Ghost in Scalar State

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An analysis is made of s-wave $\pi\pi$ scattering under the assumption of the existence of an $SU(3)$-symmetric scalar nonet satisfying the inverted equal-spacing rule. One finds that in the case of mixing this leads to a result that there is a ghost at negative squared mass, which couples strongly to pseudoscalars. The various implications of this result are analyzed. Specifically, one concludes that the $\pi\pi$ scattering length is negative and that the forward-backward symmetry in $\pi^+\pi^-$ production must change sign between 400 and 600 MeV. Scattering lengths in different isospin states are related by using a vector exchange model.

INTRODUCTION

EVEN though the experimental information about the s-wave $\pi\pi$ interaction is scanty and imprecise, the following phenomena may be ascribed to $\pi\pi$ interaction in the s-wave state.

(a) In the peripheral production process

$$\pi^- + p \rightarrow \pi^- + \pi^+ + n\,, \tag{1}$$

the angular distribution of the final pions in their center-of-mass (c.m.) system shows[1] a marked forward-backward asymmetry around the ρ meson mass. This is easily explained in terms of interference between $I=0$, s-wave state and $I=1$, p-wave state in $\pi\pi$ interaction. This explanation is further supported by the nonobservance of such an asymmetry in the production processes

$$\pi^- + p \rightarrow \pi^- + \pi^0 + p\,, \tag{2}$$

or

$$\pi^+ + p \rightarrow \pi^+ + \pi^+ + n\,. \tag{3}$$

(b) There have been two reports[2] which indicate the presence of an $I=0$ s-wave resonance at a mass of 720 MeV and having a full width of 50 MeV.

Of course the above two observations are very likely related to each other. If $SU(3)$ symmetry is assumed, as it is in this discussion, then there is an additional piece of information, namely the presence of the κ at 725 MeV[3] which may indicate that there may be an s-wave $\pi\pi$ resonance nearby. This is consistent with the previous observations.

There have been many theoretical speculations regarding the s-wave $\pi\pi$ interaction, of varying degree of interest and plausibility. One of the really striking ones is that of Chew[4] who proposes that there is a Regge ghost in the scalar state at a negative squared mass, which behaves like a bound state as far as Levinson's theorem is concerned and hence the $\pi\pi$ scattering length in the $I=0$ state is negative. Irrespective of whether one believes in Regge ghost or not, negative scattering length in $I=0$ state is a serious possibility and deserves attention.

In the following discussion, the situation is analyzed under the following assumptions. (a) $SU(3)$ symmetry is applicable to scalar mesons,[5] and these mesons satisfy the inverted equal spacing rule[5] and mix according to Schwinger's mixing formula.[6] (b) There is an $I=0$, s-wave $\pi\pi$ resonance at 720 MeV having a full width of 50 MeV. (c) The κ is an $I=\frac{1}{2}$, s-wave $K\pi$ resonance. These assumptions lead to a surprising conclusion that there is an $I=0$ scalar meson at $s=-11.9\mu^2$ where s is the squared mass of the particle and μ is the pion mass and consequently the $I=0$ $\pi\pi$ scattering length is negative. The negative scattering length then implies that if the forward-backward asymmetry mentioned before is due to s-wave and p-wave interference, the asymmetry starts as negative at the threshold but changes sign between 400 and 600 MeV thus providing a critical test for negative scattering length. The existence of a ghost may also provide an explanation for the problem of K_1-K_2 mass difference.

IMPLICATIONS OF $SU(3)$

The application of the inverted equal spacing rule which is appropriate in the present case to relate the scalar meson masses to those of pseudoscalars leads to

$$\kappa^2 - \xi^2 = \pi^2 - K^2\,, \tag{4}$$

where the symbols stand for the masses of the respective particles and ξ is the $I=1$ member[5] of the scalar octet. With κ having a mass of 725 MeV we get

$$\xi \approx 870 \text{ MeV}. \tag{5}$$

Then the Gell-Mann–Okubo mass formula implies that the $I=0$ component of the octet has a mass of 670 MeV. Since this does not agree with the experimental mass of the S meson (720 MeV), it is assumed that there is actually a nonet of scalar mesons. Calling the ninth member σ, the use of Schwinger's[6] mixing formula yields

$$(\sigma^2 - \xi^2)(S^2 - \xi^2) = \tfrac{4}{3}(\kappa^2 - \xi^2)(S^2 + \sigma^2 - 2\kappa^2)\,, \tag{6}$$

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[1] V. Hagopian and W. Selove, Phys. Rev. Letters **10**, 533 (1963); Saclay-Orsay-Bari-Bologna Collaboration, Nuovo Cimento **29**, 515 (1963).

[2] M. Feldman *et al.*, Phys. Rev. Letters **14**, 869 (1965); V. Hagopian *et al.*, *ibid.* **14**, 1077 (1965). But for evidence against see H. O. Cohn *et al.*, *ibid.* **15**, 906 (1965).

[3] G. Alexander *et al.*, Phys. Rev. Letters **8**, 447 (1962).

[4] G. F. Chew, Phys. Rev. Letters **16**, 60 (1966).

[5] L. M. Brown, Phys. Rev. Letters **14**, 836 (1965).

[6] J. Schwinger, Phys. Rev. **135**, B816 (1964).

from which one gets

$$\sigma^2 = -11.9\mu^2, \tag{7}$$

and the octet singlet mixing is given by

$$\sigma = 0.80S_1 + 0.60S_8, \tag{8}$$

$$S = 0.60S_1 - 0.80S_8. \tag{9}$$

Of course the actual values obtained should not be taken seriously though qualitatively they may be expected to be correct. With the above mixing one has

$$\Gamma_{S\to\pi\pi}/\Gamma_{\kappa\to K\pi} = 2.3(1.3-0.6x)^2/6, \tag{10}$$

where x is the ratio of singlet to octet coupling and 2.3 is the phase-space factor; furthermore, one gets

$$R_1 = (g_{\sigma\pi\pi}/g_{S\pi\pi})^2 = (1.2+0.8x)^2/(1.6-0.6x)^2, \tag{11}$$

$$R_2 = (g_{SKK}/g_{S\pi\pi})^2 = 4(0.8+0.6x)^2/3(1.6-0.6x)^2, \tag{12}$$

$$R_3 = (g_{\sigma KK}/g_{S\pi\pi})^2 = 4(0.6-0.8x)^2/3(1.6-0.6x)^2. \tag{13}$$

From experiment, $\Gamma_{S\to\pi\pi} \approx 50$ MeV but so far only the upper bound for $\Gamma_{\kappa\to K\pi} < 12$ MeV is known. However, the ratios R_1, R_2, and R_3 are rather insensitive to $\Gamma_{\kappa\to K\pi}$. Thus, for example,[7] if $\Gamma_{\kappa\to K\pi} = 10$ MeV, $R_1 = 5.6$, $R_2 = 3.5$, and $R_3 = 3.9$, whereas if $\Gamma_{\kappa\to K\pi} = 2$ MeV, $R_1 = 3.4$, $R_2 = 2.2$, and $R_3 = 3.1$. For definiteness in our discussion we will take $\Gamma_{\kappa\to K\pi} = 10$ MeV.

The most surprising thing about the above results is that σ^2 comes out as negative. But this may be an affirmation of Chew's conjecture[4] that one of the "top ranking" Regge trajectories with vacuum quantum members intersects $J=0$ at a negative squared mass and this Regge ghost leads to a negative scattering length. It is true that Chew's estimate based on the assumption of a linear trajectory leads to a value which is roughly six times the value obtained above for σ^2, but the trajectory need not be linear. Be it as it may, the consequences are in qualitative agreement with those of Chew's conjecture.

The presence of a ghost leads to a negative scattering length and decreasing phase shift as described by Chew using Levinson's theorem. But it is rather ambiguous to talk about the coupling of this ghost with other particles. This is because unitarity in crossed reactions will not allow a pole in their physical region. There are two possibilities which could make the discussion of the coupling of ghosts meaningful. One is that the ghost corresponds to a pole of the S matrix, not at a real negative squared energy, but at a complex squared energy. The other possibility is that in the context of N/D approach, the ghost corresponds to a zero of the D function; even though the N function is zero at the same position, it varies rapidly along the left-hand cut, so that when we are far away from the ghost, the ghost looks like a bona fide bound state with nonzero coupling but at a negative squared energy. Whatever may be the case, it will be assumed that our estimates of the coupling constants from $SU(3)$ give qualitatively correct results when used in the physical region for $\pi\pi$ scattering.

With the above qualifications in mind, one notes that R_2 is between 2 and 4 which implies that the S meson is coupled more strongly with $K\bar{K}$ than with $\pi\pi$. Therefore the S meson maybe looked upon more as a bound state of $K\bar{K}$ than a $\pi\pi$ resonance, in agreement[8] with the small experimental width of S.

SCATTERING LENGTHS

The scattering length for $\pi\pi$ scattering in the $I=0$ state is determined by using a simple two-pole formula for the scattering amplitude which gives

$$a_0 = \lim_{t\to 4\mu^2} \frac{e^{i\delta_0}\sin\delta_0}{\rho} = \frac{\gamma_\sigma}{m_\sigma^2-4\mu^2} + \frac{\gamma_S}{m_S^2-4\mu^2}, \tag{14}$$

where $\rho = [(t-4\mu^2)/t]^{1/2}$. From the experimental width $\Gamma_S \approx 50$ MeV, one gets $\gamma_S = 2.0$ so that taking the ratio $R_1 = 5.6$, one gets

$$a_0 = -0.6. \tag{15}$$

This number is about 2 or 3 times smaller than the value obtained by Rothe.[9] A similar calculation for $K\pi$ scattering with κ dominance gives (taking $\Gamma_{\kappa\to K\pi} = 10$ MeV),

$$a_{1/2} \approx 0.08. \tag{16}$$

To determine the scattering lengths in other isospin states, a vector exchange model of the type described by Sakurai,[10] is used. The vector exchange model essentially consists of noting that the difference in scattering lengths of π^+ and π^- scattered from any target, is dominated by the exchange of vector mesons. Then one gets

$$a_0 - \tfrac{5}{2}a_2 = (g_{\rho\pi\pi}^2/4\pi)(3\mu/m_\rho^2) \approx 0.22. \tag{17}$$

Using $\Gamma_\rho \approx 100$ MeV and $m_\rho = 750$ MeV and $a_0 = -0.6$, one has

$$a_2 = -0.33. \tag{18}$$

Similarly for $K\pi$ scattering,

$$a_{1/2} - a_{3/2} = (g_{\rho\pi\pi}g_{\rho KK}/4\pi)(3\mu/m_\rho^2(1+\mu/m_K)), \tag{19}$$

so that taking $g_{\rho KK} = g_{\rho\pi\pi}$, and using (16), one gets

$$a_{3/2} = -0.18. \tag{20}$$

ASYMMETRY IN $\pi^+\pi^-$ PRODUCTION

With the presence of S and the ghost σ in mind, the forward-backward asymmetry in reaction (1) is ex-

[7] There are two solutions for x. We choose the solution which makes $R_1 > 1$ since σ is much more tightly bound than S.

[8] Such a situation was conjectured by G. F. Chew (private communication).

[9] H. Rothe, Phys. Rev. **140**, B1421 (1965).

[10] J. J. Sakurai, Enrico Fermi International School of Physics, unpublished report.

amined. Assuming that the asymmetry is due to the interference between $I=0$ s wave and $I=1$ p wave, the asymmetry parameter is given by

$$A=(F-B)/(F+B),$$
$$\sim\frac{\sin\delta_0 \sin\delta_1 \cos(\delta_0-\delta_1)}{\sin^2\delta_0+(27/4)\sin^2\delta_1}, \quad (21)$$

where F and B are the integrated forward and backward events. In view of the negative scattering length for the $I=0$ s-wave scattering and the presence of S at 720 MeV, the phase shift δ_0 starts at 180°, and is 90° at 720 MeV. Therefore the asymmetry is negative near the threshold but changes sign somewhere between 400–600 MeV. With our crude model it is difficult to predict precisely where the change in sign takes place. Experimentally such a change in sign has been reported.[11] If confirmed, it would indeed be a strong piece of evidence in favor of negative scattering length for the $I=0$, s-wave scattering, and consequently of a ghost being present.

K_1-K_2 MASS DIFFERENCE

The K_1-K_2 mass difference provides us with a check on the s-wave $\pi\pi$ interaction. It is a reasonable assumption[12] to regard the K_1-K_2 mass difference as due to $I=0$, s-wave $\pi\pi$ interaction. Accepting this assumption, the mass difference δm is given by[12]

$$\delta m=m_{K_1}-m_{K_2}=\frac{1}{\pi m}P\int_{2\mu}^{\infty}\frac{W^2\Gamma_{2\pi}(W)}{m^2-W^2}dW, \quad (22)$$

[11] L. W. Jones *et al.*, Phys. Letters **21**, 590 (1966).

[12] K. Nishijima, Phys. Rev. Letters **12**, 39 (1964); S. H. Patil, *ibid.* **13**, 454 (1964).

where P stands for principal-value integral, m is the K mass, W is the center-of-mass energy, and $\Gamma_{2\pi}$ is the $K_1\rightarrow 2\pi$ width. For a single resonance in the $I=0$ s-wave $\pi\pi$ interaction, we have

$$\Gamma_{2\pi}(W)=\frac{(t-4\mu^2)^{1/2}}{t}\frac{C}{(t-t_r)^2+\gamma^2(t-4\mu^2)/t}, \quad (23)$$

where the parameters t_r and γ are related to the resonance mass m_r and width Γ_r by

$$t_r=m_r^2, \quad \gamma[1-(2\mu/m_r)^2]^{1/2}=m_r\Gamma_r, \quad (24)$$

and C is determined to give

$$\Gamma_{2\pi}(m)=1/\tau_1, \quad (25)$$

where τ_1 is the K_1 lifetime.

If only the S meson exists, then we find that

$$\delta m=4.2/\tau_1, \quad (26)$$

which is too large by about an order of magnitude from the experimental value[13] of $0.45/\tau_1$. This indicates that there is some strong 2π interactions in the $I=0$ s-wave state below the K mass. With both the σ and S present, it is possible to overcome this difficulty since we have an additional degree of freedom in the relative strength of the octet, singlet scalar couplings with K_1.

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[13] C. Alff-Steinberger *et al.*, CERN Report (unpublished).