However, as we have already seen in this Appendix, a Fredholm operator depending analytically upon a parameter c can have eigenvectors corresponding to unit eigenvalue for only a discrete set of values of c. Therefore, there are continuous ranges of c for which the only solutions to Eq. (C51) are $\psi_j \equiv 0$. For these continuous ranges of c, Eq. (C50) reduces to Eq. (C43), which establishes the equivalence of Eqs. (C18) and (C1).

The solutions of Eq. (C18) have been shown to be continuous functions of s, so that they obviously satisfy a Hölder condition by virtue of the mean-value theorem. The Hölder condition can be established under even weaker conditions than that of continuity.[19] Since the solutions of the Fredholm equations (C18) belong to the class of functions $\mathfrak{L}_2$, these functions must vanish for large values of s. All of the integrals in Eq. (C1) are weighted with the functions $\sigma^\alpha(s)$ which are assumed to vanish sufficiently rapidly to ensure convergence of the principal-value integrals. If this is not the case, then one simply makes enough subtractions to guarantee convergence.

[19] N. I. Muskhelishvili, Ref. 11, pp. 135–140.

PHYSICAL REVIEW VOLUME 148, NUMBER 4 26 AUGUST 1966

Ambiguity of the Meson-Baryon Couplings in a Bootstrap Static Model of $SU(6)$ Symmetry*

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(Received 8 April 1966)

In the present discussion we seek to clarify certain aspects of $SU(6)$ theory which have an important bearing on the problem of formulating a bootstrap dynamical description of symmetry breaking. We discuss here, in particular, a certain ambiguity of the meson-baryon coupling which exists even in the limit of exact $SU(6)$ symmetry, and which we call assignment mixing. It is possible to recover certain special theories, such as the so-called W-spin theory, by a particular choice of the assignment mixing angles. Bootstrap equations in the exact $SU(6)$ limit do not fix the angles, unless one also considers mesonic bootstrap equations corresponding to Fermi-Yang–type theories. It is also shown that in the exact $SU(6)$ limit, the normalization and vertex equations of the Cutkosky-Leon bootstrap method both yield the same equation, which relates the coupling constant f to the ratio of the meson mass m to the cutoff parameter k_Λ. Approximate solutions of the Bethe-Salpeter equation are obtained.

I. INTRODUCTION AND SUMMARY

It is commonly believed that $SU(6)$ theory is a closed subject in the static limit and that the only interest is in the formulation of a relativistic version of the $SU(6)$ group. However, our experience with the bootstrap version of $SU(6)$ theory has been that there are still some features of $SU(6)$ symmetry even in the nonrelativistic domain which, to the best of our knowledge, have not yet been thoroughly discussed. One of these features is the problem of assignment mixing, which we will discuss in the present paper. This mixing leads to the situation that even though there is only one $SU(6)$ Clebsch-Gordan coefficient[1] for coupling $35 \otimes 56 \supset 56$, there still remains an ambiguity in the meson-baryon couplings.

The present paper is in the first place an extension of the bootstrap version of $SU(6)$ symmetry of Capps[2] and of Belinfante and Cutkosky.[3] In addition, we intend to provide an elementary and rather explicit discussion of the model. Our emphasis therefore is not on the various successful features of the $SU(6)$ bootstrap theory, but on the conceptual problems involved in the formulation of the theory.

Since we wish to discuss low-energy meson-baryon scattering, it is reasonable to take advantage of the great simplifications which arise by making use of the static model, suitably extended to include vector mesons and spin-$\frac{3}{2}$ isobars. The simplifications include, first of all, the limitation to p-wave orbital angular momentum states. A second nice feature of the static model is that the baryon mass M disappears from the final bootstrap equations,[4] thus reducing the number of parameters in the theory. In the third place, the static model is very familiar and we have therefore the advantage of being able to build on previously acquired intuition. Finally, it is our belief that the close relation of the static model to relativistic dispersion theory[5] may help to provide a link between the present nonrelativistic theory and a relativistic $SU(6)$ theory, if such a theory exists at all. In a relativistic theory there are several vertices which

* Work supported in part by the U. S. Atomic Energy Commission.

[1] C. L. Cook and G. Murtaza, Nuovo Cimento **39**, 531 (1965).
[2] R. H. Capps, Phys. Rev. Letters **14**, 31 (1965).
[3] J. G. Belinfante and R. E. Cutkosky, Phys. Rev. Letters **14**, 33 (1965).
[4] In the case of broken symmetry, only the mass differences between the various baryons will appear.
[5] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1337 (1957).

lead to the same static limit, so that our static theory does not correspond directly to a unique relativistic theory; that is to say, the relativistic bootstrap is not uniquely determined by its static limit.

We cannot, of course, deny that there are many faults of the static model. The old original static model of pion-nucleon scattering[6] did not take into account pion-pion interactions, and we have also, for simplicity, not included the meson-exchange poles. We omit them because their inclusion would require us to discuss tri-meson vertices also, and the additional bootstrap equations which would result. These omitted meson-exchange poles would also lead to s-wave scattering, thereby further complicating the model. While these features may be necessary to make meaningful comparisons with experiment, they do not add appreciably to our understanding of either the interpretation of the $SU(6)$ group or the content of the bootstrap equations.

Baryon bootstrap models within the static model are all based on the original Chew reciprocal bootstrap model,[7] which, incidentally, did include the ρ-meson exchange poles as well as the N, N^* poles in bootstrapping the N and N^*. Extensions of this model to $SU(3)$ symmetry were studied by many people.[8–14] In our own work, the paper of Lin and Cutkosky[15] has played a major guiding role. Their model, however, fails to be an $SU(6)$ theory because the vector mesons were omitted altogether; surprisingly enough, this did not affect matters very much, and $SU(6)$-like features were discovered in their baryon bootstrap.

Our primary purpose in setting up a bootstrap $SU(6)$ theory could be achieved in various ways. We could use the Chew-Low equations and the N/D method, or the so-called $Z=0$ method[16–18] for formulating the bootstrap dynamics instead of the Bethe-Salpeter equation. These alternative approaches in general are just as adequate, but, in the particular case of static baryon-meson scattering, the N/D approach happens to have some undesirable features recently discussed by Sawyer[19]; the discussion of symmetry-breaking in particular when carried out in the N/D formalism runs counter to simple intuition, whereas the Bethe-Salpeter equation appears not to suffer in this respect. This is primarily due to the use of standard approximations and it is not a criticism of the N/D method in principle, but only of the N/D method in practice. The Bethe-Salpeter equation and the $Z=0$ methods have in common a close relation to Lagrangian field theory. We believe that calculations using the Bethe-Salpeter equation have the advantage of having a clear analog with corresponding calculations based on the Schrödinger equation, on which is based most of our intuition regarding compositeness.

The formalism developed in the present paper can be used to discuss a number of interesting questions, including the calculation of the relation between meson and baryon mass differences, and coupling constants in the case of broken symmetry. One can also calculate the ratios of magnetic moments of the baryons as well as their magnitudes, without resorting to a quark model. The ratio $-\frac{3}{2}$ for the proton to neutron magnetic moment is not automatic in bootstrap $SU(6)$ theory, and one can hope to improve upon this result. The arguments presented in the paper of Belinfante and Cutkosky[3] can actually lead to a result between $-11/8$ and $-\frac{3}{2}$, depending on what is assumed about the V-P-γ vertex (that is, the magnetic dipole transition rate for vector mesons to decay into pseudoscalar mesons). The result $-11/8$ corresponds to setting this rate equal to zero.

We now proceed to outline the contents of the remainder of the paper. In Sec. II, the vertices required in order to extend the static model to include the baryon isobars and the vector mesons are presented. It is here that the assignment ambiguity alluded to above is discussed. The construction of an $SU(6)$-invariant Lagrangian formalism from these vertices is presented in Sec. III. This Lagrangian is used in Sec. IV in deriving the Bethe-Salpeter equation for the wave function representing the baryons as bound states of the mesons and baryons. From this Bethe-Salpeter equation is derived the vertex equation, which is one of the two basic equations in the bootstrap approach of Cutkosky and Leon.[20,21] Approximate solutions of the Bethe-Salpeter vertex equation in the symmetry limit are also presented in this section, and it is shown that these solutions lead to a successful bootstrap. The other equation, the normalization equation, is derivable from the off-shell scattering amplitude, as is shown in the Appendix.

II. EXTENSION OF THE STATIC MODEL

In order to extend the static model to include baryon isobars and vector mesons, we must consider the

[6] G. F. Chew and F. E. Low, Phys. Rev. **101**, 1570 (1956).
[7] G. F. Chew, Phys. Rev. Letters **9**, 233 (1962).
[8] R. E. Cutkosky, J. Kalckar, and P. Tarjanne, Phys. Letters **1**, 93 (1962).
[9] R. E. Cutkosky, J. Kalckar, and P. Tarjanne, in *Proceedings of the International Conference on High-Energy Physics at CERN, 1962*, edited by J. Prentki (CERN, Scientific Information Service, Geneva, Switzerland, 1962), p. 653.
[10] A. W. Martin and K. C. Wali, Phys. Rev. **130**, 2445 (1963).
[11] R. E. Cutkosky, Ann. Phys. (N. Y.) **23**, 415 (1963).
[12] I. S. Gerstein and K. T. Mahanthappa, Nuovo Cimento **32**, 239 (1964).
[13] R. H. Capps, Nuovo Cimento **34**, 932 (1964).
[14] B. M. Udgaonkar, *High-Energy Physics and Elementary Particles* (International Atomic Energy Agency, Vienna, 1965), p. 791.
[15] K. Y. Lin and R. E. Cutkosky, Phys. Rev. **140**, B205 (1965).
[16] B. Jouvet, Nuovo Cimento **5**, 1 (1957).
[17] M. T. Vaughn, R. Aaron, and R. D. Amado, Phys. Rev. **124**, 1258 (1961).
[18] R. M. Rockmore, Phys. Rev. **132**, 878.
[19] R. F. Sawyer, Phys. Rev. **142**, 991 (1966).
[20] R. E. Cutkosky and M. Leon, Phys. Rev. **135**, B1445 (1964).
[21] R. E. Cutkosky and M. Leon, Phys. Rev. **138**, B667 (1965).

additional vertices required. We will describe the static spin-$\frac{1}{2}$ baryons (B) by a two-component field operator $\psi=\sum \chi_s b^s$. Here, χ_s is a Pauli spinor and b^s is the annihilation operator in the spin state s. The spin-$\frac{3}{2}$ baryon isobars (B^*) will be described by the static limit of the Rarita-Schwinger field operator.[22] This operator is a vector-spinor, $\boldsymbol{\psi}$, and satisfies the auxiliary condition

$$\boldsymbol{\sigma}\cdot\boldsymbol{\psi}=0.$$

This condition implies a number of interesting relations, of which we mention but two:

$$\boldsymbol{\psi}=i\boldsymbol{\sigma}\times\boldsymbol{\psi},$$

$$\sum_{k=1}^{3}\psi_k^\dagger\boldsymbol{\sigma}\psi_k=-i\boldsymbol{\psi}^\dagger\times\boldsymbol{\psi}.$$

The pseudoscalar (P) and vector (V) mesons are described by the field operators ϕ and $\mathbf{V}$, respectively. We will ignore any $SU(3)$ indices in this discussion of vertices.

The couplings which are possible and which eventually correspond to something easily accommodated by an $SU(6)$ model are two in number: Scalar-scalar, and axial-vector–axial-vector. There are, of course, tensor couplings which could be included, but at the expense of additional complications which we wish to avoid.

From the fields considered, one can form three independent scalars, and five axial vectors, which are listed in Table I. From these can be formed a total of

TABLE I. Scalars and axial vectors in a static model.

Type	Scalars	Axial vectors
BB	$\psi^\dagger\psi$	$\psi^\dagger\boldsymbol{\sigma}\psi$
BB^*	$\cdots$	$\psi^\dagger\boldsymbol{\psi}$
B^*B^*	$\boldsymbol{\psi}^\dagger\cdot\boldsymbol{\psi}$	$\boldsymbol{\psi}^\dagger\times\boldsymbol{\psi}$
P	$\cdots$	$\nabla\phi$
V	$\nabla\cdot\mathbf{V}$	$\nabla\times\mathbf{V}$

eight different baryon-meson vertices, which are compiled in Table II. From the latter table, it is seen that the configuration space operators $\phi(\mathbf{x})$ and $\mathbf{V}(\mathbf{x})$ appear in various combinations with the gradient ∇. It will prove to be more convenient to work in momentum space, and thus we will expand the fields ϕ and $\mathbf{V}$ in terms of plane-wave creation and annihilation operators:

$$\phi(\mathbf{x})=\int\frac{d^3k}{(2\pi)^3}\frac{1}{(2\omega_k)^{1/2}}[\varphi(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}+\text{H.c.}],$$

and

$$\mathbf{V}(\mathbf{x})=\int\frac{d^3k}{(2\pi)^3}\frac{1}{(2\omega_k)^{1/2}}[\mathbf{v}(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}+\text{H.c.}],$$

[22] W. Rarita and J. Schwinger, Phys. Rev. 60, 61 (1941); K. K. Gupta, Proc. Indian Acad. Sci. A35, 255 (1952).

TABLE II. Vertices occurring in a static model.

Type	Scalar-scalar	Axial-vector–axial-vector
BBP	$\cdots$	$\psi^\dagger\boldsymbol{\sigma}\psi\cdot\nabla\phi$
BBV	$\psi^\dagger\psi\nabla\cdot\mathbf{V}$	$\psi^\dagger\boldsymbol{\sigma}\psi\cdot\nabla\times\mathbf{V}$
BB^*P	$\cdots$	$\psi^\dagger\boldsymbol{\psi}\cdot\nabla\phi$
BB^*V	$\cdots$	$\psi^\dagger\boldsymbol{\psi}\cdot\nabla\times\mathbf{V}$
B^*B^*P	$\cdots$	$\boldsymbol{\psi}^\dagger\times\boldsymbol{\psi}\cdot\nabla\phi$
B^*B^*V	$\boldsymbol{\psi}^\dagger\cdot\boldsymbol{\psi}\nabla\cdot\mathbf{V}$	$\boldsymbol{\psi}^\dagger\times\boldsymbol{\psi}\cdot\nabla\times\mathbf{V}$

where we have set the time t equal to zero and have included normalization factors, such that the equal-time commutation relations for the creation and annihilation operators are

$$[\varphi(\mathbf{k}),\varphi^\dagger(\mathbf{k}')]=(2\pi)^3\delta(\mathbf{k}-\mathbf{k}'),$$

and

$$[\mathbf{v}(\mathbf{k}),\mathbf{v}^\dagger(\mathbf{k}')]=(2\pi)^3\delta(\mathbf{k}-\mathbf{k}')[1+\mathbf{kk}/m^2].$$

The additional factor in the commutation relation for the vector-meson operators is a familiar relativistic one, which we retain since the vector mesons are not assumed to be slowly moving.

The Fourier transform of the baryon spatial distribution function is assumed to be spherically symmetric. This being the case, the only quantities which can depend on the direction of $\mathbf{k}$ are the meson operators in combination with $\mathbf{k}$. Thus, in the interaction Lagrangian, the meson operators will only occur in the combinations

$$\mathbf{a}_P(k)=(\sqrt{\tfrac{3}{2}})\frac{1}{(2\pi)^2}\int d\Omega_k\mathbf{k}\varphi(\mathbf{k}),$$

$$\mathbf{a}_V(k)=\tfrac{1}{2}\sqrt{3}\frac{1}{(2\pi)^2}\int d\Omega_k\mathbf{k}\times\mathbf{v}(\mathbf{k}),$$

and

$$a_S(k)=\frac{m}{\omega_k}(\sqrt{\tfrac{1}{2}})\frac{1}{(2\pi)^2}\int d\Omega_k\mathbf{k}\cdot\mathbf{v}(\mathbf{k}),$$

which can be recognized as being p-wave creation and annihilation operators. The element of solid angle for the variable $\mathbf{k}$ is represented by $d\Omega_k$, while $k=|\mathbf{k}|$. The normalization factors for the pseudoscalar, transverse vector, and longitudinal vector mesons are determined by requiring that the p-wave operators satisfy commutation relations of the form

$$[a(k),a^\dagger(k')]=\delta(k-k').$$

The choice of the phases of the normalization factors is physically irrelevant. Note that the factor

$$\omega_k=(k^2+m^2)^{1/2}$$

in the normalization factor combines with $\mathbf{k}\cdot\mathbf{v}$ to give $\mathbf{k}\cdot\mathbf{v}/\omega_k$, which from the subsidiary condition $\partial_\mu V^\mu(x)=0$, is just the time component of the vector-meson annihilation operator. A consideration of the static limit of the relativistic vertex $\bar{\psi}\gamma_\mu\psi V^\mu$ indicates that it is just this component of the field which should appear.

The occurrence of p waves only is, of course, the situation in the ordinary static model and continues to be the case in this extension, since the vector mesons have the same intrinsic parity as the pseudoscalar mesons.

The purpose of developing the present formalism is to have a concrete framework in which to discuss a model of a bootstrap of the meson-baryon system. The mesons will be classified according to the quantum numbers of the $SU(6)$ 35-dimensional representation,[23] which contains the $[SU(2), SU(3)]$ submodules (1,8), (3,8), and (3,1). The first of these is an $SU(3)$ octet having total angular momentum J equal to zero and is described by the p-wave operator $a_S^\alpha(k)$, $\alpha=1\cdots 8$, being an octet index. In the case of the two $J=1$ submodules, however, there is an ambiguity, since both $\mathbf{a}_P$ and $\mathbf{a}_V$ have $J=1$. In an effort to be as general as possible, we admit a linear combination of these; thus $\mathbf{a}_P^\alpha\cos\theta_8+\mathbf{a}_V^\alpha\sin\theta_8$ will describe the (3,8), while $\mathbf{a}_P\cos\theta_1+\mathbf{a}_V\sin\theta_8$ will describe the (3,1). We have chosen to call this mixing of operators, having the same $SU(2)$ and $SU(3)$ transformation properties, assignment mixing (AM), in contradistinction to ω-ϕ mixing, which is particle mixing (PM). In the special case of $\theta_1=\theta_8=0$, the model of Capps and of Belinfante and Cutkosky (CBC)[2–3] is recovered, while the angles for which

$$\tan\theta_1=\tan\theta_8=\sqrt{2}$$

yields the meson assignment of the W spin or collinear $SU(6)$.[24] This latter set of angles is derivable by considering a Fermi-Yang model of the mesons.[25] Still another choice of mixing angles is advocated by Singh and Udgaonkar,[26] namely, $\theta_8=0$, $\theta_1=90°$. In the work of Belinfante[27] and of Renninger and Videira,[28] which were based on the CBC model, this possibility of assignment mixing was omitted.

These angles cannot be determined from the bootstrap dynamics presented here. Therefore, in order to get an idea of the magnitudes of these angles, we must compare predictions of specific quantities involving these angles with the experimental values. For instance, the angles can be determined from the magnitudes of magnetic moments and from the cross sections for baryon isobar production from a two-baryon initial state. These cross sections can be predicted by combining the Sakurai-Stodolsky model with the present model for baryon-meson interactions.[29]

[23] One could also introduce a meson singlet, which would correspond to the spatial divergence of the ω-singlet vector meson field.

[24] H. J. Lipkin and S. Meshkov, Phys. Rev. Letters **14**, 670 (1965).

[25] R. E. Cutkosky and M. Jacobs (private communication).

[26] V. Singh and B. M. Udgaonkar, Phys. Rev. **139**, B1585 (1965).

[27] J. G. Belinfante, Phys. Rev. **140**, B154.

[28] G. H. Renninger and A. L. L. Videira, Phys. Rev. **140**, B691 (1965).

[29] J. J. Sakurai and L. Stodolsky, Phys. Rev. Letters **11**, 90 (1963).

Note added in proof. A. Kanazawa, M. Saito, T. Sakuma, K. Seto, and N. Tokuda have recently obtained results similar to those presented in Sec. II of this paper. (Hokkaido University, Sapporo, Japan, unpublished report.)

III. $SU(6)$-INVARIANT LAGRANGIAN

By using the p-wave creation and annihilation operators, we can construct an $SU(6)$-invariant Lagrangian. We introduce a meson field operator ($\alpha=1\cdots 35$)

$$\phi_\alpha(k,t)=a_\alpha(k,t)+a_\alpha^\dagger(k,t)\,,$$

in terms of which we can write down a Lagrangian which is an obvious generalization of the one used to define the Chew-Low static model. In the following, $v(k)$ is the usual structure function for the baryons, which is normalized to unity at $k=0$. It is usually taken to be unity for $k<k_\Lambda$ and zero for $k>k_\Lambda$, where k_Λ is some cutoff momentum. In the limit of exact $SU(6)$, we may write the Lagrangian:

$$L=-\psi^\dagger(t)M\psi(t)+\tfrac{1}{2}i\psi^\dagger\frac{\overleftrightarrow{\partial}}{\partial t}\psi -\frac{1}{4}\int_0^\infty dk\left[\phi^\alpha(k,t)\omega_k\phi_\alpha(k,t)-\left(\frac{\partial\phi^\alpha}{\partial t}\right)\frac{1}{\omega_k}\left(\frac{\partial\phi_\alpha}{\partial t}\right)\right] -\frac{f}{m}\psi^\dagger G_\alpha\psi\int_0^\infty\frac{k^2dkv(k)}{\sqrt{\omega_k}}\phi^\alpha(k,t)\,,$$

where f is a dimensionless coupling constant, m the mesonic mass, and M the baryon mass. The G_α are the generators of $SU(6)$, which are normalized so that $G^2(56)=1$.

The entire Lagrangian, including both the kinetic part and the interaction part, is invariant under $SU(6)$. This is in contradistinction to certain earlier theories of $SU(6)$ in which it was believed that exact $SU(6)$ would require a zero meson-baryon coupling. This unpleasant feature arose from an attempt to interpret the $SU(2)$ subgroup of $SU(6)$ strictly as the rotation group, so that the generator J would always be the spin. In the model discussed in the present paper, however, this is not the case for the mesons, since the generator J here includes both the spin and the p-wave orbital angular momentum.

If one wishes to discuss broken symmetry, the coupling constants $(f/m)G_\alpha/\sqrt{\omega_k}$ and the masses M and m should be regarded as matrices, rather than as numbers. The bootstrap equations then become matrix equations and are correspondingly more difficult to solve. Since the purpose of the present paper is primarily pedagogical, we prefer not to burden the exposition with the general case of broken symmetry. However, we do wish to emphasize that our model is most useful in discussing the problem of broken symmetry, and the results obtained for symmetry breaking are less sensitive to the cutoff parameter than the results we obtain below, which are rather strongly dependent on this parameter.

Capps[30] has pointed out that further terms in an $SU(6)$ Lagrangian which describe mesonic interactions can be constructed in a fashion similar to the way presented above for the meson-baryon vertices. Here one takes the vector mesons instead of the baryons as being infinitely heavy. We shall, for the sake of simplicity, omit these further complications of the static model.

IV. BETHE-SALPETER EQUATIONS

In order to formulate the bootstrap hypothesis in a transparent way, we will write down a Bethe-Salpeter equation expressing the intuitive idea that any baryon is a bound state of baryons and mesons.

We begin by defining a vertex function Γ related to the Bethe-Salpeter amplitude Ψ by $\Psi=G_0\Gamma$, where G_0 is the product of the meson and baryon propagators. We also introduce a conjugate amplitude $\tilde{\Psi}$ and a corresponding vertex function $\tilde{\Gamma}$. These are defined as follows:

$$\frac{ik^2v(k)}{\sqrt{\omega_k}}\left(\frac{1}{\omega}\right)\left(\frac{2\omega_k}{\omega_k{}^2-\omega^2}\right)\Gamma(k,\omega)G^\alpha\chi_m =\int_{-\infty}^{\infty} dt\, e^{i\omega t}\langle 0|T\phi^\alpha(k,t)\psi(0)|B_m\rangle$$

and

$$\frac{ik^2v(k)}{\sqrt{\omega_k}}\left(\frac{1}{\omega}\right)\left(\frac{2\omega_k}{\omega_k{}^2-\omega^2}\right)\tilde{\Gamma}(k,\omega)(\chi^\dagger)^mG_\alpha =\int_{-\infty}^{\infty} dt\, e^{-i\omega t}\langle B^m|T\phi_\alpha(k,t)\psi^\dagger(0)|0\rangle.$$

In the above equations we have used the Wigner-Eckart theorem and the lack of multiplicity in the $SU(6)$ coupling $35\otimes56\supset56$ to extract the group-theoretic structure, which is summarized by the $SU(6)$ indices $\alpha=1\cdots35$ and $m=1\cdots56$. Upon examination of the Bethe-Salpeter equations, one concludes that Γ and $\tilde{\Gamma}$ do not in fact depend on k at all, and hence we may write them as functions of ω only. These functions have been normalized so that

$$\Gamma(\omega=0)=\tilde{\Gamma}(\omega=0)=f/m=g.$$

In the ladder approximation, the Bethe-Salpeter equation (see Fig. 1) for $\Gamma(\omega)$ may be written ($\Gamma=VG_0\Gamma$):

$$\Gamma(\omega)=\int_0^\infty p^4dp\, v^2(p)\frac{Rg^2}{(\omega+\omega_p)}\frac{\Gamma(\omega_p)}{\omega_p{}^2},$$

where R is a Racah coefficient having the value 11/15.

[30] R. H. Capps, Phys. Rev. Letters **14**, 842 (1965).

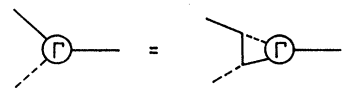

FIG. 1. Schematic representation of the Bethe-Salpeter equation. The solid lines represent baryons, while the dashed ones represent mesons carrying energy ω.

We have set $G^2(56)=1$, so that

$$\begin{aligned}G_\alpha G_\lambda G^\alpha &= G^2(56)[1-\tfrac{1}{2}G^2(35)/G^2(56)]G_\lambda\\ &=RG_\lambda.\end{aligned}$$

The Bethe-Salpeter equation for the vertex can be solved approximately. For example, if we take $v(p)=\theta(k_\Lambda-p)$ as usual, then an approximate solution can be obtained by replacing $p^4v^2(p)$ by a δ function $(k_\Lambda{}^5/5)\delta(p-k_\Lambda)$. This yields

$$\Gamma(\omega)\approx g/[1+(\omega/\omega_\Lambda)],$$

from which we find

$$\frac{1}{g^2}\approx R\int_0 \frac{p^4dp\, v^2(p)}{\omega_p{}^3}\left(\frac{\omega_\Lambda}{\omega_\Lambda+\omega_p}\right)\approx\frac{Rk_\Lambda{}^5}{10\omega_\Lambda{}^3}.$$

This is very similar to a well-known result obtained from the old-fashioned static model, except that the latter result involves the mass difference between the N^* and N, or, equivalently, the effective-range parameter of the Chew-Low theory.[31] It gives us one relation between the dimensionless coupling constant f and the dimensionless ratio m/ω_Λ. Thus, given an average meson mass and a cutoff, we can calculate the meson-baryon coupling constant.

The equation obtained here is the so-called vertex bootstrap equation in the terminology of Cutkosky and Leon.[20,21] In addition, there will be a normalization equation, which is discussed in detail in the Appendix. It is shown there that the normalization equation coincides with the vertex equation to within 1%, so that we do not get any additional condition, this being a peculiar feature of the exact symmetry limit.

APPENDIX

The derivation of the normalization equation is described in a paper by Cutkosky and Leon,[20,21] to which we have referred above. Here we shall only sketch a few of the details. The basic technique[32] is to make use of the off-shell scattering amplitude $h(\omega',\omega;\Omega)$, which is defined in the following way. From the four-point vacuum expectation value is subtracted the disconnected part. This is then Fourier transformed and the center of mass motion is extracted as $\delta(\Omega'-\Omega)$. After removing the k-dependent kinematic factors $k^2v(k)/\sqrt{\omega_k}$, the external legs of the diagram are amputated to get rid of the poles. All of these various

[31] E. M. Henley and W. Thirring, *Elementary Quantum Field Theory* (McGraw-Hill Book Company, Inc., New York, 1962), p. 213, Eq. (18.53).

[32] V. K. Agrawala, J. G. Belinfante, and G. H. Renninger, Nuovo Cimento (to be published).

operations can be summarized in a single equation defining h:

$$\int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 \int_{-\infty}^{\infty} dt_3 \int_{-\infty}^{\infty} dt_4 e^{i[\omega'(t_1-t_2)+(\Omega'+M)t_2-\omega(t_3-t_4)-(\Omega+M)t_4]}[\langle T\phi^\beta(k't_1)\psi(t_2)\phi_\alpha(kt_3)\psi^\dagger(t_4)\rangle_0-\langle T\phi\phi\rangle_0\langle T\psi\psi^\dagger\rangle_0]$$
$$=8\pi i\delta(\Omega'-\Omega)\frac{k^2k'^2v(k)v(k')(\omega_k\omega_{k'})^{1/2}}{(\omega_k{}^2-\omega^2)(\omega_{k'}{}^2-\omega'^2)(\Omega-\omega)(\Omega-\omega')}h_\alpha{}^\beta(\omega',\omega;\Omega).$$

In the limit $\omega=\omega'=\Omega$, this function $h(\omega,\omega';\Omega)$ reduces to the usual "on-shell" scattering amplitude $h(\omega)$ of the static model. In the ladder approximation, the Bethe-Salpeter equation for the function h (see Fig. 2) is

$$h_\alpha{}^\beta(\omega',\omega;\Omega)=\frac{g^2G_\alpha G^\beta}{\omega+\omega'-\Omega}+\int_0^\infty p^4dp\, v^2(p)\times\frac{g^2G_\lambda G^\beta h_\alpha{}^\lambda(\omega_p,\omega;\Omega)}{\omega_p(\omega_p-\Omega-i\epsilon)(\omega_p+\omega'-\Omega-i\epsilon)}.$$

We can write the off-shell scattering amplitude as a sum over bound states and resonances, plus a term which is analytic in Ω, by inserting a sum over intermediate states in the four-point expectation value. In particular, if we isolate the baryon contribution, we find

$$h_\alpha{}^\beta(\omega',\omega;\Omega)=-G^\beta G_\alpha\Gamma(\omega')\tilde{\Gamma}(\omega)/\Omega+\cdots.$$

We introduce an auxiliary eigenvalue problem by making an off-shell extension of the Bethe-Salpeter equation for $\Gamma(\omega)$:

$$\Gamma_n(\omega,\Omega)=\lambda_n(\Omega)Rg^2\int_0^\infty\frac{p^4dpv^2(p)\Gamma_n(\omega_p,\Omega)}{\omega_p(\omega_p-\Omega-i\epsilon)(\omega_p+\omega-\Omega-i\epsilon)},$$

where $\lambda_n(\Omega)$ is the eigenvalue, which is unity in the limit $\Omega\to 0$. Using the completeness of the eigenvectors Γ_n, we expand h as a linear combination of the Γ_n.

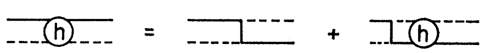

FIG. 2. Bethe-Salpeter equation for the scattering amplitude extrapolated off the baryon mass shell.

The coefficients are then determined by using the Bethe-Salpeter equations, and we find

$$h_\alpha{}^\beta(\omega',\omega;\Omega)=G^\beta G_\alpha\sum_n\frac{\Gamma_n(\omega',\Omega)\tilde{\Gamma}_n(\omega,\Omega)}{[\lambda_n(\Omega)-1]\langle\tilde{\Gamma}_n|G_0|\Gamma_n\rangle}+\cdots.$$

Comparing this with our other formula in the limit $\Omega\to 0$, we find, upon eliminating $\lambda_n'(0)$, the normalization equation

$$1=\langle\tilde{\Gamma}|G_0'|\Gamma\rangle+\langle\tilde{\Gamma}|G_0V'G_0|\Gamma\rangle,$$

where V is the potential, and primes denote differentiation with respect to Ω, evaluated at $\Omega=0$. More explicitly, the normalization equation is

$$1=\int_0^\infty k^4dk\, v^2(k)\frac{\tilde{\Gamma}(\omega_k)\Gamma(\omega_k)}{\omega_k{}^3}+Rg^2\int_0^\infty\int_0^\infty\frac{k^4dk\, v^2(k)p^4dpv^2(p)}{\omega_k{}^2\omega_p{}^2}\frac{\tilde{\Gamma}(\omega_k)\Gamma(\omega_p)}{(\omega_k+\omega_p)^2}.$$

Inserting our approximate solution and using the δ-function approximation for $k^4v^2(k)$, we obtain finally the quadratic equation

$$1=x+Rx^2,$$

where $x=k_\Lambda{}^5g^2/20\omega_\Lambda{}^3$. We recall that the vertex equation gave

$$x=1/2R.$$

These two equations yield exactly the same value for x provided that $R=\frac{3}{4}$. In our case, the Racah coefficient is $R=11/15\approx 0.733$, which is close to $\frac{3}{4}$, and hence the values of x obtained from the two equations are essentially the same.