Dispersive Magneto-Optical Phenomena in Semiconductors in Crossed Magnetic and Electric Fields

W. ZAWADZKI,* Q. H. F. VREHEN,† AND B. LAX
National Magnet Laboratory,‡ Massachusetts Institute of Technology, Cambridge, Massachusetts
(Received 14 March 1966)

The semiclassical and quantum theories of the Faraday and Voigt effects due to transitions between two simple energy bands in the presence of crossed magnetic and electric fields are developed. The dispersive phenomena are found to complement and augment the information obtained from absorption. The decrease of instensity of $\Delta n=0$ transitions, the finite probability of transitions with $\Delta n\neq 0$, and the shift of transitions to lower energies are found here, as they were in the optical absorption. The absorption constant due to second-order (forbidden) transitions is calculated and anisotropy with respect to electric-field direction is predicted. The experimental results of crossed-field Faraday rotation in gemanium are presented, and comparison with the theory is carried out, taking into account the degeneracy of the valence band. Good correlation between the theory and the experiment is obtained.

I. INTRODUCTION

MAGNETO-optical phenomena in semiconductors in the presence of a transverse electric field have recently become a subject of extensive investigation. Aronov[1] first formulated a theory of interband optical absorption due to direct transitions between simple parabolic nondegenerate energy bands in the presence of external crossed fields. Experimental works of Vrehen and Lax[2] and Vrehen[3] confirmed the main predictions of the theory. Hensel and Peter[4] and Shindo[5] investigated theoretically the Stark shift of Landau levels in the degenerate valence bands of germanium. Recently Vrehen[6] investigated both theoretically and experimentally crossed-field interband optical absorption in this material. The measurement of crossed-field absorption can in principle determine separately the effective masses of carriers in both bands. The aim of the present work is to extend the investigation of crossed-field phenomena to dispersive magneto-optical effects, both theoretically and experimentally.[7] It turns out that in some experimental situations, especially in higher electric fields, the dispersive phenomena can be more easily investigated and provide more information than the absorption.[8] The semiclassical theory generalizes the approach of Kolodziejczak, Lax, and Nishina[9] (KLN) by taking into account the presence of dc electric field and avoiding the restriction of low magnetic-field strengths. This procedure makes implicit use of the effective-mass approximation for an electron in crossed fields. Limits of these approximations are briefly discussed in Sec. III, based on a recent treatment by Zak and Zawadzki.[10] Then the quantum theory for simple bands is developed using the procedure applied to magneto-optical dispersive phenomena by Boswarva, Howard, and Lidiard,[11] Halpern, Lax, and Nishina (HLN),[12] and Bennett and Stern.[13] In the final section the experimental results on the crossed-field Faraday rotation in germanium are presented and the comparison with the theory is carried out. A good agreement is obtained and the results confirm the conclusions of the crossed-field absorption investigation.

II. SEMICLASSICAL APPROACH

The dispersion of light in solids can be obtained from the absorption coefficient by use of the Kramers-Kronig dispersion relations between the real and imaginary parts of a dielectric-constant tensor. However, the dispersion and absorption can also be obtained through the application of classical equations describing the motion of a bound electron. In this section we use the equation of motion generalizing the approach of KLN in order to take into account the presence of dc electric field perpendicular to the magnetic field.[14]

---

* On leave of absence from the Institute of Physics, Polish Academy of Sciences, Warsaw, Poland.

† Present address: Philips Research Laboratories, Eindhoven, The Netherlands.

‡ Supported by the U. S. Air Force Office of Scientific Research.

[1] A. G. Aronov, Fiz. Tverd. Tela **5**, 552 (1963) [English transl.: Soviet Phys.—Solid State **5**, 402 (1963)].

[2] Q. H. F. Vrehen and B. Lax, Phys. Rev. Letters **12**, 471 (1964).

[3] Q. H. F. Vrehen, Phys. Rev. Letters **14**, 558 (1965).

[4] J. C. Hensel and M. Peter, Phys. Rev. **114**, 411 (1959).

[5] T. Shindo, J. Phys. Chem. Solids **26**, 1431 (1965).

[6] Q. H. F. Vrehen, Phys. Rev. **145**, 675 (1966).

[7] While this work was being prepared for publication a paper has appeared by S. I. Uritsky and G. V. Shuster [Phys. Status Solidi **11**, 105 (1965)] dealing with the theory of interband Faraday and Voigt effects in crossed fields. Final integrations are not performed there but the formulation is equivalent to that presented in Sec. VI of the present paper.

[8] Q. H. F. Vrehen, W. Zawadzki, and M. Reine (to be published).

[9] J. Kolodziejczak, B. Lax, and Y. Nishina, Phys. Rev. **128**, 2655 (1962).

[10] J. Zak and W. Zawadzki, Phys. Rev. **145**, 536 (1966).

[11] I. M. Boswarva, R. E. Howard, and A. B. Lidiard, Proc. Roy. Soc. (London) **A269**, 125 (1962); I. M. Boswarva and A. B. Lidiard, *ibid.* **A278**, 588 (1964).

[12] J. Halpern, B. Lax, and Y. Nishina, Phys. Rev. **134**, A140 (1964).

[13] H. S. Bennett and E. A. Stern, Phys. Rev. **137**, A448 (1965).

[14] KLN considered the case of low magnetic fields. The present procedure does not require this assumption and is valid for high magnetic fields as well (within the limits of the effective-mass approximation). This is of importance, because as follows from the criterion given in Sec. III the description based on the equation of motion ceases to be valid for low magnetic fields (for derivation and details see Ref. 10).

We consider an electron in the valence band as a classical oscillator[15] with the oscillator frequency corresponding to that of the energy associated with the particular interband transition involved. Then in the presence of magnetic and electric fields the equations of motion can be written as follows:

$$\frac{d^2\mathbf{r}_{\alpha\beta}}{dt^2}+\omega_{\alpha\beta}^2\mathbf{r}_{\alpha\beta}+\frac{1}{\tau_{\alpha\beta}}\frac{d\mathbf{r}_{\alpha\beta}}{dt}+\frac{d\mathbf{r}_{\alpha\beta}}{dt}\times\boldsymbol{\omega}_c = \frac{e\boldsymbol{\mathcal{E}}}{m}\exp(i\omega t)+\frac{e\mathbf{E}}{m}, \quad (1)$$

where $\mathbf{r}_{\alpha\beta}$ = the displacement vector; $\omega_{\alpha\beta}$ = the appropriate oscillator frequency; $\boldsymbol{\omega}_c = e\mathbf{H}/mc$ = the cyclotron frequency; m = the free-electron mass; $\mathbf{H}$ = the dc magnetic field; $\tau_{\alpha\beta}^{-1}d\mathbf{r}_{\alpha\beta}/dt$ = the damping term with $\tau_{\alpha\beta}$ = the relaxation time; $\boldsymbol{\mathcal{E}}$ = the electric vector of the optical radiation; and $\mathbf{E}$ = the external dc electric field. The subscript $\alpha\beta$ denotes the initial and final states of the electron in the valence and conduction bands, respectively.

One can easily see that the solution of Eq. (1) may be found in the form

$$\mathbf{r}_{\alpha\beta}=\mathbf{r}_{\alpha\beta}^0\exp(i\omega t)+e\mathbf{E}/m\omega_{\alpha\beta}^2, \quad (2)$$

where $\mathbf{r}_{\alpha\beta}^0$ as a function of $\omega_{\alpha\beta}$ is identical with that obtained in the absence of dc electric field $\mathbf{E}$. Thus, since the oscillator is bound, the dc electric field simply shifts the position of equilibrium.

On the other hand we can calculate the current vector due to all the transitions

$$\mathbf{J}=\sum_{\alpha\beta} e\mathbf{v}_{\alpha\beta}N_{\alpha\beta}, \quad (3)$$

where $\mathbf{v}_{\alpha\beta}=d\mathbf{r}_{\alpha\beta}/dt$ is the electron velocity, and $N_{\alpha\beta}$ is the number of transitions between the states α and β, which depends on the oscillator strength. The conductivity we look for is defined by the relation

$$\mathbf{J}=\hat{\sigma}\boldsymbol{\mathcal{E}}, \quad (4)$$

where $\hat{\sigma}$ is the complex conductivity tensor determined by the velocities $\mathbf{v}_{\alpha\beta}$. Thus the constant term $e\mathbf{E}/m\omega_{\alpha\beta}^2$ in Eq. (2) does not enter explicitly into the expressions for the conductivities. However the dc electric field does affect the oscillator frequencies $\omega_{\alpha\beta}$ and the number of transitions $N_{\alpha\beta}$.

We choose the coordinate system to have the magnetic field along the z direction and the electric field along the y direction. This physical situation does not have the rotational symmetry with respect to z direction. Thus, even for an isotropic solid in general $\sigma_{xx}\neq\sigma_{yy}$. We shall, however, show later that if the dc electric field is not very large, i.e., the energy bands are not distorted by its presence, the allowed transitions do have the rotational symmetry in respect to the z axis. This is not true for forbidden (second-order) transitions. Hence for allowed transition effects in a cubic solid we may write the conductivity tensor in the form

$$\begin{aligned}&\sigma_{xx}=\sigma_{yy}=\tfrac{1}{2}(\sigma_+ +\sigma_-),\\ &\sigma_{xy}=-\sigma_{yx}=\tfrac{1}{2}(\sigma_+ -\sigma_-),\\ &\sigma_{zz}=\sigma_0,\end{aligned} \quad (5)$$

all the other components being zero. σ_0 is the conductivity in the absence of the magnetic field and $\sigma_\pm$ are the conductivities for circularly polarized waves. They are obtained by writing the equation of motion in component form and then combining the x and y components to obtain the solutions for $J_x \pm iJ_y$. The relations

$$\sigma_\pm=\sigma_{xx}\mp i\sigma_{xy} \quad (6)$$

are fulfilled. Using Eqs. (1)–(5), we arrive at

$$\sigma_\pm=\frac{e^2}{m}\sum_{\alpha\beta}\frac{i\omega N_{\alpha\beta}^{\pm}}{\omega_{\alpha\beta}^2-\omega^2\pm\omega\omega_*+i\omega/\tau_{\alpha\beta}}, \quad (7)$$

$$\sigma_{zz}=\frac{e^2}{m}\sum_{\alpha\beta}\frac{i\omega N_{\alpha\beta}^{z}}{\omega_{\alpha\beta}^2-\omega^2+i\omega/\tau_{\alpha\beta}}, \quad (8)$$

where $N_{\alpha\beta}^{\pm}$ and $N_{\alpha\beta}^{z}$ denote the number of transitions for longitudinal ($\|\mathbf{H}$) and transverse ($\perp\mathbf{H}$) propagation, respectively. Actually by use of the equation of motion [Eq. (1)] we get in Eq. (7) $\omega_*=\omega_c$, the cyclotron frequency. This frequency depends, however, upon the model applied and we do not specify it now.

The number of transitions is proportional to the oscillator strength and statistical factors. We assume for simplicity that the valence band is completely filled and the conduction band is empty. Then

$$N_{\alpha\beta}^{\pm}=f_{\alpha\beta}^{\pm}d\alpha d\beta, \quad (9)$$

and similarly for $N_{\alpha\beta}^{z}$. The oscillator strength is given by

$$f_{\alpha\beta}^{\pm}=2|P_{\alpha\beta}^{\pm}|^2/m\hbar\omega_{\alpha\beta}^{\pm}, \quad (10)$$

where the matrix elements of momentum between the states α and β, $P_{\alpha\beta}$, and the transition frequencies $\omega_{\alpha\beta}$ have to be calculated by use of quantum methods.

The index of refraction n and the extinction constant k are connected with the conductivities by the well-known formula

$$(n-ik)^2=1+(4\pi/i\omega)\sigma_{\text{eff}}, \quad (11)$$

where σ_{eff} stands for $\sigma_\pm$, $\sigma_{\|,\perp}$; whatever the case may be. The magneto-optical phenomena can be expressed by the optical constants as follows:

$$\begin{aligned}&\alpha=(2\omega/c)k, \text{ the absorption coefficient,}\\ &\theta=(\omega/2c)(n_+-n_-), \text{ the Faraday rotation,}\\ &\Delta=(\omega/2c)(k_+-k_-), \text{ the ellipticity,}\\ &\delta=(\omega/c)(n_{\|}-n_{\perp}), \text{ the Voigt phase shift.}\end{aligned} \quad (12)$$

[15] Bennett and Stern (Ref. 13) call this model a Hookean solid.

Using Eqs. (6)–(12) general formulas for the magneto-optical phenomena in crossed fields can be written. In order to proceed further we have to specify our model.

We assume that the transitions take place between two spherical parabolic energy bands with twofold spin degeneracy in the absence of the magnetic field. The selection rules for right and left circularly polarized light are determined by spin and the frequencies corresponding to interband transitions are

$$\omega_{\alpha\beta}{}^{\pm}=\omega_{kk'}\pm\gamma H \tag{13}$$

with

$$\gamma=(1/2h)\mu_B(g_c+g_v),$$

where μ_B is the Bohr magneton and g_c and g_v are the effective spin splitting factors for electrons in the conduction and valence bands, respectively. Furthermore we consider only one nonshifted by spin frequency for light polarized in the z direction, in analogy to the classical Lorentz triplet in the atomic Zeeman effect.

$$\omega_{\alpha\beta}{}^{z}=\omega_{kk'}. \tag{14}$$

Here k and k' denote the values of wave vector of an electron in the valence and conduction bands, respectively.

The conductivities [Eq. (7)] should have maxima for frequency in the vicinity of $\omega=\omega_{kk'}\pm\gamma H$ (the term $i\omega/\tau_{\alpha\beta}$ removes the singularities and shifts the maximum positions toward higher frequencies). In the approximation to terms $\omega_*{}^2$ this leads to

$$\omega_*=2\gamma H. \tag{15}$$

The number of transitions [Eq. (9)] for our model is given by

$$N_{kk'}{}^{\pm}=(4/(2\pi)^6)f_{kk'}{}^{\pm}d^3kd^3k'. \tag{16}$$

We now assume, following KLN approach, that in the semiclassical case the oscillator strengths are equal for all types of transitions.

$$\frac{|P_{kk'}{}^{+}|^2}{\omega_{kk'}{}^{+}}=\frac{|P_{kk'}{}^{-}|^2}{\omega_{kk'}{}^{-}}=\frac{|P_{kk'}{}^{z}|^2}{\omega_{kk'}{}^{z}}=\frac{|P_{kk'}|^2}{\omega_{kk'}}. \tag{17}$$

By use of Eqs. (7)–(17) the magneto-optical effects in crossed fields take the following form:

$$\alpha=-4A\omega\,\mathrm{Im}\int\int\frac{|P_{kk'}|^2}{\omega_{kk'}}\left[\frac{1}{\omega_{kk'}{}^2-\Omega_{\mathrm{eff}}{}^2}\right]d^3kd^3k', \tag{18}$$

where

$$\Omega_{\mathrm{eff}}=\Omega_{\pm}\quad\text{for}\quad\alpha_{\pm}$$
$$=\Omega\quad\text{for}\quad\alpha_{\parallel},$$

and for $\alpha_{\perp}$ the expression in brackets is to be replaced by

$$\frac{1}{2}\left[\frac{1}{\omega_{kk'}{}^2-\Omega_{-}{}^2}\right]+\left[\frac{1}{\omega_{kk'}{}^2-\Omega_{+}{}^2}\right],$$

$$\theta=A\omega\,\mathrm{Re}\int\int\frac{|P_{kk'}|^2}{\omega_{kk'}}\left[\frac{1}{\omega_{kk'}{}^2-\Omega_{-}{}^2}-\frac{1}{\omega_{kk'}{}^2-\Omega_{+}{}^2}\right]d^3kd^3k', \tag{19}$$

$$\Delta=A\omega\,\mathrm{Im}\int\int\frac{|P_{kk'}|^2}{\omega_{kk'}}\left[\frac{1}{\omega_{kk'}{}^2-\Omega_{-}{}^2}-\frac{1}{\omega_{kk'}{}^2-\Omega_{+}{}^2}\right]d^3kd^3k, \tag{20}$$

$$\delta=A\omega\,\mathrm{Re}\int\int\frac{|P_{kk'}|^2}{\omega_{kk'}}\left[\frac{1}{\omega_{kk'}{}^2-\Omega_{-}{}^2}+\frac{1}{\omega_{kk'}{}^2-\Omega_{+}{}^2}-\frac{2}{\omega_{kk'}{}^2-\Omega^2}\right]d^3kd^3k', \tag{21}$$

where

$$\Omega_{\pm}=[\omega(\omega\pm2\gamma H-i/\tau)]^{1/2},$$
$$\Omega=[\omega(\omega-i/\tau)]^{1/2},$$

and

$$A=e^2/8\pi^5m^2\hbar cn_0,$$

n_0 is the index of refraction in the absence of external fields.

The formulas (18)–(21) are valid for both direct and indirect allowed transitions and frequencies $\omega<\omega_g$ and $\omega>\omega_g$, if appropriate matrix elements, densities of states, and transition frequencies are introduced in every particular case.

It can easily be verified from the above formulas that for $\omega\ll\omega_{kk'}$ (long-wavelength limit), the Faraday rotation θ and the ellipticity Δ are proportional to ω^2 and the Voigt phase shift to ω^3.

III. HAMILTONIAN AND THE EFFECTIVE-MASS APPROXIMATION

In the one-electron approximation the Hamiltonian for an electron in a solid in the presence of dc magnetic and electric fields and electromagnetic radiation takes the form

$$\mathcal{H}=\frac{1}{2m}\left(\mathbf{p}+\frac{e}{c}\mathbf{A}\right)^2+V(\mathbf{r})+e\mathbf{E}\cdot\mathbf{r}+\frac{\hbar}{4m^2c^2}$$
$$\times[\boldsymbol{\sigma}\times(\nabla V+e\mathbf{E})]\left(\mathbf{p}+\frac{e}{c}\mathbf{A}\right)+\frac{e\hbar}{2mc}\boldsymbol{\sigma}(\mathbf{H}+\mathbf{H}'), \tag{22}$$

where $V(\mathbf{r})$ is a periodic potential of the solid, $\mathbf{A}$ is the vector potential that takes into account the dc magnetic field and the electric field of the electromagnetic radiation. The fourth term accounts for the spin-orbit interaction, $\boldsymbol{\sigma}$ being the spin-vector operator, and the fifth term describes the interaction of spin with the dc magnetic field H and with that of the electromagnetic wave H'. The Hamiltonian can be conveniently divided into two parts,

$$\mathcal{H}=\mathcal{H}^0+\mathcal{H}', \tag{23}$$

where

$$\mathcal{H}^0=\frac{\mathbf{P}^2}{2m}+V(\mathbf{r})+e\mathbf{E}\cdot\mathbf{r}+\frac{\hbar}{4m^2c^2}\boldsymbol{\sigma}\times(\boldsymbol{\nabla}V+e\mathbf{E})\mathbf{P}+\frac{c\hbar}{2mc}\boldsymbol{\sigma}\cdot\mathbf{H} \tag{24}$$

with

$$\mathbf{P}=\mathbf{p}+(e/2c)(\mathbf{H}\times\mathbf{r}) \tag{25}$$

describes the system without radiation, and

$$\mathcal{H}'=(e/c)\mathbf{A}'\boldsymbol{v}, \tag{26}$$

where

$$\mathbf{A}'=(ic/\omega)\boldsymbol{\mathcal{E}}_0\exp i(\omega t-\mathbf{kr})+\text{conjugate} \tag{27}$$

vector potential of the electric radiation field, and

$$\mathbf{v}=\mathbf{P}/m+(\hbar/4m^2c^2)\boldsymbol{\sigma}\times(\boldsymbol{\nabla}V+e\mathbf{E}) \tag{28}$$

the velocity operator. The time-dependent part of the Hamiltonian, Eq. (26), describes the interaction of the system with the electromagnetic wave. In Eqs. (24) and (26) we have neglected terms proportional to A'^2 and $\boldsymbol{\sigma}H'$ as small. Practically achievable dc electric fields that can be applied externally to the solid are much smaller than the internal atomic electric fields so that eE can always be neglected in comparison with $\boldsymbol{\nabla}V$. This is equivalent to the assumption that the external electric field does not distort the energy bands. Thus the perturbation (time-dependent part of the Hamiltonian) does not contain the dc electric field which affects only the transition frequencies and matrix elements, just as in the semiclassical procedure.

It has been shown recently[10] that the Hamiltonian $\mathcal{H}^0$ [Eq. (24)] for an electron in a periodic potential in the presence of crossed electric and magnetic fields ($\mathbf{E}\perp\mathbf{H}$) can be approximated by the effective-mass Hamiltonian provided the two following conditions are satisfied:

$$(2n+1)^{1/2}a/L\ll 1 \tag{29}$$

and

$$eEa/\hbar\omega_c\ll 1, \tag{30}$$

where a is a typical lattice constant and $L=(\hbar c/eH)^{1/2}$ is the radius of the first cyclotron orbit. n indicates the principal quantum number of a Landau state that can still be described by the effective-mass approximation and $\omega_c=eH/m^*c$ is the effective cyclotron frequency for the electron in a given band. Equation (30) shows that for a given electric-field strength the magnetic field cannot be too small for the effective-mass equation to be valid. For example in germanium $a\sim 5\times10^{-8}$ cm, and for the magnetic fields $H\sim10^5$ G only the first few Landau levels may be described by use of the effective-mass equation. For electric fields of order 5×10^4 V/cm, both the conduction and light hole bands can be described by the effective-mass approximation ($m^*\simeq 0.04m_0$ and $eEa/\hbar\omega_c\simeq 0.1$) whereas for the heavy holes with the effective mass much larger than that of the light holes the effective-mass formalism cannot be applied. With the above restrictions the effective-mass Hamiltonian for simple parabolic spherical and nondegenerate (except for the spin) band is

$$\mathcal{H}^0=(1/2m^*)\mathbf{P}^2+e\mathbf{E}\cdot\mathbf{r}+\tfrac{1}{2}\mu_B g^*\boldsymbol{\sigma}\cdot\mathbf{H}, \tag{31}$$

where g^* is the effective spectroscopic splitting factor. This Hamiltonian may be used to calculate frequencies and matrix elements associated with the interband transitions.

IV. CONDUCTIVITY-TENSOR RELATIONS

Time-dependent perturbation theory can be applied to the Hamiltonian [Eq. (23)] in order to carry out a quantum-mechanical calculation of the conductivity tensor. There are two methods that may be used to find the imaginary and real parts of the tensor. The first method incorporates the calculation of the absorptive (imaginary) part of the conductivity, which is proportional to the probability per unit time for the absorption processes to occur, and the application of the Kramers-Kronig dispersion relations in order to find the dispersive (real) part of the tensor. In the second method, one explicitly evaluates the expression for the current density when charged particles move in an electromagnetic field, and the conductivity tensor is given by the Kramers-Heisenberg formulas. This is in analogy to the semiclassical treatment given in Sec. II and we proceed using the second approach.[16]

The conductivity tensor for a solid having cubic symmetry [Eq. (5)] takes the form

$$\sigma_{ii}=-\frac{ie^2}{m^2\hbar}\sum_{\alpha\beta}\frac{1}{\omega_{\alpha\beta}}\left\{\frac{\pi_{\alpha\beta}{}^i\pi_{\beta\alpha}{}^i}{\omega+\omega_{\alpha\beta}}+\frac{\pi_{\alpha\beta}{}^i\pi_{\beta\alpha}{}^i}{\omega-\omega_{\alpha\beta}}\right\}, \tag{32}$$

$$\sigma_{ij}=\frac{ie^2}{m^2\hbar\omega}\sum_{\alpha\beta}\left\{\frac{\pi_{\alpha\beta}{}^i\pi_{\beta\alpha}{}^j}{\omega+\omega_{\alpha\beta}}-\frac{\pi_{\alpha\beta}{}^j\pi_{\beta\alpha}{}^i}{\omega-\omega_{\alpha\beta}}\right\}, \tag{33}$$

where

$$\pi_{\alpha\beta}=\langle\alpha|m\mathbf{v}|\beta\rangle,$$

and the velocity operator $\mathbf{v}$ is given by Eq. (28). By use of Eq. (6) $\sigma_\pm$ can be calculated and then applying Eqs. (11) and (12) the Faraday rotation is obtained in

[16] Both methods lead to identical results.

the form

$$\theta=-\frac{\pi^2e^2}{m^2chn_0}\sum_{\alpha\beta}\left\{\frac{|\pi_{\alpha\beta}{}^+|^2}{\omega_{\alpha\beta}{}^2}-\frac{|\pi_{\alpha\beta}{}^-|^2}{\omega_{\alpha\beta}{}^2}\right\}-\frac{\pi^2e^2}{m^2chn_0}\omega^2$$
$$\times\sum_{\alpha\beta}\left\{\frac{|\pi_{\alpha\beta}{}^+|^2}{\omega_{\alpha\beta}{}^2(\omega_{\alpha\beta}{}^2-\omega^2)}-\frac{|\pi_{\alpha\beta}{}^-|^2}{\omega_{\alpha\beta}{}^2(\omega_{\alpha\beta}{}^2-\omega^2)}\right\}, \quad (34)$$

where $\pi_{\alpha\beta}{}^{\pm}=\pi_{\alpha\beta}{}^x\pm i\pi_{\alpha\beta}{}^y$. Bennet and Stern[12] showed recently that the first term on the right-hand side is identically equal to zero in the general case if only the interband transitions are taken into account. In other words the Faraday rotation for $\omega\ll\omega_{\alpha\beta}$ goes to zero like ω^2 if no free carriers are present. Thus we are left only with the second term.

Similarly the Voigt phase shift by use of Eqs. (6) (11), and (12) can be obtained in the form

$$\delta=\frac{4\pi e^2}{m^2hcn_0}\omega\sum_{\alpha\beta}\left\{\frac{|\pi_{\alpha\beta}{}^x|^2}{\omega_{\alpha\beta}{}^3}-\frac{|\pi_{\alpha\beta}{}^z|^2}{\omega_{\alpha\beta}{}^2}\right\}+\frac{4\pi e^2}{m^2hcn_0}\omega^3$$
$$\times\sum_{\alpha\beta}\left\{\frac{|\pi_{\alpha\beta}{}^x|^2}{\omega_{\alpha\beta}{}^2(\omega_{\alpha\beta}{}^2-\omega^2)}-\frac{|\pi_{\alpha\beta}{}^z|^2}{\omega_{\alpha\beta}{}^2(\omega_{\alpha\beta}{}^2-\omega^2)}\right\}. \quad (35)$$

We have not been able to prove that the term proportional to ω is equal to zero. However, we assume this in order to simplify the final formulas and to make the quantum treatment consistent with the semiclassical approach.

V. TRANSITION FREQUENCIES AND MATRIX ELEMENTS

For the ideal model of two spherical parabolic nondegenerate bands (except for twofold spin degeneracy) the eigenfunctions of the problem are given in first approximation by a product of a periodic part of the Bloch functions (with the spin parts included) and the solutions of the effective-mass Hamiltonian [Eq. (31)]. In the Landau gauge

$$\mathbf{A}=(-Hy,0,0) \quad \text{and} \quad \mathbf{E}=(0,E,0)$$

the spatial envelope functions for the valence band are

$$F_v(\mathbf{r})=C\exp i(k_xx+k_zz)\psi_n(y/L-Lk_x-eEL/\hbar\omega_{c2}), \quad (36)$$

where L is the radius of first cyclotron orbit defined before, C the normalization coefficeint, and

$$\omega_{c2}=eH/m_2c$$

is the effective cyclotron frequency, m_2 being the effective mass of holes. $\psi_n(\xi)$ is the harmonic-oscillator function. For electrons in the conduction band the solutions are given by similar functions, the argument being different however: $\psi_{n'}(y/L-Lk_x'+eEL/\hbar\omega_{c1})$. Frequency corresponding to the energy of the transition between two modified Landau levels in different bands is

$$\omega_{\alpha\beta}=\omega_g+\omega_{c1}(n'+\tfrac{1}{2})+\omega_{c2}(n+\tfrac{1}{2})+\hbar k_z{}'^2/2m_1+\hbar k_z{}^2/2m_2$$
$$+\hbar^{-1}eEL^2k_x'-\hbar^{-1}eEL^2k_x-(m_1+m_2)c^2E^2/2hH^2$$
$$+\mu_Bm_j'g_cH-\mu_Bm_jg_vH, \quad (37)$$

where ω_g is the frequency corresponding to the energy gap, and m_j, m_j' are magnetic quantum numbers, which for our ideal model can take two values $\pm\frac{1}{2}$.

It is convenient to use the cylindrical coordinate in the momentum space with variables n, φ, k_z instead of k_x, k_y, k_z. In this system

$$k_x=L^{-1}(2n+1)^{1/2}\cos\varphi;$$
$$k_x'=L^{-1}(2n'+1)^{1/2}\cos\varphi', \quad (38)$$

and the density of states

$$d^3k=L^{-2}dnd\varphi dk_z, \quad (39)$$

and similarly for the conduction band.

The theory of interband optical transitions in the presence of the magnetic field can be directly adopted to the crossed-fields case. Boswarve and Lidiard[10] showed that the matrix elements of $\mathbf{P}=m\mathbf{v}$ (with spin-orbit interaction neglected) for direct allowed transitions between valence and conduction bands are given by

$$P_{\alpha\beta}{}^{\pm}=(1\pm\omega_c{}^0/\omega_g)\langle u_v|p^{\pm}|u_c\rangle\int F_v{}^*F_cd^3r, \quad (40)$$

$$P_{\alpha\beta}{}^z=\langle u_v|p^z|u_c\rangle\int F_v{}^*F_cd^3r, \quad (40a)$$

where $\omega_c{}^0$ is the free-electron cyclotron frequency, $p^{\pm}=p^x\pm ip^y$, and u_v, u_c are the periodic parts of the Bloch functions with spin parts included. As noted by Aronov[1] the envelope functions (36) in conduction and valence bands are not orthogonal for $n\neq n'$ because of the different arguments for electrons and holes; taking into account the selection rules determined by the spin parts we finally get

$$|P_{\alpha\beta}{}^{\pm}|^2=(1\pm\omega_c{}^0/\omega_g)^2|p_{vc}|^2L^2|\Phi(n,n')|^2$$
$$\times\delta(\varphi_n-\varphi_{n'}{}')\delta(k_z-k_z')\delta_{mj,mj'\mp1}, \quad (41)$$

$$|P_{\alpha\beta}{}^z|^2=L^2|p_{cv}|^2|\Phi(n,n')|^2\delta(\varphi_n-\varphi_{n'}{}')$$
$$\times\delta(k_z-k_z')\delta_{mj,mj'}, \quad (41a)$$

where φ_n and $\varphi_{n'}{}'$ satisfy the relation

$$(2n+1)^{1/2}\cos\varphi_n=(2n'+1)^{1/2}\cos\varphi_{n'}{}',$$

which is equivalent to the conservation of k_x in the

transition, and[17]

$$\Phi(n+l,n')=(2^{n'+n}n'!n!)^{-1/2}\exp(-a^2/4)\sum_{m=0}^{n+l}(-1)^{n+l-n'+m}a^{n+l-n'+2m}2^{n'-m}\times\frac{n'(n+l)!}{(n+l-n'+m)!(n-m)!m!}\quad\text{for}\quad n'\leq n+l$$

$$=(2^{n'+n}n'!n!)^{-1/2}\exp(-a^2/4)\sum_{m=0}^{n'}(-1)^{m}a^{n'-n-l+2m}\;2^{n+l-m}\times\frac{n'!(n+l)!}{(n'-n-l+m)!(n+l-m)!m!}\quad\text{for}\quad n'\geq n+l,\quad(42)$$

where $a=eEL/\hbar\omega_c$ and $\omega_c=eH/c(m_1+m_2)$. For the electric field equal to zero

$$|\Phi(n,n')|^2\delta(\varphi_n-\varphi_{n'})\rightarrow\delta_{nn'}\delta(\varphi-\varphi'),$$

the matrix-elements properties reduce to those in the magnetic-field case.

When the direct optical transitions between the two bands are only allowed to second order one has to include the first-order corrections in the wave functions.

The matrix elements for second-order (forbidden) transitions are[18]

$$\boldsymbol{\alpha}\cdot\mathbf{P}_{cv}=-i\hbar\mathbf{M}_{cv}\cdot\int F_c^*\nabla F_v dr-(eH/c)M_{cv}^x\int F_c^* y F_v dr,\quad(43)$$

where

$$\mathbf{M}_{cv}=\frac{1}{m}\sum_l\left[\frac{\mathbf{p}_{cl}(\boldsymbol{\alpha}\mathbf{p}_{lv})}{\mathcal{E}_c-\mathcal{E}_l}-\frac{(\boldsymbol{\alpha}\mathbf{p}_{cl})\mathbf{p}_{lv}}{\mathcal{E}_l-\mathcal{E}_c}\right].$$

The sum is over all bands excluding the conduction band. $\boldsymbol{\alpha}$ is a polarization vector of the radiation. All the matrix elements and the energies are taken at $k=0$. Using the envelope functions (36) the matrix element for the forbidden direct transition between levels n and n' becomes

$$\boldsymbol{\alpha}\cdot\mathbf{P}_{cv}=\hbar k_z L M_{cv}^z\Phi(n,n')\delta(k_x-k_x')(k_z-k_z')-\hbar\{M_{cv}^-\Phi(n+1,n')+M_{cv}^+\Phi(n-1,n')+(eEL/\hbar\omega_{c1})M_{cv}^x\Phi(n,n')\}\times\delta(k_x-k_x')\delta(k_z-k_z'),\quad(44)$$

where $M_{cv}^{\pm}=M_{cv}^x\pm iM_{cv}^y$. For $E=0$ the result (44) reduces to

$$\boldsymbol{\alpha}\cdot\mathbf{P}_{cv}=\hbar k_z L M_{cv}^z\delta(k_x-k_x')\delta(k_z-k_z')-(\hbar/\sqrt{2})(M_{cv}^-(n+1)^{1/2}\delta_{n+1,n'}+M_{cv}^+n^{1/2}\delta_{n-1,n'})\times\delta(k_x-k_x')\delta(k_z-k_z')\quad(45)$$

which is equivalent to Elliott's result[17] (we use a different gauge).

For a cubic solid $p_{cv}^x=p_{cv}^y=p_{cv}^z$, $M_{cv}^x=M_{cv}^y=M_{cv}^z$, and M_{cv} is parallel to $\boldsymbol{\alpha}$. When the light is propagated along the magnetic-field direction z, only p_{cv}^x and p_{cv}^y come into play. It can be seen then from Eq. (41) that for the allowed transitions in crossed fields the square of the matrix elements does not depend on the orientation of polarization with respect to the dc electric field. For example, for polarization in the x direction ($\boldsymbol{\alpha}\perp\mathbf{E}$) and in the y direction ($\boldsymbol{\alpha}\|\mathbf{E}$) the squares of the matrix elements are the same, i.e., the absorption constant is the same. This is of course also the case in the absence of the electric field. For the forbidden transitions in the presence of the magnetic field alone the same symmetry is valid, as it can be seen from Eq. (45). [The mixed term $M_{cv}^-M_{cv}^+$ in Eq. (45) vanishes because of the delta functions, and $|M_{cv}^+|^2$ and $|M_{cv}^-|^2$ do not depend on orientation.] The forbidden transitions in crossed fields [Eq. (44)], however, depend upon the orientation of the polarization in respect to the electric field, i.e., $|P_{cv}^x|^2_{\text{forb}}\neq|P_{cv}^y|^2_{\text{forb}}$. Thus the symmetry of the conductivity tensor [Eq. (5)] does not hold for the forbidden transitions in crossed fields.

VI. MAGNETO-OPTICAL EFFECTS

Now we apply the general expressions given in Sec. IV to calculate the interband magneto-optical effects using the idealized model of two spherical, parabolic, nondegenerate energy bands. We mentioned already that in formula (34) for the Faraday rotation the first (independent of ω) term can be shown to be identically zero.[13] Halpern, Lax, and Nishina[12] (HLN) assumed, moreover, that every term in the sum is equal to zero. To examine this assumption we notice that by use of Eqs. (37) and (41) the assumption can be rewritten in the form

$$(1+\omega_c^0/\omega_g)/(1+\gamma H/\omega_{kk'})\approx(1-\omega_c^0/\omega_g)/(1-\gamma H/\omega_{kk'}),\quad(46)$$

where γ is defined in Eq. (13). For completely free electrons ($g_c=g_v=2$) this relation is indeed true,

[17] For $l=0$ Eq. (42) is equivalent to Aronov's result (with a few corrections, as indicated in Ref. 6).

[18] We follow the notation of R. J. Elliott, T. P. McLean, and G. G. Macfarlane, Proc. Phys. Soc. (London) **72**, 553 (1958).

especially for lower transitions with $\omega_{kk'}\approx\omega_g$. Thus making this assumption we neglect terms of higher order due to spin-orbit interaction which result in anomalous values of g factors. Hence we assume after HLN

$$(P_{kk'}{}^{+}/\omega_{kk'}{}^{+})^2=(P_{kk'}{}^{-}/\omega_{kk'}{}^{-})^2=(P_{kk'}/\omega_{kk'})^2. \tag{47}$$

For light polarized parallel to a magnetic field we have the selection rule $\Delta m_j=0$ [Eq. (41a)]. If the g factors differ for electrons in the conduction and valence band we have in general two transitions with frequencies $\omega_{kk'}{}^{z\pm}$. The difference $\omega_{kk'}{}^{z+}-\omega_{kk'}{}^{z-}$ is, however, again different from zero due to spin-orbit interaction and we neglect this assuming $\omega_{kk'}{}^{z+}=\omega_{kk'}{}^{z-}$. Furthermore, if we assume that in the first term on the right-hand side of Eq. (35) $P_{\alpha\beta}{}^{z^2}/\omega_{\alpha\beta}{}^{z^3}=2P_{\alpha\beta}{}^{z^2}/\omega_{\alpha\beta}{}^{z^3}$ for all transitions $\alpha\beta$, this can be most easily satisfied by the HLN assumption:

$$2|P_{kk'}{}^{z}|^2/\omega_{kk'}{}^{z}=|P_{kk'}|^2/\omega_{kk'} \tag{48}$$

and

$$\omega_{kk'}{}^{z^2}=\omega_{kk'}{}^{2}-(\gamma H)^2. \tag{49}$$

Using the above simplifying assumptions the conductivities can be written in the form

$$\sigma_{\text{eff}}=\frac{ie^2\omega}{m^2h}\sum_{kk'}\frac{|P_{kk'}|^2}{\omega_{kk'}}\left(\frac{1}{\omega_{kk'}{}^{2}-\Omega_{\text{eff}}{}^{2}}\right), \tag{50}$$

where for $\sigma_{\pm}$: $\Omega_{\text{eff}}=\Omega_{\pm}$, for $\sigma_{zz}=\sigma_{\parallel}$: $\Omega_{\text{eff}}=\Omega$, and for $\sigma_{xx}=\sigma_{\perp}$ the expression in the parentheses is to be substituted by

$$\tfrac{1}{2}[(\omega_{kk'}{}^{2}-\Omega_{+}{}^{2})^{-1}+(\omega_{kk'}{}^{2}-\Omega_{-}{}^{2})^{-1}].$$

Here

$$\Omega_{\pm}=\omega\pm\gamma H,\quad \Omega^2=\omega^2+(\gamma H)^2 \tag{51}$$

and the matrix elements for allowed direct transitions are

$$|P_{kk'}|^2_{\text{dir}}=L^2|p_{cv}|^2|\Phi(n,n')|^2\times\delta(\varphi_n-\varphi_{n'})\delta(k_z-k_{z'}). \tag{52}$$

For direct-transition, terms linear in k_x and k_x' in a transition frequency Eq. (37) cancel out. Using density of states (39) the absorption coefficient can be calculated to give

$$\alpha_{\text{eff}}=4BH\omega\Omega_{\text{eff}}{}^{-2}\sum_{nn'}|\Phi(n,n')|^2(\omega_{nn'}-\Omega_{\text{eff}})^{-1/2}, \tag{53}$$

where the transition frequencies between the modified Landau levels are

$$\omega_{nn'}=\omega_g+\omega_{c2}(n+\tfrac{1}{2})+\omega_{c1}(n'+\tfrac{1}{2})-(m_1+m_2)c^2E^2/2\hbar H^2 \tag{54}$$

and

$$B=e^3(2\mu)^{1/2}|p_{cv}|^2/8\pi^2c^2m^2\hbar^{5/2}n_0;$$

$1/\mu=1/m_1+1/m_2$ is the reduced effective mass.

If spin effects are neglected ($\gamma=0$) the absorption (53) reduces to Aronov's result. To avoid the singularities in the vicinity of the transition frequencies $\omega_{nn'}$ a relaxation time can be introduced in analogy to the semiclassical approach by putting $\omega\rightarrow\omega-i/\tau$ and taking a real part of the resulting expression.

The absorption due to forbidden transitions can also be calculated using the matrix elements given by Eq. (44). For linearly polarized light propagating along the magnetic field ($\mathfrak{E}\perp\mathbf{H}$) we get (spin effects are omitted)

$$\alpha_{\text{forb.}}=\frac{e^4(2\mu)^{1/2}}{2\pi^2c^3m^2\hbar^{3/2}n_0}\frac{H^2}{\omega}\sum_{nn'}\{\quad\}^2(\omega_{nn'}-\omega)^{-1/2}, \tag{55}$$

where the expression in the braces is defined in Eq. (44). As we mentioned before, this absorption constant depends on the angle between the dc electric field and the polarization vector. For light propagating perpendicularly to the magnetic field and polarized with $\mathfrak{E}\|\mathbf{H}$ the absorption constant does not show the oscillatory behavior because of the k_z term in the matrix element (44).

The Faraday rotation for allowed transitions can be calculated on the basis of Eqs. (34), (37), (39), (42), and (50) to give

$$\theta=BH\omega\sum_{nn'}|\Phi(n,n')|^2(F_{nn'}{}^{+}-F_{nn'}{}^{-}), \tag{56}$$

where

$$F_{nn'}{}^{\pm}=\Omega_{\pm}{}^{-2}[-2\omega_{nn'}{}^{-1/2}+(\omega_{nn'}+\Omega_{\pm})^{-1/2}+(\omega_{nn'}-\Omega_{\pm})^{-1/2}]; \tag{57}$$

$\omega_{nn'}$ are defined by Eq. (54), and $\Omega_{\pm}$ by Eq. (51).

The Voigt phase shift given by the second term of Eq. (35) becomes

$$\delta=-BH\omega\sum_{nn'}|\Phi(n,n')|^2(F_{nn'}{}^{+}+F_{nn'}{}^{-}-2F_{nn'}), \tag{58}$$

where $F_{nn'}$ is defined by Eq. (57) with $\Omega_{\pm}$ replaced by Ω. The third term in Eq. (57) is singular whenever $\omega_{nn'}=\Omega_{\pm}$, so that all the effects, as in the case of magnetic field alone exhibit oscillatory behavior. If a phenomenological relaxation time is introduced the singularities go to line shapes characteristic of Landau transitions and the dominant term near the singularity is given by

$$G_{nn'}{}^{\pm}=\left\{\frac{X_{nn'}{}^{\pm}+[(X_{nn'}{}^{\pm})^2+1]^{1/2}}{(X_{nn'}{}^{\pm})^2+1}\right\}^{1/2},$$

where $X_{nn'}{}^{\pm}=(\omega_{nn'}-\Omega_{\pm})\tau$. Thus the dispersive magneto-optical effects in crossed fields exhibit behavior similar to that of absorption. Namely, with increasing electric field the transitions with $\Delta n=0$ decrease in intensity and transitions with $\Delta n\neq0$ become possible. Moreover the frequencies of transitions are shifted to lower energies by the amount $m^*c^2E^2/2H^2$.

For small electric fields the obtained formulas can be expanded, similarly to the procedure of Vrehen and Lax.[2] The shift of transitions to lower energies is negligible in this case and for the formerly allowed transitions ($\Delta n=0$) the main terms in the differential spectrum of the Faraday rotation are

$$\Delta\theta=-\frac{B}{\omega}\frac{(m_1+m_2)^2c^3}{4\sqrt{2}he}\frac{E^2}{H^2}\times\sum_{n=0}\tau^{1/2}(2n+1)(G_{nn}^{+}-G_{nn}^{-}) \quad (59)$$

and for formerly forbidden transitions ($\Delta n=\pm1$)

$$\Delta\theta=+\frac{B}{\omega}\frac{(m_1+m_2)^2c^3E^2}{4\sqrt{2}he}\frac{E^2}{H^2}\times\sum_{n=0}\tau^{1/2}m(G_{nn'}^{+}-G_{nn'}^{-}), \quad (60)$$

where

$$m=n+1 \quad \text{for} \quad n'=n+1$$
$$=n \quad \text{for} \quad n'=n-1.$$

It can be seen from these expressions that in the differential-spectrum transitions with $\Delta n=0$ correspond to negative peaks, whereas the transitions with $\Delta n=\pm1$ correspond to the positive ones as related to the spectrum without an electric field.

VII. EXPERIMENTAL RESULTS FOR CROSSED-FIELD FARADAY ROTATION

The measurements of the crossed-field Faraday rotation were made on a sample of intrinsic germanium 7 μ thick, freely mounted, the same one that had been used by Vrehen in the crossed-field absorption investigation. The experimental techniques are described in Ref. 6. The basic procedure is to investigate the differential spectra. In the presence of a strong external magnetic field a transverse electric field is applied. The electric field consists of two parts: a dc field and a smaller ac modulation. The differential spectra are then obtained by measuring the modulation in the transmitted intensity ΔI due to the oscillating component. This method proved to be very sensitive in the crossed-field absorption investigation.

To determine the usual Faraday rotation in the absence of an electric field one has to measure two transmission intensities: I_1 with a linear analyzer forming an angle of $+45°$ (counterclockwise) to linear polarizer, and I_2 with an angle of $-45°$ to the polarizer, (looking in a direction opposite to that of the magnetic field). Then, if ellipticity is neglected, the Faraday rotation θ is given by the simple formula

$$\sin2\theta_0=(I_1-I_2)/(I_1+I_2). \quad (61)$$

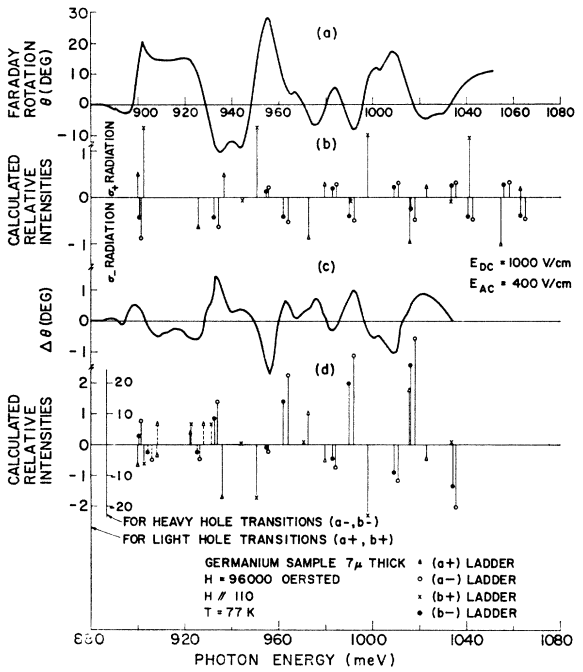

FIG. 1. The Faraday rotation and cross-field differential rotation in "strain-free" germanium at 77°K and 96-kG, H||[110]. (a) The Faraday rotation, (b) calculated transitions, (c) cross-field differential spectrum measured with $E_{dc}=1000$ V/cm and $E_{ac}=400$ V/cm (rms) and, (d) calculated differential spectrum. The solid lines denote the formerly allowed transitions, the dashed line corresponds to formerly forbidden transitions (only those below 930 meV are indicated). Note the difference in scale for light- and heavy-hole transitions in the calculated differential spectrum (d).

We assume now that the Faraday rotation and the intensities I are functions of the electric field. Then the change in the rotation due to the electric field is

$$\Delta\theta=\frac{1}{2\cos2\theta_0}\left(\frac{\Delta I_1-\Delta I_2}{I_1+I_2}-\sin2\theta_0\frac{\Delta I_1+\Delta I_2}{I_1+I_2}\right). \quad (62)$$

I_1 and I_2 are transmitted intensities for zero electric field, ΔI_1 and ΔI_2 are the modulations in the intensities due to the oscillating component of the electric field.

Figures 1(a) and 1(c) present the interband Faraday rotation and the differential-rotation spectrum as measured in germanium in a magnetic field of 96 kG for a strain-free sample and H||[110] at 77°K. The differential curve was measured with $E_{dc}=1000$ V/cm and $E_{ac}=400$ V/cm (rms).

The general theory presented in Sec. VI cannot be applied directly to interpretation of the experimental results because the valence band of germanium is degenerate. However, the main features of the spectra are in agreement with the theoretical predictions. First, the peaks in the Faraday rotation are observed in the vicinity of the frequencies corresponding to the interband transition energies. Secondly, the differential spectrum [Fig. 1(c)] is, save for a few details, a mirror

reflection of the normal rotation [Fig. 1(a)], which is predicted by the general theory if we assume that the differential rotation is dominated by the decrease of intensity of the allowed ($\Delta n=0$) transitions [Eqs. (56) and (59)].

To carry out a more detailed comparison of the experiment with the theory the actual band structure of Ge in a magnetic field has to be considered. Figure 1(b) presents the energies and the strengths of the transitions for σ_+ and σ_- radiation calculated by Vrehen[6] for a magnetic field of 96 kG along the [110] direction, on the basis of Roth's theory,[19] including effects of higher than second order in k in the effective-mass Hamiltonian. These are allowed transitions with the selection rules $\Delta n=0$ and $\Delta n=-2$. In an electric field the intensity of those transitions is decreased whereas formerly forbidden transitions with $\Delta n=-3, -1, +1$ now have a finite transition probability. The decrease in the intensity of allowed transitions gives rise to peaks in the differential spectrum in the direction opposite to those in the $E=0$ rotation, similar to the results of the theory for simple bands. The formerly forbidden transitions give rise to positive or negative peaks in the differential spectrum according to whether they occur for σ_+ or σ_- radiation, respectively. In Fig. 1(d) we present intensities due to decrease of formerly allowed transitions (solid lines) as calculated in Ref. 6 by use of perturbation theory. For the electric field used (1000 V/cm) the shift of the transition energy can be shown to be negligible. The forbidden transitions for energies lower than 930 meV are also shown (dashed lines).

The interband Faraday rotation presented in Fig. 1(a) is in agreement with results of Nishina *et al.*[20] and Mitchell and Wallis.[21] All the main peaks in the rotation can be well understood in terms of Landau transitions given in Fig. 1(b). An origin of a broad shoulder of the first positive peak is not clear, but it presumably is due to the group of four close transitions in the vicinity of 900 meV. The main feature of the differential spectrum Fig. 1(c) is that it resembles the mirror reflection of the rotation 1(a). As we have already mentioned this indicates that the main effects are due to electric-field-induced decrease of the intensity of the formerly allowed transitions. This is confirmed by a more detailed comparison with intensities due to the formerly allowed transitions plotted in Fig. 1(d). Furthermore, it can be seen that the differential spectrum is primarily governed by the heavy-hole transitions. This is due to the fact that allowed heavy-hole transitions are relatively stronger in the differential spectrum than in the normal-relation spectrum, when compared with the allowed light-hole transitions. For example in the normal rotation the predominance of light-hole transitions (especially b^+) at the energy just above 900 meV results in the large positive peak. In the differential spectrum, however, the heavy-hole transitions are much stronger than the light-hole ones (note the difference in the scale), and again the positive peak is produced, which at first seems to be in contradiction with the general reflection character of this spectrum. In the energy region 903–932 meV there are no formerly allowed transitions and it can be seen that two negative peaks in the differential curve are due to two pairs of forbidden heavy-hole transitions in the vicinity of 905 and 926 meV, respectively. These forbidden transitions can also be quite clearly visible in the differential absorption spectrum of σ_- radiation.[6] In general all the main features of the differential dispersion spectrum are in agreement with the crossed-field absorption data.

ACKNOWLEDGMENTS

This work was started when one of us (W.Z.) was at the Institute of Physics, Polish Academy of Sciences. He would like to thank Dr. J. Kolodziejczak, Professor L. Sosnowski, and Professor M. Suffczynski for helpful discussions and encouragement.

[19] Laura M. Roth, B. Lax, and S. Zwerdling, Phys. Rev. **114**, 90 (1959).
[20] Y. Nishina, J. Kolodziejczak, and B. Lax, Phys. Rev. Letters **9**, 55 (1962).
[21] D. L. Mitchell and R. F. Wallis, Phys. Rev. **131**, 1965 (1963).