One-Graviton Exchange Interaction of Elementary Particles*

BRUCE M. BARKER,† SURAJ N. GUPTA, AND RICHARD D. HARACZ‡
Department of Physics, Wayne State University, Detroit, Michigan
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The interaction of elementary particles of several different spins due to the exchange of a graviton is investigated. The exact one-graviton exchange potentials are obtained, and then simplified by making various approximations. The gravitational interaction is found to be independent of the spins of the particles, provided that the distance between them is large compared with their Compton wavelengths. It is pointed out that the spin-dependent interaction terms, whose range is of the order of the Compton wavelength, may be of interest in the theory of gravitational collapse. It is also shown that the gravitational particle-antiparticle interaction differs from the corresponding particle-particle interaction only by contact interaction terms.

1. INTRODUCTION

IT is possible to interpret Einstein's theory of gravitation as a flat-space field theory, and then quantize it in analogy with the electromagnetic field.[1] It would be interesting to apply the techniques of quantum field theory to the gravitational interaction of particles of various spins. It would also be instructive to see whether the quantum theory leads to any difference in the gravitational particle-particle and particle-antiparticle interactions.

Because of the smallness of the gravitational coupling constant, the one-graviton exchange contributions will provide highly accurate results for our purpose, and we need consider the coupling of the gravitational and "matter" fields only in the linear approximation. Thus, we express the gravitational interaction terms for neutral particles of spin 0 as

$$H_{\text{int}}=\tfrac{1}{2}\kappa h_{\mu\nu}:\left[\frac{\partial U_0}{\partial x_\mu}\frac{\partial U_0}{\partial x_\nu}-\tfrac{1}{2}\delta_{\mu\nu}\left(\frac{\partial U_0}{\partial x_\rho}\right)^2-\tfrac{1}{2}\delta_{\mu\nu}\lambda^2U_0{}^2\right]:+O(\kappa^2), \quad (1)$$

for charged particles of spin 0 as

$$H_{\text{int}}=\tfrac{1}{2}\kappa h_{\mu\nu}:\left[\frac{\partial U^*}{\partial x_\mu}\frac{\partial U}{\partial x_\nu}+\frac{\partial U^*}{\partial x_\nu}\frac{\partial U}{\partial x_\mu}-\delta_{\mu\nu}\frac{\partial U^*}{\partial x_\rho}\frac{\partial U}{\partial x_\rho}-\delta_{\mu\nu}\lambda^2U^*U\right]:+O(\kappa^2), \quad (2)$$

for particles of spin $\frac{1}{2}$ as

$$H_{\text{int}}=\tfrac{1}{8}\kappa ch h_{\mu\nu}:\left[\bar\psi\gamma_\mu\frac{\partial\psi}{\partial x_\nu}+\bar\psi\gamma_\nu\frac{\partial\psi}{\partial x_\mu}-\frac{\partial\bar\psi}{\partial x_\nu}\gamma_\mu\psi-\frac{\partial\bar\psi}{\partial x_\mu}\gamma_\nu\psi\right]:+O(\kappa^2), \quad (3)$$

and for photons as

$$H_{\text{int}}=\tfrac{1}{2}\kappa h_{\mu\nu}:[F_{\mu\rho}F_{\nu\rho}-\tfrac{1}{4}\delta_{\mu\nu}F_{\lambda\rho}{}^2]:+O(\kappa^2). \quad (4)$$

The contraction for the gravitational field operator $h_{\mu\nu}$, appearing in the above interaction terms, is given by

$$h^{\cdot}{}_{\mu\nu}(x)h^{\cdot}{}_{\lambda\rho}(x')=-ich(\delta_{\mu\lambda}\delta_{\nu\rho}+\delta_{\mu\rho}\delta_{\nu\lambda}-\delta_{\mu\nu}\delta_{\lambda\rho})D_F(x-x'), \quad (5)$$

where

$$D_F(x-x')=\lim_{\epsilon\to+0}\frac{1}{(2\pi)^4}\int dk\, e^{ik(x-x')}\frac{1}{k^2-i\epsilon}. \quad (6)$$

The gravitational interaction of elementary particles can be investigated by the procedure, which was applied earlier for the electromagnetic and mesonic interactions.[2] However, in the present case even the one-graviton exchange contributions turn out to be quite complicated in the exact form. We shall, therefore, derive the exact results as well as discuss their significance under various approximations.

We shall use the center-of-mass system throughout, and denote the rest mass of a particle as m, its relativistic mass as M, its momentum as $\mathbf{P}$, and its energy as E, so that

$$M=E/c^2=(m^2+\mathbf{P}^2/c^2)^{1/2}, \quad (7)$$

and, for a particle with the propagation four-vector p_μ,

$$\mathbf{P}=\hbar\mathbf{p},\quad E=c\hbar p_0,\quad p_\mu{}^2=-\lambda^2,\quad \lambda=mc/\hbar. \quad (8)$$

It should also be noted that the gravitational coupling constant κ is related to Newton's constant of gravitation G as

$$\kappa^2=16\pi G/c^4. \quad (9)$$

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† Present address: Department of Physics, University of Windsor, Windsor, Ontario.
‡ Present address: Sloane Physics Laboratory, Yale University, New Haven, Connecticut.
[1] For the Lorentz-covariant expansion and quantization of the gravitational field, see S. N. Gupta, Proc. Phys. Soc. (London) **A65**, 161 (1952); **A65**, 608 (1952). For the comparison of gravitation and electromagnetism and the necessity of a nonlinear theory of gravitation, see S. N. Gupta, Phys. Rev. **96**, 1683 (1954); Rev. Mod. Phys. **29**, 334 (1957). For a review of the above ideas, see S. N. Gupta, in *Recent Development in General Relativity* (Pergamon Press, Inc., New York, 1962), p. 251.
[2] S. N. Gupta, Nucl. Phys. **57**, 19 (1964).

2. INTERACTION OF TWO PARTICLES OF SPIN 0

Consider the gravitational scattering of two particles[3] of spin 0 and masses m_1 and m_2. Let the initial and final propagation four-vectors for the particle of mass m_1 be p and p', and those of the particle with mass m_2 be q and q', so that in the center-of-mass system

$$\mathbf{p}=-\mathbf{q}\,,\quad \mathbf{p}'=-\mathbf{q}'\,,\quad p_0'=p_0\,,\quad q_0'=q_0\,,$$
$$\mathbf{k}=\mathbf{p}'-\mathbf{p}=-(\mathbf{q}'-\mathbf{q})\,. \tag{10}$$

The scattering matrix element for the one-graviton exchange contribution is then given by

$$S_2=(-i/ch)(2\pi)^4\delta(p+q-p'-q')\times a^*(\mathbf{p}')a^*(\mathbf{q}')V(\mathbf{k})a(\mathbf{q})a(\mathbf{p})\,, \tag{11}$$

with

$$V(\mathbf{k})=-\frac{c^2h^2p_0q_0\kappa^2}{4\mathbf{k}^2}\times\left[1+\left(4+\frac{p_0}{q_0}+\frac{q_0}{p_0}\right)\frac{\mathbf{p}^2}{p_0q_0}-\frac{\mathbf{k}^2}{p_0q_0}+\frac{\mathbf{p}^4-\mathbf{p}^2\mathbf{k}^2}{p_0^2q_0^2}\right], \tag{12}$$

where a and a^* denote the annihilation and creation operators for spinless particles.[4]

The gravitational potential

$$V(\mathbf{r})=\frac{1}{(2\pi)^3}\int d\mathbf{k}\,e^{i\mathbf{k}\cdot\mathbf{r}}V(\mathbf{k}) \tag{13}$$

can be obtained from (12), and expressed as

$$V(\mathbf{r})=-\frac{GM_1M_2}{r}\times\left[1+\left(4+\frac{M_1}{M_2}+\frac{M_2}{M_1}\right)\frac{\mathbf{P}^2}{M_1M_2c^2}+\frac{\mathbf{P}^4}{M_1^2M_2^2c^4}\right]+\frac{4\pi Gh^2}{c^2}\left[1+\frac{\mathbf{P}^2}{M_1M_2c^2}\right]\delta(\mathbf{r})\,, \tag{14}$$

which evidently reduces to Newton's law in the static approximation if we drop the contact term.

It is interesting to simplify (14) by making three different approximations[5]:

(a) In the nonrelativistic approximation

$$V(\mathbf{r})=-\frac{Gm_1m_2}{r}\left[1+\left(4+\frac{3m_1}{2m_2}+\frac{3m_2}{2m_1}\right)\frac{\mathbf{P}^2}{m_1m_2c^2}\right]+\frac{4\pi Gh^2}{c^2}\delta(\mathbf{r})\,, \tag{15}$$

where we take $m_1^2\gg\mathbf{P}^2/c^2$ and $m_2^2\gg\mathbf{P}^2/c^2$, and retain the first- and second-order terms.

(b) In the large-mass approximation

$$V(\mathbf{r})=-\frac{Gm_1M_2}{\mathbf{r}}\left(1+\frac{\mathbf{P}^2}{M_2^2c^2}\right), \tag{16}$$

where we take $m_1\gg m_2$ and $m_1\gg|\mathbf{P}|/c$.

(c) In the large-distance approximation

$$V(\mathbf{r})=-\frac{GM_1M_2}{r}\left[1+\left(4+\frac{M_1}{M_2}+\frac{M_2}{M_1}\right)\times\frac{\mathbf{P}^2}{M_1M_2c^2}+\frac{\mathbf{P}^4}{M_1^2M_2^2c^4}\right], \tag{17}$$

where we take $r\gg h/m_1c$ and $r\gg h/m_2c$.

3. INTERACTION OF PARTICLES OF SPIN $\frac{1}{2}$ AND SPIN 0

Let us now consider the scattering of a particle of spin $\frac{1}{2}$ and mass m_1 with another particle of spin 0 and mass m_2. Let the initial and final propagation four-vectors for the particle of spin $\frac{1}{2}$ be p and p', and those for the particle of spin 0 be q and q'. Then, the scattering matrix element for the one-graviton exchange contribution in the center-of-mass system is given by

$$\begin{aligned}S_2=&-i(2\pi)^4\delta(p+q-p'-q')(ch\kappa^2/4q_0\mathbf{k}^2)\\&\times[\lambda_1(\lambda_2^2+2p_0q_0+2\mathbf{p}^2-\tfrac{1}{2}\mathbf{k}^2)\bar{\psi}^-(\mathbf{p}')\psi^+(\mathbf{p})\\&\times a^*(\mathbf{q}')a(\mathbf{q})-(p_0+q_0)(2p_0q_0+2\mathbf{p}^2-\tfrac{1}{2}\mathbf{k}^2)\\&\qquad\times\bar{\psi}^-(\mathbf{p}')\gamma_4\psi^+(\mathbf{p})a^*(\mathbf{q}')a(\mathbf{q})]\,.\end{aligned} \tag{18}$$

Transforming the Dirac spinors into the Pauli spinors, we can express (18) as

$$S_2=(-i/ch)(2\pi)^4\delta(p+q-p'-q')\times a^*(\mathbf{q}')\psi_L^{*-}(\mathbf{p}')V(\mathbf{k})\psi_L^+(\mathbf{p})a(\mathbf{q})\,, \tag{19}$$

with

$$V(\mathbf{k})=V_1(\mathbf{k})+iV_2(\mathbf{k})\boldsymbol{\sigma}\cdot\mathbf{k}\times\mathbf{p}\,, \tag{20}$$

where

$$V_1(\mathbf{k})=-\frac{\kappa^2c^2h^2(\lambda_1+p_0)q_0}{8\mathbf{k}^2}\times\left[\left(2+\frac{2p_0}{q_0}+\frac{4\mathbf{p}^2-\mathbf{k}^2}{2q_0^2}+\frac{4\mathbf{p}^2-\mathbf{k}^2}{2p_0q_0}\right)\left(1+\frac{2\mathbf{p}^2-\mathbf{k}^2}{2(\lambda_1+p_0)^2}\right)-\lambda_1\left(\frac{1}{p_0}+\frac{2}{q_0}+\frac{2\mathbf{p}^2-\mathbf{k}^2}{2p_0q_0^2}\right)\left(1-\frac{2\mathbf{p}^2-\mathbf{k}^2}{2(\lambda_1+p_0)^2}\right)\right], \tag{21}$$

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[3] The one-graviton exchange interaction of two spinless particles was first investigated by Corinaldesi to obtain the two-body equations of motion. See E. Corinaldesi, Proc. Phys. Soc. (London) **A69**, 189 (1956).

[4] For the Fourier expansion of the field operators we have dropped the volume factor of the box enclosing the field, which is known to cancel in the final results.

[5] Because of the presence of the function $\delta(\mathbf{r})$ in the exact form of $V(\mathbf{r})$, it is sometimes clearer to make the desired approximations in $V(\mathbf{k})$ and then obtain the corresponding $V(\mathbf{r})$.

and

$$V_2(\mathbf{k})=-\frac{\kappa^2c^2\hbar^2q_0}{8\mathbf{k}^2(\lambda_1+p_0)}\left[2+\frac{2p_0}{q_0}+\frac{4\mathbf{p}^2-\mathbf{k}^2}{2q_0^2}+\frac{4\mathbf{p}^2-\mathbf{k}^2}{2p_0q_0}+\lambda_1\left(\frac{1}{p_0}+\frac{2}{q_0}+\frac{2\mathbf{p}^2-\mathbf{k}^2}{2p_0q_0^2}\right)\right]. \tag{22}$$

The above general result can be simplified by making various approximations, and we then obtain:

(a) In the nonrelativistic approximation

$$V(\mathbf{r})=-\frac{Gm_1m_2}{r}\left[1+\left(4+\frac{3m_1}{2m_2}+\frac{3m_2}{2m_1}\right)\frac{\mathbf{P}^2}{m_1m_2c^2}\right]+\frac{4\pi G\hbar^2}{c^2}\left(1+\frac{3m_2}{8m_1}\right)\delta(\mathbf{r})+G\left(1+\frac{3m_2}{4m_1}\right)\frac{\hbar\boldsymbol{\sigma}\cdot(\mathbf{r}\times\mathbf{P})}{c^2r^3}, \tag{23}$$

which contains a spin-orbit interaction term.

(b) In the large-mass approximation, $V(\mathbf{r})$ agrees with (16) for $m_1\gg m_2$ and $m_1\gg|\mathbf{P}|/c$, while

$$V(\mathbf{r})=-\frac{GM_1m_2}{r}\left(1+\frac{\mathbf{P}^2}{M_1^2c^2}\right)+\frac{\pi G\hbar^2}{c^2}\left(\frac{m_2}{M_1}+\frac{m_2}{m_1+M_1}\right)\delta(\mathbf{r})+G\left(\frac{m_2}{M_1}+\frac{m_2}{m_1+M_1}\right)\frac{\hbar\boldsymbol{\sigma}\cdot(\mathbf{r}\times\mathbf{P})}{2c^2r^3} \tag{24}$$

for $m_2\gg m_1$ and $m_2\gg|\mathbf{P}|/c$.

(c) In the large-distance approximation, $V(\mathbf{r})$ agrees with (17).

4. INTERACTION OF TWO PARTICLES OF SPIN $\frac{1}{2}$

The scattering matrix element for the one-graviton exchange interaction of two particles of spin $\frac{1}{2}$ and masses m_1 and m_2 is given in the center-of-mass system by

$$\begin{aligned}S_2=i(2\pi)^4\delta(p+q-p'-q')(\kappa^2c\hbar/4\mathbf{k}^2)&\\ \times[(p_0q_0+\mathbf{p}^2-\tfrac{1}{4}\mathbf{k}^2):\bar{\psi}^-(\mathbf{p}')\gamma_\mu\psi^+(\mathbf{p})\bar{\psi}^-(\mathbf{q}')\gamma_\mu\psi^+(\mathbf{q}):&\\ +(p_0+q_0)^2:\bar{\psi}^-(\mathbf{p}')\gamma_4\psi^+(\mathbf{p})\bar{\psi}^-(\mathbf{q}')\gamma_4\psi^+(\mathbf{q}):&\\ -\lambda_1(p_0+q_0):\bar{\psi}^-(\mathbf{p}')\psi^+(\mathbf{p})\bar{\psi}^-(\mathbf{q}')\gamma_4\psi^+(\mathbf{q}):&\\ -\lambda_2(p_0+q_0):\bar{\psi}^-(\mathbf{p}')\gamma_4\psi^+(\mathbf{p})\bar{\psi}^-(\mathbf{q}')\psi^+(\mathbf{q}):],&\end{aligned} \tag{25}$$

where p and p' denote the initial and final propagation four-vectors for the particle of mass m_1, and q and q' denote those for the particle of mass m_2.

Transforming the Dirac spinors into the Pauli spinors, we can express (25) as

$$S_2=(-i/c\hbar)(2\pi)^4\delta(p+q-p'-q')\times\psi_L{}^{*-}(\mathbf{p}')\psi_L{}^{*-}(\mathbf{q}')V(\mathbf{k})\psi_L{}^+(\mathbf{q})\psi_L{}^+(\mathbf{p}), \tag{26}$$

with

$$\begin{aligned}V(\mathbf{k})=V_1(\mathbf{k})+V_2(\mathbf{k})\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}+iV_3(\mathbf{k})(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)})\cdot\mathbf{k}\times\mathbf{p}&\\ +iV_4(\mathbf{k})(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)})\cdot\mathbf{k}\times\mathbf{p}+V_5(\mathbf{k})(\boldsymbol{\sigma}^{(1)}\cdot\mathbf{k}\boldsymbol{\sigma}^{(2)}\cdot\mathbf{k}&\\ -\tfrac{1}{3}\mathbf{k}^2\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)})+V_6(\mathbf{k})(\boldsymbol{\sigma}^{(1)}\cdot\mathbf{s}\boldsymbol{\sigma}^{(2)}\cdot\mathbf{s}&\\ -\tfrac{1}{3}\mathbf{s}^2\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}),&\end{aligned} \tag{27}$$

where $\mathbf{s}=\mathbf{p}'+\mathbf{p}$, and

$$\begin{aligned}V_1(\mathbf{k})=-\frac{c^2\hbar^2p_0q_0\kappa^2}{4\mathbf{k}^2}\left(\frac{\lambda_1+p_0}{2p_0}\right)\left(\frac{\lambda_2+q_0}{2q_0}\right)\Bigg[\left(1+\frac{4\mathbf{p}^2-\mathbf{k}^2}{4p_0q_0}\right)\left(\frac{4\mathbf{p}^2-\mathbf{k}^2}{(\lambda_1+p_0)(\lambda_2+q_0)}\right)&\\ +\left(3+\frac{q_0+\lambda_1-\lambda_2}{p_0}+\frac{p_0+\lambda_1-\lambda_2}{q_0}+\frac{4\mathbf{p}^2-\mathbf{k}^2}{4p_0q_0}\right)\left(1+\frac{2\mathbf{p}^2-\mathbf{k}^2}{2(\lambda_1+p_0)^2}\right)+\left(3+\frac{q_0-\lambda_1+\lambda_2}{p_0}+\frac{p_0-\lambda_1+\lambda_2}{q_0}+\frac{4\mathbf{p}^2-\mathbf{k}^2}{4p_0q_0}\right)&\\ \times\left(1+\frac{2\mathbf{p}^2-\mathbf{k}^2}{2(\lambda_2+p_0)^2}\right)+\left(3+\frac{q_0+\lambda_1+\lambda_2}{p_0}+\frac{p_0+\lambda_1+\lambda_2}{q_0}+\frac{4\mathbf{p}^2-\mathbf{k}^2}{4p_0q_0}\right)\left(-1+\frac{4\mathbf{p}^4-4\mathbf{p}^2\mathbf{k}^2+\mathbf{k}^4}{4(\lambda_1+p_0)^2(\lambda_2+q_0)^2}\right)\Bigg],&\end{aligned} \tag{28}$$

$$V_2(\mathbf{k})=\frac{c^2\hbar^2p_0q_0\kappa^2}{4\mathbf{k}^2}\left[\left(1+\frac{4\mathbf{p}^2-\mathbf{k}^2}{4p_0q_0}\right)\frac{\mathbf{k}^2}{6p_0q_0}+\left(3+\frac{q_0+\lambda_1+\lambda_2}{p_0}+\frac{p_0+\lambda_1+\lambda_2}{q_0}+\frac{4\mathbf{p}^2-\mathbf{k}^2}{4p_0q_0}\right)\frac{4\mathbf{p}^2\mathbf{k}^2-\mathbf{k}^4}{48p_0q_0(\lambda_1+p_0)(\lambda_2+q_0)}\right], \tag{29}$$

$$\begin{aligned}V_3(\mathbf{k})=-\frac{c^2\hbar^2p_0q_0\kappa^2}{4\mathbf{k}^2}\Bigg[\left(1+\frac{4\mathbf{p}^2-\mathbf{k}^2}{4p_0q_0}\right)\frac{1}{2p_0q_0}+\left(3+\frac{q_0+\lambda_1-\lambda_2}{p_0}+\frac{p_0+\lambda_1-\lambda_2}{q_0}+\frac{4\mathbf{p}^2-\mathbf{k}^2}{4p_0q_0}\right)&\\ \times\frac{\lambda_2+q_0}{8p_0q_0(\lambda_1+p_0)}+\left(3+\frac{q_0-\lambda_1+\lambda_2}{p_0}+\frac{p_0-\lambda_1+\lambda_2}{q_0}+\frac{4\mathbf{p}^2-\mathbf{k}^2}{4p_0q_0}\right)\frac{\lambda_1+p_0}{8p_0q_0(\lambda_2+q_0)}&\\ +\left(3+\frac{q_0+\lambda_1+\lambda_2}{p_0}+\frac{p_0+\lambda_1+\lambda_2}{q_0}+\frac{4\mathbf{p}^2-\mathbf{k}^2}{4p_0q_0}\right)\frac{2\mathbf{p}^2-\mathbf{k}^2}{8p_0q_0(\lambda_1+p_0)(\lambda_2+q_0)}\Bigg],&\end{aligned} \tag{30}$$

$$V_4(\mathbf{k})=-\frac{c^2h^2p_0q_0\kappa^2}{4\mathbf{k}^2}\bigg[\bigg(3+\frac{q_0+\lambda_1-\lambda_2}{p_0}+\frac{p_0+\lambda_1-\lambda_2}{q_0}+\frac{4\mathbf{p}^2-\mathbf{k}^2}{4p_0q_0}\bigg)\frac{\lambda_2+q_0}{8p_0q_0(\lambda_1+p_0)}$$
$$-\bigg(3+\frac{q_0-\lambda_1+\lambda_2}{p_0}+\frac{p_0-\lambda_1+\lambda_2}{q_0}+\frac{4\mathbf{p}^2-\mathbf{k}^2}{4p_0q_0}\bigg)\frac{\lambda_1+p_0}{8p_0q_0(\lambda_2+q_0)}\bigg], \quad (31)$$

$$V_5(\mathbf{k})=-\frac{c^2h^2p_0q_0\kappa^2}{4\mathbf{k}^2}\bigg[\bigg(1+\frac{4\mathbf{p}^2-\mathbf{k}^2}{4p_0q_0}\bigg)\frac{1}{4p_0q_0}+\bigg(3+\frac{q_0+\lambda_1+\lambda_2}{p_0}+\frac{p_0+\lambda_1+\lambda_2}{q_0}+\frac{4\mathbf{p}^2-\mathbf{k}^2}{4p_0q_0}\bigg)\frac{4\mathbf{p}^2-\mathbf{k}^2}{16p_0q_0(\lambda_1+p_0)(\lambda_2+q_0)}\bigg], \quad (32)$$

$$V_6(\mathbf{k})=-\frac{c^2h^2p_0q_0\kappa^2}{4\mathbf{k}^2}\bigg(3+\frac{q_0+\lambda_1+\lambda_2}{p_0}+\frac{p_0+\lambda_1+\lambda_2}{q_0}+\frac{4\mathbf{p}^2-\mathbf{k}^2}{4p_0q_0}\bigg)\frac{\mathbf{k}^2}{16p_0q_0(\lambda_1+p_0)(\lambda_2+q_0)}. \quad (33)$$

The exact one-graviton exchange potential can be derived from the above results. These results, however, can be considerably simplified by making various approximations, and we then obtain:

(a) In the nonrelativistic approximation

$$V(\mathbf{r})=-\frac{Gm_1m_2}{r}\bigg[1+\bigg(4+\frac{3m_1}{2m_2}+\frac{3m_2}{2m_1}\bigg)\frac{\mathbf{P}^2}{m_1m_2c^2}\bigg]+\frac{4\pi Gh^2}{c^2}\bigg(1+\frac{3m_2}{8m_1}+\frac{3m_1}{8m_2}\bigg)\delta(\mathbf{r})+\frac{Gh^2}{4c^2r^3}\bigg(\frac{3(\boldsymbol{\sigma}^{(1)}\cdot\mathbf{r})(\boldsymbol{\sigma}^{(2)}\cdot\mathbf{r})}{r^2}-\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}\bigg)$$
$$+\frac{2\pi Gh^2}{3c^2}(\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)})\delta(\mathbf{r})+G\bigg(1+\frac{3m_2}{8m_1}+\frac{3m_1}{8m_2}\bigg)\frac{h(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)})\cdot(\mathbf{r}\times\mathbf{P})}{c^2r^3}+G\bigg(\frac{3m_2}{8m_1}-\frac{3m_1}{8m_2}\bigg)\frac{h(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)})\cdot(\mathbf{r}\times\mathbf{P})}{c^2r^3}, \quad (34)$$

which contains tensor and spin-orbit interaction terms.

(b) In the large-mass approximation, $V(\mathbf{r})$ agrees with (24) for $m_2\gg m_1$ and $m_2\gg|\mathbf{P}|/c$.

(c) In the large-distance approximation, $V(\mathbf{r})$ again agrees with (17).

5. INTERACTION OF PHOTON AND PARTICLE OF SPIN 0

We next consider the one-graviton exchange interaction of a photon with a particle of spin 0 and mass m. Denoting the initial and final propagation four-vectors for the photon as p and p', and those for the particle as q and q', we obtain for the scattering matrix element in the center-of-mass system

$$S_2=i(2\pi)^4\delta(p+q-p'-q')\frac{\kappa^2chp_0q_0}{4\mathbf{k}^2}a_i^*(\mathbf{p}')a_j(\mathbf{p})a^*(\mathbf{q}')a(\mathbf{q})$$
$$\times\bigg\{\delta_{ij}\bigg[1+\bigg(4+\frac{p_0}{q_0}+\frac{q_0}{p_0}\bigg)\frac{\mathbf{p}^2}{p_0q_0}-\bigg(1+\frac{p_0}{2q_0}+\frac{q_0}{2p_0}\bigg)\frac{\mathbf{k}^2}{p_0q_0}+\frac{\mathbf{p}^4}{p_0^2q_0^2}\bigg]+(k_ik_j/p_0q_0)[2+p_0/q_0+q_0/p_0]\bigg\}, \quad (35)$$

where the photon annihilation and creation operators $a_i(\mathbf{p})$ and $a_i^*(\mathbf{p}')$ satisfy the relations

$$p_ia_i(\mathbf{p})=0, \quad p_i'a_i^*(\mathbf{p}')=0. \quad (36)$$

We express $a_i(\mathbf{p})$ and $a_i^*(\mathbf{p}')$ as

$$a_i(\mathbf{p})=e_i^{(1)}(\mathbf{p})a^{(1)}(\mathbf{p})+e_i^{(2)}(\mathbf{p})a^{(2)}(\mathbf{p}),$$
$$a_i^*(\mathbf{p}')=e_i^{(1)}(\mathbf{p}')a^{(1)*}(\mathbf{p}')+e_i^{(2)}(\mathbf{p}')a^{(2)*}(\mathbf{p}'), \quad (37)$$

where $\mathbf{e}^{(1)}(\mathbf{p})$ and $\mathbf{e}^{(2)}(\mathbf{p})$ are unit orthogonal polarization vectors for initial photon, while $\mathbf{e}^{(1)}(\mathbf{p}')$ and $\mathbf{e}^{(2)}(\mathbf{p}')$ are those for the final photon. We also take

$$\mathbf{e}^{(1)}(\mathbf{p})\times\mathbf{e}^{(2)}(\mathbf{p})=\mathbf{p}/|\mathbf{p}|, \quad \mathbf{e}^{(1)}(\mathbf{p}')\times\mathbf{e}^{(2)}(\mathbf{p}')=\mathbf{p}'/|\mathbf{p}'|,$$
$$\mathbf{e}^{(1)}(\mathbf{p})=\mathbf{e}^{(1)}(\mathbf{p}')=(\mathbf{p}'\times\mathbf{p})/|\mathbf{p}'\times\mathbf{p}|, \quad (38)$$

and

$$a_+(\mathbf{p})=2^{-1/2}[a^{(1)}(\mathbf{p})-ia^{(2)}(\mathbf{p})],$$
$$a_-(\mathbf{p})=2^{-1/2}[a^{(1)}(\mathbf{p})+ia^{(2)}(\mathbf{p})],$$
$$a_+^*(\mathbf{p}')=2^{-1/2}[a^{(1)*}(\mathbf{p}')+ia^{(2)*}(\mathbf{p}')],$$
$$a_-^*(\mathbf{p}')=2^{-1/2}[a^{(1)*}(\mathbf{p}')-ia^{(2)*}(\mathbf{p}')], \quad (39)$$

where $a_+(\mathbf{p})$ and $a_+^*(\mathbf{p}')$ refer to photons with their spin axes parallel to their directions of motion, while $a_-(\mathbf{p})$ and $a_-^*(\mathbf{p}')$ refer to those with their spin axes antiparallel.

We then obtain from (35)

$$S_2=(-i/ch)(2\pi)^4\delta(p+q-p'-q')a^*(\mathbf{q}')a(\mathbf{q})$$
$$\times[a_+^*(\mathbf{p}')a_+(\mathbf{p})+a_-^*(\mathbf{p}')a_-(\mathbf{p})]V(\mathbf{k}), \quad (40)$$

where

$$V(\mathbf{k})=-\frac{c^2\hbar^2p_0q_0\kappa^2}{4\mathbf{k}^2}\left[2+\frac{4p_0}{q_0}+\frac{2p_0^2}{q_0^2}-\left(1+\frac{p_0}{2q_0}+\frac{q_0}{2p_0}\right)\frac{\mathbf{k}^2}{p_0q_0}\right]. \quad (41)$$

The above result shows that the photon preserves its spin orientation with respect to its direction of motion when it is scattered by a particle of spin 0, and the resulting gravitational potential is[6]

$$V(\mathbf{r})=-\frac{G\mu M}{r}\left(2+\frac{4\mu}{M}+\frac{2\mu^2}{M^2}\right)+\frac{4\pi G\hbar^2}{c^2}\left(1+\frac{\mu}{2M}+\frac{M}{2\mu}\right)\delta(\mathbf{r}), \quad (42)$$

where μ denotes the relativistic mass of the photon.

In the large-mass approximation, we obtain by taking $M\gg\mu$, and ignoring the $\delta(\mathbf{r})$ term,

$$V(\mathbf{r})=-2G\mu M/r, \quad (43)$$

which agrees with the well-known result that light is deflected by a heavy object by twice the amount predicted by the Newtonian theory.

In the large-distance approximation, (42) reduces to

$$V(\mathbf{r})=-\frac{G\mu M}{r}\left(2+\frac{4\mu}{M}+\frac{2\mu^2}{M^2}\right), \quad (44)$$

which agrees with (17) when $M_1=|\mathbf{P}|/c=\mu$ and $M_2=M$.

6. INTERACTION OF PHOTON AND PARTICLE OF SPIN $\frac{1}{2}$

The treatment of the preceding section can be extended to the one-graviton exchange interaction of a photon with a particle of spin $\frac{1}{2}$ and mass m, and we obtain for the scattering matrix element in the center-of-mass system

$$\begin{aligned}S_2=&i(2\pi)^4\delta(p+q-p'-q')(\kappa^2c\hbar/4p_0\mathbf{k}^2)a_i^*(\mathbf{p}')a_j(\mathbf{p})\\&\times\{[\delta_{ij}(\tfrac{1}{2}\mathbf{k}^2-2p_0^2-2p_0q_0)+p_ip_j']\bar{\psi}^-(\mathbf{q}')ip\gamma\psi^+(\mathbf{q})\\&+(p_0^2+p_0q_0)\bar{\psi}^-(\mathbf{q}')i(p_i\gamma_j+p_j'\gamma_i)\psi^+(\mathbf{q})\\&-\lambda[\tfrac{1}{2}\delta_{ij}\mathbf{k}^2+p_ip_j']\bar{\psi}^-(\mathbf{q}')\psi^+(\mathbf{q})\}, \quad (45)\end{aligned}$$

where p and p' denote the initial and final propagation four-vectors for the photon, and q and q' denote those for the particle.

Transforming the Dirac spinors into the Pauli spinors, and introducing the polarization vectors and the annihilation and creation operators for the photons in the same way as in Sec. 5, we can express (45) as

$$\begin{aligned}S_2=&(-i/c\hbar)(2\pi)^4\delta(p+q-p'-q')\\&\times\psi_L^{*-}(\mathbf{q}')[V_+(\mathbf{k})a_+^*(\mathbf{p}')a_+(\mathbf{p})\\&+V_-(\mathbf{k})a_-^*(\mathbf{p}')a_-(\mathbf{p})]\psi_L^+(\mathbf{q}), \quad (46)\end{aligned}$$

with

$$V_+(\mathbf{k})=V(\mathbf{k})+U(\mathbf{k}),\quad V_-(\mathbf{k})=V(\mathbf{k})-U(\mathbf{k}), \quad (47)$$

where

$$V(\mathbf{k})=-\frac{c^2\hbar^2p_0q_0\kappa^2}{4\mathbf{k}^2}\left(1+\frac{p_0}{q_0}\right)\left\{\left(1+\frac{q_0-\lambda}{p_0}-\frac{\mathbf{k}^2}{4p_0^2}\right)\left(\frac{q_0+\lambda}{q_0}\right)+\left(1+\frac{q_0+\lambda}{p_0}-\frac{\mathbf{k}^2}{4p_0^2}\right)\left(\frac{2\mathbf{p}^2-\mathbf{k}^2}{2q_0(q_0+\lambda)}+\frac{i\boldsymbol{\sigma}\cdot\mathbf{k}\times\mathbf{p}}{q_0(q_0+\lambda)}\right)\right\}, \quad (48)$$

$$U(\mathbf{k})=\frac{c^2\hbar^2\kappa^2}{16}\left(\frac{1}{p_0}+\frac{1}{q_0}\right)[\boldsymbol{\sigma}\cdot(\mathbf{p}'+\mathbf{p})]. \quad (49)$$

The result (46) shows that in the present case also the photon preserves its spin orientation with respect to its direction of motion during scattering.

After ignoring the contact terms we obtain for the gravitational potential

$$V_+(\mathbf{r})=V_-(\mathbf{r})=-\frac{G\mu M}{r}\left(2+\frac{4\mu}{M}+\frac{2\mu^2}{M^2}\right)+G\left(1+\frac{\mu}{M}\right)\left(1+\frac{\mu}{M+m}\right)\frac{\hbar\boldsymbol{\sigma}\cdot(\mathbf{r}\times\mathbf{P})}{c^2r^3}, \quad (50)$$

which reduces to (44) in the large-distance approximation.

7. INTERACTION OF PARTICLES AND ANTIPARTICLES

So far we have considered the interaction of two different particles. We shall now discuss the gravitational interaction of two identical particles and that of a particle and its antiparticle by considering, for instance, the π^+-π^+, π^+-π^-, and π^0-π^0 interactions.

The π^+-π^+ scattering matrix element contains a direct term and an exchange term. However, the effect of the exchange term is usually taken into account by symmetrization of the wave function, and therefore for the derivation of the gravitational potential we need

[6] The contact term in (42) appears to have an infrared catastrophe for the photon, but we have verified that the scattering cross section for the process under consideration remains finite when the photon energy tends to zero.

consider only the direct term. Hence, the one-graviton exchange potential V_{π^+,π^+} for the π^+-π^+ interaction can be obtained by putting $m_1=m_2=m$ in the results of Sec. 2, which gives us

$$V_{\pi^+,\pi^+}(\mathbf{r})=-\frac{GM^2}{r}\left(1+\frac{6\mathbf{P}^2}{M^2c^2}+\frac{\mathbf{P}^4}{M^4c^4}\right)+\frac{4\pi G\hbar^2}{c^2}\left(1+\frac{\mathbf{P}^2}{M^2c^2}\right)\delta(\mathbf{r}). \quad (51)$$

The one-graviton exchange potential V_{π^+,π^-} for the π^+-π^- interaction can be obtained by the earlier methods[2] and expressed as

$$V_{\pi^+,\pi^-}(\mathbf{r})=V_{\pi^+,\pi^+}(\mathbf{r})+V'(\mathbf{r}), \quad (52)$$

where $V'(\mathbf{r})$ represents the contribution due to the annihilation process involving the creation of a virtual graviton.[7] The scattering matrix element for the annihilation process is given in the center-of-mass system by

$$S_2'=(-i/c\hbar)(2\pi)^4\delta(p+q-p'-q')\times a^*(\mathbf{p}')a'^*(\mathbf{q}')V'(\mathbf{k})a'(\mathbf{q})a(\mathbf{p}), \quad (53)$$

where

$$V'(\mathbf{k})=-\frac{c^2\hbar^2\kappa^2}{16}\left(3-\frac{2\mathbf{p}^2}{p_0^2}-\frac{\mathbf{p}^4-2\mathbf{p}^2\mathbf{k}^2+\frac{1}{2}\mathbf{k}^4}{p_0^4}\right), \quad (54)$$

which contains only contact interaction terms. In particular, in the nonrelativistic approximation

$$V'(\mathbf{r})=-(3\pi G\hbar^2/c^2)\delta(\mathbf{r}), \quad (55)$$

which represents an attractive contact interaction.

The π^0-π^0 interaction involves two particles, which are not only identical but also behave as antiparticles of each other. In this case the one-graviton exchange potential is given by

$$V_{\pi^0,\pi^0}(\mathbf{r})=V_{\pi^+,\pi^+}(\mathbf{r})+\tfrac{1}{2}V'(\mathbf{r}), \quad (56)$$

provided that a symmetrized wave function is used for the π^0-π^0 system.

Similarly, the e^--e^- gravitational potential can be obtained from the results of Sec. 4 by equalizing the two masses, while the e^--e^+ gravitational potential contains only additional contact terms.[8]

8. CONCLUSION

By investigating the gravitational interaction of particles of several different spins, we find that in each case the one-graviton exchange potential can be expressed as

$$V(\mathbf{r})=V_S(\mathbf{r})+V_L(\mathbf{r}), \quad (57)$$

where the short-range potential $V_S(\mathbf{r})$ is different in each case and vanishes when the distance between interacting particles is large compared with their Compton wavelengths, while the long-range potential $V_L(\mathbf{r})$ has the same form in each case and is given by

$$V_L(\mathbf{r})=-\frac{GM_1M_2}{r}\left[1+\left(4+\frac{M_1}{M_2}+\frac{M_2}{M_1}\right)\times\frac{\mathbf{P}^2}{M_1M_2c^2}+\frac{\mathbf{P}^4}{M_1^2M_2^2c^4}\right]. \quad (58)$$

The short-range potential $V_S(\mathbf{r})$ contains spin effects, which have been determined in several different cases in this paper. The spin-dependent terms can be attractive or repulsive depending on spin orientations. We also find that when one of the masses is taken as very large compared with the other one, the gravitational interaction between them becomes independent of the spin of the large mass but depends on the spin of the small mass. It should be remarked that a phenomenological investigation of the spin-dependent gravitational interaction and its possible experimental detection has been discussed by several authors.[9,10] According to our results, the spin-dependent terms become comparable to the spin-independent interaction only at distances of the order of the Compton wavelengths of the interacting particles. This may be of interest in the theory of gravitational collapse.[11]

Some idea of the long-range potential $V_L(\mathbf{r})$ can be obtained by comparing it with Newton's law, which for the present purpose may be stated in terms of the relativistic masses as

$$V_{\text{Newton}}(\mathbf{r})=-GM_1M_2/r. \quad (59)$$

We then observe that:

(1) In the static limit, (58) and (59) agree with each other.

(2) In the extreme relativistic limit, (58) is 8 times larger than (59).

(3) For the scattering of a photon by a heavy mass, (58) is twice as large as (59).

Moreover, since $V_L(\mathbf{r})$ has the same form for particles of different spins, we conclude within the scope of the present investigation: *The gravitational interaction of elementary particles is independent of their spins, provided that the distance between them is large compared with their Compton wavelengths.*

Finally, we have shown that the gravitational particle-antiparticle potential differs from the corresponding particle-particle potential only by contact interaction terms, and therefore the speculation that matter and antimatter might repel each other is erroneous.

[7] The annihilation of two spinless particles with the creation of a virtual graviton is possible, because the quantization procedure of Ref. 1 allows the appearance of virtual spinless gravitons.

[8] The equivalence of the particle-particle and particle-antiparticle gravitational potentials, apart from the contact terms, is essentially a consequence of the invariance of the Lagrangian density of the matter field under particle-antiparticle conjugation.

[9] J. Leitner and S. Okubo, Phys. Rev. **136**, B1542 (1964).

[10] J. W. T. Dabbs, J. A. Harvey, D. Paya, and H. Horstmann, Phys. Rev. **139**, B756 (1965).

[11] For a comprehensive account of the theory and problems of gravitational collapse, see B. K. Harrison, K. S. Thorne, M. Wakano, and J. A. Wheeler, *Gravitation Theory and Gravitational Collapse* (University of Chicago Press, Chicago, 1965).