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Reciprocity Relations In Photopion Reactions*

N. CHRIST† AND T. D. LEE
Department of Physics, Columbia University, New York, New York
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The reciprocity relation in the photopion processes $\gamma+N \rightleftarrows \pi+N'$ as a possible test of C_{st} and T_{st} invariances of the electromagnetic interaction is discussed, where N can be any nucleus or nucleon. The sensitivity of such a test is estimated for the special case $N=n$ or p in the energy range near the $(\frac{3}{2},\frac{3}{2})$ resonance.

I. INTRODUCTION

At present, there is good experimental evidence that the strong interaction H_{st} is separately invariant under the space inversion P_{st}, the time-reversal T_{st} and the particle antiparticle conjugation C_{st}; there is also strong evidence that the electromagnetic interaction H_γ is invariant under the same space-inversion operation P_{st} and the product $(C_{st}P_{st}T_{st})$. However, it was realized quite recently[1] that there exists, as yet, no good evidence that H_γ is, or is not, invariant under C_{st} or T_{st}. Throughout this paper, for clarity we use the subscript "st" to denote the particular choices of these discrete symmetry operators that are determined by the strong interaction alone.

In a previous paper[2] (hereafter called 1), it was pointed out that a systematic way to study the question of T_{st} invariance of H_γ, over a wide range of energy momentum transfer, is to consider the inelastic scattering of a charged lepton $l^\pm$ on a polarized target nucleon (or nucleus) N:

$$l^\pm+N \rightarrow l^\pm+\Gamma\,, \qquad (1)$$

where $l=e$, or μ and $\Gamma\neq N$, but otherwise Γ can be any complex of strongly interacting particles. In this paper, we wish to call attention to a related possible test of T_{st} invariance of H_γ: the reciprocity relation for reactions

$$\gamma+N \rightarrow \pi+N' \qquad (2)$$

and

$$\gamma+N \leftarrow \pi+N'\,, \qquad (3)$$

where N and N' can be any two nuclei (or nucleons). As examples of (2) and (3), we may mention

$$\gamma+n \rightleftarrows \pi^-+p\,, \qquad (4)$$

$$\gamma+p \rightleftarrows \pi^++n\,, \qquad (5)$$

$$\gamma+\mathrm{H}^3 \rightleftarrows \pi^-+\mathrm{He}^3\,, \qquad (6)$$

and

$$\gamma+\mathrm{He}^3 \rightleftarrows \pi^++\mathrm{H}^3\,. \qquad (7)$$

That time-reversal symmetry implies reciprocity relations is, of course, well known.[3,4] The purpose of this paper is to examine the sensitivity of reciprocity relations as possible tests of T_{st} invariance of H_γ, assuming that the strong interaction is invariant under T_{st}. We observe that in photon reactions without meson production (or absorption), e.g., $\gamma+d \rightleftarrows n+p$, the nucleons are, or almost are, on the mass shell. Thus, except for those processes which actually correspond to a photopion production with the pion subsequently absorbed by the nucleon, reciprocity relations for these reactions hold to a good approximation provided that the strong interaction satisfies time-reversal invariance; such reciprocity relations are relatively insensitive to the transformation property of H_γ under time reversal.

Another well-studied reciprocity relation is reaction (4) at the zero-pion-energy limit.[5] This relation is usually expressed in terms of the Panofsky ratio, the S-wave pion-nucleon scattering lengths and the threshold limit of the photopion-production cross section. As will be shown in the next section, such a relation can be derived by using only the Hermiticity property of H_γ, and is independent of the transformation property of H_γ under time reversal, provided all higher order terms in the fine-structure constant are neglected.

However, at high energy both reactions (4) and (5) can be used as good tests of T_{st} invariance of the electromagnetic interactions. Some estimations of the sensitivity of these tests are given in Sec. III. In principle, reactions (6) and (7) can also be used as tests of T_{st} invariance; but, their cross sections are not known, and it is difficult to estimate their sensitivities.

It is clear that the matrix elements for reactions (2) and (3) are closely related to those for reaction (1). The relations between these two different tests of time reversal invariance are discussed in Sec. II.

II. GENERAL DISCUSSIONS

In this section, we will consider reactions (2) and (3). Let $\mathbf{k}$, λ and $\mathbf{k}'$, λ' be, respectively, the momentum and helicity of N and N' in the center-of-mass system, and $\mathbf{e}$ is the polarization of γ. [Helicity is defined to be

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[1] J. Bernstein, G. Feinberg and T. D. Lee, Phys. Rev. **139**, B1650 (1965).
[2] N. Christ and T. D. Lee, Phys. Rev. **143**, 1310 (1966).
[3] J. A. Wheeler, Phys. Rev. **52**, 1107 (1937).
[4] The application of time reversal to the photopion process was first discussed by K. M. Watson, Phys. Rev. **95**, 228 (1954). See also other references mentioned in Watson's paper.
[5] For a recent discussion, see V. K. Samaranayake and W. S. Woolcock, Phys. Rev. Letters **15**, 936 (1965).

the spin component in units of $\hbar$ along the direction of motion.] The differential cross sections for reactions (2) and (3) are, respectively, proportional to

$$R_{\gamma\to\pi}=\sum_{\lambda,\lambda',\mathbf{e}} |\langle(\pi N')_{\mathbf{k}',\lambda'}{}^{-}|\mathbf{e}\cdot\mathcal{J}|N_{\mathbf{k},\lambda}\rangle|^2 \quad (8)$$

and

$$R_{\pi\to\lambda}=\sum_{\lambda,\lambda',\mathbf{e}} |\langle N_{\mathbf{k},\lambda}|\mathbf{e}\cdot\mathcal{J}|(\pi N')_{\mathbf{k}',\lambda'}{}^{+}\rangle|^2, \quad (9)$$

where $\mathcal{J}$ is the space component of the electromagnetic current operator of the hadrons, $|N_{\mathbf{k},\lambda}\rangle$ is a single-particle eigenstate of H_{st}, $|(\pi N')_{\mathbf{k}',\lambda'}{}^{+}\rangle$ is the *outgoing* eigenstate of H_{st} and $|(\pi N')_{\mathbf{k}',\lambda'}{}^{-}\rangle$ the corresponding *incoming* eigenstate. For definiteness, we choose the coordinate system such that

$$\mathbf{k}\parallel z \text{ axis}$$

and

$$\mathbf{k}' \text{ is in the } (x,z) \text{ plane}. \quad (10)$$

Since H_{st} satisfies T_{st} invariance, one has

$$T_{\mathrm{st}}\exp(-i\pi J_y)|N_{\mathbf{k},\lambda}\rangle=|N_{\mathbf{k},\lambda}\rangle \quad (11)$$

and

$$T_{\mathrm{st}}\exp(-i\pi J_y)|(\pi N')_{\mathbf{k}',\lambda'}{}^{\pm}\rangle=|(\pi N')_{\mathbf{k}',\lambda'}{}^{\mp}\rangle, \quad (12)$$

where J_y is the y component of the total-angular-momentum operator.[6] Following the notations of paper 1, we may decompose the electromagnetic-current operator $\mathcal{J}_\mu$ of the hadrons into a sum

$$\mathcal{J}_\mu=J_\mu+K_\mu, \quad (13)$$

where

$$C_{\mathrm{st}}\mathcal{J}_\mu C_{\mathrm{st}}^{-1}=-J_\mu+K_\mu \quad (14)$$

and

$$T_{\mathrm{st}}\mathcal{J}_\mu T_{\mathrm{st}}^{-1}=-J_\mu+K_\mu. \quad (15)$$

The question of T_{st} (or C_{st}) invariance of H_γ is to study whether $K_\mu=0$ or not.

It is useful to define the asymmetry parameter

$$\alpha(k,\theta)=(R_{\gamma\to\pi}-R_{\pi\to\gamma}/R_{\gamma\to\pi}+R_{\pi\to\gamma}), \quad (16)$$

where θ is the angle between $\mathbf{k}$ and $\mathbf{k}'$, and k is the magnitude of $\mathbf{k}$. From the above expressions (8)–(16), it is clear that if $\alpha(k,\theta)\neq 0$, then $K_\mu\neq 0$ and, therefore, H_γ violates T_{st} and C_{st} invariances.

We note that the outgoing and incoming states of the $\pi N'$ system are related to each other by the strong-interaction S matrix:

$$\langle\mathbf{k}'',\lambda''|S|\mathbf{k}',\lambda'\rangle=\langle(\pi N')_{\mathbf{k}'',\lambda''}{}^{-}|(\pi N')_{\mathbf{k}',\lambda'}{}^{+}\rangle.$$

For the special case where N and N' in reactions (2) and (3) are both single-nucleon states, since all pion-nucleon scattering phase shifts $\to 0$ at zero energy, we find, at $\mathbf{k}'=\mathbf{k}''=0$

$$\langle 0,\lambda''|S|0,\lambda'\rangle=\delta_{\lambda\lambda'}, \quad (17)$$

where $\delta_{\lambda\lambda'}$ is the Kronecker δ symbol. Thus, at zero pion energy, $\alpha(k,\theta)=0$ for both reactions (4) and (5) independently of whether K_μ is, or is not, zero.

We will now derive the general expression of the asymmetry function $\alpha(k,\theta)$ for any nuclei (or nucleons) N, N', and at an arbitrary energy. In the collision $(\gamma+N)$, the final system Γ can be any one of the many possible states (called channels) of the strongly interacting particles. It is useful to choose $\Gamma=1, 2, 3,\cdots$ to denote the various *eigenchannels* of the strong-interaction S matrix; each eigenchannel Γ has a definite isospin I_Γ, a definite parity $\mathcal{P}_\Gamma$ and a definite total angular momentum j_Γ. The precise defintion of eigenchannel will be given in Appendix A. In terms of these eigenchannels the S matrix is a diagonal matrix and its diagonal matrix element is

$$\exp(2i\delta_\Gamma). \quad (18)$$

Just as in paper 1, for each of these channels Γ, we may select a *stationary* eigenstate of H_{st}, denoted by $|\lambda_\Gamma\rangle$ where λ_Γ is the z component of the total angular momentum. Since the strong interaction is assumed to be invariant under T_{st}, the phase factors of these eigenstates can always be chosen so that[6]

$$T_{\mathrm{st}}\exp(-i\pi J_y)|\lambda_\Gamma\rangle=|\lambda_\Gamma\rangle. \quad (19)$$

The explicit construction of $|\lambda_\Gamma\rangle$ and some of its properties are also given in Appendix A.

Let us consider first the transition:

$$\gamma+N\to\Gamma \quad (20)$$

in which N can be any nucleus (or nucleon), but its spin is assumed to be $\frac{1}{2}$. For any given final channel Γ, there are only two matrix elements [c.f. Eq. (32) of paper 1]:

$$F_\pm(\Gamma)=\mp\tfrac{1}{2}\langle\lambda_\Gamma=\tfrac{1}{2}\pm 1|\mathcal{J}_x(0)\pm i\mathcal{J}_y(0)|N_{\mathbf{k},\lambda=\frac{1}{2}}\rangle. \quad (21)$$

Through rotation invariance and space inversion, the matrix elements for transition from $\lambda=-\frac{1}{2}$ to $\lambda_\Gamma=-\frac{1}{2}\pm 1$ can be easily related to the same $F_\pm$ given by (21). If H_γ satisfies T_{st} invariance, then all $F_\pm(\Gamma)$ must be *relatively real.* The subscript $\pm$ in $F_\pm(\Gamma)$ denotes the helicity $\sigma=\pm 1$ of the photon.

From Eqs. (12) and (19), it follows that

$$\langle(\pi N')_{\mathbf{k}',\lambda'}{}^{-}|\lambda_\Gamma\rangle=\langle(\pi N')_{\mathbf{k}',\lambda'}{}^{+}|\lambda_\Gamma\rangle^* =\langle\lambda_\Gamma|(\pi N')_{\mathbf{k}',\lambda'}{}^{+}\rangle. \quad (22)$$

Furthermore, as will be shown in Appendix A, the phase of the matrix element (22) is related to the eigenvalue of the S matrix:

$$(22)=\text{real coefficient}\times\exp(i\delta_\Gamma). \quad (23)$$

Here, we have taken advantage of the special co-

[6] Throughout the paper, the relative phase between states of different helicities, or different z components of the angular momentum, is chosen according to the convention used in A. R. Edmonds, *Angular Momentum in Quantum Mechanics* (Princeton University Press, Princeton, New Jersey, 1957). We also use Edmonds' conventions for all Clebsch-Gordan coefficients.

ordinate system (10). Since the azimuthal angle ϕ of $\mathbf{k}'$ is zero, all spherical harmonics $Y_{lm}(\theta,\phi)$ are real.

By using (22), the rate $R_{\gamma\to\pi}$ can be expressed as a sum of four terms, corresponding respectively to the initial helicities $(\sigma,\lambda)=(1,\frac{1}{2})$, $(-1,\frac{1}{2})$, $(-1,-\frac{1}{2})$ and $(1,-\frac{1}{2})$:

$$2\sum_{\lambda'}\{|\sum_\Gamma \langle(\pi N')_{\mathbf{k}',\lambda'}{}^-|\lambda_\Gamma=\tfrac{3}{2}\rangle F_+(\Gamma)|^2 + |\sum_\Gamma \langle(\pi N')_{\mathbf{k}',\lambda'}{}^-|\lambda_\Gamma=-\tfrac{1}{2}\rangle F_-(\Gamma)|^2 + |\sum_\Gamma \langle(\pi N')_{\mathbf{k}',\lambda'}{}^-|\lambda_\Gamma=-\tfrac{3}{2}\rangle \eta_\Gamma F_+(\Gamma)|^2 + |\sum_\Gamma \langle(\pi N')_{\mathbf{k}',\lambda'}{}^-|\lambda_\Gamma=\tfrac{1}{2}\rangle \eta_\Gamma F_-(\Gamma)|^2\}, \quad (24)$$

where

$$\eta_\Gamma=\mathcal{P}_N\mathcal{P}_\Gamma \exp[i(j_N-j_\Gamma)],$$

$\mathcal{P}_N$ is the parity of N and $j_N=\frac{1}{2}$ is its spin. By using reflection symmetry with respect to the (x,z) plane, it can be readily verified that in (24) the first and the second terms are, respectively, identical with the third and the fourth terms. Hence,

$$R_{\pi\to\pi}=4\sum_{\lambda'}\{|\sum_\Gamma \langle(\pi N')_{\mathbf{k}',\lambda'}{}^-|\lambda_\Gamma=\tfrac{3}{2}\rangle F_+(\Gamma)|^2 + |\sum_\Gamma \langle(\pi N')_{\mathbf{k}',\lambda'}{}^-|\lambda_\Gamma=-\tfrac{1}{2}\rangle F_-(\Gamma)|^2\}. \quad (25)$$

By using (22), one finds

$R_{\pi\to\gamma}$ = same expression as (25), except changing
$$F_\pm(\Gamma)\to F_\pm{}^*(\Gamma). \quad (26)$$

It is convenient to introduce two *real* functions $r_+(\Gamma,\lambda')$ and $r_-(\Gamma,\lambda')$ which represent the magnitudes of the two matrix elements in (25):

$$\langle(\pi N')_{\mathbf{k}',\lambda'}{}^-|\lambda_\Gamma=\tfrac{3}{2}\rangle F_+(\Gamma)=r_+\exp[i(\delta_\Gamma+\xi_\Gamma{}^+)]$$

and

$$\langle(\pi N')_{\mathbf{k}',\lambda'}{}^-|\lambda_\Gamma=-\tfrac{1}{2}\rangle F_-(\Gamma)=r_-\exp[i(\delta_\Gamma+\xi_\Gamma{}^-)], \quad (27)$$

where $r_\sigma=r_\sigma(\Gamma,\lambda')$ also depends on the momentum $\mathbf{k}'$ ($\sigma=+$ or $-$). In terms of r_σ and $\xi_\Gamma{}^\sigma$, $R_{\gamma\to\pi}$ and $R_{\pi\to\gamma}$ are given by

$$(R_{\gamma\to\pi}-R_{\pi\to\gamma})=-8\sum r_\sigma(\Gamma,\lambda')r_\sigma(\Gamma',\lambda') \times\sin(\delta_\Gamma-\delta_{\Gamma'})\sin(\xi_\Gamma{}^\sigma-\xi_{\Gamma'}{}^\sigma) \quad (28)$$

and

$$(R_{\gamma\to\pi}+R_{\pi\to\gamma})=8\sum r_\sigma(\Gamma,\lambda')r_\sigma(\Gamma',\lambda') \times\cos(\delta_\Gamma-\delta_{\Gamma'})\cos(\xi_\Gamma{}^\sigma-\xi_{\Gamma'}{}^\sigma), \quad (29)$$

where the sum extends over all Γ, Γ', λ', and $\sigma=\pm$. The asymmetry function $\alpha(k,\theta)$ is given by the ratio of these two expressions. In this derivation, all higher order terms in the fine-structure constant are neglected.

Remarks

(i) If H_γ satisfies $T_{\rm st}$ invariance, then $\xi_\Gamma{}^\pm=0$, or π; therefore, $\alpha(k,\theta)=0$.

(ii) As already mentioned, for the zero-energy pion-nucleon system all phase shifts $\delta_\Gamma=0$. Thus, independent of whether $\xi_\Gamma{}^\pm=0$ or not, the asymmetry function $\alpha(k,\theta)=0$.

(iii) As discussed in Paper 1, the same functions $F_\pm(\Gamma)$, but at different (four-momentum transfer),[2] also appear in the various tests of $T_{\rm st}$ invariance for reaction (1). The introduction of these stationery states $|\lambda_\Gamma\rangle$ makes it possible to give a relatively clean separation of the effects of $T_{\rm st}$ invariance of the strong interaction from those of the electromagnetic interaction. Although (28) and (29) are derived for the special case $j_N=\frac{1}{2}$, it is clear that almost identical expressions can be obtained for any arbitrary j_N.

(iv) If in the sum (28), only one particular state Γ, say the $(\frac{3}{2},\frac{3}{2})$ pion-nucleon resonant state, contributes, then $\alpha(k,\theta)$ is also zero. It is important to note that in terms of the usual multipole moments there are two matrix elements $E_{1+}{}^{3/2}$ and $M_{1+}{}^{3/2}$ for the transition from a γ-nucleon system to the $(\frac{3}{2},\frac{3}{2})$ resonance, where $E_{1+}{}^{3/2}$ and $M_{1+}{}^{3/2}$ are, respectively, the electric quadrupole and the magnetic dipole matrix elements. [The explicit relations between these multipole moments and $F_\pm(\Gamma)$ are given by Eq. (37) in the next section.] These two matrix elements are relatively real, if H_γ satisfies $T_{\rm st}$ invariance. However, Eq. (28) shows that even though $E_{1+}{}^{3/2}$ and $M_{1+}{}^{3/2}$ may be relatively complex (which violates $T_{\rm st}$ invariance), their relative phase does *not* contribute to any violation of the reciprocity relation, provided one averages over the polarization of the photon. The same result also applies to the $T_{\rm st}$-symmetry-violating relative phase between either $E_{l-}{}^I$ and $M_{l-}{}^I$, or $E_{l+}{}^I$ and $M_{l+}{}^I$, for any orbital angular moment l and isospin I of the pion-nucleon system.

It will be shown in the next section that for the actual case of reactions (4) and (5) near the $(\frac{3}{2},\frac{3}{2})$ resonance, a magnitude of $|\alpha(k,\theta)|\sim 10\%$ is quite compatible with our present knowledge of these transition amplitudes.

(v) Reactions (4) and (5), or reactions (6) and (7), are related to each other by the charge symmetry operators $\exp(i\pi\mathbf{I}_y)$ where I_y is the y component of the total isospin operator. In order to derive the relations between the matrix elements $F_\pm$ of such related reactions, we may decompose

$$\mathcal{J}_\mu=(\mathcal{J}_\mu)_{\rm even}+(\mathcal{J}_\mu)_{\rm odd}, \quad (30)$$

where

$$\exp(i\pi\mathbf{I}_y)(\mathcal{J}_\mu)_{\rm even}\exp(-i\pi\mathbf{I}_y)=(\mathcal{J}_\mu)_{\rm even}$$

and

$$\exp(i\pi\mathbf{I}_y)(\mathcal{J}_\mu)_{\rm odd}\exp(-i\pi\mathbf{I}_y)=-(\mathcal{J}_\mu)_{\rm odd}.$$

For example, the usual isoscalar current belongs to

$(\mathcal{J}_\mu)_{\rm even}$ and the isovector current belongs to $(\mathcal{J}_\mu)_{\rm odd}$. Similarly, the matrix elements $F_\pm(\Gamma)$ can be decomposed into two terms:

$$F_\pm(\Gamma)=[F_\pm(\Gamma)]_{\rm even}+[F_\pm(\Gamma)]_{\rm odd}\,, \tag{31}$$

where $[F_\pm(\Gamma)]_{\rm even}$ and $[F_\pm(\Gamma)]_{\rm odd}$ are given by Eq. (21), but replacing $\mathcal{J}_\mu$ by, respectively, $(\mathcal{J}_\mu)_{\rm even}$ and $(\mathcal{J}_\mu)_{\rm odd}$.

To study the relations between a transition $\gamma+N\rightarrow\Gamma$ and its charge symmetrical transition

$$\gamma+N^c\rightarrow\Gamma^c\,, \tag{32}$$

where N^c and Γ^c are related to N and Γ by $\exp(i\pi\mathbf{I}_y)$, it is convenient to first fix the relative phase between the two state vectors $|I,I_z\rangle$ and $|I,-I_z\rangle$ within any isospin multiplet. Here, $I(I+1)$ and I_z denote the eigenvalues of the operators $\mathbf{I}^2$ and $\mathbf{I}_z$. We choose

$$\exp(i\pi\mathbf{I}_y)|I,I_z\rangle=\exp[i\pi(I+I_z)]|I,-I_z\rangle\,. \tag{33}$$

The matrix elements $F_\pm$ of transitions (20) and (32) are, then, related by

$$[F_\pm(\Gamma)]_{\rm even}=\zeta[F_\pm(\Gamma^c)]_{\rm even} \tag{34}$$

and

$$[F_\pm(\Gamma)]_{\rm odd}=-\zeta[F_\pm(\Gamma^c)]_{\rm odd}\,, \tag{35}$$

where ζ is a phase factor depending on the isospin eigenvalues I_N, $(I_z)_N$, I_Γ, $(I_z)_\Gamma$ of the states N and Γ,

$$\zeta=\exp\{i\pi[I_\Gamma+(I_z)_\Gamma-I_N-(I_z)_N]\}\,. \tag{36}$$

III. SPECIAL CASE: $N=n$ or p

In this section, we discuss reactions (4) and (5) within an energy range below the 2π threshold. The channel Γ denotes, then, only the pion-nucleon system in various j_Γ, $\mathcal{P}_\Gamma$, and I_Γ states. Let l be the orbital angular momentum of the pion-nucleon system. One has $j_\Gamma=l\pm\frac{1}{2}$ and $\mathcal{P}_\Gamma=-(-1)^l$, assuming that $\mathcal{P}_N=1$. Reactions (4) and (5) have been extensively studied in the literature,[7,8] and it is customary to express the differential cross sections in terms of the electric and magnetic multipole moments $E_{l\pm}{}^I$ and $M_{l\pm}{}^I$. As will be shown in Appendix B, these multipole moments are simply related to the form factors $F_\pm(\Gamma)$:

$$\begin{aligned}F_\pm(\Gamma)\exp(i\delta_\Gamma)=&(R/\pi)^{1/2}k^{3/2}(2\pi)\\ &\times(l+1)^{1/2}\{[(l+2)(l+1\mp1)]^{1/2}E_{l+}{}^I\\ &\qquad\mp[l(l+1\pm1)]^{1/2}M_{l+}{}^I\},\ \text{if } j_\Gamma=l+\tfrac{1}{2}\\ &\times l^{1/2}\{[(l-1)(l\pm1)]^{1/2}E_{l-}{}^I\pm[(l+1)(l\mp1)]^{1/2}M_{l-}{}^I\},\\ &\qquad\text{if } j_\Gamma=l-\tfrac{1}{2}\end{aligned} \tag{37}$$

where the superscript $I=I_\Gamma$ and the factor $(R/\pi)^{1/2}$ are due to our convention that all state vectors are normalized to unity.[9]

For clarity, we will assume first that only the states $l=0$, 1 and $j_\Gamma=\frac{1}{2}$ or $\frac{3}{2}$ contribute; i.e., there are six pion-nucleon states: $\Gamma=s_I$, $p_{I\frac{1}{2}}$, and $p_{I\frac{3}{2}}$, where the subscript I is the isospin ($I=\frac{1}{2}$ or $\frac{3}{2}$), and the second subscript in $p_{I\frac{1}{2}}$ and $p_{I\frac{3}{2}}$ denotes the total angular momentum j_Γ. It follows from definition (21) that $F_+(\Gamma)=0$ for $j_\Gamma=\frac{1}{2}$. Thus, we have althgether eight functions $F_\pm(\Gamma)$, which can be expressed in terms of the usual two electric dipole moments $E_{0+}{}^I$, four magnetic dipole moments $M_{1\pm}{}^I$ and two electric quadrupole moments $E_{1+}{}^I$. By using (27)–(29) and (37), $\alpha(k,\theta)$ can be expressed in terms of $M_{l\pm}{}^I$, $E_{l\pm}{}^I$ and the $T_{\rm st}$ symmetry violating phases $\xi_\Gamma{}^\pm$. The derivation of these expressions is straightforward but tedious. We may write

$$\alpha(k,\theta)=\frac{(A-A')+(B-B')\cos\theta+(C-C')\cos^2\theta}{(A+A')+(B+B')\cos\theta+(C+C')\cos^2\theta}\,, \tag{38}$$

where $(A+B\cos\theta+C\cos^2\theta)$ is proportional to $R_{\gamma\rightarrow\pi}$ and $(A'+B'\cos\theta+C'\cos^2\theta)$ is proportional to $R_{\pi\rightarrow\gamma}$. The coefficients A, B and C are given by

$$\begin{aligned}A=&\tfrac{4}{3}|E_{0+}{}^{3/2}\mp\sqrt{2}E_{0+}{}^{1/2}|^2+3|(E_{1+}{}^{3/2}-M_{1+}{}^{3/2})\\ &\mp\sqrt{2}(E_{1+}{}^{1/2}-M_{1+}{}^{1/2})|^2\\ &+\tfrac{1}{3}|(3E_{1+}{}^{3/2}+M_{1+}{}^{3/2}+2M_{1-}{}^{3/2})\\ &\mp\sqrt{2}(3E_{1+}{}^{1/2}+M_{1+}{}^{1/2}+2M_{1-}{}^{1/2})|^2,\end{aligned} \tag{39}$$

$$\begin{aligned}B=&\tfrac{4}{3}[E_{0+}{}^{3/2}\mp\sqrt{2}E_{0+}{}^{1/2}]^*[3E_{1+}{}^{3/2}+M_{1+}{}^{3/2}-M_{1-}{}^{3/2}\\ &\mp\sqrt{2}(3E_{1+}{}^{1/2}+M_{1+}{}^{1/2}-M_{1-}{}^{1/2})]+\text{c.c.}\,,\end{aligned} \tag{40}$$

and

$$\begin{aligned}C=&\tfrac{4}{3}|3E_{1+}{}^{3/2}+M_{1+}{}^{3/2}-M_{1-}{}^{3/2}\\ &\mp\sqrt{2}(3E_{1+}{}^{1/2}+M_{1+}{}^{1/2}-M_{1-}{}^{1/2})|^2\\ &-3|E_{1+}{}^{3/2}-M_{1+}{}^{3/2}\mp\sqrt{2}(E_{1+}{}^{1/2}-M_{1+}{}^{1/2})|^2\\ &-\tfrac{1}{3}|3E_{1+}{}^{3/2}+M_{1+}{}^{3/2}+2M_{1-}{}^{3/2}\\ &\mp\sqrt{2}(3E_{1+}{}^{1/2}+M_{1+}{}^{1/2}+2M_{1-}{}^{1/2})|^2,\end{aligned} \tag{41}$$

where the upper sign is for reaction (4) and the lower sign is for reaction (5). The coefficients A', B', and C' are, respectively, given by the same expressions (39), (40), and (41), provided that $E_{l\pm}{}^I$ and $M_{l\pm}{}^I$ are replaced by $E_{\pm l}{}'^I$ and $M_{l\pm}{}'^I$, respectively, where

$$\begin{aligned}E_{\pm l}{}'^I\exp(-i\delta_\Gamma)&=[E_{l\pm}{}^I\exp(-i\delta_\Gamma)]^*,\\ M_{l\pm}{}'^I\exp(-i\delta_\Gamma)&=[M_{l\pm}{}^I\exp(-i\delta_\Gamma)]^*,\end{aligned} \tag{42}$$

[7] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1345 (1957). For other references, see, e.g., M. L. Goldberger and K. M. Watson, *Collision Theory* (John Wiley & Sons Inc., 1964), Chap. 9.

[8] A more recent theoretical analysis has been made by A. Donnachie and G. Shaw, University College, London (unpublished).

[9] In order to conform to the notations of Paper 1, the physical system is assumed to be in a large volume Ω, and all state vectors are normalized to unity; e.g., $\langle\lambda_\Gamma|\lambda_\Gamma\rangle=1$ and $\langle N_{\mathbf{k},\lambda}|N_{\mathbf{k}',\lambda'}\rangle=\langle(\pi N)_{\mathbf{k},\lambda}{}^+|(\pi N)_{\mathbf{k}',\lambda'}{}^+\rangle=\langle(\pi N)_{\mathbf{k},\lambda}{}^-|(\pi N)_{\mathbf{k}',\lambda'}{}^-\rangle=\delta_{\mathbf{kk}'}\delta_{\lambda\lambda'}$ where $\delta_{\lambda\lambda'}$ and $\delta_{\mathbf{kk}'}$ are the usual Kronecker δ symbol. For states of definite angular momentum, it is convenient to assume Ω to be a sphere of radius R. By using the familiar expressions $\lim_{R\rightarrow\infty}(R/\pi)\delta_{kk'}\rightarrow\delta(k-k')$ and $\lim_{\Omega\rightarrow\infty}(\Omega/8\pi^3)\delta_{\mathbf{kk}'}\rightarrow\delta^3(\mathbf{k}-\mathbf{k}')$, it is easy to change the normalizations of these state vectors to the various Dirac δ functions, instead of unity.

and Γ denotes the pion-nucleon state with quantum numbers $j_\Gamma = l \pm 1$ and $I_\Gamma = I$. The matrix elements $E_{l\pm}{}^I$ and $M_{l\pm}{}^I$ for reactions (4) and (5) are related to each other by the charge symmetry operator $\exp(i\pi I_y)$; the explicit expressions are given by Eqs. (34)–(37).

If H_γ satisfies T_{st} invariance then $E_{l\pm}{}'^I = E_{l\pm}{}^I$, $M_{l\pm}{}'^I = M_{l\pm}{}^I$, and therefore $\alpha(k,\theta)=0$. We note that the amplitudes $F_+(\Gamma)$ and $F_-(\Gamma)$ refer to photon states of definite helicities $\sigma=+1$ and -1, but the multipole moments $E_{l\pm}{}^I$ and $M_{l\pm}{}^I$ refer to photon states of definite parities. For unpolarized photons, such as the case considered in the present paper, there is no interference term between $F_+(\Gamma)$ and $F_-(\Gamma)$, although, as is evident from Eqs. (39)–(41), there are interference terms between photon states of different parities (i.e., between $E_{l\pm}{}^I$ and $M_{l\pm}{}^I$). In this connection, it is relevant to recall remark (iv) of the preceding section. By using either Eqs. (25) and (26), or Eq. (37), the coefficients A, B, C, and A', B', C' can be easily expressible in terms of the helicity amplitudes $F_\pm(\Gamma)$; such expressions are somewhat simpler than Eqs. (39)–(41).

We will now estimate the sensitivity of $\alpha(k,\theta)$ to a violation of time reversal invariance, $\xi_\Gamma{}^\pm \neq 0$, for energies in the region of the $(\frac{3}{2},\frac{3}{2})$ resonance. In order to do this we must know the magnitudes of the multipole moments appearing in Eqs. (39)–(41). These cannot be uniquely determined from presently available experimental information and therefore our estimate of $\alpha(k,\theta)$ must be based on a theoretical model for pion photoproduction.

So far, we have considered only the $l=0$, 1 states of the pion-nucleon system. Equation (38) can be easily modified to include other higher angular momentum states. The $l>1$ states will also be taken into account in the following.

Let us first consider the case that the time-reversal noninvariant current K_μ satisfies the[10]

$$|\Delta \mathbf{I}| = 0 \tag{43}$$

rule; i.e., K_μ transforms like an isoscalar. In this case, a maximal violation of time reversal invariance presumably means that the matrix elements of K_μ are comparable in magnitude to the corresponding elements of the isoscalar part $(J_\mu)_s$ of the time-reversal invariant current J_μ. Now, in the energy range near the $(\frac{3}{2},\frac{3}{2})$ resonance, the photopion processes are dominated by the matrix elements of the isovector part $(J_\mu)_v$ of the current. Thus, even for a maximal violation of T_{st} invariance we should expect the matrix elements of K_μ to be, in general, small compared to those of $J_\mu = (J_\mu)_s + (J_\mu)_v$, provided (43) holds for K_μ. If ϵ is defined by

$$\left| \frac{\langle (\pi N')_{\mathbf{k}',\lambda'}{}^- | \mathbf{e}\cdot\mathbf{K} | N_{\mathbf{k},\lambda} \rangle}{\langle (\pi N')_{\mathbf{k}',\lambda'}{}^- | \mathbf{e}\cdot\mathbf{J} | N_{\mathbf{k},\lambda} \rangle} \right| = \epsilon \tag{44}$$

then we will retain terms proportional to ϵ and neglect ϵ^2. Consequently, to first order in ϵ, the presence of the time reversal violating K_μ affects *only* the *phase* and not the magnitude of the multipoles $M_{l\pm}{}^I$, $E_{l\pm}{}^I$; the magnitudes of $M_{l\pm}{}^I$ and $E_{l\pm}{}^I$ can be calculated from the usual T_{st}-invariant theory.

In the following, we will adopt a phenomenonogical approach, and use the results of Donnachie and Shaw[8] for the magnitudes (and signs) of the large multipole moments having $l \leqslant 1$. For all the magnitudes (and signs) of other amplitudes, $l \geqslant 2$, only the contribution of the Born terms, represented in Fig. 3, is included. [Further details will be given in Appendix C.] However, we allow the phase of each multipole to be different from that required by T_{st} invariance. These time-reversal symmetry-violating phase differences $\phi(E_{l\pm}{}^I)$ and $\phi(M_{l\pm}{}^I)$ are defined by

$$E_{l\pm}{}^I = (E_{l\pm}{}^I)^* \exp\{2i[\delta_\Gamma + \phi(E_{l\pm}{}^I)]\}$$

and $$\tag{45}$$

$$M_{l\pm}{}^I = (M_{l\pm}{}^I)^* \exp\{2i[\delta_\Gamma + \phi(E_{l\pm}{}^I)]\},$$

where δ_Γ is the π-N scattering phase shifts for the state Γ, characterized by a total isotopic spin I, an orbital angular momentum l and a total angular momentum $j = l \pm \frac{1}{2}$. The phases $\phi(E_{l\pm}{}^I)$ and $\phi(M_{l\pm}{}^I)$

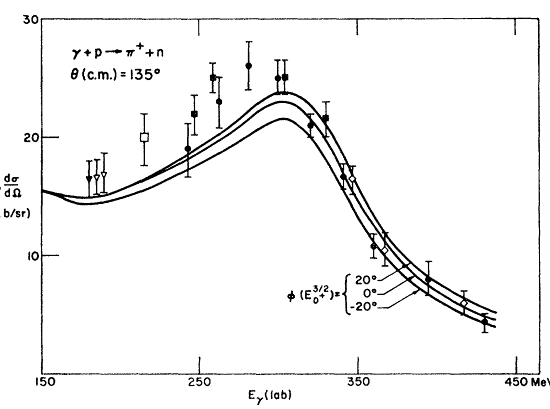

FIG. 1. Differential cross section for $\gamma + p \rightarrow \pi^+ + n$ at the center-of-mass angle 135°. The theoretical curves are calculated by assuming $\phi(E_{0+}{}^{3/2}) = 20°$, 0°, −20° while all other T_{st}-violating phases are zero. Quite similar theoretical curves would also result for different choices of $\phi(E_{l\pm}{}^I)$, or $\phi(M_{l\pm}{}^I)$, =20°, 0°, and −20°. ▼ indicates data taken from M. Bazin and J. Pine, Phys. Rev. **132**, 830 (1963); ▽ indicates data taken from M. I. Adamovich *et al.*, *Proceedings of the 1962 International Conference on High Energy Physics, CERN* (CERN, Geneva, 1962), pp. 207; ◇ indicates data taken from M. Heinberg *et al.*, Phys. Rev. **110**, 1211 (1958); □ indicates data taken from M. Beneventano *et al.*, Nuovo Cimento 4, 323 (1956); ○ indicates data taken from D. Freytag, W. J. Schwille and R. J. Wedemeyer, Z. Physik **186**, 1 (1965); and ■ indicates data taken from K. Althoff, H. Fisher and W. Paul, Z. Physik **175**, 19 (1963).

[10] T. D. Lee, Phys. Rev. **140**, B959 (1965).

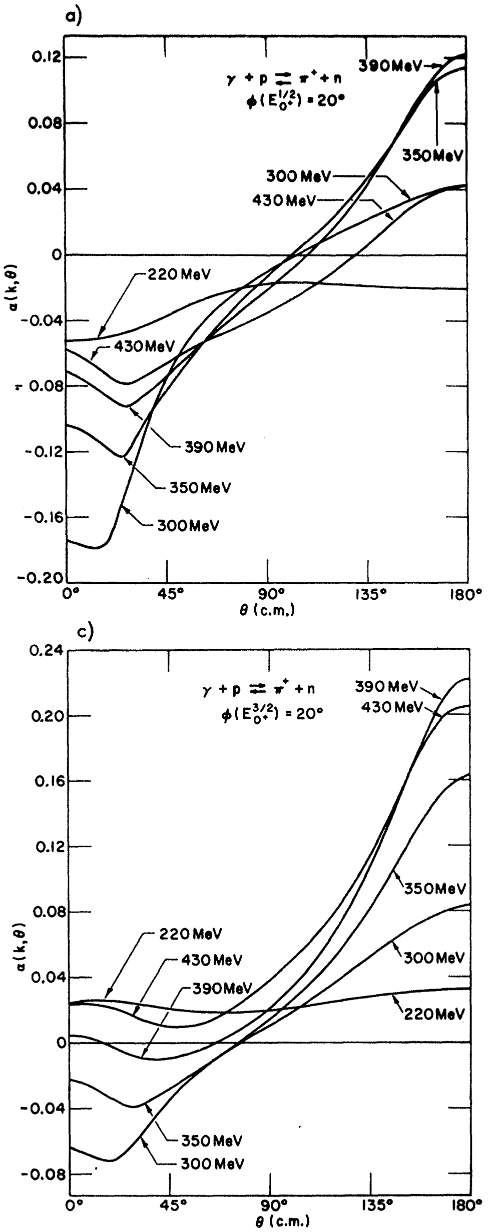

FIG. 2. The asymmetry function $a(k,\theta)$ for $\gamma+p \rightleftarrows \pi^{+}+n$. $\alpha(k,\theta)$ is displayed for (a) $\phi(E_{0+}{}^{1/2})=20°$, (b) $\phi(M_{1+}{}^{1/2})=20°$, (c) $\phi(E_{0+}{}^{3/2})=20°$, (d) $\phi(M_{1+}{}^{3/2})=20°$, while all other $T_{\rm st}$-violating phases are set equal to zero. Each curve is labeled by the γ-ray energy in the rest system of the proton.

can be related by Eq. (37) to the $\xi_{\Gamma}{}^{\sigma}$ defined before. If the selection rule (43) is valid, then only the phases $\phi(E_{l\pm}{}^{1/2})$ and $\phi(M_{l\pm}{}^{1/2})$ can be different from zero. From a phenomenological point of view, a possible *violation* of the selection rule (43) can be easily taken into account by including also the phases $\phi(E_{l\pm}{}^{3/2})$ and $\phi(M_{l\pm}{}^{3/2})$, provided ϵ^2 remains small compared to 1. If any one of these phases $\phi(E_{l\pm}{}^{I})$ and $\phi(M_{l\pm}{}^{I})$ are different from zero, then-time-reversal symmetry is violated. It is the sensitivity of $\alpha(k,\theta)$ to the presence of such phases ϕ that we wish to investigate.

Using these calculated magnitudes for $E_{l\pm}{}^{I}$ and $M_{l\pm}{}^{I}$, but assuming $T_{\rm st}$ invariance, one obtains differential cross sections for (2) in fair agreement with experiment. The size of the time reversal violating phases is restricted by the requirement that this agreement with experiment be preserved. Figure 1 compares with experiment the computed differential cross section with and without $T_{\rm st}$ violation for $\gamma+p\rightarrow n+\pi^{+}$ at a typical angle. One sees that $T_{\rm st}$-violating phases of $\sim\pm 20°$ can be allowed. Similar comparisons at other angles also lead to the same conclusion. This suggests $\epsilon\lesssim 0.3$ and $\epsilon^2\lesssim 0.1$, consistent with our assumption that ϵ^2 can be neglected with respect to 1.

Figures 2 and 3 diplay $\alpha(k,\theta)$ for reactions (4) and (5), at five values of k, and for four possible assign-

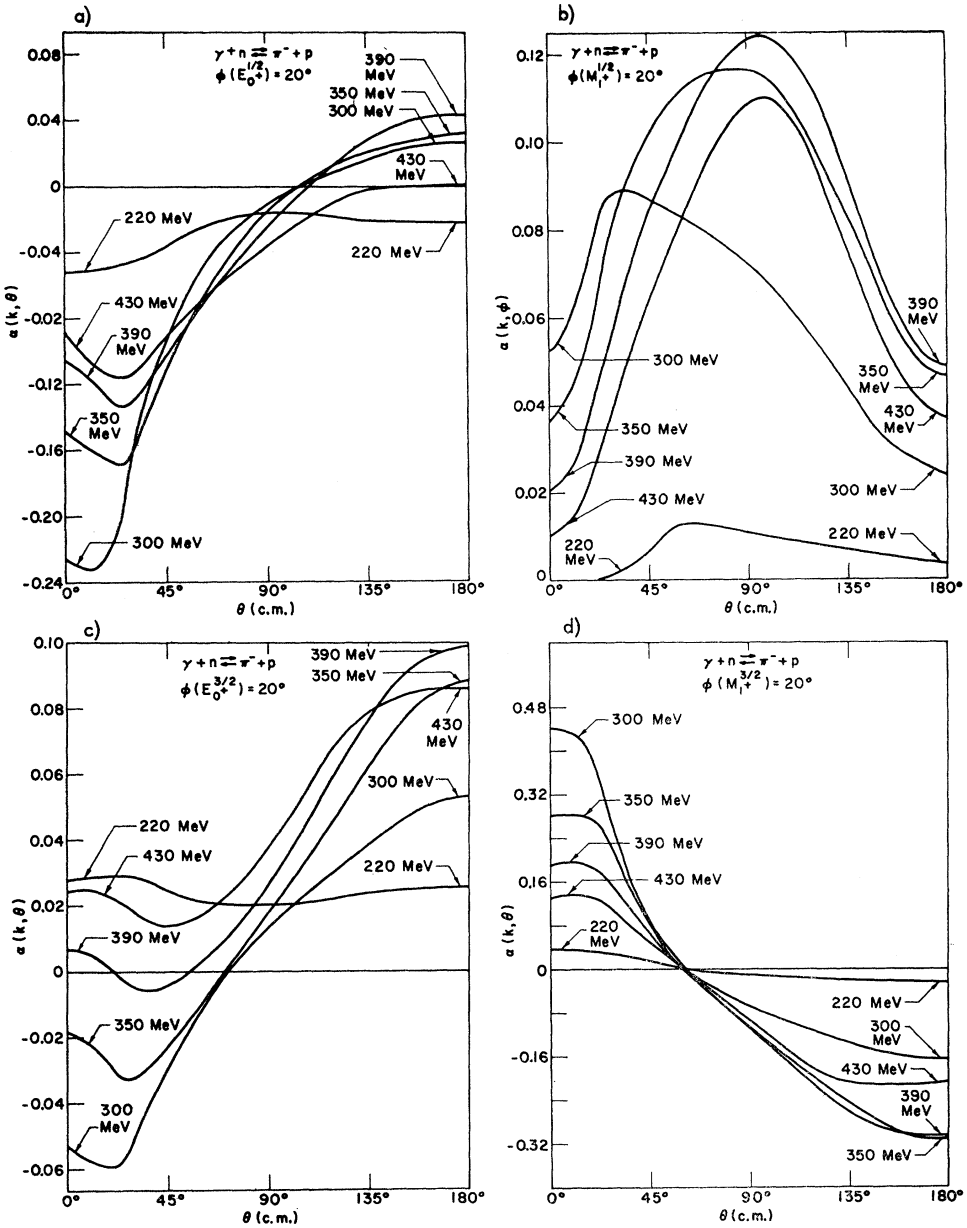

FIG. 3. The asymmetry function $\alpha(k,\theta)$ for $\gamma+n \rightleftarrows \pi^- + p$. $\alpha(k,\theta)$ is displayed for (a) $\phi(E_{0+}^{1/2})=20°$, (b) $\phi(M_{1+}^{1/2})=20°$, (c) $\phi(E_{0+}^{3/2})=20°$, (d) $\phi(M_{1+}^{3/2})=20°$, while all other T_{st}-violating phases are set equal to zero. Each curve is labeled by the γ-ray energy in the rest system of the neutron.

ments of T_{st}-violating phase. If the selection rule (43) holds, then only figures 2(a), 2(b), 3(a), and 3(b) are applicable. The presence of the factor $\sin(\delta_\Gamma - \delta_{\Gamma'})$ in Eq. (28) is revealed by the small values of $\alpha(k,\theta)$ found for energies appreciably below the $(\frac{3}{2},\frac{3}{2})$ π-N resonance, where $\delta_\Gamma \cong 0$. In our model the large multipole moments are $E_{0+}^{1/2}$, $E_{0+}^{3/2}$, and $M_{1+}^{3/2}$. Consequently, if we add a T_{st} violating phase of 20° to $E_{0+}^{1/2}$, or $E_{0+}^{3/2}$, or $M_{1+}^{3/2}$ the behavior of α is dominated by the $(B-B')\times\cos\theta$ term in Eq. (38). Hence Figs. 2(a), 2(c), 2(d), 3(a), 3(c), and 3(d) show a similar behavior. Figures 2(b) and 3(b) differ from these because of the large cross term between $M_{1+}^{1/2}$ and $M_{1+}^{3/2}$ which does not involve an interference between s and p waves. From an examination of Fig. 2 one sees that for a time-reversal violating phase $\phi=\phi(E_{l\pm}^{I})$ or $\phi(M_{l\pm}^{I})$, $\alpha(k,\theta)$ can be as large as $\sim 0.5\sin\phi$. We conclude that $\alpha(k,\theta)$, for appropriate θ, is quiet sensitive to T_{st} violation in the energy region of the $(\frac{3}{2},\frac{3}{2})$ resonance. Specifically, the asymmetry is most reliably large for $\theta \lesssim 45°$ or $\theta \gtrsim 135°$, where θ is the angle between the γ and π momenta in the center of mass system and for γ laboratory energies in reaction (2) between 300 and 390 MeV. [See, however, Figs. 2(b) and 3(b).]

In order to test time reversal by reciprocity at higher energy, these considerations would suggest that one always look near a resonant energy of the π-N system.

ACKNOWLEDGMENTS

We wish to thank A. Donnachie and G. Shaw for informing us of details of their results, and L. Lederman, L. Osborne and R. Wilson for discussions.

APPENDIX A

In this appendix, we will review some elementary properties of the stationary states $|\lambda_\Gamma\rangle$ introduced in Sec. II.

Let us consider the strong-interaction scattering process

$$\pi+N \rightleftarrows \text{various channels } c\,, \qquad (A1)$$

and also the reaction between these channels

$$c \rightleftarrows c'\,, \qquad (A2)$$

where N can be any nucleus, c (or c') $=1, 2, \cdots n$ denotes all possible channels (n may be infinite). For definiteness, we will assume that the system has zero total momentum, a definite, but arbitrary, energy E, a definite parity $\mathcal{P}$, a definite total angular momentum j and its z component j_z. Let $|c(j,j_z,\mathcal{P})^+\rangle$ and $|c(j,j_z,\mathcal{P})^-\rangle$ be, respectively, the usual outgoing and incoming eigenstates of the strong-interaction H_{st}. The elements of the strong-interaction S matrix are given by

$$\langle c'|S|c\rangle = \langle c'(j,j_z,\mathcal{P})^-|c(j,j_z,\mathcal{P})^+\rangle. \qquad (A3)$$

In (A1) and (A2), the choice of these channels is *arbitrary*, provided

$$\langle c'(j,j_z,\mathcal{P})^+|c(j,j_z,\mathcal{P})^+\rangle = \delta_{cc'}\,, \qquad (A4)$$

and likewise for the states $|c(j,j_z,\mathcal{P})^-\rangle$.

Since H_{st} satisfies T_{st} invariance, the phases of these states can be chosen to satisfy[6]

$$T_{\text{st}}|c(j,j_z,\mathcal{P})^\pm\rangle = \exp[i\pi(j+j_z)]\,|c(j,-j_z,\mathcal{P})^\mp\rangle\,. \qquad (A5)$$

Consequently, we have the well-known result[11]

$$\langle c'|S|c\rangle = \langle c|S|c'\rangle. \qquad (A6)$$

Thus, in the subspace of these n channels, the strong interaction S matrix becomes an $n\times n$ symmetric and unitary matrix. Let $\psi(\Gamma)$ be its eigencector ($\Gamma=1, 2, \cdots n$).

$$\textstyle\sum_c \langle c'|S|c\rangle\psi_c(\Gamma) = \exp(2i\delta_\Gamma)\psi_{c'}(\Gamma)\,. \qquad (A7)$$

Each of these eigenvectors is an $n\times 1$ matrix; they can be chosen to form a set of n real orthonormal vectors. From these eigenvectors and the arbitrarily chosen channels c, we can construct the *eigenchannels* $\Gamma=1, 2, \cdots n$ of the S matrix; the corresponding incoming and outgoing states are given by

$$|\Gamma(j,j_z,\mathcal{P})^\pm\rangle = \textstyle\sum_c \psi_c(\Gamma)\,|c(j,j_z,\mathcal{P})^\pm\rangle. \qquad (A8)$$

The stationary state $|\lambda_\Gamma\rangle$ used in the text is defined by

$$|\lambda_\Gamma\rangle = \tfrac{1}{2}[\exp(-i\delta_\Gamma)\,|\Gamma(j,j_z,\mathcal{P})^+\rangle + \exp(+i\delta_\Gamma)\,|\Gamma(j,j_z,\mathcal{P})^-\rangle]\,, \qquad (A9)$$

where

$$\lambda_\Gamma = j_z\,, \quad j_\Gamma = j \quad\text{and}\quad \mathcal{P}_\Gamma = \mathcal{P}. \qquad (A10)$$

These stationary states satisfy Eq. (19), and they form an orthonormal set of state vectors. Furthermore, by using (A3), (A7), and (A9), it can be easily verified that

$$\langle c(j,j_z,\mathcal{P})^\pm|\lambda_\Gamma\rangle = \exp(\mp i\,\delta_\Gamma)\psi_c(\Gamma) \qquad (A11)$$

for any arbitrarily chosen set of channels $c=1, 2, \cdots n$. Thus, Eq. (23) follows.

APPENDIX B

For completeness, the definitions of the usual multipole moments will be given in this appendix. Let us consider the reaction

$$\gamma+N \to \pi+N' \qquad (B1)$$

in the center-of-mass system, where N and N' are both nucleons. Let $\mathbf{J}$ be the total angular momentum operator. For the initial state (γN)

$$\mathbf{J} = \mathbf{j}_\gamma + \mathbf{j}_N\,, \qquad (B2)$$

where $\mathbf{j}_N$ is the nucleon-spin operator and $\mathbf{j}_\gamma$ is the angular-momentum operator of the photon (including that due to the relative momentum between γ and N). Let $J(J+1)$ and $j(j+1)$ be the eigenvalues of $\mathbf{J}^2$ and $\mathbf{j}_\gamma^2$, respectively. For any given $N=n$ or p, the initial state $|\gamma N(J,J_z,j,\mathcal{P})\rangle$ is characterized by J, J_z, j and $\mathcal{P}$ where J_z is the z component of $\mathbf{J}$ and $\mathcal{P}$ is the parity. The final ($\pi N'$) state is characterized by the total isospin I and the total orbital angular momentum l, besides J, J_z, and $\mathcal{P}$. Let $|\pi N'(J,J_z,l,\mathcal{P},I)^-\rangle$ be the corresponding incoming eigenstate of the strong interaction. The multipole moments $E_{l+}{}^I$ and $M_{l+}{}^I$ used in the text are given by

$$\begin{aligned}\langle \pi N'(J,J_z,l,\mathcal{P},I)^-|\,\mathcal{J}_\mu(0)A_\mu(0)\,|\gamma N(J,J_z,j,\mathcal{P})\rangle \\ = [j(j+1)]^{1/2}\times(\pm E_{l\pm}{}^I) \quad\text{if}\quad \mathcal{P}=(-1)^j \\ \times M_{l\pm}{}^I \quad\text{if}\quad \mathcal{P}=-(-1)^j, \qquad (B3)\end{aligned}$$

where the subscript $+$ or $-$ in $E_{l\pm}{}^I$ and $M_{l\pm}{}^I$ depends on whether $J=l+\frac{1}{2}$ or $l-\frac{1}{2}$, $A_\mu(x)$ is the electromagnetic field operator, and $\mathcal{J}_\mu(x)$ is the current operator. By using the normalization convention[9]

$$\begin{aligned}\langle \pi N'(J,J_z,l,\mathcal{P},I)^-|\pi N'(J,J_z,l,\mathcal{P},I)^-\rangle = 1\,, \\ \langle \gamma N(J,J_z,j,\mathcal{P})|\gamma N(J,J_z,j,\mathcal{P})\rangle = 1 \qquad (B4)\end{aligned}$$

and the relative-phase convention given by Ref. 6, together with Eqs. (21) and (A11), the matrix elements $F_\pm(\Gamma)$ can be easily expressed in terms of $E_{l\pm}{}^I$ and $M_{l\pm}{}^I$. The result is Eq. (37).

[11] See M. L. Goldberger and K. M. Watson, *Collision Theory* (John Wiley & Sons, Inc., New York, 1964), pp. 375.

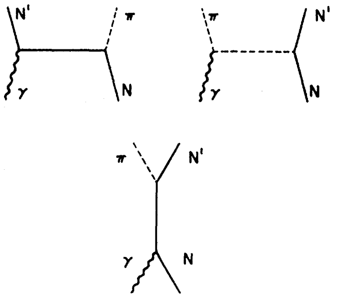

FIG. 4. Feynman diagrams representing the Born terms. These were used to compute the magnitude (and sign) of the $l \geqslant 2$ photopion production amplitudes used in our model of T_{st} violation. They were also used to obtain $E_{1+}^{1/2}$, $E_{1+}^{3/2}$, and those amplitudes arising from the isoscalar part of the electromagnetic current.

APPENDIX C

We will now discuss in detail the model for photopion production used in Sec. III to estimate the sensitivity of reactions (4) and (5) to T_{st} non-invariance. Let the electromagnetic current be split into two parts $(\mathcal{J}_\mu)_v$ and $(\mathcal{J}_\mu)_s$, the first transforming under isospin rotation as an isovector and the second as an isoscalar,

$$\mathcal{J}_\mu = (\mathcal{J}_\mu)_v + (\mathcal{J}_\mu)_s .$$

We define $E_{l\pm}{}^I(v)$, $M_{l\pm}{}^I(v)$, and $E_{l\pm}{}^I(s)$, $M_{l\pm}{}^I(s)$ by replacing $\mathcal{J}_\mu$ in Eq. (B3) by $(\mathcal{J}_\mu)_v$ and $(\mathcal{J}_\mu)_s$, respectively. For $E_{0+}{}^{1/2}(v)$, $E_{0+}{}^{3/2}(v)$, $M_{1-}{}^{1/2}(v)$, $M_{1-}{}^{3/2}(v)$, $M_{1+}{}^{1/2}(v)$, and $M_{1+}{}^{3/2}(v)$ we use the magnitudes (and signs) obtained by the fully relativistic calculation of Donnachie and Shaw[8] using the dispersion theory of Chew, Low, Goldberger, and Nambu.[7]

The magnitudes (and signs) of the other amplitudes are assumed to be that obtained from Eq. (B3) if only the contribution of the diagrams appearing in Fig. 4 is included. These multipole moments of the Born terms have been calculated previously by Schmidt and Guigay.[12]

The phases of the multipole moments in our model of T_{st} violation are given by Eq. (45), where the π-N scattering phase shifts δ_Γ are set equal to 0 for $l > 1$ and are assumed to be those obtained by Roper *et al.*[13] for $l \leqslant 1$.

It should be noted that in the limit of T_{st} invariance the photopion production differential cross section we obtain differs somewhat from that of Donnachie and Shaw. This is primarily because we have given $E_{1+}{}^{1/2}$ and $E_{1+}{}^{3/2}$ the phase δ_Γ, where $\Gamma = p_{1/2,3/2}$ and $p_{3/2,3/2}$, respectively (as is required by T_{st} invariance); Donnachie and Shaw omitted these phases. Also we do not include any approximation to the dispersion integral in the $l \geqslant 2$ amplitudes, but keep only the contribution of the Born terms.

[12] W. Schmidt and J. P. Guigay (unpublished). See Appendix II of Donnachie and Shaw, Ref. 8.

[13] L. D. Roper, R. Wright, and B. Feld, Phys. Rev. 138, B190 (1965), Table IX on p. B203.

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Long-Range Perturbations of Bound States and Resonances*

JULIAN V. NOBLE
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey
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We consider the modification of the scattering matrix in multichannel potential-scattering theory by a long-range perturbation. The perturbation of "single-particle" energy levels (bound states or resonances) is discussed by means of Weinberg's eigenvalue analysis of potential scattering. A straightforward distinction between "single-particle" and "compound" states appears in this formalism, and it leads to a proof that there are no bound states embedded in the continuum. A simple approximation for the eigenvalues corresponding to single-particle levels is developed and applied to the calculation of the energy difference between the $(\frac{1}{2}^+)$ first excited states of the C^{13}-N^{13} mirror pair.

THE purpose of this paper is to elucidate the problem of long-range perturbations in multichannel potential-scattering theory. Within this framework we discuss the modification of energy levels (bound states and resonances) by such perturbations; while doing so we evolve a novel viewpoint toward the relation between single-particle and compound resonances,[1] as well as convenient methods for doing practical calculations.

In Sec. I, we manipulate the Lippmann-Schwinger equations for multichannel scattering into a form convenient for our discussion of long-range perturbations. We treat such perturbations by formally summing an infinite (divergent) subset of the terms arising from expansion in powers of the perturbation. Section II deals with the extension of Weinberg's[2] eigenvalue analysis of

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[1] Appendix A.

[2] S. Weinberg, Phys. Rev. 131, 440 (1963).