stant. On the other hand, Kacser, Singer, and Truong¹⁶ have studied the effect of the pion-pion interactions on K_{e4} decays and have reached a contradictory conclusion. They obtain agreement with the experimental rate for K_{e4}^{+} decay when the s-wave pion-pion interaction is described by a scattering length approximation with a scattering length of $a_0 = (1 \pm 0.3)$ pion Compton wave-

¹⁶ C. Kacser, P. Singer, and T. Truong, Phys. Rev. 137, B1605 (1965).

lengths. If they assume a σ meson dominates the *s*-wave pion-pion interaction in their model, the calculated rate becomes larger than the experimental one by two orders of magnitude. The contradiction between these papers seems to arise, at least in part, from differing assumptions about the effect of the breaking of SU(3), as manifested in the large K- π mass difference, upon the relative magnitude of coupling of the σ to pions and to kaons. In any case, our conclusions support the σ model rather than the scattering-length model.

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Baryon-Antibaryon Scattering and the Collinear $U(3) \times U(3)$ Symmetry

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Baryon-antibaryon scattering processes have been considered in collinear $U(3) \times U(3)$ symmetry. Simple relations between the differential cross sections are found and have been compared with the available experimental data.

 \mathbb{C} INCE the birth of the $SU(6)_{\sigma}$ symmetry,¹ various **J** attempts have been made to obtain a Lorentzinvariant theory incorporating momentum and angular momentum of the particles.^{2–4} The \tilde{U} (12) theory which was proposed for the relativistic extension of the $SU(6)_{\sigma}$ group, seriously violated unitarity and the kineticenergy term was found noninvariant. Then a large school of thought started breaking the U(6,6) symmetry with the kineton spurion, and the $U(6) \times U(6)$, $SU(6)_W$, and the coplanar $U(3) \times U(3)$ symmetry were obtained.^{5,6} The collinear $SU(6)_W$ symmetry, which incorporates Lorentz invariance (the generators of this group commute with those of the Lorentz group in the z direction⁶), had its successes and limitations when applied to meson-baryon scattering, vertex functions, and weak decay phenomena.^{7,8}

In this note, we wish to investigate the consequences

A. Pais, 2020. 13, 175 (1904); B. Sakita, Phys. Rev. 136, B1750 (1964). ² A. Salam, R. Delbourgo, M. A. Rashid, and J. Strathdee, Proc. Roy. Soc. (London) 284A, 146 (1965); 285A, 312 (1965); B. Sakita and K. C. Wali, Phys. Rev. Letters 14, 404 (1965); Phys. Rev. 139, B1355 (1965). ³ T. Fulton and J. Wess, Phys. Letters 14, 57 (1965); 14, 334 (1965). W. Rühl, *ibid.* 13, 349 (1965); 14, 334 (1965). ⁴ K. Bardakci, J. M. Cornwall, P. G. O. Freund, and B. W. Lee, Phys. Rev. Letters 14, 48 (1965). ⁶ R. Dashen and M. Gell-Mann, Phys. Letters 17, 142 (1965). ⁶ H. Harari and H. J. Lipkin, Phys. Rev. 140, B1617 (1965); H. J. Lipkin and S. Meshkov, Phys. Rev. Letters 14, 670 (1965). See also D. Horn, lecture delivered at the Argonne National Laboratory, High Energy Physics Division, 1966 (unpublished). ⁷ D. Horn, M. Kugler, H. J. Lipkin, S. Meshkov, J. C. Carter, and J. J. Coyne, Phys. Rev. Letters 14, 717 (1965). ⁸ K. C. Tripathy, Phys. Rev. 146, 1107 (1966). This paper contains further references on the collinear $SU(6)_W$.

of the baryon-antibaryon scattering in the framework of the collinear $U(3) \times U(3)$ group of Volkov and Ruegg,^{9,10} confining ourselves only to the l=0 case. The validity of this theory for meson-baryon scattering,¹⁰ vertex functions, weak decays,¹¹ and proton-antiproton annihilation into two mesons,¹² have been considered and the successes seem to be quite promising.

Following Volkov and Ruegg,¹⁰ the 56-plet and 35plet of the U(6) reduce under this scheme to

$$56 = (10,1)_{+3/2} \oplus (1,10)_{-3/2} \oplus (6,3)_{+1/2} \oplus (3,6)_{-1/2},$$

$$35 = (8,1)_0 \oplus (1,8)_0 \oplus (1,1)_0 \oplus (3,\overline{3})_{+1} \oplus (\overline{3},3)_{-1},$$
 (1)

where the subscripts denote the helicity projections. The $\frac{3}{2}^+$ and $\frac{1}{2}^+$ baryons are described by

$$\Psi_{abc} = D_{abc}(\chi_{10})_{111},$$

$$\Psi_{\bar{a}\bar{b}\bar{c}} = D_{abc}(\chi_{10})_{222},$$

$$\Psi_{abc} = \sqrt{3} D_{abc} (\chi_{10})_{112} + (1/\sqrt{6}) (N_a{}^d \epsilon_{dbc} + N_b{}^d \epsilon_{dac}) (\chi_8)_1,$$

$$\Psi_{a\,\overline{bc}} = \sqrt{3} D_{a\,bc}(\chi_{10})_{122} + (1/\sqrt{6})(N_{b}{}^{d}\epsilon_{dac} + N_{c}{}^{d}\epsilon_{dab})(\chi_{8})_{2}.$$
 (2)

In the $SU(6)_{\sigma}$ case, we have four invariant amplitudes for baryon-antibaryon scattering, namely,

$$\overline{56} \otimes 56 = 1 \oplus 35 \oplus 405 \oplus 2695$$

Under the present scheme, we have the following

¹ F. Gürsey and L. Radicati, Phys. Rev. Letters **13**, 173 (1964); A. Pais, *ibid.* **13**, 175 (1964); B. Sakita, Phys. Rev. **136**, B1756 (1964).

⁹ D. V. Volkov, JETP Pis'mav Redaktsiyu [English transl.: JETP Letters 1, 129 (1965)]. ¹⁰ H. Ruegg and D. V. Volkov, Nuovo Cimento 43, 84 (1966). ¹¹ H. Güntor and B. Stech, Z. Physik 189, 455 (1966); F. Buccella and R. Gatto, Nuovo Cimento 40, 684 (1965). ¹² L. Schülke, Z. Physik 189, 207 (1966).

(4a)

amplitudes for the processes considered (the contribution with opposite baryon helicity can be obtained by a parity transformation):

$$\begin{aligned} \alpha \Psi^{a\,b\,c}(1) \,\Psi_{d\,e\overline{f}}(2) \,\bar{\Psi}^{d\,e\overline{f}}(3) \,\Psi_{a\,b\overline{o}}(4) \\ &+ \beta \bar{\Psi}^{a\,b\overline{c}}(1) \,\Psi_{d\,e\overline{c}}(2) \,\bar{\Psi}^{d\,e\overline{f}}(3) \,\Psi_{a\,b\overline{f}}(4) \\ &+ \delta \bar{\Psi}^{a\,b\overline{c}}(1) \,\Psi_{a\,b\overline{f}}(2) \,\bar{\Psi}^{d\,e\overline{f}}(3) \,\Psi_{d\,e\overline{r}}(4) \\ &+ \gamma \bar{\Psi}^{a\,b\overline{c}}(1) \,\Psi_{a\,b\overline{c}}(2) \,\bar{\Psi}^{d\,e\overline{f}}(3) \,\Psi_{d\,e\overline{f}}(4) \,. \end{aligned}$$

On substituting for Ψ_{abc} from (2), we find after a simple calculation that α contributes to the elastic channels only, namely, $\bar{p}p \rightarrow \bar{p}p$, and $\bar{p}n \rightarrow \bar{p}n$. The amplitudes for various processes of proton-antiproton and neutronantiproton scattering have been obtained and have been tabulated in Table I.

From Table I, we obtain in addition to the SU(3)relations13

$$3A(\bar{\Lambda}\Lambda) + A(\bar{\Sigma}^{0}\Sigma^{0}) = 2A(\bar{n}n) + 2A(\bar{\Xi}^{0}\Xi^{0}) + 2\sqrt{3}A(\bar{\Lambda}\Sigma^{0}),$$

$$A\left(\bar{\Xi}^{0}\Xi^{0}\right) = A\left(\bar{\Sigma}^{+}\Sigma^{-}\right),\tag{4b}$$

$$2A\left(\overline{\Sigma}^{0}\Sigma^{0}\right) = A\left(\overline{\Sigma}^{-}\Sigma^{+}\right) + A\left(\overline{\Sigma}^{+}\Sigma^{-}\right), \qquad (4c)$$

$$A(\bar{\Xi}^{+}\Xi^{-}) - A(\bar{\Xi}^{0}\Xi^{0}) = A(\bar{\Xi}^{0}\Xi^{-}),$$
 (4d)

$$A(\overline{\Lambda}\Sigma^{0}) = A(\overline{\Sigma}^{0}\Lambda) = A(\overline{\Lambda}\Sigma^{-})/\sqrt{2}, \qquad (4e)$$

$$A\left(\overline{\Sigma}^{-}\Sigma^{+}\right) - A\left(\overline{\Sigma}^{0}\Sigma^{0}\right) = A\left(\overline{\Sigma}^{-}\Sigma^{0}\right)/\sqrt{2}, \qquad (4f)$$

$$A(\bar{p}p) = A(\bar{n}n) + A(\bar{p}n), \qquad (4g)$$

$$A\left(\bar{\Sigma}^{0}\Sigma^{-}\right) = -A\left(\bar{\Sigma}^{-}\Sigma^{0}\right),\tag{4h}$$

$$A (\bar{\Omega}^{-} \Omega^{-}) = A (\bar{N}^{*+} N^{*-}) + 3 [A (\bar{\Xi}^{*+} \Xi^{*-}) - A (\bar{Y}^{*+} Y^{*-})], \quad (4i)$$
$$A (\bar{N}^{*--} N^{*++}) - A (\bar{N}^{*+} N^{*-})$$

$$(\bar{N}^{*++}) - A (\bar{N}^{*+}N^{*-})$$

= 3 \[A (\bar{N}^{*-}N^{*+}) - A (\bar{N}^{*0}N^{*0}) \]: (4i)

the following new relations:

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$$A(\bar{N}^{*+}N^{*-}) = 3A(\bar{\Xi}^{*+}\Xi^{*-}) = \frac{3}{2}A(\bar{\Xi}^{+}\Xi^{-}), \quad (5a)$$

$$\begin{bmatrix} A \left(\bar{Y}^{*+} Y^{*-} \right) - A \left(\bar{\Xi}^{*0} \Xi^{*0} \right) \end{bmatrix}$$

= $\frac{3}{2} \begin{bmatrix} A \left(\bar{\Xi}^{+} \Xi^{-} \right) - A \left(\bar{\Xi}^{0} \Xi^{0} \right) \end{bmatrix} = \frac{3}{2} A \left(\bar{\Xi}^{0} \Xi^{-} \right),$ (5b)

$$A(\overline{\Lambda}\Sigma^{-}) = \sqrt{3}A(\overline{\Sigma}^{0}\Sigma^{-}) = -\sqrt{3}A(\overline{\Sigma}^{-}\Sigma^{0})$$
$$= A(\overline{\Sigma}^{-}\Lambda^{0}) = 2A(\overline{\Lambda}Y^{*0}), \quad (5c)$$

$$A(\bar{p}N^{*+}) = A(\bar{n}N^{*0}) = 2A(\bar{\Sigma}^{-}V^{*+}),$$
 (5d)

$$A\left(\overline{\Lambda}Y^{*0}\right) = -\left(1/\sqrt{2}\right)A\left(\overline{\Lambda}\Sigma^{0}\right), \qquad (5e)$$

$$A\left(\overline{\Sigma}^{+}Y^{*-}\right) = A\left(\overline{\Xi}^{+}\overline{\Xi}^{*-}\right), \qquad (5f)$$

$$2A\left(\bar{\Xi}^{*0}\Xi^{*0}\right) = A\left(\bar{\Xi}^{0}\Xi^{0}\right) = A\left(\bar{\Sigma}^{+}\Sigma^{-}\right), \qquad (5g)$$

$$A\left(\bar{\Xi}^{0}\Xi^{*0}\right) = 2A\left(\bar{\Sigma}^{0}Y^{*0}\right),\tag{5h}$$

where $A(\bar{p}p)$, e.g., denotes $A(\bar{p}p \rightarrow \bar{p}p)$ etc. Relations (4a) ..., (4h) were also obtained in $\tilde{U}(12)$ and (5e)

TABLE I. $U(3) \times U(3)$ invariant amplitudes for proton-antiproton
and neutron-antiproton scattering processes.

Processes/amplitudes	α	β	δ	γ		
	(a)	$\bar{p}p \rightarrow$				
1. $p p$	16	20	20	16		
2. $\bar{n}n$	0	16	4	16		
3. $\overline{\Sigma}^{-}\Sigma^{+}$	0	4	16	16		
4. $\overline{\Sigma}^+\Sigma^-$	0	0	8	16		
5. <u>Ξ</u> °三 ⁰	0	0	8	16		
 5. 豆+豆- 	0	0	16	16		
7. $\overline{\Sigma}^0 \Sigma^0$	0	2	12	16		
8. $\overline{\Sigma}{}^{0}\Lambda^{0}$	0	$-6/\sqrt{3}$	$12/\sqrt{3}$	0		
9. $\bar{\Lambda}^0 \Sigma^0$	0	$-6/\sqrt{3}$	$12/\sqrt{3}$	0		
10. $\bar{\Lambda}^{0}\Lambda^{0}$	0	6	12	16		
11. $\bar{N}^{*}N^{*++}$	0	24	12	24		
12. $\bar{N}^{*-}N^{*+}$	0	16	12	8		
13. $\bar{N}^{*+}N^{*-}$	0	0	24	24		
14. $\bar{N}^{*0}N^{*0}$	0	8	16	8		
15. $\bar{Y}^{*-}Y^{*+}$	0	8	8	8		
16. $Y^{*-}\bar{Y}^{*+}$	0	0	16	8		
17. $\bar{Y}^{*0}Y^{*0}$	0	4	12	4		
18.	0	0	4	8		
19. Ź* +Z*-	0	0	8	8		
20. $\overline{\Omega}^+\Omega^-$	0	0	0	24		
21. $\bar{p}N^{*+}+c.c.$	0	0	$32/\sqrt{2}$	0		
22. $\bar{n}N^{*0}$ +c.c.	0	0	$32/\sqrt{2}$	0		
23. $\bar{\Sigma}^- Y^{*+} + c.c.$	0	0	$-16/\sqrt{2}$	0		
24. $\bar{\Sigma}^+ Y^{*-} + c.c.$	0	$-16/\sqrt{2}$	$-8/\sqrt{2}$	0		
25. $\bar{\Sigma}^{0}Y^{*0}$ +c.c.	0	$-12/\sqrt{2}$	$4/\sqrt{2}$	0		
26. $\bar{\Lambda}^{0} Y^{*0} + \text{c.c.}$	0	$12/\sqrt{6}$	$-24/\sqrt{6}$	0		
27. $\bar{\Xi}^{0}\Xi^{*0}$ +c.c.	0	$24/\sqrt{2}$	$-8/\sqrt{2}$	0		
28. $\bar{\Xi}^+\Xi^{*-}$ +c.c.	0	$-16/\sqrt{2}$	$-8/\sqrt{2}$	0		
(b) $\bar{p}n \rightarrow$						
29. pn	16	4	16	0		
30. ⁻ ⁻ ⁻ ⁻	0	0	8	0		
31. $\bar{\Lambda}^{0}\Sigma^{-}$	0	$-12/\sqrt{6}$	$24/\sqrt{6}$	0		
32. $\overline{\Sigma}^0\Sigma^-$	0	$-4/\sqrt{2}$	$8/\sqrt{2}$	0		
33. $\overline{\Sigma}^{-}\Sigma^{0}$	0	$\frac{1}{4/\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	0		
34. $\overline{\Sigma}^{-}\Lambda^{0}$	0	$-12/\sqrt{6}$	$24/\sqrt{6}$	0		

(5f) in $SU(6)_{W}$.^{14,15} To compare our results with the existing experimental data,^{16,17} we relate, following Meshkov et al.,¹⁸ the total cross section and the invariant amplitude squared as,

$$|A|^{2} = (E_{\text{c.m.}}^{2} \times p_{\text{in}}/p_{\text{f}})\sigma \equiv \rho\sigma \equiv \bar{\sigma} \quad (\text{say}), \quad (6)$$

where, p_{in} and p_f are the momenta of the incident and outgoing particles in the c.m. frame and $E_{e.m.}$ is the total c.m. energy. In Tables II and III, we have tabu-

¹⁴ S. Y. Lo, Phys. Rev. 140, B95 (1965).

¹⁵ Dao Wong Duc, Joint Institute for Nuclear Research, Dubna,

¹³ K. Tanaka, Phys. Rev. 135, B1186 (1964); M. Konuma and Y. Tomozawa, Phys. Rev. Letters 12, 425 (1964).

¹⁶ Dao Wong Duc, Joint Institute for Nuclear Research, Duona, 1965, unpublished report. ¹⁶ B. Musgrave *et al.*, Nuovo Cimento **35**, 735 (1965). The data \hat{p} lab momentum 3 BeV/c has been taken from this paper. ¹⁷ C. Baltay *et al.*, Phys. Rev. **140**, B1027 (1965). We have taken the data at \hat{p} lab momentum 3.7 BeV/c from this paper. See also C. Baltay *et al.*, Phys. Rev. Letters **11**, 32 (1963) and K. Röck-mann *et al.*, Phys. Letters **15**, 356 (1965) for the process $\hat{p}p \rightarrow \tilde{N}^{*}-N^{*+1}$.

¹⁸ - ⁷ + ¹⁹ S. Meshkov, G. A. Snow, and G. B. Yodh, Phys. Rev. Letters **12**, 87 (1964).

	Processes	(BeV/c)	ρ (BeV) ²	σ (mb)	$= rac{\sqrt{ar{\sigma}}}{(ho\sigma)^{1/2}}$
1.	$\bar{p}p$	1.017	7.645	21.0	12.7
2.	пn	1.017	7.645	1.3	3.15
3.	$\overline{\Sigma}^-\Sigma^+$	0.706	11.007	36×10 ³	0.63
4.	$\overline{\Sigma}^+\Sigma^-$	0.693	11.22	10×10 ⁻³	0.335
5.	$\overline{\Sigma}{}^0\Sigma^0$	0.701	11.09	18×10 3	0.447
6.	$\bar{\Lambda}\Lambda$	0.818	13.02	117×10 ⁻³	1.23
7.	$ar{\Lambda}\Sigma^0$	0.76	10.23	51×10-3	0.722
8.	壹+=-	0.411	18.92	2×10^{-3}	0.615
9.	$ar{Y}^{*-}Y^{*+}$	0.01	77.74	8×10^{-3}	0.789
10.	$\bar{Y}^{*+}Y^{*-}$	0.01	77.74	5×10-3	0.307
11.	$\bar{\Lambda} V^{*0}$	0.578	13.46	10×10-3	0.367
12.	$\overline{\Sigma}^- Y^{*+}$	0.495	15.7	11×10 ⁻³	0.421
13.	$\overline{\Sigma}{}^{_0}Y^{*0}$	0.495	15.7	5×10-3	0.281
14.	$\overline{\Sigma}^+ Y^{*-}$	0.495	15.7	1×10-3	0.396

TABLE II. Experimental data for proton-antiproton scattering at \bar{p} lab momentum 3 BeV/c and $E_{\rm e.m.} = 2.765$ BeV.

TABLE III. Experimental data for proton-antiproton scattering at \tilde{p} lab momentum 3.7 BeV/c and $E_{\rm e.m.} = 2.97$ BeV.

	Processes	(BeV/c)	$({ m BeV})^2$	σ (mb)	$\equiv \frac{\sqrt{\bar{\sigma}}}{(\rho\sigma)^{1/2}}$
1.	$ar{\Sigma}^{-}\Sigma^{+}$	0.912	11.22	44×10^{-3}	0.702
2.	$\overline{\Sigma}{}^0\Sigma^0$	0.908	11.26	$<26 \times 10^{-3}$	0.541
3.	$\bar{\Sigma}^+\Sigma^-$	0.902	11.84	8×10 ³	0.300
4.	$\bar{\Lambda}\Lambda$	1.001	10.22	82×10 ⁻³	0.915
5.	$ar{\Lambda}\Sigma^0$	0.932	10.98	34×10^{-3}	0.615
6.	<u>=+=</u>	0.708	14.44	2×10^{-3}	0.537
7.	$ar{N}^{*N^{*++}}$	0.849	12.05	1.8	4.67
8.	$ar{Y}^{*-}Y^{*+}$	0.513	19.94	8×10 ⁻³	0.4
9.	$ar{Y}^{*+}Y^{*-}$	0.513	19.94	5×10 ⁻³	0.316
10.	$ar{\Lambda} V^{st 0}$	0.8	12.8	14×10^{-3}	0.423
11.	$\overline{\Sigma}^- Y^{*+}$	0.732	13.97	18×10 ³	0.502
12.	$\overline{\Sigma}^+Y^{*-}$	0.732	13.97	5×10-3	0.264
13.	$\bar{\Xi}^0 \Xi^{*0}$	0.412	24.83	1×10-3	0.157

lated ρ , σ , and $\sqrt{\sigma}$ for those processes for which we have the experimental cross sections^{16,17} at \bar{p} lab momentum 3 BeV/*c* and 3.7 BeV/*c*.

From (4) and (5), we have the following relations between the $\bar{\sigma}$'s and the triangular inequalities:

$$2(\sqrt{\bar{\sigma}})(\bar{n}n) \leq 3(\sqrt{\bar{\sigma}})(\bar{\Lambda}\Lambda) + (\sqrt{\bar{\sigma}})(\bar{\Sigma}^{0}\Sigma^{0}) + 2(\sqrt{\bar{\sigma}})(\bar{\Xi}^{0}\Xi^{0}) + 2(\sqrt{3})(\sqrt{\bar{\sigma}})(\bar{\Lambda}\Sigma^{0}), \quad (7a)$$

$$2(\sqrt{\sigma})(\overline{\Sigma}^{0}\Sigma^{0}) \leq (\sqrt{\sigma})(\overline{\Sigma}^{-}\Sigma^{+}) + (\sqrt{\sigma})(\overline{\Sigma}^{+}\Sigma^{-}), \quad (7b)$$

$$\begin{split} \bar{\sigma}(\bar{\Lambda}\Sigma^{0}) &= \bar{\sigma}(\bar{\Sigma}^{0}\Lambda) = \frac{1}{2}\bar{\sigma}(\bar{\Lambda}\Sigma^{-}) \\ &= \frac{3}{2}\bar{\sigma}(\bar{\Sigma}^{0}\Sigma^{-}) = \frac{3}{2}\bar{\sigma}(\bar{\Sigma}^{-}\Sigma^{0}) \\ &= \frac{1}{2}\bar{\sigma}(\bar{\Sigma}^{-}\Lambda^{0}) = 2\bar{\sigma}(\bar{\Lambda}Y^{*0}) , \end{split}$$
(7c)

 $(\sqrt{\sigma})(\overline{\Xi}{}^{0}\Xi^{-}) + (\sqrt{\sigma})(\overline{\Xi}{}^{0}\Xi^{0}) \ge (\sqrt{\sigma})(\overline{\Xi}{}^{+}\Xi^{-}), \quad (7d)$

$$(\sqrt{\sigma})(\overline{\Sigma}^{-}\Sigma^{+}) \leq (\sqrt{\sigma})(\overline{\Sigma}^{0}\Sigma^{0}) + (1/\sqrt{2})(\sqrt{\sigma})(\overline{\Sigma}^{-}\Sigma^{0}), \quad (7e)$$

$$(\sqrt{\sigma})(\bar{p}p) \leq (\sqrt{\sigma})(\bar{n}n) + (\bar{\sigma})(\bar{n}p), \qquad (7f)$$

$$\bar{\sigma}(\bar{N}^{*+}N^{*-}) = 9\bar{\sigma}(\bar{\Xi}^{*+}\Xi^{*-}) = (9/4)\bar{\sigma}(\bar{\Xi}^{+}\Xi^{-}), \quad (7g)$$

$$(\sqrt{\sigma})(\bar{Y}^{*+}Y^{*-}) \leq (\sqrt{\sigma})(\bar{\Xi}^{*0}\Xi^{*0}) + \frac{3}{2}(\sqrt{\sigma})(\bar{\Xi}^{0}\Xi^{-}),$$
 (7h)

$$\bar{\sigma}(\bar{p}N^{*+}) = \bar{\sigma}(\bar{n}N^{*0}) = 4\bar{\sigma}(\bar{\Sigma}^- V^{*+}), \qquad (7i)$$

$$\bar{\sigma}(\bar{\Sigma}^+ Y^{*-}) = \bar{\sigma}(\bar{\Xi}^+ \Xi^{*-}), \qquad (7j)$$

$$4\bar{\sigma}(\bar{\Xi}^{*0}\Xi^{*0}) = \bar{\sigma}(\bar{\Xi}^{0}\Xi^{0}) = \bar{\sigma}(\bar{\Sigma}^{+}\Sigma^{-}), \qquad (7k)$$

$$4\bar{\sigma}(\bar{\Sigma}^0 Y^{*0}) = \bar{\sigma}(\bar{\Xi}^0 \Xi^{*0}). \tag{71}$$

From Tables II and III and Eqs. (7), we obtain,

$$\begin{aligned} (\sqrt{\sigma}) &(\bar{\Xi}^0 \Xi^0) = 0.335 & (\bar{p} \text{ lab momentum 3 BeV}/c) \\ &= 0.301 & (\bar{p} \text{ lab momentum 3.7 BeV}/c) , \\ (\sqrt{\sigma}) &(\bar{\Sigma}^- \Sigma^0) = 0.585 & (3 \text{ BeV}/c) , \\ &= 0.498 & (3.7 \text{ BeV}/c) . \end{aligned}$$

Substituting these values and taking the values of $\sqrt{\sigma}$ from Tables II and III, we find the inequalities 7a and 7b are very well satisfied.¹⁹ The relation $\bar{\sigma}(\bar{\Lambda}\Sigma^0)$ $=2\bar{\sigma}(\bar{\Lambda}Y^{*0})$ is highly satisfied at (3.7 BeV/c) and at (3 BeV/c) we have the left-hand side = 52.1×10^{-2} and the right-hand side = 26.9×10^{-2} . The sum rule (7k) can be tested only at a higher energy, since at 3.7 BeV/c, it is not energetically possible. We do not have presently any data on $\overline{\Omega}_+\Omega^-$ production, and so the relation (4i) can only be tested when the experimental data are available. Relations (7c), (7g), (7i) and (7j) have been predicted for experimental check. If one assumes that the amplitude β only contributes at threshold, we find that the elastic channel dominates over the other channels $\bar{p}p \rightarrow \bar{Y}Y$. This result was also obtained from $\tilde{U}(12)^{20}$ and is fairly satisfied with experiment. (From Table II, we find $\sigma_{\bar{p}p} \sim 21$ mb $\sigma_{\bar{n}n} \sim 1$ mb, and $\sigma_{\bar{Y}Y} \sim a$ few μ b.) In short, we find that most of the relations obtained under $U(3) \times U(3)$ symmetry fairly agree with the existing experimental data. In contrast to SU(3) symmetry, we have been able to relate the amplitudes for processes $\bar{p}p \rightarrow \bar{B}B$, $\overline{B}D$, and $\overline{D}D$. Further, most of the results obtained under $SU(6)_W$ and $\tilde{U}(12)$ symmetry, have been reproduced in our calculation. The successes of this theory for obtaining Johnson-Treiman relations¹⁰ [in contrast to $SU(6)_W$ is well known. Thus, we feel that the collinear $U(3) \otimes U(3)$ group has given us more promising results economically-in contrast to other hierarchies of symmetry, in a Lorentz-invariant way.

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¹⁹ We have to note here that in Tomozawa and Konuma's paper, the sum rule (7.1) was not fully satisfied. However, we find in our present calculation that the left-hand side of (7.1) is 6.3, the right-hand side being 7.3 experimentally (see Table II), which is fairly satisfactory.

²⁰ D. A. Akyeampong and R. Delbourgo, Phys. Rev. 140, B1013 (1965). This paper contains further references (especially see Refs. 23, 24, 25) on proton-antiproton scattering.