

For the η-K elastic channel of the three terms important in the π-N channel, only the nucleon exchange remains. The only meson exchanges that contribute are $I=0$, even parity and even G parity. The only low-

[5] J. S. Ball and D. Y. Wong, Phys. Rev. 133, B179 (1964).

mass candidate would seem to be the σ meson ($I=0$, s-wave π-π). This exchange will be omitted on the grounds that it is relatively unimportant in π-N scattering as well as being somewhat speculative. The baryon exchanged must have $I=\frac{1}{2}$. The next candidate after the nucleon is the $D_{3/2}$ π-N resonance. This will be omitted on two grounds: (1) it has a rather high mass, and (2) it does not seem to show up in the π-$N \rightarrow \eta$-N data, indicating that the coupling of the η-N to the $D_{3/2}$ state may be rather small.

For the production channel, the lowest mass meson exchange candidate is the A_2, which will be ignored because of the high mass of the A_2. The baryon exchanged again must have $I=\frac{1}{2}$ and for the reasons stated above, only the nucleon exchange term is included.

Finally, since we wish to compare the π-$N \rightarrow \eta$-N with experiment, no attempt will be made to produce the nucleon as a p-wave bound state of the π-N and η-N system. The direct nucleon pole is put in to all three processes with the correct position and residue.

The interaction terms produced by the direct and crossed nucleon poles are summarized as follows:

$$\frac{B_{11}}{4\pi}=\frac{3g^2}{m^2-s}+\frac{g^2}{m^2-\bar{s}_{11}},$$

$$\frac{B_{12}}{4\pi}=\frac{\sqrt{3}gg_\eta}{m^2-s}+\frac{gg_\eta}{m^2-\bar{s}_{12}},$$

$$\frac{B_{22}}{4\pi}=\frac{g_\eta^2}{m^2-s}+\frac{g_\eta^2}{m^2-\bar{s}_{22}},$$

where g_η is the η-N coupling constant, and

$$\bar{s}_{ij}=2M^2+\mu_i^2+\mu_j^2-s-t_{ij}.$$

The remainder of the interaction in the 1-1 channel due to N^* and ρ exchange is that given in BW Eqs. (6.1) and (6.2) for $I=\frac{1}{2}$.

V. NUMERICAL RESULTS

Solutions for the $J=\frac{1}{2}$ scattering amplitudes depend on two adjustable parameters W_c and g_η which must be adjusted to fit the η-production data as well as the $S_{1/2}$ pion-nucleon phase-shift analysis. The sensitivity of the solutions to these parameters separates nicely, with the π-N s-wave scattering length being determined by W_c and comparatively independent of g_η while the height of the peak in the production amplitude is sensitive to g_n and relatively insensitive to W_c. The experimental value of $a_{1/2}=0.170\pm0.005$ is used to fix W_c. The determination of g_η is more complicated as the experimental data[6] give only $\pi+N\rightarrow\eta+N\rightarrow 2\gamma+N$ and the absolute η-production rate then depends on the branching ratio,

$$\frac{\eta\rightarrow 2\gamma}{\eta\rightarrow \text{all}}\simeq 0.31 \quad \text{to} \quad 0.38,$$

about which there seems to be some uncertainty.[7] For this reason the extreme values 0.31 and 0 38 were both used to obtain two values of g_η. The resulting η-production cross section is shown in Figs. 1 and 2 for both values of the branching ratio. The behavior of the $S_{1/2}$ π-N phase shift near the η-production threshold shown in Figs. 3 and 4 must be considered a prediction of this model as W_c was adjusted to give the correct slope at the π-N threshold and has little influence on the cusp behavior of the phase shift. The agreement between the calculated phase shift and the phase-shift analysis results of Auvil *et al.*[8] is a clear indication of the relevance of this model to η production.

In Fig. 5 the calculated $P_{1/2}$ π-N phase shift is plotted and we see the rather striking agreement with the Roper phase shift[9] at low energy.

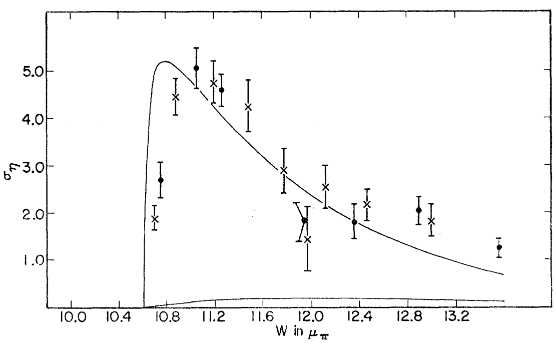

FIG. 1. The $J=\frac{1}{2}=I$ s- and p-wave production cross section with $\eta\rightarrow 2\gamma$ ratio taken to be 0.38. The values of the parameters are $g_\eta=2.6$ and $W_c=26.6\mu_\pi$. The experimental points are those of Bulos *et al.* (×) and the Berkeley-Hawaii collaboration (●) (Ref. 6).

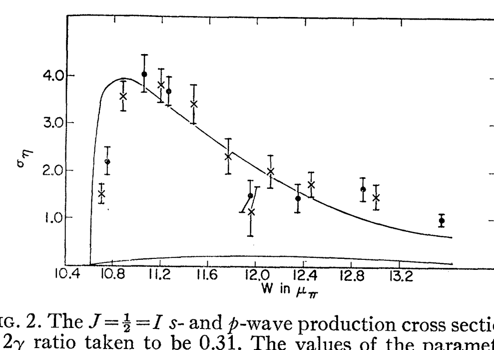

FIG. 2. The $J=\frac{1}{2}=I$ s- and p-wave production cross section with $\eta\rightarrow 2\gamma$ ratio taken to be 0.31. The values of the parameters are $g_\eta=2.8$ and $W_c=26.5\mu_\pi$. The experimental points are those of Bulos *et al.* (×) and the Berkeley-Hawaii collaboration (●) (Ref. 6).

[6] F. Bulos *et al*, Phys. Rev. Letters **13**, 486 (1964); W. Bruce Richards *et al.*, *ibid.* **16**, 1221 (1966).

[7] V. Z. Peterson (private communication).

[8] P. Auvil, A. Donnachie, A. T. Lea, and C. A. Lovelace, Phys. Letters **12**, 76 (1964).

[9] L. Roper, R. M. Wright, and B. T. Feld, Phys. Rev. **138**, B190 (1965).

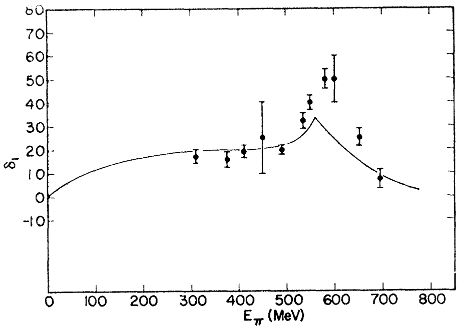

FIG. 3. The $s_{1/2}$ π-N phase shift for $g_\eta=2.6$ and $W_c=26.6$. The experimental points are those of Auvil *et al.* set *I*. (Ref. 8).

The behavior of this phase shift can be understood as follows: The direct nucleon pole in the π-N channel acts as a repulsive force, producing a negative phase shift; the inelastic process and the N, N^*, ρ-exchange terms provide attraction which dominates at higher energy causing the phase shift to change sign and become large and positive. The η production in the $p_{1/2}$ state should not be the most important inelastic channel as σ production can occur in a relative S state and should clearly dominate at low energy. The calculated phase shift rises to about 40° and presumably including the σ-N channel, which increases the attraction would bring the phase shift up to produce better agreement with the experimental phase shift.

If $SU(3)$ symmetry is assumed, the value of g_η^2 can be determined from g^2 and the D/F ratio for the pseudoscalar-baryon coupling as follows:

$$g_\eta^2=\tfrac{1}{3}[1-4F/(D+F)]^2g^2.$$

Martin and Wali[10] find that

$$F/(D+F)\simeq 0.25\,,$$

giving $g_\eta^2\simeq 0$ in contrast to the value $g_\eta^2\simeq 7$–8 as obtained in this calculation. Other determinations of g_η^2, while being not as sensitive as the present calculation, give similar results; for example Scotti and Wong[11] obtained $g_\eta^2=12$.

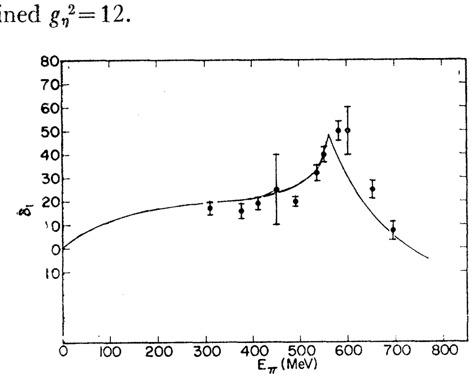

FIG. 4. The $s_{1/2}$ π-N phase shift for $g_\eta=2.8$ and $W_c=26.5$. The experimental points are those of Auvil *et al.* set *I*, (Ref. 8).

[10] A. W. Martin and K. C. Wali, Phys. Rev. **130**, 2455 (1963).
[11] A. Scotti and D. Y. Wong, Phys. Rev. **138**, B145 (1965).

The discrepancy in Figs. 1 and 2 between the calculated points near the threshold and the experimental points may be the result of a slight systematic shift in the energy scale as well as the effect of the energy spread in the incident beam. In the Berkeley-Hawaii experiment[6] the energy spread was 2.5% corresponding to $\Delta W=\pm 0.06\mu_\pi$ at the η-production threshold. In any case it is not possible for this model to produce a slow rise and maintain the height of the maximum, as the experimental points seem to require.

VI. POLES OF THE SCATTERING AMPLITUDE

Once the parameters are fixed by comparison of the solution with experiment, we then look for nearby poles on the unphysical sheets of the scattering amplitude. This is accomplished by continuing the matrix D to each of the three unphysical sheets connected to the physical sheet by the two channel unitarity cuts, and looking for zeros of the determinant of D.

From Eq. (3.2) we see that the discontinuity of D is

$$D(w+i\epsilon)-D(W-i\epsilon)=-2i\rho(W)N(W)\,. \quad (6.1)$$

The continuation of D_{11} and D_{12} through the cut

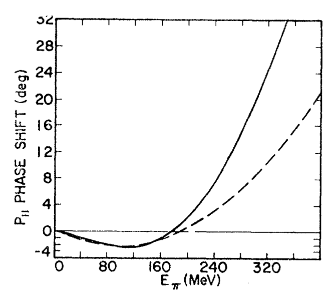

FIG. 5. The $P_{1/2}$ pion-nucleon phase shift. The solid line is the energy-dependent phase shift obtained by Roper (Ref. 9). These results are the same for both solutions for g_η and W_c. (Dashed line is the calculated phase shift.)

associated with ρ_1 is

$$D_{1i}^{\mathrm{II,III}}(w)=D_{1i}(w)+2i\rho_1N_{1i}(W)i=1,2\,, \quad (6.2)$$

where II and III denote the two sheets reached by passing through the ρ_1 cut. The continuation of D_{21} and D_{22} through the ρ_2 cut is

$$D_{2i}^{\mathrm{III,IV}}(W)=D_{2i}(W)+2i\rho_2N_{2i}(W)i=1,2\,, \quad (6.3)$$

where III and IV denote the sheets reached by passing through the ρ_2 cut.

An examination of the various sheets around the η-nucleon threshold reveals a pole in sheet IV as defined by Eqs. (6.2) and (5.3). The position of this pole is 1425 MeV for $g_\eta=2.6$ and 1466 MeV for $g_\eta=2.8$ while the displacement from the axis is 100 and 60 MeV, respectively.

Two important differences exist between the continuation of the determinant of D as carried out here and the effective-range continuation used in other treatments of η production. The effective-range procedure is the following: Consider

$$T^{-1}=M-i\rho\,;$$

then M is real in the physical region and is taken to be a constant matrix. Under these assumptions the continuation consists of taking ρ_1 and ρ_2 on each sheet. If we calculate the M matrix from the solutions obtained here we find that the individual elements of M are large and vary rapidly with energy, and M_{22} has a singularity at $W=(M^2+2\mu_\eta^2)^{1/2}=8.7\mu_\pi$ arising from the crossed nucleon pole in T_{22}. The huge size of the M's and some of the energy variation arises because $\det(N)$ is quite small, meaning that T is nearly singular. As a result of this, our solution lies somewhere between that of a Breit-Wigner resonance for which T is singular and that of constant M. The presence of the branch cut singularity in M relatively near the η threshold makes extensive continuation below the threshold nonsense if M is taken to be constant or a low-order polynomial in W.

VII. CONCLUSIONS

The dynamical model of π-N and η-N scattering which has been developed here seems to produce remarkably good agreement with experiment, particularly in view of the small number of adjustable parameters required to obtain the fits of experiment as shown in Figs. 1 and 2. Assuming that poles on all unphysical sheets are to be associated with unstable single-particle states (as is always done for poles on the second sheet), we find an odd-parity baryon of mass 1440 MeV. The only other candidate for a $J=\frac{1}{2}$ baryon of odd parity is the Y_0^*1405. These three particles could then form an $SU3$ triplet of very nearly equal mass. However, the fact that these particles are strongly coupled to the meson-baryon system which contains no triplets indicates that an octet is the lowest multiplet containing these particles. The η-N baryon is the nucleon member, while the Y_0^* corresponds to the Λ; the near equality of the masses forces the Σ member mass difference from 1400 MeV to be twice that of the Ξ member in order for the Gell-Mann–Okubo mass formula to be satisfied.

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Simple $\pi\pi\rho$ Bootstrap Model*

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A bootstrap model of the $\pi\pi\rho$ system is studied using a simple and somewhat unusual approach. An attempt is made to bootstrap both the ρ and the π; we find that self-consistency can be achieved in the ρ channel, but not in the π channel. Two adjustable (cutoff) parameters can then be used to fit both the observed ρ mass and width.

I. INTRODUCTION AND SUMMARY

It is at least conceivable that the mesons are bound states and resonances of one another. In order to become familiar with some of the dynamical problems such a bootstrap picture poses, we consider the $\pi\pi\rho$ system.

The dynamics of our model are contained in some bootstrap equations derived by considering a Bethe-Salpeter equation (ladder approximation) in each channel.[1] The mass dependence in the equations enters through some dynamical factors which contain integrals over an internal-momentum loop (Fig. 1) with Bethe-Salpeter wave functions appearing at the vertices. The simplest approximation to these factors is obtained by replacing these vertex functions by constants, corresponding to point interactions. This would be closest in spirit to the N/D method with N replaced by the Born term. However, it here results in highly divergent integrals because of the (fixed) spin of the ρ. The best approximation, presumably, would be to find and use the Bethe-Salpeter wave functions; these would provide the high-momentum damping

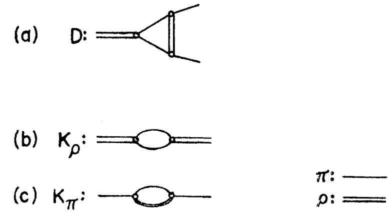

Fig. 1. Integrals containing the dynamics of the model.

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[1] This approach to bootstrap equations is to be found in (a) R. E. Cutkosky and M. Leon, Phys. Rev. **135**, B1445 (1964), and applied in (b) R. E. Cutkosky and M. Leon, *ibid.* **138**, B667 (1965), and (c) K. Y. Lin and R. E. Cutkosky, *ibid.* **140**, B205 (1965).