which is consistent with the hypothesis that this amplitude be real. The generalization of the preceding analysis to the "complete" problem involving all two-particle channels is straightforward.

Further details of this type of self-consistency problem, which we note can be considered to be a special case of the multichannel approach of Lichtenberg and Williams,[28] will be published elsewhere. We merely note here that a preliminary (spin inclusive) calculation along these lines indicated that the ω-p elastic cross section is a few mb and is characterized by a strong forward diffraction peak. These results were obtained with the same parameters used in the calculations presented in Sec. III. We are thus inclined to believe that our approach in calculating absorptive amplitudes has some merit, which leaves the problem of photo-ρ production unsolved as yet.

[28] D. B. Lichtenberg and P. K. Williams, Phys. Rev. 139, B179 (1965).

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Existence of Spurious Solutions to Many-Body Bethe-Salpeter Equations*

PAUL G. FEDERBUSH†

Department of Physics and The Laboratory of Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts

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The many-body equations studied by Weinberg are applied to a particular system of three particles interacting with separable potentials. For certain values of the energy, it is found that the homogeneous equation has extraneous nonzero solutions.

A SET of equations has been proposed by Weinberg[1] to solve for the T-matrix of the many-body Schrödinger system. The equations are integral equations of Bethe-Salpeter type, with a connected energy-dependent kernel. For local square-integrable potentials the kernel is known to be Hilbert-Schmidt if the energy is not real.[2] In a certain case, we will show the existence of solutions of the homogeneous equation for nonreal energy. These, of course, do not correspond to bound states of the Schrödinger equation; each bound state does, however, correspond to a homogeneous solution. This situation indicates the care one must exercise generally in identifying homogeneous solutions of Bethe-Salpeter equations with bound states.

We consider a system of three particles interacting with separable potentials:

$$M_1=M_3=\tfrac{1}{2}\,,\quad M_2=\infty\,,$$
$$V_{13}=0\,,\quad V_{12}=V_1=\alpha\alpha\,,\quad V_{32}=V_2=-\alpha\alpha\,. \tag{1}$$

In this situation we write Weinberg's equation

$$A=\left(\frac{1}{E-H_1}V_1\frac{1}{E-H_0}V_2+\frac{1}{E-H_2}V_2\frac{1}{E-H_0}V_1\right)A+I, \tag{2}$$

with

$$H_0=K_1^2+K_2^2\,,\quad H_1=H_0+V_1\,,\quad H_2=H_0+V_2\,. \tag{3}$$

I, the inhomogeneous term, does not concern us; A is related to the connected part of the T-matrix. We will use the familiar expression for the resolvent involving a single potential

$$\frac{1}{E-H_1}(K_1,K_2,K_1',K_2')=\delta^3(K_2-K_2')\left[\frac{1}{E-K_1^2-K_2^2}\delta^3(K_1-K_1')+\frac{1}{E-K_1^2-K_2^2}\alpha(K_1)\frac{1}{E-K_1'^2-K_2^2}\times\alpha(K_1')\frac{1}{1-h(E,K_2^2)}\right], \tag{4}$$

$$h(E,K_2^2)=\int d^3K\,\alpha(K)^2\frac{1}{E-K_2^2-K^2}\,. \tag{5}$$

Defining

$$B(K_1,K_2)=\frac{A(K_1,K_2)(E-K_1^2-K_2^2)}{\alpha(K_1)\alpha(K_2)} \tag{6}$$

(in our solution A will equal zero when $\alpha=0$), the

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[1] Steven Weinberg, Phys. Rev. 133, B232 (1964).

[2] W. Hunziker, Phys. Rev. 135, B800 (1964).

homogeneous part of Eq. (2) becomes

$$B(x,y)=-\frac{1}{1-h(E,y)}\int d^3x_1d^3y_1\frac{\alpha(x_1)^2\alpha(y_1)^2}{E-x_1^2-y^2}\times\frac{1}{E-x_1^2-y_1^2}B(x_1,y_1)-\frac{1}{1+h(E,x)}\int d^3x_1d^3y_1\times\frac{\alpha(x_1)^2\alpha(y_1)^2}{E-x^2-y_1^2}\frac{1}{E-x_1^2-y_1^2}B(x_1,y_1)\,. \quad (7)$$

$\alpha(x)$ is now further specified

$$\alpha(x)=0\,,\quad x<K_0 \quad\text{or}\quad x>K_0+a\,,$$
$$\alpha(x)=\left(\frac{\lambda}{4\pi ax^2}\right)^{1/2},\quad K_0<x<K_0+a\,. \quad (8)$$

a and λ are parameters. From now on all variables x and y are limited to the range $K_0<x$, $y<K_0+a$. As motivation, we consider the limit $\alpha\to 0$ when $\alpha(x)\to(\lambda/4\pi K_0^2)^{1/2}\delta^{1/2}(x-K_0)$. This limit does not represent a potential. Equation (7) then becomes an algebraic relation

$$1=-\frac{1}{1-\lambda/(E-2K_0^2)}\frac{\lambda^2}{E-2K_0^2}\frac{1}{E-2K_0^2}-\frac{1}{1+\lambda/(E-2K_0^2)}\frac{\lambda^2}{E-2K_0^2}\frac{1}{E-2K_0^2}\,. \quad (9)$$

There are two values of E solving this equation:

$$E=2K_0^2\pm i\lambda\,. \quad (10)$$

We now show that for any δ there is an ϵ such that Eq. (7) has a solution with $|E-2K_0^2+i\lambda|<\delta$ if $a<\epsilon$. Writing Eq. (7) as

$$KB=B\,, \quad (11)$$

we define K_0 as

$$K_0(x,y,x',y')=(1/\lambda^2)\alpha(x)\alpha(y)\alpha(x')\alpha(y') \quad (12)$$

and write Eq. (11) as

$$(K_0+\delta K)B=B \quad (13)$$

or

$$B=[1/(1-\delta K)]K_0B\,. \quad (14)$$

In any region of energy a finite distance from the real-energy axis, the operator norm of δK approaches zero as $a\to 0$, so Eq. (14) is well defined ($1-\delta K$ has an inverse). From Eq. (14) one sees that if Eq. (11) has a solution, it is of the form

$$B=C[1/(1-\delta K)]\alpha\alpha \quad (15)$$

with C an arbitrary constant. Equation (11) is then equivalent to the relation

$$1=[(1/\lambda^2)]\left(\alpha\alpha\frac{1}{1-\delta K}\alpha\alpha\right). \quad (16)$$

Equation (11) has a solution if and only if Eq. (16) has a solution.

Calling the right-hand side of Eq. (16) $R(E,a)$, one easily shows: (1) for sufficiently small a, R is analytic in E in a neighborhood N (independent of a), of $E=2K_0^2-i\lambda$. (2) $R\to F(E)$ on N uniformly as $a\to 0$, with $F(E)$ analytic, $F(2K_0^2-i\lambda)=1$, and $(dF/dE)(2K_0^2-i\lambda)\neq 0$. The existence of a solution to Eq. (16) for small enough a follows from Rouché's theorem.

The example can be generalized trivially to the case when M_2 is sufficiently large but not infinite, and V_{13} is sufficiently small but not zero, by including these terms in δK in the above argument. It is natural to seek similar examples involving only local potentials (assuming they exist). Presumably these are more difficult to find, because it would seem to require approximation of the corresponding K by a separable K_0 of rank greater than one. The generalization of Eq. (16) is

$$\mathrm{Det}(\delta_{ij}-u_i[1/(1-\delta K)]u_j)=0\,, \quad (17)$$

where the equation to be solved is $KB=B$ and $K=\Sigma u_iu_i+\delta K$. If δK is made sufficiently small, under favorable circumstances Rouché's theorem may again be used to verify the presence of a homogeneous solution. Of course, other methods may be used to find such solutions.

Finally, we should state that the presence of the homogeneous solutions we have found in no significant way limits the utility of Weinberg's equations. Coming as they do at isolated values of the energy, such solutions can be handled by a variety of techniques.