PHYSICAL REVIEW VOLUME 149, NUMBER 4 30 SEPTEMBER 1966

Photon Dissociation Model for Vector-Meson Photoproduction*

MARC ROSS
University of Michigan, Ann Arbor, Michigan

AND

LEO STODOLSKY
Brookhaven National Laboratory, Upton, New York†

and

University of Michigan, Ann Arbor, Michigan
(Received 9 August 1965; revised manuscript received 7 June 1966)

We show how the diffraction-dissociation approach can be formulated and used to yield detailed results for photoproduction of photon-like states such as ρ, ω, and φ. The model is shown to yield a coherent description of both the qualitative and quantitative aspects of the relevant experiments, including the size and behavior of the cross sections, the mass shifting of the ρ peak, and the nuclear A dependence. We are able to extract from experiment scattering cross sections for the vector mesons and their effective photon couplings which are in accord with other data, insofar as they are available. We suggest a number of interesting experiments that can be used to test the model and the φ-ω mixing theory for the SU_3 structure of the photon.

I. INTRODUCTION

IN this paper we wish to emphasize the significance of diffraction dissociation[1] for high-energy processes and to study a simple model, based on this idea, that can explain the predominant features of recent experiments on ρ and ω photoproduction, including the mass skewing of the ρ peak, the coherent behavior of the nuclear production, and the general shape, magnitude, and energy dependence of the cross sections.

Furthermore, we claim that with this model we can extract from the experiments (for the ρ): (a) the total ρ-nucleon cross section; (b) the approximate ρ-nucleon elastic cross section; and (c) the effective direct ρ-photon coupling constant. We propose that similar experiments, particularly measurements of the forward cross section on nuclei as well as hydrogen, can be performed for ω and ϕ to yield the corresponding quantities. We also suggest that electroproduction of vector mesons may be used to test some of the points of the model, including the explanation of the mass shift and the suitability of using a direct ρ-photon coupling.

A. Diffraction Dissociation

Diffraction dissociation is probably the dominant inelastic process at extremely high energy. It may be described by considering one or both colliding particles to be composite systems (e.g., the deuteron as a bound state of neutron and proton, or the pion as coupled to three pions), or to consist of several components (e.g., the photon as ρ or ω), or both :

$$d \to n+p\,,$$
$$\pi \to 3\pi\,,$$
$$\gamma \to \rho\,.$$

These "dissociations" are not, of course, realized in free space because of energy-momentum conservation, but at very high energy only a small three-momentum transfer between the colliding systems can suffice to materialize the components. The dissociation of a particle striking a target which does not become excited requires longitudinal momentum transfer

$$|\Delta| \gtrsim (M^{*2}-M^2)/2p\,,$$

where M is the mass, p is the laboratory momentum of the initial particle, and M^* is the mass of the dissociated system. (For interesting numbers such as $M^{*2}-M^2=1$ BeV^2 and $p=5$ BeV/c the minimum momentum transfer is only 100 MeV/c.)

In diffraction dissociation we have very low momentum-transfer elastic scattering between the incident particles or their constituents, leading to the materialization of the dissociated system. This elastic scattering is diffraction scattering, which is strongly peaked at small momentum transfer and predominant at high energy. We can define diffraction processes as very high-energy reactions involving the exchange of no quantum numbers such as B, Q, C, S, T, G. In other words, only a "vacuon" or "Pomeranchon" is exchanged between the colliding systems. Because elastic diffraction scattering is the underlying mechanism, no internal-symmetry quantum numbers are transferred in dissociation with one important exception: internal orbital angular momentum may be transferred to the composite particle. Since the momentum is transferred to a finite-sized system, J and P are not necessarily preserved from incident particle to dissociated system. Only "natural" parity changes are allowed, however: $\Delta P=(-1)^{\Delta L}$, e.g., the transition $0^- \to 1^+$ can occur, but not $0^- \to 1^-$.

An equivalent view of the dissociation phenomenon, which carries different connotations, is obtained by expressing the incident particles' state vector in terms of

* Work performed under the auspices of the U. S. Atomic Energy Commission.
† Present address.
[1] M. L. Good and W. D. Walker, Phys. Rev. 120, 1857 (1960).

states that scatter into themselves in the collision region (essentially states that diagonalize the interaction in the collision region). To the extent that these states scatter differently, the combination coefficients characterizing the incident pair of particles are relatively changed so that the outgoing wave will contain new components. This view is clearly developed by Good and Walker.[1] It is particularly convenient when the states diagonalizing the interaction are simple, as in the case of photon-hadron collision.

Diffraction dissociation is thus characterized by a final state of two systems carrying most of the properties of the corresponding incident particles. It is possible that dissociation dominates inelastic processes at high energy just as elastic diffraction dominates elastic scattering. In addition to large cross section and the preservation of the quantum numbers of the incident particles, there are many detailed differences between dissociation and the currently more popular particle-exchange processes. These are discussed below. We immediately note, however, that coherence on nuclear targets is one important property of the dissociation mechanism. There has been some confusion between dissociation and conventional (e.g., particle-exchange) coherent processes. Conventional coherent mechanisms involve C or G exchange, but they can be coherent because nuclear states are not eigenstates of these operators. Any quantitative discussion of coherent processes at high energy involves strong absorption inside the nucleus. Unlike conventional processes, however, dissociation occurs because of this absorption and is not most conveniently thought of in the usual way as the sum of individual nucleon amplitudes attenuated by absorption. We also distinguish the diffraction processes from the conventional ones because at high energy the diffraction processes should dominate.

Experimental evidence of diffraction dissociation has been developed for the three processes listed above. Deuteron dissociation has been discussed before.[2] The richer example $\pi \rightarrow 3\pi$ has been subject to some beautiful experimental study.[3] The theory of this, or any elementary-particle dissociation involving hadrons only, is not, however, at all in a satisfactory state, although there exist a variety of calculations related to the dissociation process.[4] The reasons for this unsatisfactory situation are both the fundamental difficulties and the difficulties of a many-body calculation. In this paper we will consider the $\gamma \rightarrow \rho$ process, on which there is considerable experimental evidence, and which is blessed with some very simple theoretical aspects.

[2] E. L. Feinberg and I. Pomeranchuk, Nuovo Cimento Suppl. 3, 652 (1956).

[3] J. F. Allard *et al.*, Phys. Letters **19**, 431 (1965).

[4] S. D. Drell and K. Hiida, Phys. Rev. Letters **7**, 199 (1961); P. T. Matthews and A. Salam, Nuovo Cimento **12**, 126 (1961); M. Ross, Phys. Rev. **131**, 2678 (1963); R. T. Deck, Phys. Rev. Letters **13**, 169 (1964); U. Maor and T. O'Halloran, Phys. Letters **15**, 281 (1965); and L. Resnick (private communication).

B. Qualitative Evidence for a Diffraction Process in ρ Photoproduction

Various aspects of the reactions

$$\gamma + p \rightarrow p + (\pi^+ + \pi^-)$$
$$\gamma + A \rightarrow A + (\pi^+ + \pi^-)$$

(where A stands for a nucleus) have been measured at CEA[5,6,9] and DESY[7,8] for γ energies up to about 5 BeV. The ρ^0 resonance is very prominent in the data. The experimenters have discussed their results in terms of two mechanisms: diffraction dissociation and one-pion exchange, and have shown that the data overwhelmingly favor the former. We here review selected results from the four experiments and the reasons favoring a diffraction-dissociation picture. The various points are presented in order of their significance for distinguishing dissociation from other mechanisms.

(*1*) *Dependence of the total production cross section*, $\sigma_{\gamma\rho}$ *on total energy*. Above the immediate threshold region the dissociation process should be roughly independent of s. The pion-exchange mechanism predicts $\sigma_{\gamma\rho} \propto s^{-2}$ or p_{lab}^{-2}. The results are the same for $(d\sigma/dt)_{0°}$. s and t are the usual squares of the total center-of-mass energy and invariant momentum transfer, respectively. The experiments show a gradual fall or constancy with increasing energy, compatible with dissociation, and disagreeing strikingly with pion exchange.

(*2*) *Dependence of* $d\sigma/dt$ *on* t, *for small* t, *on various nuclear targets* (*not hydrogen*). This would be a very sensitive method for distinguishing coherent processes such as dissociation from noncoherent processes such as pion exchange. The dependence should be the same as in, say, π- or p-elastic scattering (see below). This experiment has not been done, but there is preliminary experimental evidence[9] on carbon for the very fast decrease of the cross section as $|t|$ increases, consistent with this nuclear-radius effect.

(*3*) *Dependence of* $\sigma_{\gamma\rho}$ *and* $(d\sigma_{\gamma\rho}/dt)_{0°}$ *on* A (*the nuclear mass number*). The experiment of Pipkin and collaborators[5] shows that roughly $d\sigma/dt \propto A^{1.6}$. This crude power law applies up to about $A \approx 50$. We note for the moment that this coherent behavior is completely incompatible with a mechanism like π^0 exchange, involving T exchange,[10] while it is reasonable for a diffraction process, which behaves like elastic scattering.

[5] L. J. Lanzerotti *et al.*, Phys. Rev. Letters **15**, 210 (1965). We thank F. M. Pipkin for providing us with a prepublication report.

[6] Brown-CEA-Harvard-MIT-Padova-Weizmann Institute Collaboration (to be published). We would like to thank B. Feld and U. Maor for preliminary data.

[7] Aachen-Berlin-Bonn-Hamburg-Heidelberg-München Collaboration (to be published). We would like to thank E. Lohrmann for providing us with prepublication reports.

[8] H. Blechschmidt, B. Elsner, K. Heinloth, A. Ladage, J. Rathje, and D. Schmidt (to be published).

[9] F. M. Pipkin (private communication).

[10] L. Stodolsky, Phys. Rev. **144**, 1145 (1966); A. S. Goldhaber and M. Goldhaber in *Preludes in Theoretical Physics*, edited by A. de-Shalit, H. Feshbach, and L. Van Hove (North-Holland Publishing Company, Amsterdam, 1966).

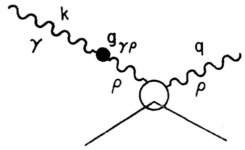


FIG. 1. Dissociation diagram for ρ photoproduction.

(4) The absolute cross section. Diffraction dissociation of the photon should be a strong process. We will show that production of an important resonance like $\gamma \to \rho$ should have a cross section perhaps just one order of magnitude below $\alpha\pi R^2$, where α is the fine-structure constant and R the nucleon or nuclear radius. According to this, tens of microbarns would be reasonable for $\gamma+p \to \rho+p$. On the other hand, detailed calculation[6,7] indicates a much smaller cross section, at (say) 4 BeV for π exchange. Both calculations, of course, involve some assumptions which may not be completely convincing. The experimental results for $\sigma(\gamma p \to \rho p)$ vary between 11.5 and 17.6 μb. This large cross section is quite compatible with dissociation and is hard to understand on the basis of π exchange.

(5) Dependence of $d\sigma/dt$ on t for hydrogen, for small t. In the dissociation process we expect essentially the same t dependence as in elastic diffraction scattering. The data on $\gamma+p \to \rho+p$ show a diffraction-like peak, $d\sigma/dt \propto e^{at}$, with (crudely) $a \approx 10$–12 BeV^{-2},[5–7] which is a somewhat steeper slope than that found in π or p elastic scattering. If we think of the steepness of the peak as related to the size of the scatterer, then this value of a fits in nicely with our deduction (see below) that the total ρp cross section is ≈ 50 mb, larger than that for πp and pp. This rapid falloff of the differential cross section is not so easily explained by π exchange, even with absorption effects.[7]

(6) The decay angular distribution of the ρ. This is a difficult test for distinguishing between models because the ρ is produced strongly forward where, neglecting nucleon spin-flip, its decay distribution must be $\sin^2\theta$ (θ = angle between decay-π and incident-photon directions) regardless of the production mechanism, since at 0° the polarization remains the same as that of the γ. According to the diffraction picture in which the $\gamma+p \to \rho+p$ amplitude is proportional to the ρp-elastic scattering, one predicts that at high energy and off the forward direction the process is helicity-preserving. This is not inconsistent with rough results of the CEA hydrogen bubble chamber group.[11]

The DESY and CEA hydrogen bubble chamber groups[6,7] have evaluated the decay distribution in the π-exchange model, following the Jackson prescription, and compared it with their data. The agreement is poor.

(7) The mass distribution of the ρ. We discuss below how the diffraction process skews the ρ distribution towards low mass, in qualitative agreement with all four experiments. This effect also is difficult to explain in terms of conventional models.

II. PHOTOPRODUCTION OF ρ

A. Theory of Photodissociation of ρ

Photoproduction of ρ's on protons has been previously calculated by Berman and Drell[12] using the diffraction-dissociation mechanism. Their calculation is based in detail on the multiperipheral model of high-energy scattering. Here we would like to propose a simpler model of the diffraction production process, one which emphasizes the special characteristics of production processes in which the scattering system does not change its quantum numbers.

We introduce a phenomenological γ-ρ coupling,[13] $V = g_{\gamma\rho} m_\rho^2 A_\mu B_\mu$, which simply changes a photon to a ρ with a certain coefficient while not changing the "wave function."[14] See Fig. 1.

To first order in V the matrix element for the transition is

$$\langle \psi_q^{(-)\rho} | V | \psi_k^{(+)\gamma} \rangle, \tag{1}$$

where $\psi_k^{(+)\gamma}$ is the wave function of the incoming photon of momentum k (which we may take to be a plane wave, since the photon interacts only electromagnetically) and $\psi_q^{(-)\rho}$ is the wave function of the outgoing ρ,

[11] U. Maor (private communication).

[12] S. M. Berman and S. D. Drell, Phys. Rev. **133**, B791 (1964). Several groups have recently been examining various aspects of this mechanism for ρ photoproduction: A. Brandstetter and A. Levy; S. Drell and J. Trefil; P. G. O. Freund; and U. Maor and P. Yock.

[13] We can adopt the view explicated by M. Gell-Mann [Phys. Rev. **125**, 1067 (1962)] that one can write an unsubtracted dispersion relation with respect to the photon four-momentum squared, and that the intermediate ρ dominates the amplitude in question. In the present problem we assume that the ρ-N elastic scattering is nonhelicity-flip and calculate the amplitude for ρ helicities $\lambda = \pm 1$. This amplitude, using the $\gamma\rho$ coupling, is gauge-invariant for forward production of ρ's. The gauge we use (radiation gauge in the lab frame) seems appropriate because it implies no production of longitudinal ρ's. It should also be remembered that if the ρ couples to a conserved current, then gauge invariance automatically holds [L. Stodolsky, Phys. Rev. **134**, B1099 (1964), Ref. 15]. Thus, at least in this limit, there is no problem in formulating our theory. A different view of this formal problem may be obtained from examination of the regeneration picture of Sec. V of this paper. One of us (M.R.) would like to thank M. Nauenberg for discussion of the gauge invariance problem. Other discussion can be found in footnote of Ref. 12, S. Berman, Proceedings of the Conference on Photon Interactions in the BeV Range, Massachusetts Institute of Technology, 1963 (unpublished), and G. Feldman and P. T. Matthews, Phys. Rev. **132**, 823 (1963).

[14] In terms of a Feynman diagram (see Fig. 1) the amplitude is

$$g_{\gamma\rho} m_\rho^2 [1/(-m_{\pi\pi}^2)] M(k,q)$$

where $M(k,q)$ is the off-the-mass-shell invariant amplitude. We use the actual $\pi\pi$ final mass, since we can argue that the mass tends to be preserved by the diffraction scattering. The argument is as follows: Suppose that the nucleon has a diameter d_N perhaps a little larger than the ρ diameter d_ρ. The collision time of the ρ and nucleon is, say, d_N/c, while the natural period of the ρ is, say, $\gamma d_\rho/c$, where γ is the relativistic γ for the ρ in the lab frame. The natural period for the ρ to establish its identity is thus longer than the collision time by a factor $\gamma d_\rho/d_N$ which is distinctly larger than 1 in typical experiments. We show above that this rather shaky argument is not necessary to justify the $1/m_{\pi\pi}^2$ factor if the amplitude is evaluated in a more physical way.

asymptotically of momentum q. This $\psi_q^{(-)\rho}$ is the ρp component of the full state vector. We are thus neglecting any diffraction-dissociation process such as

$$\gamma \to K\bar{K}, \quad (K\bar{K})+p \to \rho p.$$

By arguments similar to those given just below, it may be seen that this corresponds to the neglect of a sum of inelastic ρ scattering amplitudes connecting the ρ to the various photon-like states ($K\bar{K}$, etc.).

By using the effective γ-ρ coupling for V, the matrix element is found to be

$$g_{\gamma\rho}m_\rho^2[2k2\omega_\rho(k)]^{-1/2}\langle\psi_q^{(-)}(x)|e^{i\mathbf{k}\cdot\mathbf{x}}\rangle. \tag{2}$$

To evaluate the overlap integral appearing here, we use the fact that the elastic-scattering wave function $\psi_q^{(-)}$ must obey the integral equation

$$\psi_q^{(-)}(x)=e^{i\mathbf{q}\cdot\mathbf{x}}-(H_0-E(q)-i\epsilon)^{-1}U\psi_q^{(-)}(x), \tag{3}$$

where U is the complex effective potential for ρ-elastic scattering on the target particle A. The energy of the whole system is

$$\omega_\rho+p^2/2M_A\approx\omega_\rho,$$

since at small momentum transfer the energy of A will be negligible. Thus

$$\langle e^{i\mathbf{k}\cdot\mathbf{x}}|\psi_q^{(-)}(x)\rangle=-1/(\omega_\rho(k)-\omega_\rho(q)) \times\langle e^{i\mathbf{k}\cdot\mathbf{x}}|U|\psi_q^{(-)}(x)\rangle. \tag{4}$$

We note that if we had $|\mathbf{k}|=|\mathbf{q}|$, the matrix element on the right-hand side would be the matrix element for elastic scattering of ρ's. The off-energy-shell effect is small if $(k-q)R\ll 1$, where R is the range of U. Now

$$\begin{aligned}\omega_\rho(k)-\omega_\rho(q)=(k^2+m_\rho^2)^{1/2}-(q^2+m_\rho)^{1/2} &\simeq(k-q)(1+m_\rho^2/2kq)\\ &\approx k-q\simeq m_{\pi\pi}^2/2k,\end{aligned}$$

in terms of the effective mass of the final $\pi\pi$ system.

These relations established, we henceforth neglect the small differences between ω_ρ, k, and q. Now since

$$\frac{d\sigma(\gamma\to\rho)}{d\Omega}=\frac{1}{(2\pi)^2}|\langle\psi^{(-)\rho}|V|\psi^{(+)\gamma}\rangle|^2q^2, \tag{5}$$

and since for resonance production we can introduce the dependence on actual mass by means of the one-level resonance formula, near the resonance,

$$\frac{d}{dm}\leftrightarrow\frac{1}{\pi}\frac{\Gamma/2}{(m_{\pi\pi}-m_\rho)^2+\Gamma^2/4},$$

we then have

$$\frac{d^2\sigma(\gamma\to\rho)}{dm_{\pi\pi}d\Omega}=g_{\gamma\rho}^2\left(\frac{m_\rho^4}{m_{\pi\pi}^4}\right)\frac{d\sigma(\rho\to\rho;\,q,k)}{d\Omega}\times\frac{1}{\pi}\frac{\Gamma/2}{(m_{\pi\pi}-m_\rho)^2+\Gamma^2/4}. \tag{6}$$

TABLE I. Observed ρ^0 mass in photoproduction.

Expt.	CEA (5)	CEA (6)	DESY (7)	DESY (8)
	Counter	Bubble chamber	Bubble chamber	Spark chamber
M_ρ (MeV)	740	728	729	720

The two momentum labels in the cross section on the right-hand side remind us that we must deal with the off-the-mass-shell scattering amplitude $f(\theta)=-q\langle\psi_q^{(-)}|U|e^{i\mathbf{k}\cdot\mathbf{x}}\rangle/2\pi$. For hydrogen, the off-shell effect should be very small. Strictly speaking, this ρ scattering cross section refers only to $\pm$ helicity,[15] of course.

Integrating (6), we have

$$\frac{d\sigma}{d\Omega}(\gamma\to\rho)=g_{\gamma\rho}^2\frac{d\sigma}{d\Omega}(\rho\to\rho;\,q,k). \tag{7}$$

Integrating again, we have

$$\sigma(\gamma\to\rho)=g_{\gamma\rho}^2\sigma_E, \tag{8}$$

where σ_E is the total-elastic-scattering cross section for ρ's on the target.

If we assume the ρ-elastic scattering amplitude is predominantly imaginary at 0°, we obtain, using the optical therem,

$$\frac{d\sigma}{d\Omega}(\gamma\to\rho,0°)=g_{\gamma\rho}^2\left(\frac{k}{\pi}\right)^2\sigma_T^2, \tag{9}$$

when σ_T is the total cross section for ρ's on the target. This is our most useful result.

B. The Mass Shift

We predict that the mass of the state produced in diffraction dissociation will be shifted downward by the factor $m_\rho^4/m_{\pi\pi}^4$ in (6). The effect is only likely to be observable in the case of a wide resonance such as the ρ. We find that there is a downward shift in the photoproduced ρ of about $\Gamma^2/2m_\rho\approx 10$ MeV compared with the ρ produced in the πN collisions ($m_\rho=760$ MeV, $\Gamma=120$ MeV). The shape of the ρ is also quite skewed.

The experimental data show a noticeable skewing qualitatively similar to that calculated, and a downward shift of 20–30 MeV as indicated in Table I. The presently observed shift thus seems larger than 10 MeV, but not necessarily in bad disagreement with our prediction because the experimental errors are large.

Another mechanism has been suggested by Söding[16] for a downward shift. Since π pairs can be produced without first producing a ρ, there will be a background of nonresonant p-wave π pairs. This background will

[15] S. M. Berman, Phys. Rev. Letters **11**, 220 (1963).
[16] P. Söding, Phys. Letters **19**, 702 (1965).

interfere with the pions produced by ρ decay, resulting in a mass shift. There is a well established example of this type of phenomenon: the shift of the N_{33}^* as seen in $\gamma+p\rightarrow\pi^++n$.[17] The effect, a downward shift of some 30 MeV, is large because, qualitatively speaking, the π^+n state is readily produced even in the absence of the N^*.

This interference process is, however, rather subtle,[18] and a convincing calculation is difficult. In any case there should be some other experimental effects of the Söding mechanism since the π's not produced via the ρ will be in all waves, and they can interfere with the ρ decay. If this background plays an important role, there should be, for example, a $\cos^4\theta$ term in the 2π decay angular distribution which varies with the ρ "mass" $m_{\pi\pi}$ in the region of the ρ peak. There would also be an easier-to-observe $\cos\theta$ term, but this is likely to be small except at small $m_{\pi\pi}$ because π^+p and π^-p scattering amplitudes are very similar, so that the initial $T=1$ pion pair will remain mainly $T=1$.

Furthermore, we would like to suggest that our explanation of the mass shift can be tested experimentally by electroproduction of ρ's at high energy. Here the incoming photon has effectively a negative $(\text{mass})^2$ equal to t, the invariant momentum transfer squared to the electron. Thus, as may be seen either from the "wave-function" approach [Eq. (1)] or the Feynman-diagram point of view, the mass-skewing factor changes from $(m_{\pi\pi})^{-4}$ to $(-t+m_{\pi\pi}^2)^{-2}$. Thus at high $(-t)$, the mass shift should tend to disappear, essentially because the range of energies of the incoming virtual photons necessary to produce the range of $m_{\pi\pi}$ in the ρ peak will correspond to only a small variation in k in the overlap integral in Eq. (2).

C. Dependence of $d\sigma/d\Omega$ on Nuclear Mass

The derivation of Eq. (6) applies directly to any target. We need not adopt any assumption that the ρ is produced on a nucleon in the nucleus and travels out, but rather may assume that the diffraction is on the nucleus as a whole. The distinction between conventional coherent mechanisms and our model can be emphasized by thinking of the effect of increasing the absorption of ρ's: In the conventional case the production rate goes down because of absorption; in the diffraction mechanism it goes up because of the resultant larger ρ cross section.

The relative cross sections for forward ρ photoproduction on various nuclei will be independent of $g_{\gamma\rho}$ and depend on ρ-nuclear total cross sections. We can calculate the ρ-nuclear total cross section in terms of σ_t, the proton total cross section, following the methods of the optical model,[19] and evaluate σ_t by comparison with the experimental A dependence of the forward photoproduction.[5] We will test this method and the nuclear parameters we use by first comparing our deductions with total-cross-section measurements of 3.0-BeV/c pions on nuclei by Longo and Moyer,[20] and of 19.3-BeV/c protons on nuclei at CERN.[21]

The optical potential for elastic scattering of ρ's, p's, or π's on nuclei (not hydrogen) has strength (integrated over the nuclear volume)

$$U_0=\tfrac{1}{2}A\sigma_t.$$

In this equation and below, we omit momentum-dependent factors which differ from unity at high energies by less than about 2%. For the moment we also neglect the effect of nucleon correlations associated with the Pauli principle, which are conventionally introduced here, but about which there is some controversy. Define Q by $U_0Q(\sigma_T,\mathbf{k},\mathbf{q})=2\pi f(\theta)/\omega$. The laboratory cross section on a nucleus is

$$(d\sigma/d\Omega)_A=KA^2|Q|^2(d\sigma/d\Omega)_H. \qquad (10)$$

The constant K is evaluated by comparison with the π and p data. We find that K is insensitive to σ_T. Presumably K represents the various inadequacies of the calculation.

For ease of calculation, we evaluate Q by assuming that the optical potential is a square well and that,

---

[17] The effect can be seen in any photoproduction calculation which includes both Born and isobar contributions. For an explicit presentation see M. Ross, Phys. Rev. **103**, 760 (1956), Sec. IIIC.

[18] We sketch the relevant final-state interaction theory. [J. Gillespie, *Final State Interactions* (Holden Day, San Francisco, 1964), Chap. 7, and N. Zigury (to be published).] Consider a weak or electromagnetic hadron-production amplitude, to be specific, ρ^0 photoproduction. Let the elastic scattering of $\pi^+\pi^-$ in the resonance p state be given by $h(\omega)=e^{i\delta}\sin\delta$. The production amplitude G of p-wave pion pairs as a function of their energy can be written

$$G=B+\frac{1}{\pi}\int d\omega'\frac{h^*G}{\omega'-\omega},$$

where B contains all singularity structure except that from the final-state $\pi\pi$, i.e., ρ scattering. (Problems arise here because of the presence of the third strongly interacting particle in the final state which we feel we can neglect.) The solution of this Omne's-type equation can be written

$$G=e^{i\delta}B\cos\delta+\frac{1}{D}\frac{P}{\pi}\int\frac{\rho BNd\omega'}{\omega'-\omega},$$

where $\rho=k^3/N$ and $h/\rho=N/D$ in the usual way. Now $1/D$ and G have the phase of $e^{i\delta}$. We can write

$$G(\omega)=e^{i\delta}[B'(\omega)\cos\delta+C\sin\delta],$$

where C is real and independent of ω, and B' is real and probably varies smoothly in the resonance region. The Watson final-state approximation is usually considered to involve neglecting B'. Consider two cases: (a) if $B(\omega)$ has the same left-hand singularities as those which generate $h(\omega)$, then $G\propto h$ and B' is very small. (b) If B has very different left-hand singularities, then, for example, B' may be comparable with B. In case (a), as in π^+n photoproduction, the shift in resonance mass is likely to be downward because C and B are likely to be of the same sign.

[19] M. L. Goldberger and K. M. Watson, *Collision Theory* (John Wiley & Sons, Inc., New York, 1964); R. Lipperheide and D. Saxon, Phys. Rev. **120**, 1458 (1960); R. Frank, J. Gammel and K. Watson, *ibid.* **102** 1157 (1956); K. Watson and C. Zemach, Nuovo Cimento **10**, 452 (1958); R. J. Glauber, in *Lectures in Theoretical Physics*, edited by W. E. Brittin and Lita G. Dunham (Interscience Publishers, Inc., New York, 1959).

[20] M. Longo and B. Moyer, Phys. Rev. **125**, 701 (1962).

[21] Belletini *et al.*, Nucl. Phys. **79**, 609 (1966). We assume their $d\sigma/dt$ for carbon measured at 21.5 BeV/c would be the same at 19.3 BeV/c.

within the sphere of radius $R=r_0A^{1/3}$, the wave function is

$$\psi^{(+)}(\mathbf{x})=\exp\left(i\mathbf{q}\cdot\mathbf{x}-\frac{z+(R^2-\rho^2)^{1/2}}{2}\frac{3\sigma_t}{4\pi r_0^3}\right),$$

where z and ρ are cylindrical coordinates in the nucleus. Thus Q is a function of momentum transfer $\mathbf{\Delta}=\mathbf{q}-\mathbf{k}$ and is determined by integrating over the sphere $|\mathbf{x}|\leq R$:

$$Q(\sigma_T,\mathbf{\Delta})=\frac{3}{4\pi r_0^3A}\int d\mathbf{x}$$
$$\times\exp\left(i\mathbf{\Delta}\cdot\mathbf{x}-\frac{z+(R^2-\rho^2)^{1/2}}{2}\frac{3\sigma_T}{4\pi r_0^3}\right), \quad (11)$$

where we use $r_0=1.2\times10^{-13}$ cm.

For forward π and p scattering, $\Delta=0$ and we obtain the results shown in Fig. 2. The value of K used is 1.37. We now apply the expressions to forward ρ production. We fix the hydrogen point from the data, and find, using the same value of K, that $\sigma_T(\rho p)=50$ mb yields a good fit to both the shape and magnitude of the nuclear production data. To show the sensitivity of this type of determination, the curve for $\sigma_T=40$ mb is also shown.

This determination would become almost completely empirical and rather accurate if $\bar{p}$-nucleus total cross sections were also measured at high energy, so that we could bracket the cross section from above.

We conclude that $\sigma_T(\rho p)=50\pm5$ mb.

D. ρ Scattering and Coupling Parameters

With this value for $\sigma_T(\rho p)$, we can determine $g_{\gamma\rho}$. Now Ref. 5 gives, at 4.4 BeV, for $\gamma+p\to\rho^0+p$ at 0°:

$$d\sigma/d\Omega=1.36\pm0.20 \text{ mb/sr};$$

thus Eq. (9) gives $g_{\gamma\rho}\sigma_T=2.10$ μb, and so we have

$$g_{\gamma\rho}^2=0.24\alpha(1\pm0.3),$$

where $\alpha=1/137$, the fine-structure constant. The errors quoted are our estimates based only on the quoted errors in the data,[5] which are essentially statistical. The determination of σ_T is the more dependable because it depends on the relative cross sections, while determination of $g_{\gamma\rho}^2$ involves the absolute cross section on hydrogen, which may be slightly high, according to bubble-chamber results. No error is included for theoretical assumptions such as use of the optical theorem to give the forward amplitude and neglect of other processes.

Knowing $g_{\gamma\rho}^2$, we can find the approximate elastic-ρp cross section from Eq. (8). The experiments appear to disagree slightly on $\sigma(\gamma\to\rho)$. In hydrogen bubble

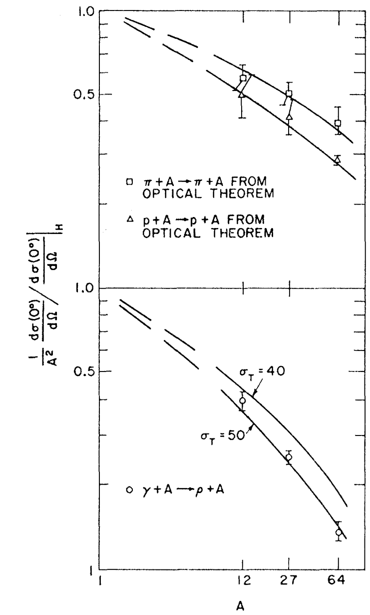


FIG. 2. The ratio of experimental $(1/A^2)(d\sigma/d\Omega)_{0°}$ to that for hydrogen, evaluated by neglecting $\mathrm{Re}\,f(0°)$ and obtaining $\mathrm{Im}\,f(0°)$ from the total-cross-section measurements of Refs. 20 and 21, is compared with $1.37|Q|^2$ in the upper graph. The factor 1.37 was chosen to make a good fit. For π's on hydrogen at 3.0 BeV/c, a cross section $\sigma_T=31$ mb was used. For the p's at 19.3 BeV/c, a cross section $\sigma_T=39.5$ mb was used. In the lower graph the A dependence of the ρ production at 0° measured in Ref. 5 is compared with $1.37|Q|^2$ for $\sigma_T=40$ and 50 mb. (The experimental points shown reflect certain small additional refinements graciously communicated by Professor F. M. Pipkin.)

chambers (DESY[7] and CEA,[6] respectively), we have

$$\sigma_{\gamma\rho}=11.5\pm3\ \mu b,\quad E_\gamma\approx4.5 \text{ BeV},\quad |t_0|<0.5 \text{ BeV}^2 \quad \text{(DESY)}$$

$$\sigma_{\gamma\rho}=15.4\pm2.2\ \mu b,\quad E_\gamma\approx4.5 \text{ BeV},\quad |t_0|<0.4 \quad \text{(CEA)}.$$

The experimenters integrated over all but large angles as shown by the limit on the square of the four-momentum transfer, t_0. Meanwhile, from the counter experiments,[5] we can estimate the production cross section by using

$$\sigma_{\gamma\rho}=\int_{t_{\min}}^{t_{\max}} e^{12t}\frac{d\sigma}{dt}(0°)dt.$$

From the measured cross section in hydrogen of $(d\sigma/d\Omega)_{0°}=1.36\pm0.20$ μb/sr in the lab, we get

$$\sigma_{\gamma\rho}=17.6\pm2\ \mu\text{b},\quad E_\gamma\approx4.4 \text{ BeV},\quad |t_0|<0.4 \text{ BeV}^2.$$

Here the error indicates the error in $(d\sigma/dt)_{0°}$ and not the possible error in the guessed value (see below), 12 BeV^{-2}, for the parameter governing the t dependence. The CEA bubble chamber result contains a subtraction of perhaps 10%, and the DESY bubble chamber results a subtraction of about $\frac{1}{3}$, of the events in the ρ region as a phase-space background, while the counter experiment involves a very small subtraction;

TABLE II. Differential scattering on protons. The slope $a=(d/dt)\ln(d\sigma/dt)$ at $t=-0.3$ BeV^2.

Particle	K^+	K^-	π^+	π^-	p(6–14 BeV)	p(14–22 BeV)	$\bar{p}$
a (BeV^{-2})	6.05	7.32	7.40	7.82	8.23	8.72	12.6
σ_T (mb)	17.2	22.5	24.4	25.6	39.8	38.7	54.1

so these results are closer than indicated. If we take $\sigma_{\gamma\rho}\approx 16\,\mu$b we find then $\sigma_E(\rho p)\approx 8.4$ mb, or $\sigma_E/\sigma_T\approx 17\%$.

A different way to obtain the elastic cross section is from the observed shape of $d\sigma/d\Omega(\gamma\to\rho)$. Assume the shape is $d\sigma/dt\propto e^{at}$ for small momentum transfer t. The elastic scattering of ρ's has the same shape, according to our model, so

$$\sigma_E=\int dt\frac{d\sigma}{dt}(0°)e^{at}=\frac{1}{16\pi}\sigma_T^2\frac{1}{a},$$

using the optical-theorem assumption. Unfortunately, the observations do not yield a good value for a. Instead we can deduce its value from an empirical relation between a and σ_T, since a varies rather regularly from particle to particle with the observed value of σ_T as shown in Table II.[22] Using our value of $\sigma_T(\rho p)=50$ μb, we obtain $a\approx 11$ BeV^{-2} at $t=-0.3$ BeV^2. This number depends sensitively on the $\bar{p}p$ scattering. We estimate from the average behavior of K's, π's, and p's on protons that, for $t\approx 0$, $a\approx 12$ BeV^{-2} for ρp scattering. These a's are in rough agreement with data.[5–7] The value $a=12$ BeV^{-2} then yields $\sigma_E=10.3$ mb.

E. Discussion of σ_T and $g_{\gamma\rho}^2$

The total ρ-p cross section at 4.4 BeV/c, $\sigma_T=50$ mb, can be compared with analyses of ρ production involving final-state absorption. Jackson *et al.*[23] determine final-state scattering parameters to achieve a fit to the experimental data on $\pi^\pm p\to\rho N$ at 4 BeV/c. Using their simpler form [Eq. (3) of Ref. 23], the ρN cross section (in their notation) is $\sigma_T=4\pi C_-/2q_-^2\gamma_-$. Their fit, $C_-=1.0$ and $\gamma_-=3.0\times10^{-2}$, yields $\sigma_T(\rho N)=58$ mb. Considering the relative insensitivity of their results to σ_T (see their Fig. 2 showing two curves which correspond to considerably smaller σ_T but which are almost as satisfactory), we find this to be good agreement. It is fair to say that Jackson *et al.* tend to confirm that $\sigma_T(\rho N)>\sigma_T(\pi N)\approx 30\,\mu$b. Byers and Yang[24] also claim on the basis of their droplet model that $\sigma_T(\rho N)>\sigma_T(\pi N)$.

The ρ-γ coupling strength, $g_{\gamma\rho}^2=0.24\alpha$, can be compared, in principle, with the charge of the pion. If we can write an unsubtracted dispersion relation and if the ρ dominates,[25] we have $e^2=g_{\gamma\rho}^2f_{\rho\pi\pi}^2$. Since (for a ρ width of 120 MeV) $f_{\rho\pi\pi}^2=2.4$ (the true ρ width and thus $f_{\rho\pi\pi}^2$ may be somewhat smaller), this yields $g_{\gamma\rho}^2=0.42\alpha$.

This argument is usually made in terms of the nucleon form factor, where the weight of the ρ in the nucleon isovector form factor is given essentially by $g_{\gamma\rho}f_{\rho nn}$. Now usually form-factor analyses[26] in which the ρ is given its actual mass give the ρ contribution a weight >1 with a negative background (or "ρ'") term; this would be compatible with our $g_{\gamma\rho}^2$ if $f_{\rho nn}^2$ were larger than the universal isospin coupling value $f_{\rho\pi\pi}^2$, a possibility certainly not ruled out by nuclear-force calculations.

Our main point here is the good agreement between $g_{\gamma\rho}^2$ deduced from our model of ρ photoproduction and that found from ρ dominance of the form factor. If we go further and neglect possible errors in the ρ-photoproduction determination, it is seen that diffraction dissociation of the photon could provide important constraints on the form-factor analysis. Repetition and refinement of the ρ-photoproduction experiment, at 0° on nuclei as well as hydrogen, would be essential, especially to reduce the possibility of systematic errors.

Similar information is available in the quantitative analysis of electromagnetic ρ decay. Our $g_{\gamma\rho}$ yields an e- or μ-pair branching ratio of 0.28×10^{-4},[27] in excellent agreement with the experimental[28] value of

$$(0.33_{-0.07}^{+0.16})\times10^{-4}.$$

III. PHOTOPRODUCTION OF ω AND φ

The methods used above apply equally well to the production of the photon-like states ω and φ. The most striking aspect of the data thus far is that while ω production[6,7] relative to ρ production is down by perhaps $\frac{1}{5}$, φ production is almost totally absent[29] (perhaps $\lesssim 1/50$). Now according to the usual φ-ω mixing theory, the photon couples to

$$\rho^0+(1/\sqrt{3})\varphi_0,\quad \varphi_0\cong(\sqrt{2}\varphi+\omega)/\sqrt{3}\text{ (the unitary octet).}$$

We note that the fact that ω and φ production is smaller than ρ production is in qualitative agreement with our model in this context. Now for ω and φ, the theory

[22] The data on σ_T are from W. Galbraith *et al.*, Phys. Rev. **138**, B913 (1965). The data on a are from K. J. Foley *et al.*, Phys. Rev. Letters **15**, 45 (1965), and previous experiments by the same group referred to therein.

[23] J. D. Jackson *et al.*, Phys. Rev. **139**, B428 (1965).

[24] N. Byers and C. N. Yang, Phys. Rev. **142**, 976 (1966).

[25] J. J. Sakurai, in *Proceedings of the International School of Physics "Enrico Fermi", Varenna, Italy, XXVI Corso* (Academic Press Inc., New York, 1963); M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).

[26] R. Wilson, in *Proceedings of the International Symposium on Electron and Photon Interactions at High Energies, Hamburg* (Springer-Verlag, Berlin, 1966).

[27] The theoretical expression is contained in M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters **8**, 261 (1962).

[28] J. K. dePagter *et al.*, Phys. Rev. Letters **16**, 35 (1966); see also R. A. Zdanis *et al.*, *ibid.* **14**, 721 (1965).

[29] This is our estimate based on preliminary evidence from the CEA bubble chamber group (see Ref. 6). We would like to thank V. Fischer of the group for information on photoproduction of strange particles.

given above leads to

$$\begin{pmatrix} A_\omega \\ A_\phi \end{pmatrix} = \frac{g m_\rho^2}{\sqrt{3}} \begin{pmatrix} f_{\omega\omega} & f_{\omega\varphi} \\ f_{\omega\varphi} & f_{\varphi\varphi} \end{pmatrix} \begin{pmatrix} 1/m_\omega^2 & 0 \\ 0 & 1/m_\varphi^2 \end{pmatrix} \begin{pmatrix} (\frac{1}{3})^{1/2} \\ (\frac{2}{3})^{1/2} \end{pmatrix}, \qquad (12)$$

where for the $\gamma \to \rho$ amplitude we had $A_\rho = g f_{\rho\rho}$. Here f_{ij} are the "elastic" scattering amplitudes for vector meson i into j on protons. Note that the mass differences introduce an ω_0 (unitary singlet) component "before" the scattering. There is some ambiguity about this point. Some authors[30] construct the effective couplings in just such a way that we would have pure φ_0, even with the actual masses. It is a difficult point, and for definiteness we proceed with Eq. (12) in mind. There are at least three simple possibilities that may account for the small φ production. (1) Unitary-spin exchange may be negligible, with the diagonal amplitudes for the unitary octet φ_0 and unitary singlet ω_0 combining in such a way that the coherent combination in (12) cancels the φ production. (2) The scattering may be φ-ω diagonal and the φ scattering is small. (3) Finally, of course, something may be radically wrong with the mixing formulation so that the photon does not couple to φ. Now in the first case, ω_0 and φ_0 scatter diagonally with amplitudes f_1 and f_8, with $f_8 = f_\rho$, since φ_0 and ρ are in the same octet. This gives

$$\frac{\sigma(\gamma \to \omega)}{\sigma(\gamma \to \rho)} = \left(\frac{2.20 + 0.80 f_1/f_8}{9}\right)^2,$$

$$\frac{\sigma(\gamma \to \varphi)}{\sigma(\gamma \to \rho)} = \left(\frac{3.11 - 0.56 f_1/f_8}{9}\right)^2.$$

A satisfactory fit ($\frac{1}{4}$ and 1/40 for these ratios, respectively), is obtained for $f_1 \approx 3 f_8$. This is an interesting result. It implies a very large ω_0-nuclear cross section, $\sigma(\omega_0) \approx 150$ mb. If the other coupling scheme is correct and we simply have φ_0 "before" scattering, then this φ_0-ω_0 diagonal scattering clearly cannot lead to small φ production.

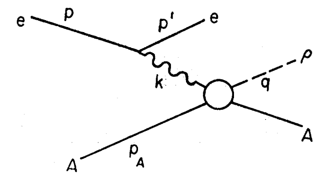
On the other hand, if we simply have the physical ω and φ scattering diagonally, then $\sigma(\gamma \to \omega)/\sigma(\gamma \to \rho) \sim \frac{1}{5}$ and $\sigma(\gamma \to \varphi)/\sigma(\gamma \to \rho) \lesssim 1/50$ give $f_\omega \approx (3/\sqrt{5}) f_\rho$ and $f_\varphi \leq (3/10)(m_\varphi^2/m_\rho^2) f_\rho$, and since the cross sections for $\omega\rho$ and $\varphi\rho$ are proportional by the optical theorem to these forward diffraction amplitudes,

$$\sigma_T(\omega) = \frac{3}{\sqrt{5}} \sigma_T(\rho) \approx 67 \text{ mb}$$
$$\sigma_T(\varphi) \lesssim \frac{3}{10} \frac{m_\varphi^2}{m_\rho^2} \sigma_T(\rho) \approx 27 \text{ mb}, \qquad (13)$$

with appropriate adjustment for the other coupling scheme.

[30] R. F. Dashen and D. H. Sharp, Phys. Rev. **133**, B1585 (1964); J. J. Sakurai, Nuovo Cimento **34**, 1582 (1964).

FIG. 3. Diagram for electroproduction of ρ's on target A. In terms of the four-momenta P_A, P, P', q, k, we define $s = (P + P_A)^2$, $s_0 = (k + P_A)^2$, $t = (p' - p)^2 = k^2$, and $t_0 = (k - q)^2$.



The nuclear production here is quite interesting because the φ-ω mixing and the near momentum degeneracy of the φ and ω energy in high-energy production suggest the possibility that there can be "regeneration" effects in coherent nuclear production quite analogous to the famous K^0-meson phenomena. A discussion of this possibility is to be published by us elsewhere (Phys. Rev. Letters, to be published).

IV. PROPOSED EXPERIMENTS

We briefly summarize the possibilities for a number of interesting experiments suggested by our discussion:

(A) Electroproduction of vector mesons, particularly ρ^0, provides a profound test of the theory. The electroproduction amplitude can be written as a sum of products[31]: a factor depending on the electron times a photoproduction amplitude $A_{\lambda\lambda'}(s_0 t_0; t)$. (See Fig. 3 for definition of the variables.) These $\lambda\lambda'$ are the intermeditate-photon and final-ρ helicities, respectively. For example, if the azimuthal angle and the spins are summed over, the electroproduction cross section is

$$\frac{d^3\sigma_{\lambda'}(e + A \to e + \rho + A)}{dt\, ds_0\, dt_0} = \sum_\lambda f_\lambda \frac{d\sigma_{\lambda\lambda'}(\gamma \to \rho)}{dt_0}, \qquad (14)$$

where according to the diffraction-dissociation model,

$$\frac{d\sigma_{\lambda\lambda'}}{dt_0} = g_{\gamma\rho}^2 \frac{m_\rho^2}{-t + m_{\pi\pi}^2} \frac{d\sigma_{\lambda\lambda'}}{dt_0}(\rho \to \rho; s_0 t_0) \qquad (15)$$

and

$$\frac{d\sigma_{\lambda\lambda'}}{dt_0}(\rho \to \rho) \propto \delta_{\lambda\lambda'}.$$

The f_λ are explicit functions of s, s_0, t, and t_0. There seem to be no experimental problems, except diminishing cross section, in varying the observed electron energy and angle so that $-t_0$ remains small while $-t$ varies from near zero to values $> m_\rho^2$. Thus one can test whether $d\sigma_{\pm 1, \pm 1}(\gamma \to \rho)/dt$ follows the $1/(-t + m_{\pi\pi}^2)$ dependence, whether the production of longitudinal ρ's grows from zero as $-t$ grows from zero, and whether the mass shift and skewing of the ρ disappear for large $-t$.

Finally, behavior of the cross sections according to the simple formulas (14) and (15) would help to show that there are no anomalies connected with the use of the direct ρ-γ coupling $g_{\gamma\rho}$.

[31] R. H. Dalitz and D. Yennie, Phys. Rev. **105**, 1598 (1957); M. Gourdin, Nuovo Cimento **21**, 1094 (1961); L. Hand, Phys. Rev. **129**, 1834 (1963); and S. Berman, *ibid.* **135**, 1249 (1964).

(B) Obviously it is desirable to improve the ρ-photoproduction measurements on hydrogen and nuclei and, when possible, to do the experiment at higher total energy. Of immediate interest would be study of the ρ decay with reference to helicity states to determine the degree to which $\lambda=0$ is missing, and study of the decay angular distribution of the ρ in photoproduction versus energy across the resonance, which can test whether other partial waves are present as in the Söding mechanism (see Sec. IIB).

(C) The forward photoproduction of ω and φ on nuclei can be used in conjunction with data on hydrogen to help determine $\sigma_{T,E}(\omega p)$, $g_{\gamma\omega}^2$, $\sigma_{T,E}(\varphi p)$, and $g_{\gamma\varphi}^2$ by the method of Sec. III, and to see if regeneration effects exist.

V. ALTERNATIVE DISCUSSION OF DIFFRACTION DISSOCIATION IN TERMS OF REGENERATION

In order to elucidate the derivation of our result Eq. (7) we calculate $\gamma+A \to \rho+A$ in yet another way. Consider a target which is a plane absorbing slab of nuclear matter. Our model is that there are two states, γ_0 and ρ_0, which are diagonal in the target material, i.e., they either scatter elastically or are absorbed in the sense of exciting the target. In particular, the γ_0 does not interact with the target, while the ρ_0 interaction is characterized by an absorption parameter λ (for $0<x<a$). We assume a unitary (and real) transformation from the state vectors γ, ρ that are diagonal in vacuo to the state vectors γ_0, ρ_0 that are diagonal in the medium:

$$\gamma=(1-\epsilon^2)^{1/2}\gamma_0+\epsilon\rho_0\,,$$
$$\rho=-\epsilon\gamma_0+(1+\epsilon^2)^{1/2}\rho_0.$$

The intuitive concept is that the electromagnetic part of the photon state vector (bare-photon, lepton pairs, etc.) interacts weakly with the nuclear medium, while the hadron component of the photon state vector is strongly absorbed.

We can, at high energy, neglect reflected waves and use continuity of the waves, and not of their derivatives, on the boundaries. Thus the wave functions are

$$\begin{array}{ll} e^{ikx}\gamma\,, & x<0 \\ \alpha_\gamma e^{ikx}\gamma_0+\alpha_\rho e^{i\kappa x-\lambda x}\rho_0\,, & 0<x<a \\ t_\gamma e^{ikx}\gamma+t_\rho e^{iqx}\rho\,, & x>a \end{array}$$

where the α's and t's are constants and κ is a real momentum, presumably very close in value to k and q. The ρ-production amplitude t_ρ is immediately found to be

$$t_\rho=-e^{i(q-k)a}\epsilon(1-\epsilon^2)^{1/2}(1-e^{i(\kappa-k)a-\lambda a})\,.$$

We interpret ϵ as follows: The interaction

$$V=gm_\rho^2A_\mu^0B_\mu^0$$

in vacuum leads to the 2×2 Hamiltonian (labeled by γ_0 and ρ_0 states)

$$H=\begin{pmatrix} k & g[m_\rho^2/2(k\omega(k))^{1/2}] \\ g[m_\rho^2/2(k\omega(k))^{1/2}] & \omega(k) \end{pmatrix},$$

where $\omega(k)=(k^2+m^2)^{1/2}\approx k+[m^2/2(k\omega)^{1/2}]$. We neglect corrections to the diagonal elements of order g^2. Note the distinction between the actual mass m of the hadron component and the nominal mass m_ρ conventionally used in the definition of V. Constructing the eigenstates of H, we find

$$\epsilon=\frac{m^2}{2gm_\rho^2}\left[1-\left(1+\frac{4g^2m_\rho^4}{m^4}\right)^{1/2}\right]$$
$$\simeq g(m_\rho^2/m^2) \quad \text{where} \quad \epsilon\ll1\,.$$

Thus we finally obtain:

$$|t_\rho|=g(m_\rho^2/m^2)\,|1-e^{i(\kappa-k)a-\lambda a}|\,.$$

Since $e^{ika}-e^{i\kappa a-\lambda a}$ is the form of the amplitude on the back or outgoing surface in elastic diffraction scattering, this result for t_ρ is the same factorable amplitude for photoproduction as obtained above by other methods. It is of special interest in relation to our explanation of the mass shift that, in the case of large absorption ($\lambda a\gtrsim1$), the $1/m^2$ factor here arises from matching the ρ components on the back surface and so is directly associated with the actual mass m of the ρ produced. Also, for large λa, note that the total cross section for $\gamma\to\rho$ is

$$\sigma_{\gamma\rho}=Ag^2$$

in agreement with (6), since the total elastic cross section is equal to the area A of the absorber.

By extension, we see that as the γ energy is increased and as more and more channels open up, the cross section into each channel goes to an energy-independent value. The sum of all these cross sections converges to a finite constant even as the numbers of channels $\to\infty$ with energy, because the total probability of all the components in the state vector is finite. In other words, there is a sum rule on the coupling constants $g_{\gamma X}^2$ which guarantees that the total photo-cross-section goes to a constant at high energy.

Let us use this approach to make a general estimate of the photoproduction cross section for hadrons. The reaction $\gamma+A\to X+A$ is likely to be dominated by diffraction dissociation of the photon (when the quantum numbers of X allow). (This claim is made for high energy; see below). If this is true, the situation is interesting for the theorist because he can make an estimate of the total photon-dissociation cross section on simple physical grounds. Let the total probability for all hadrons in the γ state vector be P_h. Then at high energy the diffraction-dissociation cross section into hadrons will be

$$\sigma = P_h \pi R^2 .$$

A reasonable radius for the nucleon is 0.8 to 0.9 F. The basic question is the order of magnitude of P_h. We will guess that $P_h \approx \alpha$, the fine structure constant. We are then in a position to estimate the production of particular hadrons by a photon beam at high energy without making an error of more than an order of magnitude.

As an example, we estimate the cross section $\sigma(\gamma \to K)$ for photoproduction of K's and K^*'s. We consider photons of 16 BeV/c laboratory momentum, high enough that the momentum transfer satisfies

$$\Delta = m^2/2k \ll 1/R .$$

(If m is the mass of two K^*'s relatively at rest and R is a reasonable nucleon radius, say 0.8 F, then $k \gg 6$ BeV/c is satisfactory.) To estimate the percentage of K's in the hadron component we use the fraction of K production to total absorption in pp reactions at 24 BeV/c, roughly 5%.[32] Using this number we estimate

$$\sigma(\gamma \to K\text{'s}) \approx 0.05\alpha(\pi R^2) \approx 7\ \mu\text{b} .$$

This is the total cross section.[33]

ACKNOWLEDGMENTS

One of us (M.R.) would like to acknowledge the hospitality of the Stanford Linear Accelerator and the University of Washington Physics Department, where some of the preliminary work was done, and also more recently the hospitality of the Physics Department, MIT. We would also like to thank Dr. R. Deck for his generous assistance with the numerical work.

[32] A. N. Diddens *et al.*, Nuovo Cimento **31**, 961 (1964).

[33] This calculation may be compared with a conventional peripheral calculation (vector-meson exchange) which we feel is subject to very large uncertainties: S. Drell and M. Jacob, Phys. Rev. **138**, B1312 (1965).

PHYSICAL REVIEW VOLUME 149, NUMBER 4 30 SEPTEMBER 1966

Modified Effective-Range Approximation Based on Regge Poles. Application to Low-Energy π-N Phase Shifts

U. Trivedi
Tata Institute of Fundamental Research, Bombay, India

(Received 21 April 1966)

An improved form of the Khuri-Udgaonkar ansatz is presented. Its justification has been sought in the domain of scattering by Yukawa potentials. The ansatz is then applied to P_{11} and P_{33} amplitudes of the π-N scattering for low energies, assuming that the principal contributions arise from nucleon, N^*, and ρ exchanges. It is found that the ρ-exchange contribution is responsible for a turning of the P_{11} phase shift.

I. INTRODUCTION

An effective-range approximation has been proposed by Khuri and Udgaonkar[1] based on a modified Regge representation given by Khuri.[2] This effective-range approximation starts with an ansatz,

$$a(J,W) = \tfrac{1}{2}\beta(W) \frac{e^{-[J-\alpha(W)]\xi_t(W)} + e^{-[J-\alpha(W)]\xi_u(W)}}{J-\alpha(W)}, \quad (1.1)$$

for the contribution of a single Regge pole. Here ξ_t and ξ_u give rise to the force cuts arising from the lowest mass exchanges in the t and u channels, respectively. This ansatz was applied to the P_{11} amplitude of πN scattering, with the hypothesis that the nucleon lies on a Regge trajectory, the lowest mass exchanges being the nucleon exchange in the u channel and 2π exchange in the t channel. The results thus obtained were an improvement over those of Chew and Low, and of Balázs.[1]

However, the expression in Eq. (1.1) has certain shortcomings. Firstly, it gives formally an equal weight to the t- and u-channel exchanges, a situation which is not at all expected in general; indeed in the $(\frac{3}{2},\frac{3}{2})$ partial wave of the π-N system it is the u-channel exchanges which dominate over the t-channel ones. Secondly, it does not exhibit cuts due to higher mass exchanges. We suggest that both these shortcomings may be removed if one uses the alternative ansatz

$$a(J,W) = \sum_{j=1}^{M} \frac{\beta_j(W) e^{-[J-\alpha(W)]\xi_j(W)}}{J-\alpha(W)}, \quad (1.2)$$

wherein $\xi_j(W)$, $j=1, 2, \cdots, M$ give rise to higher mass exchange cuts, each such exchange being given a different weight.

Section II gives a justification for this ansatz by considering scattering by Yukawa potentials. In Sec. III we apply it to the P_{11} and P_{33} amplitudes of π-N scattering. Section IV contains the results which in Sec. V are then discussed and compared with earlier effective-range calculations.

[1] N. N Khuri and B. M. Udgaonkar, Phys. Rev. Letters **10**, 172 (1963), and references cited therein.

[2] N. N. Khuri, Phys. Rev. **130**, 429 (1963).