and μ_i assumed to vary as

$$u_i, \mu_i \sim e^{i[KN_i a_i - \omega t]},$$

H_i^e is given to first order as

$$\begin{aligned} H_1^e = & H_{01} + \bar{H}_1' \\ & - \frac{1}{M_s}[2b_{11}\bar{\alpha}_1 u_a + b_{44}(\bar{\alpha}_2(u_b+v_a)+\bar{\alpha}_3(u_c+w_a)] \\ & - 4\pi M_s N_1[N_j(\mu_j - \bar{\alpha}_j(u_a+v_b+w_c)] \\ & + 4\pi\lambda\nabla_a^2 m_1, \end{aligned} \tag{A5}$$

with

N_i = unit vector in propagation direction,

H_{0i} = external field,

$\bar{H}_i'$ = static dipolar field.

Equations for H_2^e and H_3^e are obtained from Eq. (A5) by cyclic permutation. The third right-hand-side term in Eq. (A5) is the effective magnetoelastic coupling field linear in $\partial u_i/\partial a_j$. A similar term of the form $\mu_i(\partial u_j'/\partial a_k)$ has been dropped from Eq. (A5) because it is quite small. The fourth and fifth right-hand-side terms in Eq. (A5) are the dipolar (including the dilatational dipolar field) and exchange fields. Using Eq. (2.5), the linearized equations of motion are

$$\dot{\mu}_i = \gamma_0 \epsilon_{ijk}(\bar{\alpha}_j h_k^e + \mu_j \bar{H}_k^e), \tag{A6}$$

with h_k^e and $\bar{H}_k^e$ defined as the spatially varying and spatially nonvarying portions of H_k^e in Eq. (A5).

PHYSICAL REVIEW VOLUME 148, NUMBER 2 12 AUGUST 1966

Possible Experimental Test of the Band Theory of Magnetism

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(Received 4 February 1966)

We suggest an experiment which may allow the band theory of ferromagnetism to be tested in a direct way. If a dc electric field is applied to a sample, the magnetic electrons will drift. The frequency of a spin wave of given wave vector, when viewed in the laboratory frame, will suffer a Doppler shift when compared with the case when the electric field is zero. This Doppler shift, although small, is considerably larger for the band model than the electric field dependence of the spin-wave dispersion relation for a localized spin model. We discuss the possibility of detecting the Doppler shift by measuring the phase velocity of a coupled spin–transverse-phonon wave. Similar measurements have been performed on insulators doped with paramagnetic impurities. In these measurements, high precision has been obtained by employing an interference technique which allows a null experiment to be performed.

I. INTRODUCTION

VARIOUS models have been proposed to describe the magnetic properties of transition metals. While these models assume very different mechanisms are responsible for the magnetically ordered state, they nonetheless predict elementary excitation spectra and thermodynamic properties which are qualitatively similar. As a consequence, it is difficult to decide from experimental measurements which of the models is most suited to describe the magnetism of the transition metals.

Historically, the first approach to the problem was by Heisenberg,[1] who employed a model of localized spins, each coupled to its nearest-neighbor spins by means of the exchange interaction which results from the overlap of atomic orbitals. If the exchange integral has the appropriate sign, the ground state of the system is ferromagnetic. The elementary excitations of the system are spin waves,[2] and one finds that as the temperature is increased from zero, the change in magnetization varies as $T^{3/2}$, in agreement with experimental observations. This model has been studied extensively. A number of other magnetic properties of transition metals, such as the variation of the magnetic susceptibility with temperature just above the Curie point,[3] and the magnetic critical scattering observed in neutron diffraction experiments[4] may be accounted for with this theory. In the Heisenberg model, the interaction between the localized spins is short-ranged, since the wave function of a given spin overlaps appreciably only with its nearest neighbors. The conduction electrons (s electrons) play no role, as far as the magnetic properties of the system are concerned.

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[1] We refer the reader to the review article by J. H. Van Vleck, Rev. Mod. Phys. **17**, 27 (1945).

[2] F. Bloch, Z. Physik **61**, 206 (1930).

[3] See for instance, M. E. Fisher, in *Proceedings of the International Conference on Magnetism, Nottingham, 1964* (Institute of Physics and the Physical Society, London 1965), p. 79.

[4] R. J. Elliott and W. Marshall, Rev. Mod. Phys. **30**, 75 (1958).

It has been shown by Ruderman and Kittel,[5] and Yosida[6] that the presence of the conduction electrons can lead to a long-range oscillatory exchange coupling between localized spins. This interaction certainly plays a dominant role in the magnetism of most rare-earth metals, and has been invoked to discuss the magnetism of transition metals well.[7] There is experimental evidence which may be used to support the presence of such a long-range exchange interaction.[8,9]

However, on the basis of these models, it is difficult to understand the nonintegral number of Bohr magnetons per atomic site observed in the transition metals of the first series, the contribution of the d electrons to the electronic specific heat, and the disappearance of the local moment above the Néel point in some of these metals.[10]

These properties suggest that the d electrons are not localized. As a consequence, a theory of magnetism in these metals based on an itinerant model has been developed by Stoner[11] and pursued by a number of workers.[12–16] although this theory involves different assumptions and methods, nonetheless one obtains from the theory magnetic excitations of spin-wave character,[16] a $T^{3/2}$ demagnetization law, and a description of magnetic critical scattering. The band theory is not inconsistent with the description provided by a theory with localized moments coupled by a long-range exchange interaction.[15]

Since all of the theories mentioned above provide a description of the magnetic properties of transition metals which is qualitatively reasonable, it is not clear which of these theories is most applicable.

In this paper, we would like to suggest an experiment which we feel may be possible with present day techniques, and which would allow one to determine whether the itinerant model or a localized spin model provides the appropriate description of the magnetic state of transition metals.

Suppose we place the metal in a uniform dc electric field. If the band picture is valid, the d electrons will drift in this field with some drift velocity $\mathbf{v}_d$, which may be determined from the d-electron mobility. If we observe the frequency of a spin wave of given wave vector $\mathbf{k}\|\mathbf{v}_d$ in the laboratory frame, this frequency will suffer a Doppler shift if compared to the frequency of a wave with the same $\mathbf{k}$ when the d electrons are at rest.

On the other hand, if the magnetic electrons are localized on the ionic sites, such a Doppler shift cannot occur. The electric field may affect the spin-wave dispersion relation, but in the Heisenberg model with nearest-neighbor coupling, we will see the effect is much smaller than in the band model. If the coupling between the localized spins is by means of the Ruderman-Kittel-Yosida (RKY) interaction, we will find no shift to first order in the electric field.

Even in the band model, the Doppler shift is small. If the d electrons drift with a drift velocity of 1 cm/sec, the fractional change in frequency of a spin wave of wave vector 10^5 cm^{-1} will be 10^{-5} to 10^{-6}. It appears difficult to observe this small shift by a ferromagnetic-resonance experiment, or by a neutron-diffraction experiment.

We suggest that it may be possible to observe this shift by measuring the electric field dependence of the phase velocity of an acoustical wave with a frequency in the vicinity of 10 kMc/sec. It was first shown by Kittel[17] that because of the coupling between the sound wave and the spin motion, the normal modes of a ferromagnet have mixed spin wave and phonon character. Consequently, the ultrasonic wave will be accompanied by spin motion as it propagates in the crystal. If the magnetic electrons are described by the band picture, the change in the spin-wave dispersion law produced by the Doppler effect will alter the phase velocity of the coupled spin-phonon wave. As we shall see, under optimum conditions, the change in phase velocity from this effect will be very small, perhaps one part in 10^5 or 10^6.

Recently a series of experiments in which the magnetic field dependence of the phase velocity of ultrasonic waves propagating in an insulator doped with a small concentration ($\sim 10^{-6}$) of paramagnetic impurities have been performed by Joffrin, Guermeur, Levelut, and Penné.[18] In these experiments, an interference technique was employed which allowed measurements of changes in phase velocity as small as one part in 10^7.

Very high sensitivity was obtained in this work because the interference technique employed by Joffrin *et al.* allowed a null experiment to be performed.

Unfortunately, to carry out such an experiment in a metal will be more difficult, primarily because the propagating wave will be damped much more strongly than in the experiments cited above. In the experiments of Joffrin *et al.*, the waves had a mean free path the order of a centimeter. The estimates presented below indicate that in a metal the mean free path will be only a fraction of a millimeter.

In Sec. II, we discuss briefly the properties of the spin waves associated with the models mentioned above.

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[18] R. Guermeur, J. Joffrin, A. Levelut, and J. Penné, Compt. Rend. **260**, 108 (1965).

In Sec. III, we present a short summary of the properties of coupled spin-phonon waves following the phenomenological treatment of Kittel,[17] and in Sec. IV we estimate the mean free path of the waves. Section V contains some comments concerning the experimental arrangement.

II. PROPERTIES OF THE SPIN-WAVE EXCITATIONS—ELECTRIC FIELD DEPENDENCE

Let us first consider the band model, in which the d electrons are assumed to move through the lattice. In most treatments, one avoids the complications introduced by the presence of orbital degeneracy, and assumes the existence of only a single nondegenerate d band. One often uses a tight-binding approximation, in which two electrons interact only if they find themselves on the same atomic site, although this is not a necessary restriction.[19]

In a single-band, tight-binding approximation, the paramagnetic state is unstable with respect to a ferromagnetic state if $Un(\epsilon_F)>1$, where $n(\epsilon_F)$ is the density of states at the Fermi level, and U is the Coulomb repulsion between two electrons on the same atomic site. If the Coulomb repulsion U is sufficiently strong, the ground state will be completely polarized, with the spins of all d electrons aligned.

In the magnetic state, the system has elementary excitations of spin-wave character.[20] The dispersion relation may be shown to have the form (for small values of $\mathbf{k}$)

$$\omega_s(\mathbf{k})=\omega_0+Dk^2, \tag{1}$$

where ω_0 is the Larmor precession frequency of the electron spin in the external magnetic field, and $\mathbf{k}$ the wave vector of the spin wave. For nickel, one finds $D\cong 10^{-1}$ rad cm^2/sec.

Now suppose we apply a uniform electric field to the system, so that the d electrons drift with some drift velocity $\mathbf{v}_d$. If we make a Gallilean transformation into the frame moving with the drifting electrons, the system viewed from this frame will have spin-wave excitations described by the dispersion relation of Eq. (1).[21]

The frequency of a spin wave of wave vector $\mathbf{k}$ when viewed from the laboratory frame will be Doppler shifted by an amount

$$\Delta\omega_s/\omega_s=(\hat{k}\cdot\mathbf{v}_d)/v_p, \tag{2}$$

where $v_p=\omega_s(\mathbf{k})/k$ is the phase velocity of the spin wave.

Suppose the angular frequency of the spin wave is 6×10^{10} rad/sec., and its wave vector 2×10^5 cm^{-1}. Then $v_p=3\times10^5$ cm/sec. If we assume v_d is 1 cm/sec and is parallel to $\mathbf{k}$, then we find $\Delta\omega_s/\omega_s=1/3\times10^{-5}$.

The drift velocity $\mathbf{v}_d$ is related to the electric field by the equation $\mathbf{v}_d=\mu_d\mathbf{E}$, where μ_d is the mobility of the d electrons. If m_d is the effective mass of the d electrons, and τ_d the relaxation time, then $\mu_d=e\tau_d/m_d$. Typically $m_d\cong5$ free-electron masses. The relaxation time τ_d is difficult to estimate reliably or determine experimentally, since the transport properties of transition metals are dominated by the s electrons.

We shall assume[22] that the d-electron relaxation time is comparable to that of the s electrons. Then we take $\tau_d\cong10^{-14}$ sec, which is the order of magnitude of the room-temperature s-electron relaxation time. We shall see later that it will be advantageous to carry out the experiment at room temperature. If we use the numbers mentioned above, a drift velocity of 1 cm/sec can be produced by an electric field of the order of $\frac{1}{2}$ V/cm. Sample heating may be avoided by employing pulsed electric fields. We shall estimate in Sec. V that a field of this magnitude may be imposed on the sample for as long as a tenth of a millisecond.

We consider next the Heisenberg model, in which a given localized spin interacts with its nearest neighbors through the exchange interaction produced by the overlap of the atomic wave functions. For this model, the dispersion relation for the spin waves is

$$\omega_s(k)=\omega_0+\{J(\mathbf{k})-J(0)\}, \tag{3}$$

where $J(\mathbf{k})=\sum_\delta J_\delta\exp(i\mathbf{k}\cdot\boldsymbol{\delta})$, with J_δ being the exchange integral between an ion and its nearest neighbor at the site $\boldsymbol{\delta}$ relative to the ion.

Let $|0\rangle$ be the ground-state wave function of the ion in the crystalline electric field. We assume $|0\rangle$ nondegenerate, and that the crystalline field has an inversion center. Then in the presence of the E field, the perturbed ground-state wave function will be given by

$$|0'\rangle=|0\rangle+e\sum_n\frac{|n\rangle\langle n|\mathbf{r}|0\rangle}{E_n-E_0}\cdot\mathbf{E}.$$

We can expect the fractional change in the exchange integral to be $\Delta J/J\sim eEr_{0n}|\Delta$, where Δ is the energy difference between the ground state and the first excited state of opposite parity. Then if $\Delta=1$ eV, and $r_{0n}=1$ Bohr radius, we find $\Delta J/J\cong10^{-9}$ for $E=\frac{1}{2}$ V/cm.

Thus the Doppler shift discussed above for the band model is several orders of magnitude larger than the corresponding shift for the Heisenberg model. The shift in frequency upon application of the electric field appears too small in the latter model to be detected by

[19] J. Hubbard, Proc. Roy. Soc. (London) **A276**, 238 (1963).
[20] K. Kawasaki, Phys. Rev. **135**, A1371 (1964).
[21] This argument is not strictly rigorous, even in a one-band model. In the moving frame, one will see a current density due to the positive background and this current density will give rise to a magnetic field, which will affect $\omega_s(k)$. In a transition metal, the magnetic field which arises from the motion of the s electrons will be larger than this last mentioned field. In Sec. V, we suggest a geometry which will reduce the effect of these fields so that the Doppler shift will dominate the electric field dependence of $\omega_s(\mathbf{k})$.
[22] See the discussion by N. F. Mott and H. Jones, *Metals and Alloys* (Oxford University Press, New York, 1936), p. 267.

the method discussed in this paper, with presently available techniques.

Finally, we mention the properties of a system of localized spins exchange coupled by the RKY interaction mentioned in the introductory section. The spin-wave dispersion relation for this model has the form of Eq. (3), with

$$J(\mathbf{k})=\frac{\mathcal{J}_{s\text{-}d}^2}{N}\sum_{\mathbf{k}'}\frac{n_\uparrow(\mathbf{k}')-n_\downarrow(\mathbf{k}'+\mathbf{k})}{\epsilon_\downarrow(\mathbf{k}'+\mathbf{k})-\epsilon_\uparrow(\mathbf{k})}, \tag{4}$$

where $\mathcal{J}_{s\text{-}d}$ is the s-d exchange interaction, and $n_\uparrow(\mathbf{k})$ and $n_\downarrow(\mathbf{k})$ are the up and down spin s-electron distribution functions. The number of unit cells in the crystal is N, and $\epsilon_\downarrow(\mathbf{k})$ describes the s-electron energy bands.

Let us assume the s electrons are unpolarized $[n_\uparrow(k)=n_\downarrow(k)]$. If we apply an electric field, the s electron Fermi sphere will be displaced from the origin in $\mathbf{k}$ space by an amount $\mathbf{k}_s$, so that $n(\mathbf{k})=n_0(\mathbf{k}-\mathbf{k}_s)$, where $n_0(\mathbf{k})$ is the distribution function for $\mathbf{E}=0$. Then, to first order in $\mathbf{E}$, one may write the change in $J(\mathbf{k})$ in the form

$$\Delta J(\mathbf{k})=\mathbf{k}_s\cdot\frac{\mathcal{J}_{s\text{-}d}^2}{N}\sum_{\mathbf{k}'}\left\{\frac{1}{\epsilon(\mathbf{k}'+\mathbf{k})-\epsilon(\mathbf{k}')}+\frac{1}{\epsilon(\mathbf{k}'-\mathbf{k})-\epsilon(\mathbf{k}')}\right\}\cdot\nabla_{\mathbf{k}'}n_0(\mathbf{k}').$$

To derive this result, we have used $\epsilon(\mathbf{k})=\epsilon(-\mathbf{k})$, which is a consequence of time-reversal invariance. The integrand is the product of an even function of $\mathbf{k}'$ multiplied by an odd function of $\mathbf{k}'$, so the sum vanishes. Thus the correction to $J(\mathbf{k})$ linear in $\mathbf{E}$ vanishes. The first correction term will be quadratic in the electric field.

We see that of the three models mentioned in the Introduction, the band model exhibits a shift in the frequency of a spin wave of given wave vector upon application of an electric field which is linear in $\mathbf{E}$, and larger than the shift found for the Heisenberg model by several orders of magnitude. A model based on localized spins exchange coupled by the RKY interaction exhibits no shift linear in $\mathbf{E}$, if the s-band is unpolarized.

III. THE COUPLED SPIN-PHONON EXCITATIONS

In this section, we discuss briefly the properties of the coupled spin-phonon modes, following the phenomenological treatment of Kittel.[17] We then estimate the electric field dependence of the phase velocity of the coupled mode.

The Hamiltonian of the system may be written as a sum of three terms,

$$H=H_M+H_P+H_{MP}, \tag{5}$$

where H_M describes the magnetic degrees of freedom, H_P the lattice motion, and the term H_{MP} the coupling between the lattice and spin motion.

The term H_M consists of a Zeeman term and the exchange energy, which may be represented in the form (with $\hbar=1$)

$$H_M=H_z+H_{\rm ex}=\omega_0\int M_z d\tau+\frac{A}{M_s^2}\int\{|\nabla M_x|^2+|\nabla M_y|^2\}d\tau.$$

Let us consider the propagation of an ultrasonic shear wave, with displacement parallel to the z axis (we assume the magnetization is also directed parallel to the z axis), and with wave vector parallel to the x axis. Then if ρ is the density of the crystal,

$$H_P=\int\{\tfrac{1}{2}\rho(\partial R/\partial t)^2+\tfrac{1}{2}\rho v^2(\partial R/\partial x)^2\}d\tau,$$

where R is the displacement associated with the wave and v the velocity. Also,

$$H_{MP}=\frac{b_2}{M_s}\int M_x(\partial R/\partial x)d\tau,$$

where the magnetoelastic coupling constant $b_2=10^8$ erg/cm^3 for Ni.

Following Kittel, we may derive the secular equation from which the dispersion relations for the coupled modes may be found. We linearize the equation by replacing M_z by the saturation magnetization M_s. We then find the secular equation

$$[\Omega^2-v^2k^2][\Omega^2-\omega_s^2(k)]+\gamma b_2^2\omega_s(k)k^2/\rho M_s=0, \tag{6}$$

where γ is the magnetogyric ratio defined by Kittel.

The phase velocity v_p of the wave is $v_p=\Omega(k)/k$, where $\Omega(k)$ is the solution of Eq. (6) for the wave vector k. The change in phase velocity which results from a change in $\omega_s(k)$ is given by

$$\Delta v_p/v_p=-\frac{1}{4}\frac{\gamma b_2^2}{\rho v^2 M_s}\frac{\omega_0}{[\Omega-\omega_s(k)]^2}\frac{\Delta\omega_s}{\omega_s}, \tag{7}$$

provided that $|\Delta\omega_s|\ll|\Omega-\omega_s(k)|$. We have assumed we will be interested only in waves with a frequency not far from the Larmor frequency.

Kittel has pointed out[17] that the strongly mixed modes which have frequencies in the vicinity of the crossover of the unperturbed spin wave and phonon branches are heavily damped as a consequence of the damping of the transverse motion of the spins. This leads to a mean free path of the order of one wavelength.

In order to obtain a mean free path sufficiently long to permit study of the coupled modes, it will be necessary to perform the measurement at a frequency some

distance off the crossover frequency. We shall see in the next section that as a consequence of the damping of the lattice motion by the electron-phonon interaction, waves with a frequency in the range of 10^{10} rad/sec will be damped in a distance the order of a few tenths of a millimeter. If we shoose $[\Omega-\omega_s(k)]\approx 3\times 10^9$ rad/sec, we will see that the mean free path from the damping of the spin motion will be comparable to that from the electron-phonon interaction.

We now estimate $\Delta v_p/v_p$ from Eq. (7), assuming $[\Omega-\omega_s(k)]=3\times 10^9$ rad/sec and $\Delta\omega_s/\omega_s=1/3\times 10^{-5}$, as in Sec. II for the itinerant model.

With $v=3\times 10^5$ cm/sec, $M_s=500$ Oe, $\gamma=2\times 10^7$ (Oe sec)$^{-1}$, $\rho=6$ g/cm^3, and $b_2=10^8$ erg/cm^3, we find

$$\Delta v_p/v_p\cong 0.6\times 10^{-5}.$$

If the accuracy of the experiments performed by Joffrin *et al.* can be maintained for the system under consideration in the present paper, this shift would be observable.

IV. ESTIMATE OF THE MEAN FREE PATH

Until now, we have neglected all processes which damp the motion of the lattice or the spins. Consequently the coupled spin-phonon modes discussed in the last section are eigenmodes of the Hamiltonian and have an infinite mean free path.

In metals, the presence of the conduction electrons will damp the lattice motion, while the transverse motion of the spins will also decay in time.

Let us first examine the attenuation of an acoustical wave by the conduction electrons. As mentioned earlier, we shall be interested in temperatures near room temperature. The wave will be damped principally by the s electrons, which at this temperature have a mean free path short compared to the wavelength of the acoustical wave. As the lattice moves with the electron gas, it will be damped by the viscosity of the electrons. In this limit, we have the attenuation coefficient α_p given by[23]

$$\alpha_p=(8/15)(n\epsilon_F\tau_s\Omega^2/\rho v^3)\ (\mathrm{cm}^{-1}).$$

If $n=10^{23}$ cm^{-3}, $\epsilon_F=10$ V, $v=3\times 10^5$ cm/sec, $\rho=6$ g/cm^3 and, as above, $\Omega=6\times 10^{10}$ rad/sec and $\tau_s=10^{-14}$ sec, we find the mean free path to be about 0.1 mm. In the present discussion, we are concerned with the mean free path of a coupled spin-phonon mode rather than a pure acoustical wave. But, as mentioned above, we will be interested in frequencies sufficiently far from the crossover frequency of the unperturbed spin-wave and phonon branches that the mode will consist predominantly of lattice motion. Then the above estimate should not be seriously in error.

The transverse motion of the spins will also be damped. One may compute the mean free path which results from the spin damping by replacing $[\Omega^2-\omega_s^2(k)]$ in Eq. (6) by $[(\Omega+i\Gamma)^2-\omega_s^2(k)]$, where Γ is the rate of decay of the transverse spin amplitude. If

$$\Gamma\ll|\Omega-\omega_s(k)|\ ,\quad \text{and}\quad \Omega\cong\omega_0\ ,$$

(with $k\cong\omega_0/v$) one finds the contribution to the attenuation rate of the coupled wave is

$$\alpha_s=-\frac{1}{4}\frac{\gamma b_2^2\omega_0}{\rho v^3 M_s}\frac{\Gamma}{[\Omega-\omega_s(k)]^2}\ (\mathrm{cm}^{-1}).$$

If $\Gamma=10^8$ sec^{-1}, and $(\Omega-\omega_s)=3\times 10^9$ rad/sec, with other quantities given above, one finds the mean free path to be approximately 0.1 mm from this source.

One can make the attenuation coefficient smaller by working farther from the crossover, so $[\Omega-\omega_s(k)]$ will be larger in magnitude. As a consequence, the fractional change in phase velocity for a given electric field will be smaller. Even if α_s is decreased in this manner, the mean free path would still be limited by the interaction of the lattice motion with the electrons.

We are driving the spins with a wave of frequency Ω and wave vector $k\cong\Omega/v$. Kittel[17] has shown that for this situation, the transverse motion of the spins is attenuated at the rate $\Gamma\cong(10^{19}/\Omega)$ sec^{-1} from eddy currents induced in the metal by the motion of the magnetization. To obtain this result, the conductivity of the metal was assumed to be $\sigma=10^{17}$ esu, typical of iron at room temperature. If $\Omega=6\times 10^{10}$ rad/sec, as assumed above, then from this mechanism, $\Gamma\cong 10^8$ sec^{-1}.

The eddy-current attenuation rate is proportional to the conductivity. If we choose to carry out the experiment at low temperatures (say at helium temperatures), then the eddy-current damping would become very severe for a wave of the frequency and wave vector considered here. This is the primary reason why we have suggested the experiment be carried out at room temperature. From the above estimates, it appears necessary to work with a mean free path the order of a fraction of a millimeter.

V. GENERAL COMMENTS

In this section, we consider a number of points which we feel will be relevant to the experiment.

In order to obtain an effect large enough to be observed with an experiment of the accuracy of that of Joffrin *et al.*, we assumed an electric field of approximately $\frac{1}{2}$ V/cm was applied to the sample. If the conductivity of the sample is 10^{17} esu, then the power dissipated is approximately 4×10^{11} erg/cm^3 sec. If the field is applied for 10^{-4} sec, then the rise in temperature of the sample may be estimated to be about 1°K, if none of the energy is conducted away from the sample. We have assumed the specific heat to be the order of $3nk_b$ erg/cm^3 °K, where k_b is Boltzmann's constant.

[23] C. Kittel, *Quantum Theory of Solids* (John Wiley & Sons, Inc., New York, 1963), p. 310.

Thus it appears as if the electric field of the required magnitude may be applied to the sample for about a tenth of a millisecond, without sample heating of any significant kind. If a sample with a thickness the order of a millimeter is employed, then a wave train of roughly 10^6 wavelengths will traverse the sample in this period.

The preceding discussion indicates that an electric field of the order of magnitude assumed in the first sections of the paper may be applied to the sample for a time long enough for many wave trains to traverse the crystal. There is however, one difficulty associated with the high (principally s-electron) current density which will result from the application of the electric field. The current density will produce a magnetic field h_c. The magnetic electrons will see the field h_c in addition to the external field and the demagnetization field. The presence of h_c will alter the spin-wave dispersion relation in a complicated way, since h_c will vary with position. Since h_c is proportional to the current density, and consequently to $\mathbf{E}$, we will find a second contribution to the electric field dependence of ω_s in addition to the Doppler effect.

The effect of an h_c of a given magnitude may be minimized by driving the current density parallel to M_s. (We assume any external magnetic field is applied parallel to M_s, and that the sample shape is such that the demagnetization field is parallel to the external field.) If the current density is parallel to M_s, then h_c will be perpendicular to the external field.

Let us assume the sample has a rectangular cross section of width a and thickness d, with $d \ll a$. Then the field h_c will be a maximum at the surface, assuming M_s and the current density perpendicular to the cross section. At the surface, $h_c = 2\pi jd/c$, with all quantities expressed in cgs units. By making the thickness d small, one may make h_c sufficiently small that its effect may be neglected. If we choose d equal to one micron, and the current density $j = 10^5$ A/cm^2 (consistent with the numbers given above), then we find $h_c = 5$ Oe at the surface. Since we assume the Larmor frequency of the spins to be 6×10^{10} rad/sec, and a Landé g factor of 2, the fractional change in the magnitude of the magnetic field at the surface will be one part in 10^6. This is at the surface of the sample, where h_c is largest. It appears from the above estimates that the sample geometry may be chosen so that the effect of h_c will be dominated by the Doppler shift.

In contrast to the discussion of Sec. III, application of the electric field parallel to M_s will require the coupled wave to be propagated parallel to M_s, in order to obtain the maximum Doppler shift for a given field. This will not alter the numerical estimates of Sec. III in any significant way. As Kittel has shown, the normal modes of the system then involve lattice motion which is circularly polarized, rather than plane polarized as we discussed above. If we send a plane polarized wave parallel to M_s, its plane of polarization will be rotated as it proceeds down the crystal, but each circularly polarized component will suffer the same Doppler shift to a very good approximation.

In this paper, we have considered the propagation of transverse ultrasonic waves in the crystal.

Let us suppose for the moment that we treat the conduction electrons as a gas of free electrons. There is no change in the charge density of the positive background associated with a transverse wave, to first order in the amplitude. The electron gas will then feel the lattice motion only through the current associated with the motion of the positive ions. The electric-field dependence of the response functions of the electron gas, in this approximation, will produce a negligible change in the phase velocity.

Recent work[24] has shown that an important, and perhaps the dominant, contribution to the elastic constants of the simple metals comes from the indirect interaction of the positive ions through the conduction electrons. In a manner similar to the discussion in Sec. II of the electric field dependence of the RKY exchange integral, one may show that there is no change in the velocity of sound of a transverse wave, to first order in the field.

ACKNOWLEDGMENTS

During the course of this work, we have enjoyed conversations with many persons. The comments of P. G. de Gennes, P. Nozières, A. Blandin, and J. Friedel have been particularly helpful, and a lengthy discussion with J. Joffrin, A. Levelut, R. Guermeur, J. Penné, and Y. Le Corre proved most useful.

[24] J. Friedel and J. L. Deplanté (private communication).