PHYSICAL REVIEW VOLUME 149, NUMBER 4 30 SEPTEMBER 1966

Baryon Mass Differences in a Model of Broken Unitary Symmetry*

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(Received 25 April 1966)

Proceeding from the assumption that the breaking of unitary symmetry within the baryon octet can be described as a low-energy effect generated by the pseudoscalar-meson mass differences, a model is proposed which relates the departure from full SU_3 symmetry within these two octets. Although it is assumed that the corrections to the low-energy behavior of the baryon two-point functions are dominated by the least massive two-particle intermediate states, no condition is required on the strength of the baryon-meson coupling. Using the experimentally observed mass differences, one can calculate within the framework of this model the SU_3-invariant baryon-meson coupling constants, the results being in reasonable agreement with currently accepted values.

I. INTRODUCTION

DURING the past several years, calculations of symmetry-breaking effects have been of considerable interest in the context of the proposed strong-interaction symmetries such as SU_3, SU_6, etc. The greater part of this work is based on the assumption that the interaction Lagrangian can be split into 2 parts: a very strong (V.S.) interaction which is invariant under the group and a medium-strong (M.S.) interaction which breaks the symmetry. In particular, the problem of the determination of mass differences induced among the members of a given irreducible representation of SU_3 by the M.S. interactions has been treated with appreciable success by the methods of the tensor calculus.[1] This has led to the derivation of the Gell-Mann–Okubo (GMO) mass relation which is satisfied by the experimentally observed masses of the lowest lying hadron states to a surprising degree of accuracy. Since, however, the explicit structure of the M.S. interaction is not yet known, these mass formulas have made use only of the assumed tensor transformation properties of the M.S. interaction and consequently consist of expressions for the mass of each member of a multiplet as a function of a finite number of parameters related to the matrix elements of the M.S. interaction. In this way, one gets nontrivial mass relations only when the number of such parameters is less than or equal to the number of particles in a multiplet which acquire different masses as a consequence of the M.S. interactions. Because these parameters cannot, by existing techniques, be written in terms of experimentally observable quantities such as coupling constants, f/d ratio, etc., this approach unfortunately sheds little light on the explicit mechanism for the breakdown of the symmetry in question.

It is the intent of this paper to propose a model which is soluble in some sense and for which the GMO mass relations for the baryon octet may be understood in terms of experimentally measurable parameters. One such effort in this direction has been made recently by Pietschmann[2] who derives the GMO mass formula for the pseudoscalar octet and the vector nonet of SU_3 by using $\omega-\varphi$ mixing as the proposed symmetry–breaking term. This work, as well as the present paper, uses an SU_3-invariant formulation of the Zachariasen model.[3] Our motivation here is, however, considerably different from that of Pietschmann inasmuch as we endeavor to demonstrate that the mass splitting in the pseudoscalar-meson octet can induce the observed mass splitting in the baryon octet even if one assumes the latter to have a common bare mass and an SU_3-invariant coupling to the meson fields.

Although the usual divergences inherent in field-theoretical calculations make it virtually impossible to obtain reliable estimates of mass-renormalization effects, the problem of the determination of mass differences within a given multiplet seems considerably less sensitive to the high-energy behavior of the two-point functions. Thus it is not unreasonable to hope (as in this paper) that the divergence problems of field theory can be largely circumvented in a calculation of mass differences.[4] The reasonable agreement of the coupling constant and f/d ratio obtained here with the somewhat uncertain experimental results seems to lend some support to this view.

II. THE MODEL

The usual Yukawa-type SU_3-invariant interaction between the baryon and pseudoscalar-meson octet can be written in the form

$$\mathfrak{L}_{\text{int}}=(1-f)g_0\,\mathrm{Tr}[\bar{B}\gamma_5BP+\bar{B}\gamma_5PB] +fg_0\,\mathrm{Tr}[\bar{B}\gamma_5BP-\bar{B}\gamma_5PB], \quad (1)$$

where the bare SU_3 coupling constant g_0 has been

* Research supported in part by the Atomic Energy Commission.

[1] S. Okubo, Progr. Theoret. Phys. (Kyoto) **27**, 949 (1962).

[2] H. V. R. Pietschmann, Phys. Rev. **139**, B446 (1965).

[3] F. Zachariasen, Phys. Rev. **121**, 1851 (1961). This model consists of taking only a simple bubble diagram for the baryon self-energy Σ in the integral equation for the baryon two-point function S: $S=S_0+S_0\Sigma S$.

[4] Another approach to this problem has been proposed by J. Moffat, Phys. Rev. **145**, 1177 (1966). This author assumes, however, that the leading perturbative contributions to the mass differences cancel despite the fact that they are logarithmically divergent. The view taken in this paper is that these divergent terms must give the dominant effect and can be consistently calculated by a cutoff technique of the type discussed in Sec. III.

chosen so that $f+d=1$. In Eq. (1), B is the usual 3×3 matrix for the baryon octet and P the corresponding 3×3 matrix for the pseudoscalar octet.[5] Using Eq. (1) and the expressions for B and P, one can write down the baryon-meson coupling constants predicted by SU_3.[6] These are listed in Table I.

We make the assumption that the phenomenologically determined coupling constants of broken SU_3 at least approximately satisfy the SU_3 predictions of Table I and that the mass splitting in the baryon octet due to M.S. interactions can be understood to be primarily a low-energy effect arising as a consequence of the mass splitting of the pseudoscalar meson octet. This latter set of states has been singled out by virtue of its being the least massive multiplet to which the baryons are strongly coupled. One thus writes

$$M_i=M_i(\mu_1,\mu_2,\mu_3)\,, \tag{2}$$

where M_i is the mass of the ith baryon in the baryon octet and μ_1, μ_2, μ_3 are the masses of the pseudoscalar octet (M_i, $i=1$, 2, 3, 4 stands for the N, Σ, Ξ, and Λ, respectively; μ_α, $\alpha=1$, 2, 3 stands for the π, K, and η, respectively). In the SU_3 limit where $\mu_1=\mu_2=\mu_3=\mu$ the SU_3-meson central mass, one clearly has the result $M_1=M_2=M_3=M_4=M$ the SU_3-baryon central mass. This suggests the utility of attempting to include the effect of M.S. interactions on the physical masses of the baryons by expanding M_i in a Taylor series in the μ_α's around $\mu_\alpha=\mu$ and retaining terms to first order in the meson mass splittings, i.e.,

$$M_i=M+\sum_{\alpha=1}^{3}\left[\frac{\partial M_i(\mu_1\cdots\mu_3)}{\partial\mu_\alpha^2}\right]_{\mu,M}(\mu_\alpha^2-\mu^2)\,. \tag{3}$$

Since the meson mass differences transform as T_3^3 in unitary spin space, the validity of this approximation receives strong support from the well-known success of the first-order GMO mass formula.

To find an expression for the coefficients $[\partial M_i/\partial\mu_\alpha^2]_{\mu,M}$, we write down the SU_3-invariant dispersion relation for the inverse two-point function $S^{-1}(p)$ of the baryon octet

$$S^{-1}(p)=\gamma\cdot p+M_0+\frac{1}{\pi}\int_{M+\mu}^{\infty}\left\{\frac{\operatorname{Im}[S^{-1}(\gamma\cdot p=-m)]}{\gamma\cdot p+m}+\frac{\operatorname{Im}[S^{-1}(\gamma\cdot p=+m)]}{\gamma\cdot p-m}\right\}dm\,, \tag{4}$$

where the unitary spin indices have been suppressed. In order to have a particle of mass M in the spectrum, one must have $S^{-1}(\gamma\cdot p=-M)=0$. Imposing this condition, Eq. (4) becomes

$$M=M_0+\frac{1}{\pi}\int_{M+\mu}^{\infty}\left\{\frac{\operatorname{Im}[S^{-1}(\gamma\cdot p=-m)]}{m-M}-\frac{\operatorname{Im}[S^{-1}(\gamma\cdot p=m)]}{m+M}\right\}dm\,, \tag{5}$$

which, using the shorthand notations $r_{\text{V.S.}}^{\pm}(M,\mu,m)=-(1/\pi)\operatorname{Im}[S^{-1}(\gamma\cdot p=\mp m)]$, may be written in the form

$$M=M_0-\int_{M+\mu}^{\infty}\left[\frac{r_{\text{V.S.}}^{+}(M,\mu,m)}{m-M}-\frac{r_{\text{V.S.}}^{-}(M,\mu,m)}{m+M}\right]dm\,. \tag{5'}$$

Although this expression includes only SU_3-invariant terms, one can immediately generalize this result to include M.S. interaction effects to find

$$M_i=M_0-\sum_{j,\beta}\int_{M_j+\mu_\beta}^{\infty}\left[\frac{r^{+}(M_j,\mu_\beta,m)}{m-M_i}-\frac{r^{-}(M_j,\mu_\beta,m)}{m+M_i}\right]dm\,, \tag{6}$$

where $r^{\pm}=r_{\text{V.S.}}^{\pm}+r_{\text{M.S.}}^{\pm}$ ($r_{\text{M.S.}}^{\pm}$ being the M.S. contribution) and we have assumed a common bare mass M_0 for the entire baryon multiplet. The summation in Eq. (6) is to include all intermediate states containing M_j and μ_β which are coupled to M_i. Applying $\partial/\partial\mu_\alpha^2$ gives

$$\frac{\partial M_i}{\partial\mu_\alpha^2}=-\sum_{j,\beta}\int_{M_j+\mu_\beta}^{\infty}\left\{\left[\frac{r^{+}}{(m-M_i)^2}+\frac{r^{-}}{(m+M_i)^2}\right]\frac{\partial M_i}{\partial\mu_\alpha^2}+\left[\frac{\partial r^{+}/\partial\mu_\alpha^2}{m-M_i}-\frac{\partial r^{-}/\partial\mu_\alpha^2}{m+M_i}\right]\right\}dm\,, \tag{7}$$

the contribution to the derivative from the lower limit vanishing as a consequence of the threshold behavior of the spectral function $r^{\pm}$. Using the chain rule, one writes

$$\frac{\partial r^{\pm}(M_j,\mu_\beta,m)}{\partial\mu_\alpha^2}=\frac{\partial r^{\pm}}{\partial\mu_\beta^2}\delta_{\alpha\beta}+\frac{\partial r^{\pm}}{\partial M_j}\frac{\partial M_j}{\partial\mu_\alpha^2}$$

TABLE I. The SU_3 predictions for the baryon-meson coupling constants.

Coupling constant	SU_3 predictions
$g_{N\pi}^2/4\pi$	$g^2/4\pi$
$g_{\Lambda\pi}^2/4\pi$	$\frac{4}{3}(1-f)^2g^2/4\pi$
$g_{\Sigma\pi}^2/4\pi$	$4f^2g^2/4\pi$
$g_{\Xi\pi}^2/4\pi$	$(1-2f)^2g^2/4\pi$
$g_{\Lambda K}^2/4\pi$	$\frac{1}{3}(1+2f)^2g^2/4\pi$
$g_{\Sigma K}^2/4\pi$	$(1-2f)^2g^2/4\pi$
$h_{\Lambda K}^2/4\pi$	$\frac{1}{3}(1-4f)^2g^2/4\pi$
$h_{\Sigma K}^2/4\pi$	$g^2/4\pi$
$g_{N\eta}^2/4\pi$	$\frac{1}{3}(1-4f)^2g^2/4\pi$
$g_{\Lambda\eta}^2/4\pi$	$\frac{4}{3}(1-f)^2g^2/4\pi$
$g_{\Sigma\eta}^2/4\pi$	$\frac{4}{3}(1-f)^2g^2/4\pi$
$g_{\Xi\eta}^2/4\pi$	$\frac{1}{3}(1+2f)^2g^2/4\pi$

[5] S. Coleman and S. Glashow, Phys. Rev. Letters **6**, 423 (1961).
[6] A. W. Martin and K. C. Wali, Phys. Rev. **130**, 2466 (1963).

and Eq. (7) becomes

$$\left\{1+\sum_{j,\beta}\int_{M_j+\mu_\beta}^{\infty}\left[\frac{r^+}{(m-M_i)^2}+\frac{r^-}{(m+M_i)^2}\right]dm\right\}\frac{\partial M_i}{\partial\mu_\alpha^2}=-\sum_{j,\beta}\int_{M_j+\mu_\beta}^{\infty}\left\{\left[\frac{\partial r_{\rm M.S.}^+/\partial\mu_\beta^2}{(m-M_i)}-\frac{\partial r_{\rm M.S.}^-/\partial\mu_\beta^2}{(m+M_i)}\right]\delta_{\alpha\beta}+\left[\frac{\partial r_{\rm M.S.}^+/\partial M_j}{(m-M_i)}-\frac{\partial r_{\rm M.S.}^-/\partial M_j}{(m+M_i)}\right]\frac{\partial M_j}{\partial\mu_\beta^2}\right\}dm,\qquad(8)$$

where we have used $\partial r_{\rm VS}/\partial\mu_\beta^2=\partial r_{\rm V.S.}/\partial M_i=0$. The quantity in brackets on the left side of Eq. (8) multiplying $\partial M_i/\partial\mu_\alpha^2$ is seen to be equal to Z_{2i}^{-1}, the wave-function renormalization for the ith baryon. Rewriting Eq. (8) gives

$$\left[\frac{\partial M_i}{\partial\mu_\alpha^2}\right]_{\mu,M}=-Z_2\left\{\sum_{j,\beta}\int_{M+\mu}^{\infty}dm\left[\frac{\partial r_{\rm M.S.}^+/\partial\mu_\alpha^2}{(m-M)}-\frac{\partial r_{\rm M.S.}^-/\partial\mu_\alpha^2}{(m+M)}\right]+\sum_{j,\beta}\int_{M+\mu}^{\infty}dm\left[\frac{\partial r_{\rm M.S.}^+/\partial M_j}{m-M}-\frac{\partial r_{\rm M.S.}^-/\partial M_j}{m+M}\right]\frac{\partial M_j}{\partial\mu_\alpha^2}\right\}_{M,\mu},\qquad(8')$$

where we have used the fact that Z_{2i} becomes independent of i on letting $M_i\to M$.

We divide the spectral function $r^\pm$ into the contribution from the lowest two-particle state and the higher order terms. The contribution from the simple bubble diagram can be calculated in second-order perturbation theory to yield[7]

$$Z_{2i}r^\pm(M_j,\mu_\beta,m)=a_{ij\beta}\frac{g_{ij\beta}^2}{32\pi^2}\,\frac{[m^2-(M_j+\mu_\beta)^2]^{1/2}[m^2-(M_j-\mu_\beta)^2]^{1/2}(m\mp M_j-\mu_\beta)(m\mp M_j+\mu_\beta)}{m^3},\qquad(9)$$

where the indices refer to the diagram of Fig. 1 and $a_{ij\beta}$ is the isospin factor[6] coming from the charge independence of M.S. interactions. Taking the indicated derivatives in (9) and inserting into Eq. (8′) one gets

$$\left[\frac{\partial M_i}{\partial\mu_\alpha^2}\right]_{M,\mu}=\frac{1}{16\pi^2}\left\{\mathcal{I}_1\sum_j a_{ij\alpha}g_{ij\alpha}^2-\mathcal{I}_2\sum_{j,\beta}a_{ij\beta}g_{ij\beta}^2\left[\frac{\partial M_j}{\partial\mu_\alpha^2}\right]_{M,\mu}\right\}+\mathcal{K}_{i\alpha},\qquad(10)$$

$\mathcal{K}_{i\alpha}$ representing the higher order contributions in $r_{\rm M.S.}^\pm$. The integrals $\mathcal{I}_1$ and $\mathcal{I}_2$ come from the differentiation of the intermediate state with respect to μ_α^2 and M_j, respectively, and are given by

$$\mathcal{I}_1=\int_{M+\mu}^{\infty}\frac{dm}{m^3}\left\{\frac{m^4+M^4+\mu^4-2\mu^2(m^2+M^2)-mM(m^2+M^2-\mu^2)}{(m-M)[m^2-(M+\mu)^2]^{1/2}[m^2-(M-\mu)^2]^{1/2}}-\frac{m^4+M^4+\mu^4-2\mu^2(m^2+M^2)+mM(m^2+M^2-\mu^2)}{(m+M)[m^2-(M+\mu)^2]^{1/2}[m^2-(M-\mu)^2]^{1/2}}\right\},\qquad(10a)$$

$$\mathcal{I}_2=\int_{M+\mu}^{\Lambda(M+\mu)}\frac{dm}{m^3}\left\{\frac{(m^2+M^2-\mu^2-2mM)(M^2-m^2-\mu^2)M+(M-m)(m^4+M^4+\mu^4-2m^2\mu^2-2m^2M^2-2M^2\mu^2)}{(m-M)[m^2-(M+\mu)^2]^{1/2}[m^2-(M-\mu)^2]^{1/2}}-\frac{(m^2+M^2-\mu^2+2mM)(M^2-m^2-\mu^2)M+(M+m)(m^4+M^4+\mu^4-2m^2\mu^2-2m^2M^2-2M^2\mu^2)}{(m+M)[m^2-(M+\mu)^2]^{1/2}[m^2-(M-\mu)^2]^{1/2}}\right\}.\qquad(10b)$$

We note that $\mathcal{I}_2$ is logarithmically divergent and hence one must introduce a cutoff Λ, thereby increasing the number of independent parameters in the model to three (g^2,f,Λ). Using the shorthand notation $Q_{i\alpha}=[\partial M_i/\partial\mu_\alpha^2]_{\mu,M}$,

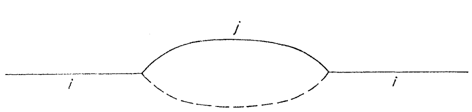

FIG. 1. The second-order bubble diagram of the model.

[7] C. R. Hagen, Phys. Letters 13, 165 (1964).

$P_{i\alpha}=\sum_j a_{ij\alpha}g_{ij\alpha}^2$, and $R_{ij}=\sum_\beta a_{ij\beta}g_{ij\beta}^2$, Eq. (10) may be rewritten

$$Q_{i\alpha}=\frac{\mathcal{I}_1}{16\pi^2}(P_{i\alpha}-a\sum_j R_{ij}Q_{j\alpha})+\mathcal{H}_{i\alpha}, \tag{10'}$$

where $a=\mathcal{I}_1/\mathcal{I}_2$. Thus Eq. (3) has the form

$$M_i=M+\sum_\alpha \mathcal{H}_{i\alpha}(\mu_\alpha^2-\mu^2)+\sum_\alpha Q_{i\alpha}(\mu_\alpha^2-\mu^2), \tag{11}$$

or alternatively

$$M_i=M+\mathcal{H}_i+\sum_\alpha Q_{i\alpha}(\mu_\alpha^2-\mu^2). \tag{11'}$$

Since we are assuming throughout that the baryon mass differences are essentially a low-energy effect, we ignore variations in $\mathcal{H}_i$ and consistently replace it by some suitable average $\mathcal{H}$. This leads to the form

$$M_i=\bar{M}_0+\sum_\alpha Q_{i\alpha}(\mu_\alpha^2-\mu^2), \tag{11''}$$

where we have defined $\bar{M}_0=M+\mathcal{H}$. One can now consider Eq. (10′) as a matrix equation and write the formal solution

$$Q=(1+bR)^{-1}(\mathcal{I}_1/16\pi^2)P, \tag{10''}$$

where we have set $b=\mathcal{I}_1a/16\pi^2=\mathcal{I}_2/16\pi^2$. In order to carry out the indicated inversion of the matrix $1+bR$ we use the SU_3 predictions for the coupling constants $g_{ij\alpha}^2$ so that one can write R and P in terms of the two parameters f and g^2

$$R=g^2\begin{bmatrix} \frac{2}{3}(8f^2-4f+5) & 3(4f^2-4f+1) & 0 & \frac{1}{3}(4f^2+4f+1) \\ 2(4f^2-4f+1) & \frac{4}{3}(7f^2-2f+1) & 2 & \frac{4}{3}(f^2-2f+1) \\ 0 & 3 & \frac{2}{3}(20f^2-16f+5) & \frac{1}{3}(16f^2-8f+1) \\ \frac{2}{3}(4f^2+4f+1) & 4(f^2-2f+1) & \frac{2}{3}(16f^2-8f+1) & \frac{4}{3}(f^2-2f+1) \end{bmatrix} \tag{12a}$$

$$P=g^2\begin{bmatrix} 3 & \frac{2}{3}(20f^2-16f+5) & \frac{1}{3}(16f^2-8f+1) \\ \frac{4}{3}(7f^2-2f+1) & 4(2f^2-2f+1) & \frac{4}{3}(f^2-2f+1) \\ 3(4f^2-4f+1) & \frac{2}{3}(8f^2-4f+5) & \frac{1}{3}(4f^2+4f+1) \\ 4(f^2-2f+1) & \frac{4}{3}(10f^2-2f+1) & \frac{4}{3}(f^2-2f+1) \end{bmatrix}. \tag{12b}$$

It is convenient to take the GMO mass formula for the meson octet in the form[1]

$$\mu_\alpha^2=\mu^2+C[\tfrac{1}{4}Y^2-I(I+1)], \tag{13}$$

so that $\mu_\eta^2=\mu^2$. In this case only $\mu_\pi^2-\mu^2$ and $\mu_K^2-\mu^2$ contribute to the sum in Eq. (11) and one need not calculate the four terms Q_{i3}. Having chosen μ, we find that $(\mu_\pi^2-\mu^2)/(\mu_K^2-\mu^2)=4$ from Eq. (13); this is considerably smaller than the physical value of 5.3 and for reasons which will later become clear, we must choose the physical value 5.3.

Inverting the matrix $1+bR$ of Eq. (10″) and inserting the expression for Q into Eq. (11), one gets the following expression for the baryon masses[8] (we have indicated the mass of a particle by its symbol):

$$N=\bar{M}_0-\frac{\mathcal{I}_1Cg^2}{48\pi^2D}\Big[(40f^2-32f+57.88)+\tfrac{2}{3}\gamma(-1880.96f^4+2263.04f^3+720f^2+846.4f+337.24)$$
$$+\frac{2}{9}\gamma^2(-109558f^6+245161f^5-193290f^4+46020f^3+17682f^2-10141f+238.4)+\frac{8}{27}\gamma^3(246651f^8-240691f^7$$
$$-300157f^6+485783f^5-250840f^4+19914f^3+27696f^2-5251f-600)\Big], \tag{14a}$$

[8] It is to be noted that any attempt to neglect the terms bR in Eq. (10″) leads to a negative value for g^2. This is not surprising as $bR\sim g^2$ and we cannot expect to be able to consistently neglect terms of order g^2 in a strong coupling theory.

$$\Sigma=\bar{M}_0-\frac{g_1Cg^2}{48\pi^2D}\Big[4(43.24f^2-16.64f+8.32)+\tfrac{2}{3}\gamma(4501.1f^4-4597.1f^3+1964.2f^2+334.4f-83.6)$$
$$+\frac{4}{9}\gamma^2(6914.6f^6-23048f^5+12065f^4+14566f^3-22525f^2+11330f-1888.4)+\frac{2}{27}\gamma^3(-442982f^8-155812f^7$$
$$+671969f^6-491458f^5-209679f^4+485530f^3-319258f^2+102144f-12768)\Big], \quad (14b)$$

$$\Xi=\bar{M}_0-\frac{g_1Cg^2}{48\pi^2D}\Big[(207.52f^2-199.52f+57.88)+\tfrac{2}{3}\gamma(4149.8f^4-5777.9f^3+3735.4f^2-1851.5f+337.2)$$
$$+\frac{2}{9}\gamma^2(-50591f^6+175473f^5-246897f^4+180036f^3-69428f^2+7280.6f+238.4)+\frac{8}{27}\gamma^3(21504f^8-305019f^7$$
$$+230546f^6+308882f^5-532274f^4+357634f^3-113021f^2+14851f-600)\Big], \quad (14c)$$

$$\Lambda=\bar{M}_0-\frac{g_1Cg^2}{48\pi^2D}\Big[4(25.96f^2-33.92f+16.96)+\tfrac{2}{3}\gamma(-156.8f^4-895.36f^3+2442.2f^2-1994.6f+488.69)$$
$$+\frac{4}{9}\gamma^2(-57321f^6+146711f^5-149779f^4+70046f^3-95395f^2-4783.2f+797.2)+\frac{4}{27}\gamma^3(450294f^8-771932f^7$$
$$-192983f^6+1222226f^5-1109652f^4+482458f^3-80410f^2)\Big], \quad (14d)$$

where

$$\gamma=g^2b, \quad C=\mu^2-\mu_K{}^2,$$

and

$$D=1+\tfrac{4}{3}\gamma(22f^2-14f+7)+\frac{2}{9}\gamma^2(544f^4-448f^3+456f^2-232f+58)+\frac{4}{27}\gamma^3(-9088f^6+22656f^5-29184f^4$$
$$+20096f^3-7824f^2+1680f-280)+\frac{4}{81}\gamma^4(43008f^8-159744f^7+26880f^6+183552f^5-228288f^4+136320f^3$$
$$-45120f^2+9600f-1200). \quad (14e)$$

These lead to the expressions

$$\Sigma-N=-\frac{g_1Cg^2}{48\pi^2D}\Big[(132.96f^2-34.56f-24.60)+\tfrac{2}{3}\gamma(6382.1f^4-6860.1f^3+244.2f^2+1180.8f-420.8)$$
$$+\frac{2}{9}\gamma^2(123387f^6-291257f^5+217420f^4-16888f^3-62732f^2+32801f-4015.2)+\frac{8}{27}\gamma^3(-357396f^8$$
$$+201738f^7+468149f^6-608648f^5+198420f^4+101468f^3-107510f^2+30787f-2592)\Big], \quad (15a)$$

$$\Xi-N=-\frac{g_1Cg^2}{48\pi^2D}\Big[(167.52f^2-167.52f)+\tfrac{2}{3}\gamma(6030.8f^4-8040.9f^3+3015.4f^2+1005.1f)$$
$$+\frac{2}{9}\gamma^2(58967f^6-69688f^5-53607f^4+134016f^3-87110f^2+17421.6f)+\frac{8}{27}\gamma^3(-225147f^8-64328f^7$$
$$+530703f^6-176901f^5-281434f^4+337720f^3-140717f^2+20102f)\Big], \quad (15b)$$

$$\Lambda-N=-\frac{\mathcal{I}_1Cg^2}{48\pi^2D}\Big[(63.84f^2-103.7f+9.96)+\tfrac{2}{3}\gamma(1724.2f^4-3158.8f^3+1722.2f^2-1148.2f+161.4)$$
$$+\frac{2}{9}\gamma^2(-5084f^6+48261f^5-106268f^4+94072f^3-36761f^2+574.6f+1356)+\frac{8}{27}\gamma^3(-21504f^8-145275f^7$$
$$+203666f^6+125330f^5-303986f^4+221315f^3-67902f^2+5251f+600)\Big],\quad (15c)$$

for the mass differences.

Inserting the experimental values of these masses, one can solve Eqs. (15) for the three unknowns g^2, f, and γ. In this connection, it was noted that if one uses $(\mu^2-\mu_\pi^2)/(\mu^2-\mu_K^2)=4$, as predicted by Eq. (13), the expressions for the baryon masses given by Eq. (14) identically satisfy the GMO relation for all g^2, f, and γ thereby providing an important check on the detailed algebra. This, however, leaves us with only two independent mass differences in Eqs. [15(a)–15(c)] which are consequently insufficient to obtain the three parameters of the model, a situation which does not occur in the case $(\mu^2-\mu_\pi^2)/(\mu^2-\mu_K^2)\neq 4$. Although there are many solutions to Eqs. (15), we find only one which gives g^2 positive and f real. Taking M and μ to be the Λ and η masses respectively, and calculating $\mathcal{I}_1$ to be -0.551, one has that $g^2/4\pi=7.31$, $f=0.185$, and $\gamma=-0.504$. From these results, the cutoff for the divergent integral $\mathcal{I}_2$ is found to be $\Lambda=2.8$.

III. DISCUSSION AND RESULTS

In order to determine the effect of the parameter $\Delta=(\mu^2-\mu_\alpha^2)/(\mu^2-\mu_K^2)$ on the solution, the calculation was repeated using $\Delta=5.0$ and $\Delta=4.0$ (in the latter case, the cutoff for the 5.0 case was used in order to calculate $\mathcal{I}_2$ thus leaving only the two independent parameters f and g^2). The results are shown in Table II. We see that g^2 is not greatly dependent on Δ and all values of f are within acceptable limits. The cutoff is furthermore small enough to be qualitatively consistent with our assumption that the mass differences are essentially a low-energy effect.

To assess the computed value of g^2 it is useful to prepare a table of the SU_3 predictions for the coupling together with some of the experimental data presently available. With the exception of Ref. 9, these values of $g_{\Lambda K}^2/4\pi$ and $g_{\Sigma K}^2/4\pi$ in Table III[9–14] were calculated from the same photoproduction experiment, the numbers given depending upon which intermediate states the authors used in their analysis. We thus see that the calculated value of g^2 gives a reasonably good fit to $g_{\Lambda K}^2/4\pi$ and $g_{\Sigma K}^2/4\pi$ although it gives a π–N coupling constant too small by a factor of two. This is qualitatively what one would expect, however, as the fact that all the observed coupling constants except $g_{\mu\pi}^2/4\pi$ are smaller than those predicted by unitary symmetry is an effect which tends to lower the calculated value of g^2.[15] There is also a calculation by Jarlskog and Pilkuhn[16] in which they find $g_{\Lambda K}^2/g_{\Sigma K}^2=5_{-2}^{+4}$. This would seem to be rather larger than that indicated by the results given in Table III.[17]

The value of f we have calculated here is well within the limits given in Ref. 6. On the other hand, it has been argued by Lee and Sakurai[18] that the f/d ratio in V.S. interactions should be equal to $(f/d)_A$, the f/d ratio of the

TABLE II. Variation of Λ, $g^2/4\pi$, and f with different values of Δ.

Δ	Λ	$g^2/4\pi$	f
5.3	2.8	7.31	0.185
5.0	2.7	9.57	0.200
4.0	2.7	9.42	0.106

TABLE III. Comparison of coupling constants calculated from Table I with experimental value.

Coupling constant	(Table I) SU_3	Experimental values			
$g_{\mu\pi}^2/4\pi$	7.31		13.5–15		
$g_{\Lambda K}^2/4\pi$	4.57	4.0[a]	5–6[b]	5.8[c]	1.4[d]
$g_{\Sigma K}^2/4\pi$	2.90	4.5[e]	1.5–3[f]		
$g_{\mu\eta}^2/4\pi$	.164	1–2[f]			

[a] See Ref. 10. [b] See Ref. 11. [c] See Ref. 12. [d] See Ref. 13. [e] See Ref. 14. [f] See Ref. 9.

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axial-vector current of weak interactions. The value of $(f/d)_A$ has been measured by Willis,[19] using the leptonic decays of the Σ, to be 0.58 and by Brene,[20] also using hyperon decays, to be 0.50. If one accepts the arguments of Ref. 18, one therefore sees that the discrepancy in our f/d ratio is roughly of the same order as that encountered in the calculation of the pion-nucleon coupling constant.

One can estimate the effects of deviations of $\mathcal{H}_i$ from $\mathcal{H}$ by noticing that such terms would tend to change the effective values of the mass differences in Eqs. (15) [e.g., $\Sigma-N=0.252-(\mathcal{H}_1-\mathcal{H}_2)$, etc.]. Hence by calculating the variations in the mass differences with respect to g^2, f, and γ, one can estimate how much our results might be changed by these deviations. One thus writes

$$\frac{\Delta M}{M}=\frac{\partial M}{\partial g^2}\frac{\Delta g^2}{M}+\frac{\partial M}{\partial f}\frac{\Delta f}{M}+\frac{\partial M}{\partial \gamma}\frac{\Delta \gamma}{M}, \tag{16}$$

where we choose to look at the particular mass difference $M=\Sigma-N$. In the neighborhood of the set of values $f=0.185$, $g^2/4\pi=7.31$, $\gamma=-0.504$ one finds that M behaves like

$$M \propto g^2 f^{2.8}\gamma^{3.4}. \tag{17}$$

One sees that, holding two of the variables on the right constant, a 10% change in M implies a 10% change in in g^2, a 3.6% change in f or a 2.9% change in γ. Thus one can say that the substitution of $\mathcal{H}$ for the $\mathcal{H}_i$ seems admissible here if the resultant change in M induced by higher order terms is of the order of 10% or less.

Finally, we consider the expression for Z_{2i}

$$Z_{2i}=1-\sum_{j,\beta}\int_{M_j+\mu_\beta}^{\infty}\left[\frac{\bar{r}_{\text{V.S.}}^{+}}{(m-M_i)^2}+\frac{\bar{r}_{\text{V.S.}}^{-}}{(m+M_i)^2}\right]dm+\int_{M_j+\mu_\beta}^{\infty}\left[\frac{\bar{r}_{\text{M.S.}}^{+}}{(m-M_i)^2}+\frac{\bar{r}_{\text{M.S.}}^{-}}{(m+M_i)^2}\right]dm, \tag{18}$$

where $\bar{r}^{\pm}=Z_2\, r^{\pm}$. Defining the function $w^{\pm}$ by [c.f. Eq. (9)]

$$w_{\text{V.S.}}^{\pm}=\left[\frac{a_{ij\beta}g_{ij\beta}^2}{32\pi^2}\right]^{-1}\bar{r}_{\text{V.S.}}^{\pm},\quad w_{\text{M.S.}}^{\pm}=\left[\frac{a_{ij\beta}g_{ij\beta}^2}{32\pi^2}\right]^{-1}\bar{r}_{\text{M.S.}}^{\pm}, \tag{19}$$

one can write Eq. (18) as

$$Z_{2i}=1-(14f^2-10|+5)\frac{g^2}{24\pi^2}\int_{M+\mu}^{\infty}dm\left[\frac{w_{\text{V.S.}}^{+}(M,\mu,m)}{(m-M)^2}+\frac{w_{\text{V.S.}}^{-}(M,\mu,m)}{(m+M_i)^2}\right]$$
$$-\sum_{j,\beta}\frac{a_{ij\beta}g_{ij\beta}^2}{32\pi^2}\int_{M_j+\mu_\beta}^{\infty}\left[\frac{w_{\text{M.S.}}^{+}(M_j,\mu_\beta,m)}{(m-M_i)^2}+\frac{w_{\text{M.S.}}^{-}(M_j,\mu_\beta,m)}{(m+M_i)^2}\right]dm=1-\mathfrak{z}_2^{\text{V.S.}}-\mathfrak{z}_{2i}^{\text{M.S.}}. \tag{18'}$$

If the computed values of g^2, f, and Λ are used together with Eq. (9), one finds $\mathfrak{z}_2^{\text{VS}}=+1.95$ which would seem to imply a negative value for Z_{2i}. However, the cutoff which has been calculated here is a consequence of M.S. interactions alone, having originated in the differentiation of $\bar{r}_{\text{MS}}^{\pm}$. Hence, it cannot be used in integrals involving V.S. as well as M.S. effects (the cutoff for V.S. integrals is unknown inasmuch as $\partial\bar{r}_{\text{VS}}^{\pm}/\partial\mu_\alpha^2=0$). One can consequently only apply the cutoff Λ to the integral $\mathfrak{z}_{2i}^{\text{M.S.}}$ which we cannot estimate unless the physical values of the $g_{ij\beta}^2$ are known [physical values because M.S. renormalization effects must be taken into account; c.f. Ref. 15] and the cutoff we have calculated is taken to be the same for all integrals. Crudely speaking, however, the assumption that M.S. interactions are considerably weaker that V.S. interactions (e.g., as evidenced by the success of first order perturbation theory in deriving the GMO relations) lends credence to the hope that $\mathfrak{z}_{2i}^{\text{M.S.}}$ would indeed be smaller than 1 as required.

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