expressions involving $\phi(\{v_\lambda\})$ obviously has to be interpreted with some care when $\phi(\{v_\lambda\})$ is singular. The precise sense in which the expressions are then to be understood has recently been found.[14–16] However, with this understanding the various relations we have obtained [such as Eqs. (8), (12), (17), (23), (24)] all remain valid. In other words, even when $\phi(\{v_\lambda\})$ is negative and singular, we may continue to use the formalims of the semiclassical method of calculation *as though* $\phi(\{v_\lambda\})$ *were a probability*, and obtain the correct result. Once this is understood, it becomes clear that the semiclassical method is of very great generality.

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Matter Traversed by Low-Energy Cosmic-Ray Nuclei in Space

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The amount of matter traversed in space by the primary cosmic-ray nuclei of energy 50–150, 200–500, 500–1000, and >1500 MeV/nucleon are obtained as 5.5±1.4, 9.1±0.9, 5.3±0.5, and 3.1±0.4 g cm^{-2} of hydrogen, respectively, using recent measurements of spallation cross sections with proton beams from accelerators and Li/M ratios of cosmic-ray nuclei (where M denotes nuclei with $6\leqslant Z\leqslant 9$). These results and other evidence seem to suggest the hypothesis of two distinct sources (or propagation paths) of cosmic-ray nuclei, one responsible for nuclei of energy roughly 50–150 MeV/nucleon and the other of energies greater than this.

THE amount of matter traversed by primary cosmic-ray nuclei can be deduced from a determination of the ratio[1] L/M or L/S assuming that the L nuclei are absent in the source and that they are produced by fragmentations of heavier nuclei in collisions with hydrogen in space during their propagation. Such data available at kinetic energy $E>1.5$ BeV/nucleon show that the matter traversed is 2–3 g cm^{-2} of hydrogen.[2,3] At lower energies, statistically significant results on the energy dependence of L/M have been obtained only recently[4–8]; these are summarized in Fig. 1(a). From this figure it is seen that L/M has a maximum value of $\sim$0.5 at 200–500 MeV/nucleon which drops to $\sim$0.3 at 50–150 MeV/nucleon and to 0.25 at $E>1.5$ BeV/nucleon. It has not been known so far whether this effect is due to the variation of cross sections with energy or to the variation of the amount of matter traversed with energy. The object of this work is to determine the amount of matter traversed by the low-energy cosmic-ray nuclei as a function of energy, using the recently available cross-section data and the experimentally measured ratios of the cosmic-ray nuclei.

The evaluation of the production rate of L nuclei is difficult at present because we need cross sections for the production of a large number of isotopes, both radioactive and stable, of Li, Be, and B for a number of targets, each bombarded by protons of various energies. These data are not available at present. To circumvent this difficulty we make use of the experimental Li/M ratios [Fig. 1(b)] and the recent data on the cross sections for the production of Li isotopes.[9,10]

CROSS-SECTION DATA

The cross sections used by us are given in Table I. The values of σ for $C^{12}(p,x)Li^6$, $C^{12}(p,x)Li^7$, $O^{16}(p,x)Li^6$, and $O^{16}(p,x)Li^7$ have been obtained from recent measurements[9,10]; these are shown in Fig. 2. σ for $N^{14}(p,x)Li^6$ and $N^{14}(p,x)Li^7$ are estimated from the measured cross section[11] of 3 μb at 150 MeV for $N^{14}(p,x)Li^9$ and by

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[1] Heavy nuclei of the primary cosmic rays are generally classified into the following groups: L: $3\leqslant Z\leqslant 5$; M: $6\leqslant Z\leqslant 9$; H_1: $10\leqslant Z\leqslant 14$; H_2: $15\leqslant Z\leqslant 19$; H_3: $20\leqslant Z\leqslant 28$; and S: $Z\geqslant 6$. Al is chosen as representative of (H_1+H_2) groups and Fe of H_3 groups.

[2] G. D. Badhwar, R. R. Daniel, and B. Vijayalakshmi, Progr. Theoret. Phys. (Kyoto) **30**, 615 (1963).

[3] W. R. Webber, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1966), 46/2.

[4] G. M. Comstock, C. Y. Fan, and J. A. Simpson, in *Proceedings of the International Conference on Cosmic Rays, London, 1965* (The Institute of Physics and the Physical Society, London, 1966), Vol. 1, p. 383.

[5] V. K. Balasubrahmanyan, D. E. Hagge, G. H. Ludwig, and F. B. McDonald, J. Geophys. Res. **71**, 1771 (1966).

[6] C. E. Fichtel, D. E. Guss, K. A. Neelakantan, and D. V. Reames, in *Proceedings of the International Conference on Cosmic Rays, London, 1965* (The Institute of Physics and the Physical Society, London, 1966), Vol. 1, p. 400.

[7] W. R. Webber, in *Proceedings of the International Conference on Cosmic Rays, London, 1965* (The Institute of Physics and the Physical Society, London, 1966), Vol. 1, p. 403.

[8] K. C. Anand, S. Biswas, P. J. Lavakare, S. Ramadurai, N. Sreenivasan, V. S. Bhatia, V. S. Chohan, and S. D. Pabbi, J. Geophys. Res. (to be published).

[9] R. Bernas, M. Epherre, E. Gradsztajn, R. Klapisch, and F. Yiou, Phys. Letters **15**, 147 (1965).

[10] R. Bernas, E. Gradsztajn, H. Reeves, and E. Schatzman (unpublished).

[11] M. Lefort (private communication).

TABLE I. Cross-section values in millibarns.

	Energy intervals in MeV					Badhwar *et al.*[b]
	50–100	100–200	200–500	500–1000	3000	>1.5 BeV
$C^{12}(p,x)Li^a$	18	18	16	14	14	40
$O^{16}(p,x)Li^a$	14	28	33.5	41	41	16.5
$N^{14}(p,x)Li^a$	0	0	0.2	0.5	0.5	40
$Al^{27}(p,x)Li^a$	0	4.4	7.0	24.5	44	43
$Fe^{56}(p,x)Li^a$	0	0	2.8	14.2	57	35
$Al(p,x)M$	0	115	134	154	134	132
$Fe(p,x)M$	0	0	1.5	7	54	64

[a] Li includes Li^6, Li^7, and He^6 (β^-, $T_{1/2}=0.8$ sec).
[b] Reference 2.

using the Rudstam relation.[12] There may be considerable uncertainty in the estimation of the cross section for the production of Li from N^{14}, but this does not affect the final results because of the small relative abundance of nitrogen. The values of σ for $Al^{27}(p,x)Li^6$ and $Al^{27}(p,x)Li^7$ are obtained from the measured cross sections of $Al^{27}(p,x)Be^7$ of 1.0 mb,[13] 1.5 mb,[14] and 5.0 mb[15] at 150, 350, and 750 MeV, respectively, and by using the Rudstam relation. The values of σ for $Fe^{56}(p,x)Li^6$ and $Fe^{56}(p,x)Li^7$ are obtained from that for $Fe^{56}(p,x)Be^7$ of 2.9 mb at 730 MeV,[16] the excitation function for the production of Be^7 on an Fe target,[17]

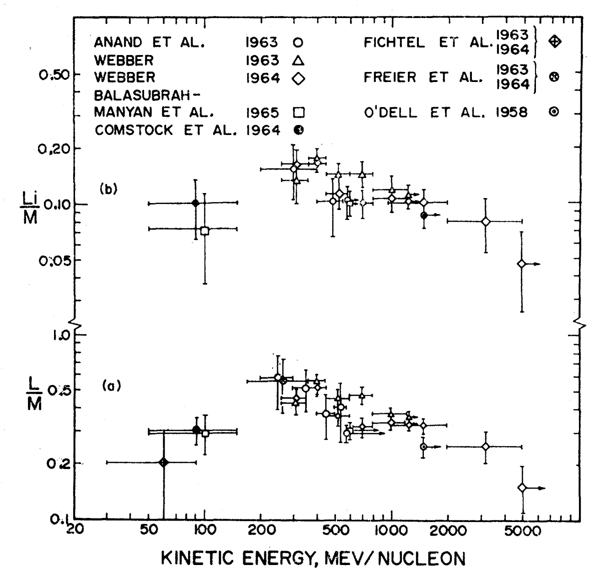

FIG. 1. Experimental values of (a) L/M, and (b) Li/M, as a function of kinetic energy as determined in 1963–65, together with the values of O'Dell *et al.* [J. Phys. Soc. Japan **17**, Suppl. AIII, 23 (1962)].

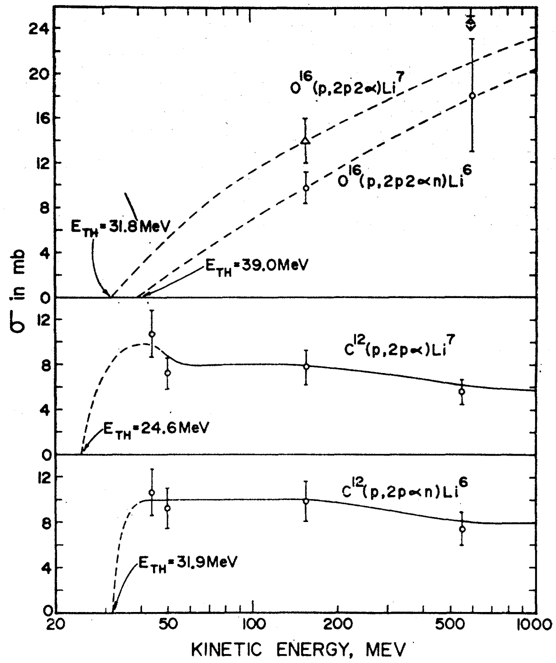

FIG. 2. Experimental values of cross-sections as a function of energy (Refs. 9 and 10). Li^6 includes He^6. At 600 MeV, only upper limit is known for $\sigma(O^{16}\rightarrow Li^7)$. We have estimated this value from the measured $\sigma(O^{16}\rightarrow Li^7)/\sigma(O^{16}\rightarrow Li^6)$ at lower energy. Dashed lines represent the approximate nature of the variation.

and the Rudstam relation. In the estimates for Al and Fe targets, we have used the values of parameters and the energy-dependent term as given by Arnold *et al.*[17] This procedure, we believe, gives us the required cross sections for Al and Fe targets with reasonable accuracy. We wish to emphasize here that in the energy interval of 50–1000 MeV/nucleon, the spallation of C and O nuclei constitutes the dominating source of the production of Li nuclei. Since these cross sections are now fairly well determined, the amount of matter traversed can be estimated with reasonably good accuracy. For example, in the energy intervals 50–200, 200–500, and 500–1000 MeV/nucleon, contributions to Li nuclei from the H_1 group of nuclei are only about 9%, 16%, and 26%, and from the H_3 group 3%, 8%, and 10%, respectively. Hence an uncertainty of even 50% in the spallation cross sections of the Al and Fe groups of nuclei would introduce an error of only about 6%, 12%, and 18% in the calculated amount of matter traversed in the above three intervals. In calculating these cross sections, radioactive decay of nuclei[2] has been taken into account.

Other cross sections of less importance needed in the diffusion equation are calculated from a similar procedure and are shown in Table I. The values of $\sigma(M\rightarrow M)$, $\sigma((H_1+H_2)\rightarrow(H_1+H_2))$, $\sigma(H_3\rightarrow H_3)$, and $\sigma(H_3\rightarrow(H_1+H_2))$ are obtained as 85, 110, 490, and 107 mb, respectively at 1 BeV, and the energy dependences of these last four parameters have been

[12] G. Rudstam, thesis, Uppsala (1956) (unpublished).
[13] M. Ligonniere, B. Vassent, and R. Bernas, Compt. Rend. **259**, 1406 (1964).
[14] G. Friedlander, J. Hudis, and R. Wolfgang, Phys. Rev. **99**, 263 (1955).
[15] L. Marquez, Phys. Rev. **86**, 405 (1952).
[16] M. Honda and D. Lal, Nucl. Phys. **51**, 363 (1964).
[17] J. R. Arnold, M. Honda, and D. Lal, J. Geophys. Res. **69**, 3519 (1961).

neglected. The total inelastic cross sections σ_t of Li, M, (H_1+H_2), and H_3 nuclei in hydrogen are taken as 140, 264, 415, and 665 mb, respectively.[18] Comparing the cross sections we obtained at 3 BeV with those obtained by Badhwar *et al.*,[2] we find that in general there is fair agreement between the two except for the first three values. Since $\sigma(M\rightarrow \text{Li})$ remains nearly the same in both the cases, these disagreements do not alter the final result at $E>1.5$ BeV, as shown later.

DIFFUSION EQUATIONS

In order to calculate the flux of Li nuclei produced from the fragmentations of heavier nuclei, we consider the diffusion equation which takes into account ionization loss and fragmentation, as given by Apparao.[19] Let $N_i(\epsilon_1,x)d\epsilon_1$ be the number of nuclei in the ith group with energies between ϵ_1 and $\epsilon_1+d\epsilon_1$ at a distance x g cm^{-2} from the source. If this group of nuclei pass through Δx g cm^{-2} of hydrogen, they will emerge with energies between ϵ_2 and $\epsilon_2+d\epsilon_2$. Thus the differential spectrum is given by

$$N_i'(\epsilon_2, x+\Delta x)d\epsilon_2 = N_i(\epsilon_1,x)(1-\Delta x/\Lambda_i)d\epsilon_1+\sum_{j\neq i} N_j(\epsilon_2,x)d\epsilon_2 \times(\Delta x/m)\sigma_{ji}(\epsilon_2),$$

where Λ_i is the absorption mean free path of ith group nuclei as given by $\Lambda_i=\lambda_i/(1-P_{ii})$, λ_i is the interaction mean free path in gcm^{-2}, P_{ji} is the fragmentation probability of jth-group nuclei going into ith-group nuclei in an interaction, $\sigma_{ji}(\epsilon)=\sigma_t P_{ji}(\epsilon)$, and $\lambda_i=m/\sigma_t$, where m is the proton mass in grams. We use the range-energy relation of the form $\epsilon-1=Cx^n$ where ϵ is the total energy in units of rest mass, x is in g cm^{-2} of neutral hydrogen, and C and n are constants for a particular group of nuclei and particular energy ranges. We have derived four equations for Li, M, (H_1+H_2), and H_3 groups of nuclei including the variation of σ_{ji} with energy. These are similar in form to the equations 5(a,b,c) of Apparao.[19]

We have assumed the source spectrum to be of the form $N^S(\epsilon,A,Z)$. $d\epsilon=K(A,Z)/\epsilon^\gamma$ where K is a constant, depending on A and Z. For the value of γ we assume two forms: (1) $\gamma=2.5$ in the entire energy interval, and (2) $\gamma=4$ for $\epsilon\leqslant 1.2$ and $\gamma=2.5$ for $\epsilon>1.2$. The reasons for making the second choice are discussed later.

The relative abundances of heavy nuclei in the "source" region are assumed as follows: Li:C:N:(O+F):(H_1+H_2):H_3=0:100:15:85:(75+5):50. These relative abundances are similar to those obtained in the recent satellite experiments.[4,5]

The calculations were done with a CDC-3600 computer, and iterations were done at every 0.1 g cm^{-2} interval for the amount of matter traversed from 0 to 15 g cm^{-2}. For each given energy interval, the variation

[18] N. Durgaprasad, thesis, Bombay, 1964 (unpublished).
[19] M. V. K. Apparao, Nuovo Cimento **32**, 1158 (1964).

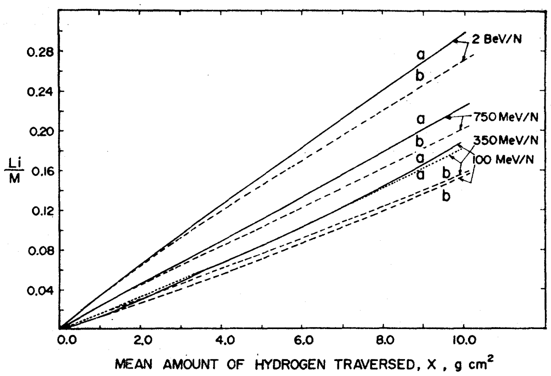

FIG. 3. The ratio Li/M as a function of x, the mean amount of hydrogen traversed in space for different energies at the earth. The set of curves (a) are calculated without taking into account the distribution in path lengths, and curves (b) with the distribution function proposed by Balasubrahmanyan *et al.* (Ref. 20).

of Li/M was calculated as a function of x; the results are plotted in Fig. 3. We have also calculated a similar set of curves (denoted by b in Fig. 3), taking into account the distribution of the path lengths about the mean value according to the distribution function proposed by Balasubrahmanyan *et al.*[20] We then made use of the measured values of Li/M to read off from this figure the corresponding values of x, the amount of matter traversed; these values are summarized in Table II.

TABLE II. The amount of matter traversed as a function of energy.

Kinetic energy interval (MeV/nucleon at the earth)	Experimental Li/M ratio	Amount of hydrogen traversed[a] (g cm^{-2}) Source spectrum (1) b	c	Source spectrum (2)
50–150	0.092±0.027	5.5±1.4	6.4±1.7	5.4±1.4
200–500	0.164±0.018	9.1±0.9	10.7±1.1	8.9±0.9
500–1000	0.117±0.010	5.3±0.5	5.7±0.5	5.3±0.5
>1500	0.099±0.012	3.1±0.4	3.3±0.4	3.1±0.4

[a] The errors in the path length are calculated from the errors in the experimental Li/M ratios.
[b] Without assuming the distribution in the path length.
[c] Assuming the distribution in the path length as given in Ref. 20.

DISCUSSION

From the calculations made above the following conclusions are derived:

(a) The amount of matter traversed at each of the energy interval does not depend significantly on whether or not one takes into account the distributions in the path lengths about the mean value.

(b) The amount of matter traversed is insensitive to the form of the source spectrum. However, the assumed spectral shape (2) results in a spectrum for M nuclei at the earth of the form $E^{0.66}$ ($50\leqslant E\leqslant 150$ MeV/nucleon)

[20] V. K. Balasubrahmanyan, E. Boldt, and R. A. R. Palmeira, Phys. Rev. **140**, B1157 (1965).

which is closer to the observed flat spectrum in this region.[4] Since we have neglected solar modulation, the source spectrum is expected to be at least as steep as $(1+E)^{-4}$ in this energy region.

(c) The value of $x=3.1$ g cm^{-2} for $E>1.5$ BeV/nucleon obtained by us is in agreement with previous calculations.[2]

(d) At $E=200$–500 MeV/nucleon, the path length $x=9.1\pm0.9$ g cm^{-2} is considerably higher than that for $E>1.5$ BeV/nucleon.

(e) At the lowest energy 50–150 MeV/nucleon, the path length $x=5.5\pm1.4$ g cm^{-2} seems to be smaller than that at 200–500 MeV/nucleon, although with the present uncertainties we cannot completely rule out the possibility that the path length is the same. However, an examination of Fig. 3 and the experimental data as given in Fig. 1(b) seems to indicate that the difference in the path length is probably real. We wish to point out here that recent results on the $He^3/(He^3+He^4)$ ratio in the primary cosmic rays in the energy interval 80–150 MeV/nucleon yield a path length of 6.5 ± 1.5 g cm^{-2},[21] consistent with what is obtained here.

[21] D. J. Hoffmann and J. R. Winckler, Phys. Rev. Letters **16**, 109 (1966).

Thus we find that the path increases from 3 to $\sim$9 g cm^{-2} as the energy of the particles decreases from 1500 to $\sim$200 MeV/nucleon; the path then decreases to $\sim$5.5 g cm^{-2} at still lower energies, 50–150 MeV/nucleon. These observations suggest the hypothesis of two distinct sources (or propagation paths), one responsible for nuclei of energy roughly between 50–150 MeV/nucleon and the other for nuclei of all energies greater than this. Other properties of these low-energy nuclei which seem to support this hypothesis are: (1) the "source" spectrum is at least as steep as $(1+E)^{-4}$, which follows from the experimentally measured flat spectrum[4,5] near the earth, and (2) the high H_3/C ratio, near the earth, is[4] 50:100 as compared to 15:100 for $E>1.5$ BeV/nucleon.[2,3]

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Geometry and Newtonian Physics

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The question is raised whether there is any physical content in the requirement that freely moving particles move along geodesics in 4-space. The question is partially answered in the affirmative by demonstrating that some motions cannot be expressed as geodesics in any 4-space under certain conditions. In particular, it is shown that the trajectories of the Newtonian equations of motion for any type of nonvanishing conservative force field cannot be expressed as geodesics in any static 4-space when it is required that the metric have the diagonal form (1,1,1, constant) at infinity.

I. INTRODUCTION

THE present mood in general-relativity physics is to regard the field equations (or a suitable action principle) as basic, whereas the statement that freely moving observers comprise local geodesic systems (and therefore move along geodesics in space-time) is considered as secondary. This view is no doubt the result of Einstein's derivation of the geodesic equations of motion from the field equations.

However, in a recent paper by the author considerations were the other way around.[1] In that work two geometrical statements were taken as the basis of the theory. Briefly, the assumptions were that (1) freely moving observers comprise locally geodesic coordinate systems in space-time, and (2) the geometry of space-time must be unique. It was then demonstrated that these assumptions did, to some extent, influence the form of the field equations. Therefore, one concludes that these geometrical statements have physical content as well. This type of formulation raises the general question of when requirements of a geometrical form have physical content. The following specific question can be raised at this point: Is there any physical significance to the statement that freely moving particles must move along geodesics in 4-space? That is, given certain motions of test particles (say those given by Newtonian theory) can one always find a 4-space such that in this space the motions are geodesics? If this is the case, then the requirement that particles move along geodesics could not influence the prescribed mo-

[1] J. Cohn (to be published).