The crossing relation between the g's of Eq. (2.7) and the Wolfenstein parameters is

$$\begin{pmatrix} a(\theta) \\ m(\theta) \\ c(\theta) \\ g(\theta) \\ h(\theta) \end{pmatrix} = \frac{1}{2s^{1/2}} \begin{pmatrix} 1+\alpha(\theta) & -1+\alpha(\theta) & 2\beta(\theta) & 0 & 0 \\ -1+\alpha(\theta) & 1+\alpha(\theta) & 2\beta(\theta) & 0 & 0 \\ i\beta(\theta) & i\beta(\theta) & -2i\alpha(\theta) & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} g_1 \\ g_2 \\ g_3 \\ g_4 \\ g_5 \end{pmatrix},$$

where the Wolfenstein parameters are now expressed in terms of the analytically continued g's. The functions $\alpha(\theta)$ and $\beta(\theta)$ were given in Eq. (2.12).

The crossing relation and Stapp's Table I can be used to rederive Eq. (2.11), and the triple-correlation parameters, if needed.

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Study of the ρ and B Mesons in π-ω Scattering*

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The reciprocal bootstrap idea is applied to the ρ and B mesons in π-ω scattering. The ρ appears as a 1^- bound state in the π-ω system, while arguments favoring a 2^- p-wave B meson are presented. Our method of calculation follows a previously developed general theoretical framework which treats the interaction of pseudoscalar and vector-meson systems. Because of the divergence difficulties associated with higher spin particles and the eventual necessity for an additional parameter in the form of a cutoff, we demand only that the masses of the ρ and B mesons be self-consistent. Values of the $\pi\rho\omega$ and $\pi B\omega$ coupling constants required to give this self-consistency are compared with the corresponding output parameters characterizing these solutions. Our results show that the ρ can be produced self-consistently at its experimental mass, although the B width and $\pi\rho\omega$ coupling constant required to do this are somewhat larger than what experiment indicates. On the other hand, a completely self-consistent B meson can be obtained in reasonable agreement with experiment only if the distant singularities associated with ρ exchange are neglected. Otherwise, ρ exchange is effectively repulsive in the 2^- amplitude because of the change in sign of the Born term, thus requiring a large B width to produce a resonance at the experimental mass of the B.

I. INTRODUCTION

THE problem of π-ω scattering has recently attracted the attention of many authors[1-6] with the discovery of a possible resonance in the π-ω system.[7,8] This resonance, called the B meson, has a mass of 1220 MeV and full width of 125 MeV; however, its spin and parity have yet to be determined experimentally. A model in which ρ exchange is the dominant force has been used to determine the spin and parity of the B on theoretical grounds.[1-3] Using this model in a nonrelativistic calculation, Peierls[1] has obtained a resonance, with J^P of 2^-, as the analogy of the N^* in π-N scattering. Kuo[2] found the same result in a relativistic calculation. Abers,[3] however, demonstrated that a 1^+ resonance is also possible in this model. All of these previous authors considered forces from ρ exchange only and the effects due to B exchange were absent.

More recently, in another nonrelativistic calculation, one of us[4] showed the existence of a reciprocal bootstrap between the ρ and a 2^- B meson, in analogy with the reciprocal bootstrap of the N and N^* in π-N scattering, proposed by Chew.[9] In this scheme one takes the ex-

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[1] R. F. Peierls, Phys. Rev. Letters **12**, 50 (1964); **12**, 119(E) (1964).
[2] T. K. Kuo, Phys. Rev. Letters **12**, 465 (1964).
[3] E. Abers, Phys. Rev. Letters **12**, 55 (1964).
[4] J. Franklin, Phys. Rev. **137**, B994 (1965).
[5] K. Kang, Phys. Rev. **140**, B1629 (1965). This work will be referred to as K in the text.
[6] R. Atkinson, III, Phys. Rev. **142**, 1154 (1966).
[7] M. Abolins, R. L. Lander, W. A. W. Mehlhop, N. Xuong, and P. M. Yager, Phys. Rev. Letters **11**, 381 (1963).
[8] G. Goldhaber, S. Goldhaber, J. A. Kadyk, and B. C. Shen, Phys. Rev. Letters **15**, 118 (1965), have analyzed data which suggest that the B is not a resonance in the π-ω system.
[9] G. F. Chew, Phys. Rev. Letters **9**, 233 (1962).

change of both ρ and B mesons to provide the force responsible for low-energy scattering. Clearly, one has to calculate forces not only from ρ exchange but also from B exchange in order to determine the preferable quantum state of the B meson.

A general theoretical framework for treating the interaction of any pseudoscalar meson and vector-meson system has recently been given in K.[5] This formalism, based on the S-matrix point of view, gives a convenient procedure for calculating the Born term for the exchange of a particle of arbitrary spin and parity. A model calculation, which considers both 1^+ and 2^- assignments for the B and which takes into account both ρ and B exchange, prefers a 2^- B meson.

Atkinson[6] has also considered the effects upon π-ω scattering of B exchange, calculated in a field-theoretic model, and found a 2^- assignment for the B most likely.

In the present paper we re-examine the question of the quantum numbers of the B meson, and we perform a relativistic calculation of π-ω scattering in an effort to obtain both the ρ and B mesons via the reciprocal-bootstrap mechanism. The concept of the reciprocal bootstrap arose from a study of π-N scattering. It had been known from the Chew-Low model[10] that the exchange of a nucleon provides a sufficiently strong attractive force to give a resonance in the $P_{3/2,3/2}$ state of π-N scattering, the N^* resonance. It was then observed[9] that the exchange of the N^* is sufficient to produce a bound state in the $P_{1/2,1/2}$ amplitude of π-N scattering, identified as the nucleon itself. On the other hand, nucleon exchange gives a weak, attractive force to its own state, while the exchange of the N^* is negligible in its own state.[11] Thus, taken together, the nucleon and N^* provide the forces necessary for the existence of each other, although neither can reproduce itself, as the ρ can in π-π scattering.

We apply the reciprocal-bootstrap idea to the ρ and B mesons in π-ω scattering. The forces for π-ω scattering are given by both ρ and B exchange, as illustrated in Fig. 1. We then search for both a bound state and a resonance, the bound state in the 1^- amplitude, to represent the ρ, and the resonant state in whichever amplitude we choose for the B. The diagrams we wish to calculate are illustrated in Fig. 2. It is not inconsistent with the reciprocal-bootstrap concept for the individual

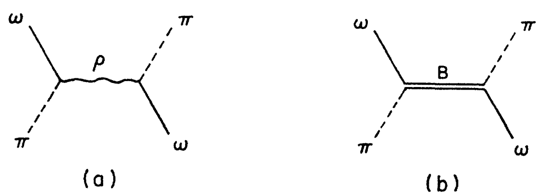

FIG. 1. π-ω scattering diagrams, showing ρ exchange and B exchange.

[10] G. F. Chew and F. E. Low, Phys. Rev. **101**, 1570 (1956).

[11] E. Abers and C. Zemach, Phys. Rev. **131**, 2305 (1963).

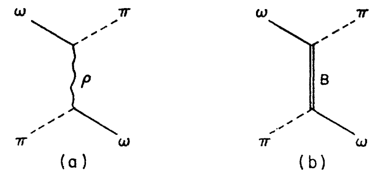

FIG. 2. π-ω scattering diagrams for a ρ meson and a B meson in the direct channel.

forces of ρ and B exchange in either state of interest to be of the same order of magnitude. The important point is that, to the extent the two particles form a closed system, both are necessary to explain each other.

The present calculation should be considered as complementary to the more usual procedure of trying to produce the ρ as a bootstrap resonance in π-π scattering.[12] These calculations generally predict too low a ρ mass and require too large a reduced width. Calculations have been made coupling the π-ω channel to the π-π channel, but these have included only ρ exchange and do not appreciably help the width problem.[13] It is known that if a "resonance" is predominantly a bound state in a higher threshold channel, the predicted width can be appreciably smaller than if it is predominantly a scattering resonance.[14] With this in mind, we investigate the π-ω channel including the B exchange force (which does not exist in the π-π channel) and considering the π-π channel only as it manifests itself by the ρ meson in the $I=1$, π-π state. This shows up in π-ω scattering as the ρ-exchange force and a bound-state pole in the direct π-ω channel.[15]

In order to achieve a completely self-consistent solution, the masses of the ρ and B exchanged in the diagrams of Fig. 1 (input masses) must equal the masses of the ρ and B found by solving the partial-wave dispersion relations to obtain the diagrams of Fig. 2 (output masses). Secondly, the input coupling constants of Fig. 1 must reproduce themselves in Fig. 2. In the present calculation we do not try to achieve complete self-consistency. As in most numerical calculations involving vector mesons, some form of cutoff is necessary in order that the dispersion integrals converge. In a reciprocal-bootstrap approach to the N and N^*, the suggestion[16] has been made to adopt the same value of cutoff for both the N and N^* amplitudes, the cutoff being determined to give the N^* at its correct mass. However, there is no reason why the cutoff should be the same in two different partial-wave amplitudes. Because of this ambiguity, we shall adopt the simpler approach of choosing

[12] See, for instance, Refs. 26 and 27, the second and third papers of Ref. 29, and further references in those papers.

[13] J. R. Fulco, G. L. Shaw, and D. Y. Wong, Phys. Rev. **137**, B1242 (1964); P. W. Coulter and G. L. Shaw, *ibid.* **138**, B1273 (1965); see also Refs. 27 and 39.

[14] This was first pointed out by R. H. Capps, Phys. Rev. **131**, 1307 (1963), in connection with the η-K channel and the K^* resonance. The idea was further developed by M. Bander, P. W. Coulter, and G. L. Shaw, Phys. Rev. Letters **14**, 270 (1965), who emphasize the importance of whether the calculation of a higher threshold channel can produce a bound state.

[15] The validity of this procedure is discussed in Ref. 1.

[16] J. S. Ball and D. Y. Wong, Phys. Rev. **133**, B179 (1964).

particular values of the mass for the ρ and B mesons; we then determine values of the $\pi\rho\omega$ and $\pi B\omega$ coupling constants, in order that the output mass of each particle equal its input mass. In addition, we specify for each particle the value of its self-consistant coupling constant, wherever possible.

Next we consider the question of the quantun numbers for the B meson. Scattering in the π-ω system can take place in every spin-parity state except 0^+. We consider all possible quantum numbers for the B with $J\leq 2$.

0^-. From an examination of the Born terms, it is seen that ρ exchange gives a repulsive force to the 0^- state. In addition, the exchange of a 0^- particle is repulsive in the ρ state. Hence, in this model, this assignment is ruled out.

1^-. Experimentally, the decays $B\to 2\pi$, $K\bar{K}$ have not been observed, whereas they would be expected if the B were 1^-. We do not consider this possibility.

1^+. The exchange of a 1^+ meson is repulsive in the ρ state, so that this possibility must be ruled out in a reciprocal-bootstrap model. It has been noted that ρ exchange produces a sufficiently strong attraction in the 1^+ state to produce a resonance.[3] However, as happens in π-N scattering, the long-range part of this force is negligible in the s-wave state; hence, this force is mainly of short range, and since short-range effects have been neglected in this calculation, it would be inconsistent to consider this force seriously. We discuss this point in greater detail later.

2^+. ρ exchange in the 2^+ state is repulsive, which is inconsistent with the reciprocal bootstrap idea.

2^-. The exchange of a 2^- resonance gives an attractive force both to the 1^- amplitude as well as to itself. In addition, ρ exchange is attractive in the 2^- state. Hence, this state is the only one consistent with the reciprocal-bootstrap idea.

Thus, we perform a calculation for the 1^- and 2^- scattering states of the π-ω system. We solve the partial-wave dispersion relations by the well-known N/D method,[17] under the assumption of elastic unitarity. The experimental absence of the decay $B\to\pi+\phi$ allows the unitarity condition to be satisfied up to higher energies without the need for a multichannel calculation. In addition, we shall see that f-wave mixing in the 2^- amplitude is quite small, so we consider only 2^- p-wave scattering by performing effectively a single-channel calculation.

In Sec. II we discuss the kinematics of π-ω scattering and the partial-wave decomposition. In Sec. III we write down the Born terms and examine them in some detail to verify statements made concerning their order of magnitude and relative signs. In Sec. IV we present our calculations and discuss the solutions. For numerical purposes we approximate the Born terms by poles, which allow an exact solution to be given for the partial-wave amplitudes by the N/D method (with a cutoff imposed on all integrals). Two different approximations are made for ρ exchange: (1) the Born term, as it is given by theory, is approximated by one or two poles, and (2) the long-range part of the Born term only (the short cut) is approximated by one pole. Conditions under which the ρ and B mesons reproduce themselves are presented for both cases. Some general discussion and conclusions are given in Sec. V.

[17] G. F. Chew and S. Mandelstam, Phys. Rev. **119**, 467 (1960).

II. KINEMATICS AND ANGULAR MOMENTUM DECOMPOSITION

The kinematics of the reaction $\pi+\omega\to\pi+\omega$ are illustrated in Fig. 3; p, k (p',k') are the initial (final) momentum four-vectors for the π and ω, respectively, and ϵ (ϵ') is the initial (final) polarization vector of the ω. We introduce the usual kinematic variables s, t, u by

$$\begin{aligned} s&=-(p+k)^2=2k^2+m_\omega^2+\mu^2+2(k^2+m_\omega^2)^{1/2}(k^2+\mu^2)^{1/2},\\ t&=-(p-p')^2=-2k^2(1-\cos\theta),\\ u&=-(p-k')^2=2m_\omega^2+2\mu^2-s-t, \end{aligned}\tag{2.1}$$

where $k=|\mathbf{k}|=|\mathbf{p}|$ is the magnitude of the spatial momentum in the barycentric system. θ is the scattering angle, m_ω is the ω mass, and μ is the pion mass.

One can easily see that a description of π-ω scattering requires four independent amplitudes. One choice of invariant amplitudes which are free from kinematic singularities and which satisfy the Mandelstam representation is given by Eq. (19) of K:

$$\begin{aligned} T(s,t,u)=\epsilon'\cdot\epsilon F_1(s,t,u)&+(\epsilon'\cdot P)(\epsilon\cdot P)F_2(s,t,u)\\ &+(\epsilon'\cdot R)(\epsilon\cdot R)F_3(s,t,u)\\ &+\tfrac{1}{2}[(\epsilon'\cdot P)(\epsilon\cdot R)+(\epsilon'\cdot R)(\epsilon\cdot P)]F_4(s,t,u), \end{aligned}\tag{2.2}$$

where

$$P=p+k,\qquad R=k-p'.$$

By appropriate choices of the ϵ, ϵ', which are given in the Appendix, one can construct helicity amplitudes, $t_{\lambda\mu}(s,t,u)$, where $\lambda(\mu)$ is the helicity of the initial (final) ω. It is convenient to define parity-conserving amplitudes which in the present problem are given by Eqs. (52)–(55) of K[18]:

$$\begin{aligned} f_{++}^-&=(1+\cos\theta)^{-1}t_{++}-(1-\cos\theta)^{-1}t_{+-},\\ f_{++}^+&=(1+\cos\theta)^{-1}t_{++}+(1-\cos\theta)^{-1}t_{+-},\\ f_{+0}^+&=2(\sin\theta)^{-1}t_{+0},\\ f_{00}^+&=2t_{00}. \end{aligned}\tag{2.3}$$

In terms of these amplitudes, partial-wave helicity amplitudes of definite parity P and total angular momentum J can be found easily.[19] These amplitudes,

[18] These expressions are based on the definition of parity-conserving amplitudes, given by M. Gell-Mann, M. L. Goldberger, F. E. Low, E. Marx, and F. Zachariasen, Phys. Rev. **133**, B145 (1964).

[19] M. Jacob and G. C. Wick, Ann. Phys. (N. Y.) **7**, 404 (1959).

denoted by $t_{\lambda\mu}{}^{J\pm}(s)$, are given by Eqs. (56)–(59) of K:

$$
\begin{aligned}
t_{++}{}^{J-}(s)&=\frac{1}{2}\int_{-1}^{+1} dz\left[P_J(z)f_{++}{}^{+}(s,z)+\frac{JP_{J+1}(z)+(J+1)P_{J-1}(z)}{2J+1}f_{++}{}^{-}(s,z)\right],\\
t_{++}{}^{J+}(s)&=\frac{1}{2}\int_{-1}^{+1} dz\left[\frac{JP_{J+1}(z)+(J+1)P_{J-1}(z)}{2J+1}f_{++}{}^{+}(s,z)+P_J(z)f_{++}{}^{-}(s,z)\right],\\
t_{+0}{}^{J+}(s)&=\frac{1}{2}\int_{-1}^{+1} dz\frac{[J(J+1)]^{1/2}}{2J+1}[P_{J+1}(z)-P_{J-1}(z)]f_{+0}{}^{+}(s,z)(m_\omega/W),\\
t_{00}{}^{J+}(s)&=\frac{1}{2}\int_{-1}^{+1} dz\,P_J(z)f_{00}{}^{+}(s,z)(m_\omega/W)^2,
\end{aligned}
\tag{2.4}
$$

where $W=(k^2+m_\omega{}^2)^{1/2}$. Here the amplitude $t_{++}{}^{J-}(s)$ has parity $(-1)^J$, while the others $t_{\lambda\mu}{}^{J+}(s)$ have parity $-(-1)^J$.

In the calculations of this paper, it is convenient to work in terms of transition amplitudes between states of definite orbital angular momentum L. These amplitudes are denoted by $t^{JP}(L\rightarrow L')$. Since the pion has spin zero and the ω has spin 1, one can have $L=J$, $J\pm1$. The following transitions are allowed by parity conservation: $L=J\rightarrow L'=J$, with parity $(-1)^J$ and $L=J-1\rightarrow L'=J-1$, $L=J-1\rightarrow L'=J+1$, and $L=J+1\rightarrow L'=J+1$ all having parity $-(-1)^J$. Time-reversal invariance insures that the two transitions $L=J-1\rightarrow L'=J+1$ and $L=J+1\rightarrow L'=J-1$ be the same. The transformation between the two sets of amplitudes, $t_{\lambda\mu}{}^{J\pm}$ and $t^{JP}(L\rightarrow L')$, is given by Eqs. (62)–(65) of K, by using a formula given in Appendix B of Jacob and Wick.[19] The results for $J=1$ and $J=2$ are stated explicitly in the Appendix.

Finally, we note the relationship between the invariant amplitude $T(s,t,u)$ and the differential cross section:

$$\left|\frac{T(s,t,u)}{8\pi s^{1/2}}\right|^2=\frac{d\sigma}{d\Omega}. \tag{2.5}$$

III. BORN TERMS

The driving forces for low-energy π-ω scattering are taken to be the exchange of the ρ and B mesons. The general method for discussing the exchange of any spin-parity state has been given by K, and the reader is advised to consult this paper for details. Here we wish to point out certain features.

The first step of this procedure is to express the invariant amplitudes $F_i(s,t,u)$ in terms of the particular spin-parity state in question in the direct channel, as illustrated in Fig. 2. Here a distinction is drawn between the role of the ρ and the B. Since the mass of the ρ lies below the π-ω threshold, it is a bound state in π-ω scattering and its amplitude is given exactly by a pole. On the other hand, the B is a resonance in π-ω scattering; to determine its amplitude, we assume that the B saturates π-ω scattering in the low-energy region and we express the absorptive part of the invariant amplitudes in terms of a delta function which is obtained from the Breit-Wigner resonance form in the narrow-width approximation. Thus, we have the following expressions for the ρ and B amplitudes:

$$
\begin{aligned}
t_\rho(s)&=\frac{f_\rho(s)}{m_\rho{}^2-s},\\
\mathrm{Im}t_B(s)&=f_B(s)\delta(s-m_B{}^2).
\end{aligned}
\tag{3.1}
$$

The two functions $f_\rho(s)$ and $f_B(s)$ are chosen to guarantee that the invariant amplitudes $F_i(s,t,u)$ are free from kinematic singularities, when expressed in terms of the ρ- and B-meson amplitudes, Eq. (3.1). One finds that $f_\rho\sim sk^2$, while $f_B\sim s^2k^2$ for a 2^- p-wave B meson and $f_B\sim s$ for a 1^+ s-wave meson, denoted by A, and if for convenience we express t_B as a pole, keeping explicitly the factor $f_B(s)$, we have

$$
\begin{aligned}
t_\rho{}^{1-}(s)&=\tfrac{2}{3}\gamma_\rho[sk^2/(m_\rho{}^2-s)],\\
t_B{}^{2-}(s)&=\tfrac{2}{3}\gamma_B[s^2k^2/(m_B{}^2-s)],\\
t_A{}^{1+}(s)&=\gamma_A[s/(m_A{}^2-s)].
\end{aligned}
\tag{3.2}
$$

These expressions serve to define the $\pi\rho\omega$, $\pi B\omega$, and $\pi A\omega$ coupling constants, γ_ρ, γ_B, and γ_A, respectively, used in the present calculation. We note that $\gamma_\rho/4\pi=f^2/4\pi$ in the conventional notation, where $f^2/4\pi=0.35$,[20] for an ω width of 8 MeV.

Finally, to obtain the Born terms for ρ and B exchange, we use the crossing properties of π-ω scattering. For ρ exchange, crossing is applied directly to the in-

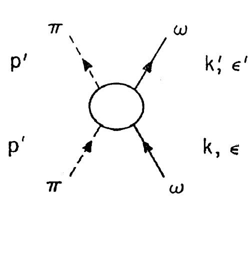

FIG. 3. Kinematics of π-ω scattering.

[20] M. Gell-Mann, D. Sharp, and W. Wagner, Phys. Rev. Letters 8, 261 (1962).

variant amplitudes to yield the Born terms of Fig. 1(a); for B exchange, crossing is applied to the absorptive part of the invariant amplitudes, $\mathrm{Im}F_i(s,t,u)$, the Born terms of Fig. 1(b) being determined from the dispersion integral,

$$F_i(u,t,s)=\frac{1}{\pi}\int du'\frac{\mathrm{Im}F_i(u',t,s)}{u'-u}. \tag{3.3}$$

Having obtained the quantities F_i, we can write the amplitudes $f_{\lambda\mu}^{\pm}$ at once. The Born terms for ρ exchange and for the exchange of a 2^- p-wave and 1^+ s-wave B meson have been presented in K.

With the Born terms in hand, the next step is to project out partial waves. This can readily be done by Eq. (2.4) to yield helicity amplitudes of given total angular momentum and parity. We have calculated the $J=1$ and $J=2$ projections of ρ exchange and of the exchange of a 2^- p-wave B meson and also the $J=1$ projections of the exchange of a 1^+ s-wave B meson. (Note that the $J=2$ projections are irrelevant for 1^+ B exchange.) These results are presented in the Appendix. We note from these formulas that the paritalwave projections of both the ρ- and B-exchange Born terms have the asymptotic behavior $s\ln s$. The Born terms for transition amplitudes between states of definite orbital angular momentum can be found from (2) of the Appendix and are given explicitly in Appendix I of K. In Fig. 4 we present graphs of the $J=1$ and $J=2$

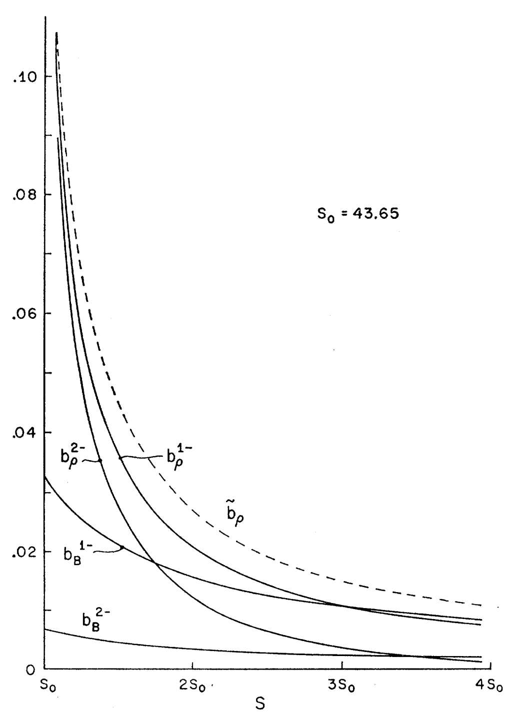

FIG. 4. Born terms for ρ exchange and B exchange in the 1^- and 2^- p-wave states. The solid curves show the actual Born terms while the dashed curve shows the pole term representing the short cut alone for ρ exchange. We used the values $m_\rho=769$ MeV, $m_B=1220$ MeV, $\gamma_\rho/4\pi=0.35$, and $\gamma_B=3.08\times10^{-2}$.

TABLE I. Threshold values of $t^{JP}(L\rightarrow L')/k^Lk^{L'}$ for the $J=1$ and $J=2$ Born terms for ρ exchange and for 2^- p-wave and 1^+ s-wave B exchange.

Angular-momentum state		ρ exchange	2^- p-wave B exchange	1^+ s-wave B exchange
$J=1$	$p\rightarrow p$	5.28	1.42	-0.204
	$s\rightarrow s$	0.00	-71.00	12.0
	$s\rightarrow d$	1.27	0.68	-0.19×10^{-1}
	$d\rightarrow d$	0.37	-0.83×10^{-1}	0.47×10^{-2}
$J=2$	$p\rightarrow p$	4.95	0.286	
	$p\rightarrow f$	-0.17×10^{-1}	-0.86×10^{-2}	
	$f\rightarrow f$	-0.57×10^{-1}	0.89×10^{-3}	
	$d\rightarrow d$	-0.57	-0.16×10^{-1}	

p-wave projections of the ρ-exchange and 2^- p-wave B-exchange Born terms.

Next we examine the sign and order of magnitude of the partial-wave Born terms for $J=1$ and $J=2$. For this purpose we follow Abers and Zemach,[11] and list the quantities $t^{JP}(L\rightarrow L')/k^Lk^{L'}$, evaluated at threshold. These results appear in Table I.[21]

We may now verify our previous remarks concerning the effects of ρ and B exchange. We find from this table that ρ exchange and 2^- p-wave B exchange give rise to strong attractive forces in both $J=1$ and $J=2$ p-wave amplitudes. We also note that 1^+ s-wave exchange gives repulsion to the ρ; although the absolute magnitude of this force at threshold is weaker by an order of magnitude than the attractive force from ρ exchange, the ρ meson, as we shall see, is unable to bootstrap itself as a bound state. Hence, in a reciprocal bootstrap of the ρ and B mesons, a 1^+ B is not possible. We further note from Table I, that the exchange of either the ρ or the 2^- p-wave B is very weak in both p-f and f-f $J=2$ transitions. Therefore, it is consistent to treat a 2^- B meson as pure p-wave in evaluating the exchange term; in addition, this allows us to solve for the 2^- amplitude in a single-channel calculation which neglects the p-f and f-f transitions completely.

The contribution of ρ exchange to the 1^+ s-wave state requires further consideration. Near threshold, this term behaves as k^2, although it represents s-wave scattering. If we wish to compare the effect of a particular exchange in different angular momentum states, we might do this by comparing the Born terms with the threshold-momentum dependence removed. Applying this criterion, we find that ρ exchange contributes forces of the same order of magnitude to 1^+ s-wave and 2^- p-wave scattering in the low-energy region. It is not surprising that ρ exchange produces a 1^+ resonance.

However, the scattering produced by ρ exchange is of different character in these two amplitudes, as we can

[21] All numerical calculations in this paper are performed with the following mass values in MeV: $\mu=139.6$, $m_\omega=782.7$, $m_\rho=769$, $m_B=1220$. In Table I, the following coupling constants were used: $\gamma_\rho=4.4$, from $f^2/4\pi=0.35$; $\gamma_B=3.08\times10^{-2}$, for a B width of 125 MeV (cf. Ref. 33); and $\gamma_A=9.26$.

determine by studying the singularities of ρ exchange in the partial-wave amplitudes. We find that, in the s plane, ρ exchange gives rise to a short cut along the real axis, from $(m_\omega^2-\mu^2)^2/m_\rho^2$ to $2m_\omega^2+2\mu^2-m_\rho^2$, as well as to a semi-infinite cut, also on the real axis, starting at $s=0$ and going out to $-\infty$.

It can be shown that the effects of the short cut is of order μ/m_ω, compared with unity, in s-wave states, and hence s-wave scattering is due mainly to the more distant cut starting at $s=0$. On the other hand, the short cut is quite important for p-wave states. It is part of present-day S-matrix philosophy that low-energy scattering arises mainly from the nearby singularities of the scattering amplitude and that distant singularities are less important.[22] Furthermore, there are many contributions to the distant singularities and for the most part we do not know how to include them. Thus, we feel that it would be inconsistent to take seriously only one of the distant singularities while ignoring the rest.

IV. CALCULATIONS AND NUMERICAL RESULTS

Having obtained the Born terms for ρ and B exchange, we are now ready to perform the reciprocal-bootstrap calculation for these two particles. We let $A_J(s)$ be the p-wave amplitude for the total angular momentum $J=1$, 2 and $B_J(s)$ the complete p-wave Born term. $A_J(s)$ is normalized so that

$$A_J(s)=(8\pi s^{1/2}/k)e^{i\delta_J}\sin\delta_J. \tag{4.1}$$

We next define a new amplitude and new Born term:

$$a_J(s)=A_J(s)/f_J(s), \quad b_J(s)=B_J(s)/f_J(s). \tag{4.2}$$

Here $a_J(s)$ is an amplitude free from kinematic singularities and zeros; the factor $f_J(s)$ is chosen to accomplish this and to guarantee that $A_J(s)$ have correct threshold behavior. The form of the direct-channel pole terms, Eq. (3.2), suggests that we choose $f_1=sk^2$ and $f_2=s^2k^2$.[23]

The scattering amplitude is determined from the partial-wave dispersion relation

$$a_J(s)=\frac{1}{\pi}\int_{s_0}^{\infty} ds'\frac{\mathrm{Im}\,a_J(s')}{s'-s}+b_J(s), \tag{4.3}$$

where $s_0=(m_\omega+\mu)^2$, together with the unitarity condition

$$\mathrm{Im}\,a_J(s)=\rho_J(s)\,|a_J(s)|^2; \tag{4.4}$$

$\rho_J(s)$ is given by

$$\rho_J(s)=-\mathrm{Im}[a_J(s)]^{-1}=\frac{1}{8\pi}\frac{k}{s^{1/2}}f_J(s). \tag{4.5}$$

The Born term $b_J(s)$ appearing in Eq. (4.3) represents the contribution of the unphysical singularities to $a_J(s)$ and can be put in the form of a dispersion integral

$$b_J(s)=\frac{1}{\pi}\int_L ds'\frac{\mathrm{Im}\,a_J(s')}{s'-s}, \tag{4.6}$$

where L denotes the unphysical cuts. Although it is generally most convenient when calculating the effect of single-particle exchanges to evaluate the Born term directly rather than through the integral in Eq. (4.6), this form will be useful later.

To solve Eq. (4.3), we use the well-known N/D method.[17] We set

$$a_J(s)=N_J(s)/D_J(s); \tag{4.7}$$

then $N_J(s)$ and $D_J(s)$ are given by the equations[24]

$$N_J(s)=b_J(s)+\frac{1}{\pi}\int_{s_0}^{\infty} ds'\rho_J(s')N_J(s')\times\frac{b_J(s')-[(s-a)/(s'-a)]b_J(s)}{s'-s}, \tag{4.8a}$$

$$D_J(s)=1-\frac{s-a}{\pi}\int_{s_0}^{\infty} ds'\frac{\rho_J(s')N_J(s')}{(s'-a)(s'-s)}, \tag{4.8b}$$

where a is the point at which $D_J(s)$ is set equal to unity.

Because of the well-known divergence problem one encounters when dealing with vector mesons, a rigorous solution of Eq. (4.3) does not exist. One can see this by examining the convergence of the integral equation for $N_J(s)$, Eq. (4.8a). Since $B_J(s)\sim s\ln s$, $b_J(s)\sim \ln s/s$ for $f_1(s)=sk^2$, and $b_2(s)\sim\ln s/s^2$ for $f_2(s)=s^2k^2$. However, the integral $\int ds'\rho_J(s')N_J(s')b_J(s')/(s'-s)$ generally does not decrease faster than $1/s$, so that $N_J(s)\sim 1/s$, at best.[25] Thus, the integral in Eq. (4.8a) diverges at least logarithmically for $J=1$ and linearly for $J=2$.[23]

Despite this general failing of the theory, several methods[11,16,26–30] have been proposed for extracting information concerning low-energy scattering in numerical calculations. If a straight cutoff[11,16,27,28] is imposed on all integrals, the integral equation for $N_J(s)$ becomes Fredholm and a unique solution exists. However, one generally cannot obtain the analytic solution in closed form so that one must resort to a lengthy computer solution. To avoid this, several approximate solutions

[22] G. F. Chew, *S-Matrix Theory of Strong Interactions* (W. A. Benjamin, Inc., New York, 1961).

[23] In the calculations of this paper we choose $f_1=f_2=sk^2$, for reasons discussed later in the text.

[24] A. W. Martin and J. L. Uretsky, Phys. Rev. **135**, B803 (1964).

[25] Cf. Ref. 11 for a similar discussion on this point.

[26] G. F. Chew and S. Mandelstam, Nuovo Cimento **19**, 752 (1961); L. Balazs, Phys. Rev. **128**, 1939 (1962); J. Franklin, D. J. Land, and R. Pinon, *ibid.* **137**, B172 (1965).

[27] F. Zachariasen and C. Zemach, Phys. Rev. **128**, 849 (1962).

[28] M. Bander and G. L. Shaw, Ann. Phys. (N. Y.) **31**, 506 (1965).

[29] G. L. Shaw, Phys. Rev. Letters **12**, 345 (1964); G. Q. Hassoun and K. Kang, Phys. Rev. **137**, B955 (1965); J. Franklin, *ibid.* **139**, B192 (1965); H. R. Pagels, *ibid.* **140**, B1599 (1965).

[30] S. C. Frautschi and J. D. Walecka, Phys. Rev. **120**, 1486 (1960); A. W. Martin, *ibid.* **135**, B975 (1964).

of Eq. (4.3) have been suggested.[29] These are generally designed to yield solutions which depend only on the evaluation of integrals, while keeping the exact Born terms. Nevertheless, most of these methods would require some form of cutoff if applied to the present problem. There exists a third school of thought[30] which suggests that the Born terms be approximated by a small number of first- and second-order poles. In this case, an exact analytic solution is possible (with presumably a cutoff in some cases), since the kernal of the integral in Eq. (4.8a) becomes a separable function of s and s'.

In the present calculation we have chosen to approximate the Born terms for ρ and B exchange, $b_\rho(s)$ and $b_B(s)$, by a few poles. Because the exact equations of the theory have no solution without imposing a cutoff, and because we lack all knowledge of the distant singularities and of the asymptotic behavior of the partial-wave amplitudes, we feel that this approach is as likely to yield valid semiquantitative results as any other method.[31] Furthermore, the integral in the equation for $N_J(s)$ will not depend sensitively, for values of s in the low-energy region, on the exact form of $b(s')$ for large s', provided $b(s')$ does not oscillate. In other words, the integral is essentially independent of small s for s' above some value Λ' around the cutoffs we are using. Thus, by using a form of $b(s)$ which agrees with the exact Born term in the low-energy region, we obtain results which will differ from those obtained from use of the exact Born term only in the cutoff value needed to reproduce a particular resonance for a given set of input parameters. Moreover, it is possible to obtain a fairly accurate representation for the Born terms throughout the low-energy region with only one or two poles.

It should be noted that, after replacing the Born term by poles, we change the asymptotic behavior of the Born term by eliminating the factor $\ln s$. This generally has the effect of reducing the degree of divergence of the integrals by one power, and subsequently it reduces somewhat the dependence of the physical results on the cutoff.[32] In the present problem one finds that the integrals diverge logarithmically for $J=1$ and linearly for $J=2$, if the pole approximation is used.

In a bootstrap calculation of this sort, it is somewhat disturbing to find that two partial waves of interest have a rather different dependence on the cutoff. In addition to this, a linear divergence in itself makes results difficult to interpret, since they are apt to depend rather sensitively on the cutoff values. It is possible in the present problem to reduce the linear divergence in the $J=2$ wave to a logarithmic one by choosing $f_2=sk^2$ rather than s^2k^2. This comes about because the Born terms in both the $J=1$ and $J=2$ amplitudes have the same behavior in both the low-energy region as well as in the asymptotic region. Furthermore, we note that the point $s=0$ corresponds to the asymptotic limit of scattering in the u channel and therefore the high-energy behavior of the scattering amplitude is required in order to investigate this point fully. For these reasons we choose $f_2=sk^2$ in the calculations.

Finally we discuss the criteria by which poles were chosen to represent the Born terms. For both ρ and B exchange, pole terms were chosen to reproduce the exact Born term from threshold to an energy of about 2700 MeV with an average accuracy of 95%. We remind the reader that threshold occurs at 922 MeV and that B mass is 1220 MeV. If we achieve this accuracy within this energy region, the Born term will be given sufficiently well for our purposes. In fact, we were able to achieve an average accuracy of about 98% throughout most of this interval with one pole for all Born terms except the $J=2$ p-wave projection of ρ exchange, which required two poles because of the zero in the Born term. The largest errors occurred, naturally enough, at the limits of the aforementioned interval. The expressions we have obtained for the Born terms are the following:

ρ exchange:

$$J=1,\quad b_\rho^{1-}(s)=\gamma_\rho\frac{0.2360}{s-37.02};\tag{4.9a}$$

$$J=2,\quad b_\rho^{2-}(s)=\gamma_\rho\left[\frac{0.3008}{s-35}-\frac{0.3618}{s+30}\right];\tag{4.9b}$$

B exchange:

$$J=1,\quad b_B^{1-}(s)=\gamma_B\frac{45.0}{s};\tag{4.10a}$$

$$J=2,\quad b_B^{2-}(s)=\gamma_B\frac{12.76}{s+21.57}.\tag{4.10b}$$

To illustrate the accuracy of the pole-term approximation, we present in Table II values of $b_\rho^{1-}(s)/\gamma_\rho$ and $b_\rho^{2-}(s)/\gamma_\rho$ given by the exact form of the Born terms and by the above forms, Eqs. (4.9a) and (4.9b). We see from the table that these Born terms are reproduced according to the criteria stated earlier in the interval from $s=43.65$ to $s=382$, which corresponds to the energy interval 922–2700 MeV. The ρ-exchange Born term in the $J=2$ p-wave state has a sign change at $s=305$. The pole approximation for this term continues to follow the exact Born term up to about $s=1200$, although numerically it is about a third smaller above $s=305$. We would therefore expect that the repulsive effect of this sign change would be underestimated in our solutions.

[31] Cf. Ref. 11, where a comparison is made between two solutions for the $p_{3/2,3/2}$ state in π-N scattering: in one case, the exact Born term is used with a cutoff and, in the other, the Born term is approximated by poles. The two solutions are quite similar.

[32] We make this statement from experience with calculations of the ρ meson in π-π scattering. The exact N/D p-wave equation is singular and does not possess a solution; if the Born term is approximated by poles, thereby eliminating the $\ln s$ factor from its asymptotic behavior, an exact solution is possible without the need for a cutoff. Cf. Ref. 17.

TABLE II. Comparison between the exact form of the ρ-exchange Born term and the pole approximation for the $J=1$ and $J=2$ p-wave projections.

s	$B_\rho^{1-}(s)/sk^2$ Exact	$B_\rho^{1-}(s)/sk^2$ Eq. (4.9a) pole	$B_\rho^{2}(s)/sk^2$ Exact	$B_\rho^{2}(s)/sk^2$ Eq. (4.9b) pole
43.65	2.799×10^{-2}	3.560×10^{-2}	2.631×10^{-2}	2.986×10^{-2}
67.06	0.7864×10^{-2}	0.7856×10^{-2}	0.5812×10^{-2}	0.5655×10^{-2}
86.11	0.4851×10^{-2}	0.4807×10^{-2}	0.2873×10^{-2}	0.2769×10^{-2}
103.96	0.3550×10^{-2}	0.3526×10^{-2}	0.1712×10^{-2}	0.1661×10^{-2}
121.23	0.2812×10^{-2}	0.2803×10^{-2}	0.1114×10^{-2}	0.1096×10^{-2}
138.17	0.2333×10^{-2}	0.2333×10^{-2}	0.7640×10^{-3}	0.7642×10^{-3}
154.89	0.1994×10^{-2}	0.2002×10^{-2}	0.5383×10^{-3}	0.5521×10^{-3}
187.94	0.1544×10^{-2}	0.1564×10^{-2}	0.2803×10^{-3}	0.2939×10^{-3}
220.67	0.1259×10^{-2}	0.1285×10^{-2}	0.1453×10^{-3}	0.1767×10^{-3}
261.33	0.1021×10^{-2}	0.1052×10^{-2}	0.5366×10^{-4}	0.8714×10^{-4}
301.80	0.8565×10^{-3}	0.8912×10^{-3}	0.0341×10^{-4}	0.3702×10^{-4}
342.16	0.7373×10^{-3}	0.7735×10^{-3}	-0.2583×10^{-4}	0.0713×10^{-4}
382.45	0.6458×10^{-3}	0.6830×10^{-3}	-0.4344×10^{-4}	-0.1146×10^{-4}
462.9	0.5163×10^{-3}	0.5542×10^{-3}	-0.6090×10^{-4}	-0.3106×10^{-4}
623.4	0.3659×10^{-3}	0.4025×10^{-3}	-0.6850×10^{-4}	-0.4250×10^{-4}
863.8	0.2523×10^{-3}	0.2854×10^{-3}	-0.6364×10^{-4}	-0.4185×10^{-4}
1229.	0.1644×10^{-3}	0.1923×10^{-3}	-0.5206×10^{-4}	-0.3485×10^{-4}
1664.	0.1211×10^{-3}	0.1451×10^{-3}	-0.4323×10^{-4}	-0.2892×10^{-4}
2465.	0.7870×10^{-4}	0.9720×10^{-4}	-0.3201×10^{-4}	-0.2123×10^{-4}
3265.	0.5800×10^{-4}	0.7311×10^{-4}	-0.2536×10^{-4}	-0.1668×10^{-4}

By way of comparison, a second approximate form is considered for ρ exchange. This is a pole, chosen to reproduce the effect of the short cut only, representing the long-range part of the total force. From the integral representation of $b_J(s)$, Eq. (4.6), this force is given by the equation

$$\tilde{b}_\rho(s)=\frac{1}{2\pi i}\int_{(m_\omega^2-\mu^2)^2/m_\rho^2}^{2m_\omega^2+2\mu^2-m_\rho^2} ds'\frac{\mathrm{disc}b(s')}{(s'-s)}. \quad (4.11)$$

If, for values of s in the physical region, we neglect the variation of the denominator along the cut and evaluate s' at the center of the cut s_c, we get

$$\tilde{b}_\rho(s)=\frac{1}{2\pi i}\int_{(m_\omega^2-\mu^2)^2/m_\rho^2}^{2m_\omega^2+2\mu^2-m_\rho^2} ds'\,\mathrm{disc}b(s')\Big/(s_c-s). \quad (4.12)$$

A rough evaluation of the integral which neglects the variation in s' of disc$b(s')$ along the cut and which neglects terms of order μ/m_ω gives, for either $J=1$ or $J=2$, the result

$$\tilde{b}_\rho(s)=\gamma_\rho\frac{0.3333}{s-32.55}. \quad (4.13)$$

We note that the residue one obtains is exactly the residue of the pole in the static model.[1] It is interesting to compare the two results we have for the ρ-exchange Born term, Eqs. (4.13) and (4.9). For $J=1$, the two forms are numerically quite similar near threshold, but the form representing the complete Born term falls off somewhat more sharply. The exact Born term for this wave actually has a zero for $s\sim6400$; thus, the distant cut, starting at $s=0$, contributes a weak repulsive force, which is not effective at low energies. On the other hand, the short-cut contribution for $J=2$ agrees rather closely with the first term in Eq. (4.9b); hence, the distant cut contributes a rather strong repulsive force to this wave, as evidenced by the existence of the second term in Eq. (4.9b). A considerable difference in the solutions obtained from each of these forms would be expected in this case. Figure 4 also presents a plot of the pole term, Eq. (4.13), representing the effect of the short cut of ρ exchange.

The solution for the amplitudes $a_J(s)$ can be obtained by algebraic means, when the Born term is expressed as a sum of poles. If the Born term is written as

$$b(s)=\sum_i\frac{r_i}{s-\alpha_i}, \quad (4.14)$$

where the sum is over all poles, then we have the following expressions for $N(s)$ and $D(s)$:

$$N(s)=\sum_i\frac{r_i}{s-\alpha_i}[1+(a-\alpha_i)k_i], \quad (4.15)$$

$$D(s)=1-(s-a)\sum_i \mathcal{G}_i(s)r_i[1+(a-\alpha_i)k_i], \quad (4.16)$$

where k_i is the solution of the following set of algebraic equations:

$$\sum_j\{\delta_{ij}-\mathcal{G}_{ij}r_j(a-\alpha_j)\}k_j=\sum_j \mathcal{G}_{ij}r_j, \quad (4.17)$$

and

$$\mathcal{G}_{ij}=\frac{1}{\pi}\int_{s_0}^{\Lambda} ds'\frac{\rho(s')}{(s'-a)(s'-\alpha_i)(s'-\alpha_j)}, \quad (4.18a)$$

$$\mathcal{G}_i(s)=\frac{1}{\pi}\int_{s_0}^{\Lambda} ds'\frac{\rho(s')}{(s'-a)(s'-\alpha_i)(s'-s)}. \quad (4.18b)$$

The existence of a bound or resonant state at energy squared s_R is determined by the condition[11,27]

$$D(s_R)=0, \quad \text{or} \quad \mathrm{Re}D(s_R)=0. \quad (4.19)$$

The $\pi\rho\omega$ coupling constant is self-consistent if it satisfies the condition

$$\gamma_\rho^{(\mathrm{in})}=\gamma_\rho^{(\mathrm{out})}=-\frac{3}{2}\frac{N(s)}{dD(s)/ds}\bigg|_{s=s_R}, \quad (4.20)$$

while we have for the $\pi B\omega$ coupling constant

$$\gamma_B^{(\mathrm{in})}=\gamma_B^{(\mathrm{out})}=-\frac{3}{2}\frac{1}{s_R}\frac{N(s)}{d\,\mathrm{Re}D(s)/ds}\bigg|_{s=s_R}. \quad (4.21)$$

Experimentally, the $\pi\rho\omega$ coupling constant has the value $\gamma_\rho/4\pi=0.35$, while the $\pi B\omega$ coupling constant takes the value $\gamma_B=3.08\times10^{-2}$ for a B-meson width of 125 MeV.[33]

We proceed now to a discussion of the numerical

[33] The relation between the B width Γ_B and γ_B is

$$\gamma_B=12\pi\Gamma_B/m_B^2k_B^3.$$

results of our calculations. The $J=1$ and $J=2$ p-wave states for the ρ and B mesons will be considered separately. As is obvious from the above discussion, there are five parameters in each amplitude among which γ_ρ and m_ρ are associated with ρ exchange and γ_B and m_B come from B exchange. A cutoff parameter is also present in both amplitudes.

A. The ρ Meson

Several situations are studied for the 1^- amplitude to determine self-consistent parameters for the ρ meson. First, by taking their experimental values as the input masses for the ρ and B mesons, the quantity $\gamma_B/8\pi$ is varied to produce a bound state in $a_1(s)$ at the experimental mass of the ρ for a fixed value of cutoff and of $\gamma_\rho/4\pi$. Values of the $\pi\rho\omega$ coupling constant $\gamma_\rho/4\pi$ are chosen from 0.0 (for which no ρ-exchange contributes) to 1.40, which is about four times the value deduced by Gell-Mann, Sharp, and Wagner.[20] Cutoff values are taken at $s=800$, 1600, and 6400, corresponding to energies of $5m_\omega$, $7m_\omega$, and $14m_\omega$, respectively. Calculations are performed both for the case where the actual Born term for ρ exchange is approximated by the pole term of Eq. (4.9a), and for the case where only the short-cut contribution of ρ exchange is approximated by Eq. (4.13). In both cases B exchange is approximated by Eq. (4.10a).

The results of the calculation are presented in Fig. 5. We observe first that the values of $\gamma_B/8\pi$ needed to reproduce the ρ at its experimental mass range from 4 to 10 times larger than the experimental value ($\gamma_B/8\pi=0.12\times10^{-2}$), depending on the cutoff. In order to bring $\gamma_B/8\pi$ down to the experimental value, we need an extremely large cutoff. Furthermore, it is impossible to obtain a self-consistent mass for the ρ when B exchange is completely neglected. Secondly, the effect of ρ exchange in reproducing itself in the direct channel is quite small and, in fact, appears to be repulsive in the pole approximation. The reason for this is obvious in the sense that we are trying to obtain the ρ as a bound state at the position $s=30.35$, which is below the interaction-pole position, $s=37.02$, of Eq. (4.9a), representing ρ exchange. As a result of this aspect of the pole approximation we would require a larger value of $\gamma_B/8\pi$ in order to obtain a self-consistent ρ mass. We notice again that if B exchange were not taken into account, it would be impossible to achieve a bound state for $s\leq 37.02$.

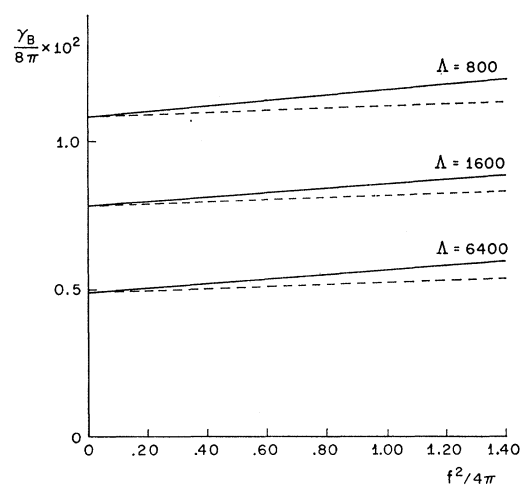

FIG. 5. Values of $\gamma_B/8\pi$ versus $f^2/4\pi$ which produce the ρ meson with a self-consistent mass of 769 MeV. $m_B=1220$ MeV. The solid curves show the results when the entire ρ-exchange Born term was used in the calculation, while the dashed curves show the results when only the short cut for ρ exchange was considered.

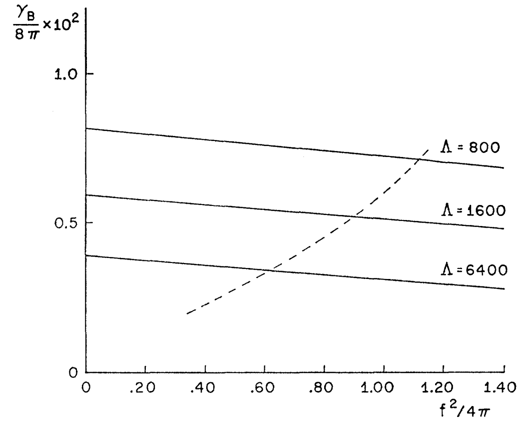

FIG. 6. Values of $\gamma_B/8\pi$ versus $f^2/4\pi$ which produce the ρ meson with a self-consistent mass of 859 MeV. $m_B=1220$ MeV. The dashed curve indicates complete self-consistency for the ρ meson.

This calculation is repeated for a ρ mass of 859 MeV. Here we consider only the case of approximating the entire Born term for ρ exchange by a single pole, given by the form

$$b_\rho^{1-}(s)=\gamma_\rho\frac{0.2384}{s-28.65}.$$

B exchange is again given by Eq. (4.10a). These results are presented in Fig. 6. We notice first that the values of $\gamma_B/8\pi$ are smaller than before. Secondly, ρ exchange behaves as a weak, attractive force. This is due to the fact that we are now seeking the ρ at the position $s=37.83$, which is higher in energy than the pole due to ρ exchange at $s=28.65$, from the above equation. Unlike the previous case, we can, by enhancing the effect of ρ exchange, obtain the ρ meson at its self-consistent mass for smaller values of $\gamma_B/8\pi$. Thirdly, for all three values of cutoff, a completely self-consistent ρ is found and is indicated by the dashed curve in Fig. 6.

Finally we consider the effect of varying the B mass for both values of the ρ mass previously considered, taking only one cutoff at 6400. These results are presented in Fig. 7; the crosses in the figure indicate complete self-consistency for the ρ meson. We summarize the parameters which yield a self-consistent ρ in Table III.

TABLE III. Values of $f^2/4\pi$ and B width Γ_B which give a completely self-consistent ρ meson.

B mass (MeV) / ρ mass (MeV)	1220		1400	
	$f^2/4\pi$	Γ_B	$f^2/4\pi$	Γ_B
769	1.10	588 MeV	0.88	1210 MeV
859	0.62	346 MeV	0.51	785 MeV

From this table we see that the larger mass, representing a smaller binding energy in the π-ω system, requires smaller values of $f^2/4\pi$ and B width Γ_B, as would be expected. On the other hand, the smaller value of m_B yields the smaller value of Γ_B, nearer the experimental value, while the larger m_B yields the value of $f^2/4\pi$ more consistent with the experimental value.

B. The B Meson

Calculations similar to the above are performed for the $J=2$ p-wave amplitude in order to determine self-consistent parameters for the B meson. First, taking the physical masses for the ρ and B mesons as input and varying $f^2/4\pi$ from 0.0 to 1.40, we determine the value of $\gamma_B/8\pi$ which gives a resonance at the position of the B meson. Three values of cutoff are considered, as before. Figure 8 shows the results when the entire ρ-exchange Born term is approximated by two poles, Eq. (4.9b), while Fig. 9 presents the same results when only the short cut is kept for ρ exchange. Equation (4.10b) is used in both cases for B exchange. The difference between these two cases is now quite striking. When the entire ρ-exchange Born term is used, the force is effectively repulsive, as we can anticipate from the strong repulsive pole term in Eq. (4.9b). Output B widths, which are somewhat larger than the experimental values, are nevertheless considerably smaller than the input values needed to produce a resonance at the B mass. Some numerical results are presented in Table IV. On the other hand, when the short cut only is maintained for ρ exchange, a completely self-consistent B meson is obtained for each cutoff value. Again, this situation is indicated by the dashed curve in Fig. 9. For $\Lambda=6400$, we obtain a self-consistent B at 1220 MeV with a width of 180 MeV, in reasonable agreement with experiment; the input ρ parameters are given by their experimental values.

We also consider the effect of varying the mass of the B. Several values of mass, for which the Born terms are approximated by the pole parameters given in Table V, are taken as input at constant $\gamma_B/8\pi$. The output masses

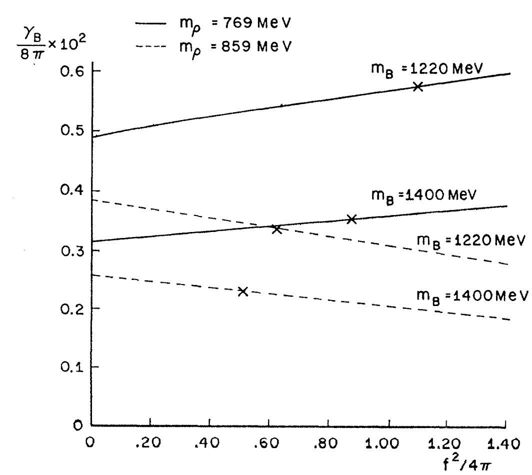

FIG. 7. Comparison of values of $\gamma_B/8\pi$ versus $f^2/4\pi$, which produce the ρ meson with a self-consistent mass, for two values of ρ mass and for two values of B mass. A single cutoff at 6400 was used. **X** indicates complete self-consistency for the ρ meson.

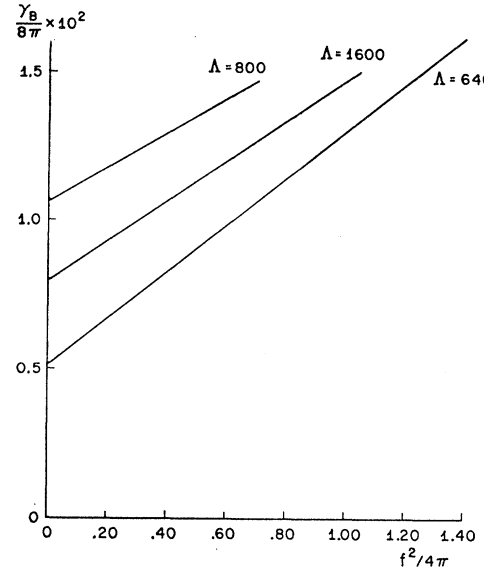

FIG. 8. Values of $\gamma_B/8\pi$ versus $f^2/4\pi$ which produce the B meson with a self-consistent mass of 1220 MeV, for the case in which the entire ρ-exchange force was used in the calculation. $m_\rho=769$ MeV.

TABLE IV. Input and output values of Γ_B in MeV, to produce a resonance at 1200 MeV in the $J=2$ p-wave amplitude. $m_\rho=769$ MeV.

	$\Lambda=800$		$\Lambda=1600$		$\Lambda=6400$	
$f^2/4\pi$	$\Gamma_B^{(\text{in})}$	$\Gamma_B^{(\text{out})}$	$\Gamma_B^{(\text{in})}$	$\Gamma_B^{(\text{out})}$	$\Gamma_B^{(\text{in})}$	$\Gamma_B^{(\text{out})}$
0.0	1072	214	803	165	522	113
0.35	1290	293	1044	242	815	193
0.70	1500	389	1290	334	1090	281

TABLE V. Parameters of the pole terms for B exchange in the $J=2$ p-wave state, where $b_B^{2-}(s)=\gamma_B\beta/(s-\alpha)$.

B mass (MeV)	α	β
1040	−6.96	8.06
1150	−16.61	10.98
1280	−31.90	15.14
1400	−39.18	19.12

TABLE VI. Input and output values of $\gamma_B/8\pi$ to produce a resonance in the $J=2$ p-wave amplitude with a self-consistent mass. $m_\rho=769$ MeV and $f^2/4\pi=0.35$.

B mass (MeV)	$\gamma_B{}^{(\text{in})}/8\pi$	$\Gamma_B{}^{(\text{in})}$ (MeV)	$\gamma_B{}^{(\text{out})}/8\pi$	$\Gamma_B{}^{(\text{out})}$ (MeV)
1150	0.0100	542	0.0024	130
1220	0.0080	815	0.0019	193
1280	0.0060	980	0.0015	245
1400	0.0040	1360	0.0012	410

at which resonances occur for these parameters are presented in Fig. 10. Also shown are the output values of $\gamma_B/8\pi$ when the mass is self-consistent. We note, as mentioned previously, that the output widths are smaller than the input widths, but are still larger than the experimental width. These results are summarized in Table VI.

Finally we note that the same calculations as those of Fig. 8 are performed with $m_\rho=859$ MeV. The results are essentially identical with those given in Fig. 8 for which $m_\rho=769$ MeV. The entire driving force due to ρ exchange, Eq. (4.9b), in our model is effectively repulsive in the $J=2$ p-wave state and therefore a larger Γ_B is needed when ρ exchange is enhanced by increasing $\gamma_\rho/4\pi$, in order to get the output m_B at the experimental value. This behavior arises from the sign change of the ρ-exchange Born term at $s=305$. We mention once again that Eq. (4.9b) reproduces the actual Born term closely up to $s=1000$, as is shown from Table II.

On the other hand, the driving force coming from the short cut of ρ exchange is attractive in this state, as is clear from Fig. 9, and this force gives a completely self-consistent B meson at the experimental mass, but with a somewhat larger width. This explains why calculations based on the static model[1,4] or on the low-energy behavior of the Born term[2,5] give resonant solutions containing the parameters associated with the B meson.

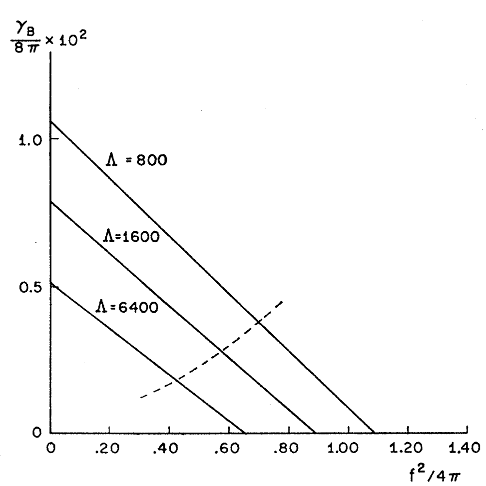

FIG. 9. Values of $\gamma_B/8\pi$ versus $f^2/4\pi$ which produce the B meson with a self-consistent mass of 1220 MeV, for the case in which only the short cut arising from ρ exchange was considered. $m_\rho=769$ MeV.

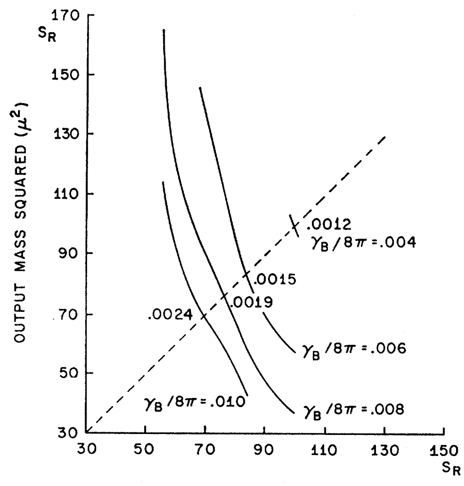

FIG. 10. Output mass squared versus input mass squared for the B meson, for several values of $\gamma_B/8\pi$. A single cutoff at 6400 was used. The output values of $\gamma_B/8\pi$, when the mass of the B meson is self-consistent, are also indicated on the graph. $m_\rho=769$ MeV. The entire ρ-exchange Born term was considered in the calculation.

V. DISCUSSION

In this section, we review and discuss the main results of the present work, in the light of the experimental status of the B meson, which is by no means clear today. Recently, Goldhaber *et al.*,[8] in a study of the ω's in the decay of $B\to\pi+\omega$, speculated that the B may not be a resonance in the π-ω system. They considered the possibility that the B may be either a four-pion resonance or a resonance between a pion and a three-pion state with $J^P\neq 1^-$, but no evidence could be found to support these conjectures. It has also been pointed out[34] that the B can be explained on theoretical grounds as a kinematic enhancement of the π-ω system in the reaction $\pi+p\to p+\omega+\pi$, according to the Deck mechanism.[35] Recent analyses[36,37] of the experimental data, however, have not produced conclusive results.

In the present work we find that the parameters of the B meson depend quite critically on the manner in which ρ exchange is treated. If the nearby cut, representing the long-range part of this force, is kept and the distant cut neglected, parameters for the B are found in close agreement with experiment. However, the distant cut which does not appear in a nonrelativistic calculation is effectively repulsive and strong in our model, thus requiring a large B width and favoring a small $\pi\rho\omega$ coupling constant to produce a resonance at the experimental position of the B. Output widths are

[34] U. Maor and T. A. O'Halloran, Phys. Letters **15**, 281 (1965).
[35] R. T. Deck, Phys. Rev. Letters **13**, 169 (1964).
[36] M. A. Abolins, D. D. Carmony, R. L. Lander, N. Xuong, and P. M. Yager, in *Proceedings of the Second Topical Conference on Recently Discovered Resonant Particles* (Ohio University, Athens, Ohio, 1965).
[37] S. U. Chung, M. Neveu-René, O. I. Dahl, J. Kirz, D. H. Miller, and Z. G. T. Guiragossian, Phys. Rev. Letters **16**, 481 (1966); **16**, 635(E) (1966).

comparable with the experimental value but are not self-consistent. In this work we adopt the existence of the B meson and try to explain the ρ and a 2^- p-wave B as a reciprocal bootstrap in the π-ω system. The possibility that the B does not exist as a π-ω resonance is not excluded, however, from our numerical calculations. We remark once again that the 1^+ assignment for the B meson is ruled out from the discussion given in Sec. I.

Thus, we find two alternative interpretations of these results. From the solutions we have obtained when the complete ρ-exchange Born term is considered, we might conclude that the B meson does not exist as a π-ω resonance. If this conclusion is borne out experimentally, one might have some trust in the qualitative results of relativistic calculations which take single-particle exchanges to provide the forces. If, on the other hand, the B meson remains as a bona fide resonance as described in this paper, one might believe the qualitative results of nonrelativistic calculations, where applicable, or of relativistic calculations which consider only the forces of longest range. In either event, as emphasized earlier, some assessment of the importance of multiparticle and higher mass states, which are usually nelgected (due to our lack of knowledge) in discussions of the low-energy resonances, must be made before reliable quantitative calculations can be performed.

We remark in passing that ambiguities similar to those encountered here, when a division of the singularities arising from single-particle exchange is made, have been seen in the calculation of the N^* in π-N scattering.[38] The short cut of nucleon exchange is, by itself, not sufficient to produce the N^*; however, when the distant singularities of nucleon exchange are also considered, the N^* is obtained as a bound state, indicating that the total force is actually too strong to produce the physical N^*. Somehow, the effect of these cuts must be suppressed. In our calculation of π-ω scattering, the total force of ρ exchange is weakened by the addition of the distant cut. If the same situation as that of π-N scattering holds as to suppressing the single-particle exchange force by the short-range type of force coming from higher mass exchanges which we do not know how to handle, the possibility of having the B meson as a π-ω resonance is somewhat remote.

Turning to the calculation of the ρ meson, we find that ρ exchange has a very small effect on itself and, if taken alone, it cannot reproduce itself at its physical mass. On the other hand, the exchange of the physical B meson is not strong enough to bind the π-ω system at the ρ mass, but, if enhanced somewhat, this force does give rise to a self-consistent ρ meson at its experimental mass but with an output $\pi\rho\omega$ coupling constant larger than the experimental value.[39] We therefore conclude that the ρ meson cannot be considered primarily as a dynamical bound state in the π-ω system. The force from B exchange in the 1^- amplitude is nevertheless strong and attractive and could play an important role in producing the ρ in a calculation which couples the π-π and π-ω channels.

[38] A. W. Martin and K. C. Wali, Phys. Rev. **130**, 2455 (1963).

[39] R. Blankenbecler and R. Sugar, Phys. Rev. **142**, 1051 (1966), found in their calculation of the ρ meson in π-π scattering that the effect of the higher mass π-ω channel had to be greatly exaggerated in order to produce the ρ with its physical parameters, for small values of the cutoff parameter.

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APPENDIX

We present in this Appendix several formulas which supplement material in the text.

(1) Polarization Vectors for Helicity States

Helicity amplitudes are readily obtained from the general expression for the invariant amplitude for π-ω scattering, $T(s,t,u)$, given by Eq. (2.2) by use of the following polarization four-vectors:

$$\begin{array}{lll}
\text{Helicity state} & \epsilon & \epsilon' \\
+ & \left(-\dfrac{\hat{\imath}+i\hat{\jmath}}{\sqrt{2}},0\right) & \left(-\dfrac{\hat{\imath}\cos\theta+i\hat{\jmath}-\hat{k}\sin\theta}{\sqrt{2}},0\right), \\
0 & \left(\dfrac{W}{m_\omega}\hat{k},\dfrac{k}{m_\omega}\right) & \left(\dfrac{W}{m_\omega}(\hat{\imath}\sin\theta+\hat{k}\cos\theta),\dfrac{k}{m_\omega}\right), \\
- & \left(\dfrac{\hat{\imath}-i\hat{\jmath}}{\sqrt{2}},0\right) & \left(\dfrac{\hat{\imath}\cos\theta-i\hat{\jmath}-\hat{k}\sin\theta}{\sqrt{2}},0\right).
\end{array}$$

These expressions are derived by assuming that the incident momentum of ω meson is in the z direction. We thus have the following momentum vectors:

$$\begin{array}{lll}
\text{Particle} & & \\
\omega & k=(k\hat{k},W), & k'=[\hat{\imath}k\sin\theta+\hat{k}k\cos\theta,\,W], \\
\pi & p=(-k\hat{k},E), & p'=[-\hat{\imath}k\sin\theta-\hat{k}k\cos\theta,\,E].
\end{array}$$

Here, $W=(k^2+m_\omega^2)^{1/2}$, $E=(k^2+\mu^2)^{1/2}$, and $\hat{\imath}$, $\hat{\jmath}$, $\hat{k}$ are unit vectors along the x, y, z directions, respectively.

(2) Relation between $t_{\lambda\mu}{}^{J\pm}$ and $t^{JP}(L\to L')$ for $J=1, 2$

We state the relation between helicity amplitudes of given total angular momentum and parity and orbital angular-momentum amplitudes:

$$J=1$$

$$t^{1-}(1\to 1)=t_{++}{}^{1-},$$

$$t^{1+}(0\to 0)=\tfrac{2}{3}t_{++}{}^{1+}+\tfrac{2}{3}t_{+0}{}^{1+}+\tfrac{1}{6}t_{00}{}^{1+},$$

$$t^{1+}(0\to 2)=\tfrac{1}{3}\sqrt{2}t_{++}{}^{1+}-\tfrac{1}{6}\sqrt{2}t_{+0}{}^{1+}-\tfrac{1}{6}\sqrt{2}t_{00}{}^{1+},$$

$$t^{1+}(2\to 2)=\tfrac{1}{3}t_{++}{}^{1+}-\tfrac{2}{3}t_{+0}{}^{1+}+\tfrac{1}{3}t_{00}{}^{1+}.$$

$$J=2$$

$$t^{2+}(2\to 2)=t_{++}{}^{2-},$$
$$t^{2-}(1\to 1)=\tfrac{3}{5}t_{++}{}^{2+}+\tfrac{2}{5}\sqrt{3}t_{+0}{}^{2+}+\tfrac{1}{5}t_{00}{}^{2+},$$
$$t^{2-}(1\to 3)=\tfrac{1}{5}\sqrt{6}t_{++}{}^{2+}-\tfrac{1}{10}\sqrt{2}t_{+0}{}^{2+}-\tfrac{1}{10}\sqrt{6}t_{00}{}^{2+},$$
$$t^{2-}(3\to 3)=\tfrac{2}{5}t_{++}{}^{2+}-\tfrac{2}{5}\sqrt{3}t_{+0}{}^{2+}+\tfrac{3}{10}t_{00}{}^{2+}.$$

(3) Partial-Wave Amplitudes for ρ and B Exchange

We present here the partial-wave helicity amplitudes, $t_{\lambda\mu}{}^{J\pm}$ for ρ and 2^- p-wave B exchange in the $J=1$ and $J=2$ states and for the exchange of a 1^+ s-wave meson, denoted by A, in the $J=1$ state. In these equations, $Q_l(a)$ is the Legendre function of the second kind and

$$a=1+\frac{2m_\omega{}^2+2\mu^2-m_R{}^2-s}{2k^2},$$

where R stands for the ρ, B, or A particles.

(a) ρ Exchange

$$J=1$$

$$t_{++}{}^{1-}(s)=\tfrac{1}{2}\gamma_\rho\{\tfrac{1}{5}k^2Q_3(a)-(k^2+\tfrac{2}{3}m_\omega{}^2+\tfrac{1}{2}\mu^2)Q_2(a)+((9/5)k^2+m_\omega{}^2+\mu^2-s)Q_1(a)-(k^2+\tfrac{1}{3}m_\omega{}^2+\tfrac{2}{3}\mu^2)Q_0(a)\},$$

$$t_{++}{}^{1+}(s)=\tfrac{1}{2}\gamma_\rho\{-\tfrac{1}{5}(k^2+m_\omega{}^2)Q_3(a)+(k^2+\tfrac{1}{3}m_\omega{}^2+\tfrac{1}{3}\mu^2-\tfrac{1}{3}s)Q_2(a)$$
$$-((9/5)k^2+\tfrac{4}{5}m_\omega{}^2+\mu^2)Q_1(a)+(k^2+\tfrac{2}{3}m_\omega{}^2+\tfrac{2}{3}\mu^2-\tfrac{2}{3}s)Q_0(a)\},$$

$$t_{+0}{}^{1+}(s)=\gamma_\rho m_\omega{}^2\left\{\tfrac{1}{5}(Q_3(a)-Q_1(a))+\frac{1}{3}\frac{s-m_\omega{}^2+\mu^2}{s+m_\omega{}^2-\mu^2}(Q_2(a)-Q_0(a))\right\},$$

$$t_{00}{}^{1+}(s)=-\frac{8}{5}\frac{\gamma_\rho m_\omega{}^4 s}{(s+m_\omega{}^2-\mu^2)^2}(Q_3(a)-Q_1(a)).$$

$$J=2$$

$$t_{++}{}^{2+}(s)=\tfrac{1}{2}\gamma_\rho\{-(8/35)(k^2+m_\omega{}^2)Q_4(a)+(k^2+\tfrac{2}{5}m_\omega{}^2+\tfrac{2}{5}\mu^2-\tfrac{2}{5}s)Q_3(a)$$
$$-((11/7)k^2+(4/7)m_\omega{}^2+\mu^2)Q_2(a)+(k^2+\tfrac{3}{5}m_\omega{}^2+\tfrac{3}{5}\mu^2-\tfrac{3}{5}s)Q_1(a)-\tfrac{1}{5}(k^2+m_\omega{}^2)Q_0(a)\},$$

$$t_{+0}{}^{2+}(s)=\frac{\sqrt{3}}{2}\gamma_\rho m_\omega{}^2\left\{\left(\frac{8}{35}Q_4(a)-\frac{2}{21}Q_2(a)-\frac{2}{15}Q_0(a)\right)+\frac{2}{5}\frac{s-m_\omega{}^2+\mu^2}{s+m_\omega{}^2-\mu^2}(Q_3(a)-Q_1(a))\right\},$$

$$t_{00}{}^{2+}(s)=-\frac{2\gamma_\rho m_\omega{}^4 s}{(s+m_\omega{}^2-\mu^2)^2}\left\{\frac{24}{35}Q_4(a)-\frac{20}{21}Q_2(a)+\frac{4}{15}Q_0(a)\right\},$$

$$t_{++}{}^{2-}(s)=\tfrac{1}{2}\gamma_\rho\{(8/35)k^2Q_4(a)-(\tfrac{4}{5}k^2+\tfrac{3}{5}m_\omega{}^3+\tfrac{2}{5}\mu^2)Q_3(a)$$
$$+((11/7)k^2+m_\omega{}^2+\mu^2-s)Q_2(a)-((7/5)k^2+\tfrac{2}{5}m_\omega{}^2+\tfrac{3}{5}\mu^2)Q_1(a)+\tfrac{1}{5}k^2Q_0(a)\}.$$

(b) 2⁻ p-Wave B Exchange

Here we use the notation

$$\mathfrak{F}_1=m_B{}^2(k_B{}^2+m_\omega{}^2+\mu^2-\tfrac{1}{2}m_B{}^2-\tfrac{1}{2}s),$$
$$\mathfrak{F}_2=m_B{}^2,$$
$$\mathfrak{F}_3=\tfrac{4}{3}k_B{}^2+\tfrac{1}{3}m_\omega{}^2+2\mu^2-\tfrac{1}{2}m_B{}^2-\tfrac{1}{2}s),$$
$$\mathfrak{F}_4=-(m_B{}^2-m_\omega{}^2+\mu^2),$$
$$k_B{}^2=(1/4m_B{}^2)[m_B{}^2-(m_\omega+\mu)^2][m_B{}^2-(m_\omega-\mu)^2].$$

$$J=1$$

$$t_{++}{}^{1-}(s)=-\tfrac{1}{2}\gamma_B m_B{}^2\{(1/k^2)Q_1(a)\mathfrak{F}_1+\tfrac{1}{3}(Q_2(a)-Q_0(a))\mathfrak{F}_3\},$$

$$t_{++}{}^{1+}(s)=-\tfrac{1}{2}\gamma_B m_B{}^2\{(1/k^2)(\tfrac{1}{3}Q_2(a)+\tfrac{2}{3}Q_0(a))\mathfrak{F}_1+(\tfrac{1}{5}Q_3(a)-\tfrac{1}{5}Q_1(a))\mathfrak{F}_3\},$$

$$t_{+0}{}^{1+}(s)=\gamma_B m_B{}^2\left\{\frac{1}{3k^2}(Q_2(a)-Q_0(a))\left(\mathfrak{F}_1+k^2\frac{(s-m_\omega{}^2+\mu^2)\mathfrak{F}_3+s\mathfrak{F}_4}{s+m_\omega{}^2-\mu^2}\right)+\tfrac{1}{5}(Q_3(a)-Q_1(a))\mathfrak{F}_3\right\},$$

$$t_{00}{}^{1+}(s)=-\gamma_B m_B{}^2\left\{(\tfrac{2}{5}Q_3(a)+\tfrac{3}{5}Q_1(a))\mathfrak{F}_3+\frac{1}{k^2}(\tfrac{2}{3}Q_2(a)+\tfrac{1}{3}Q_0(a))\left(\mathfrak{F}_1+2k^2\frac{(s-m_\omega{}^2+\mu^2)\mathfrak{F}_3+s\mathfrak{F}_4}{s+m_\omega{}^2-\mu^2}\right)\right.$$
$$\left.+\frac{Q_1(a)}{(s+m_\omega{}^2-\mu^2)^2}(-4s\mathfrak{F}_1+4s^2\mathfrak{F}_2+(s-m_\omega{}^2+\mu^2)^2\mathfrak{F}_3+2s(s-m_\omega{}^2+\mu^2)\mathfrak{F}_4)\right\}.$$

$J=2$

$$t_{++}{}^{2+}(s)=-\tfrac{1}{2}\gamma_B m_B{}^2\{(1/k^2)(\tfrac{2}{5}Q_3(a)+\tfrac{3}{5}Q_1(a))\mathfrak{F}_1+((8/35)Q_4(a)-(3/7)Q_2(a)+\tfrac{1}{5}Q_0(a))\mathfrak{F}_3\},$$

$$t_{+0}{}^{2+}(s)=\sqrt{3}\gamma_B m_B{}^2\left\{\frac{1}{5k^2}(Q_3(a)-Q_1(a))\left(\mathfrak{F}_1+k^2\frac{(s-m_\omega{}^2+\mu^2)\mathfrak{F}_3+s\mathfrak{F}_4}{s+m_\omega{}^2-\mu^2}\right)+\left(\frac{4}{35}Q_4(a)-\frac{1}{21}Q_2(a)-\frac{1}{15}Q_0(a)\right)\mathfrak{F}_3\right\},$$

$$t_{00}{}^{2+}(s)=-\gamma_B m_B{}^2\left\{\left(\frac{12}{35}Q_4(a)+\frac{11}{21}Q_2(a)+\frac{2}{15}Q_0(a)\right)\mathfrak{F}_3+\frac{1}{k^2}(\tfrac{3}{5}Q_3(a)+\tfrac{2}{5}Q_1(a))\left(\mathfrak{F}_1+2k^2\frac{(s-m_\omega{}^2+\mu^2)\mathfrak{F}_3+s\mathfrak{F}_4}{s+m_\omega{}^2-\mu^2}\right)\right.$$
$$\left.+\frac{Q_2(a)}{(s+m_\omega{}^2-\mu^2)^2}(-4s\mathfrak{F}_1+4s^2\mathfrak{F}_2+(s-m_\omega{}^2+\mu^2)^2\mathfrak{F}_3+2s(s-m_\omega{}^2+\mu^2)\mathfrak{F}_4)\right\},$$

$$t_{++}{}^{2-}(s)=-\tfrac{1}{2}\gamma_B m_B{}^2\{(1/k^2)Q_2(a)\mathfrak{F}_1+\tfrac{1}{5}(Q_3(a)-Q_1(a))\mathfrak{F}_3\}.$$

(c) 1⁺ s-Wave A Exchange

$$t_{++}{}^{1-}(s)=-\tfrac{1}{2}\gamma_A\{(1/k^2)m_A{}^2Q_1(a)+\tfrac{1}{3}(Q_2(a)-Q_0(a))\},$$

$$t_{++}{}^{1+}(s)=-\tfrac{1}{2}\gamma_A\{(1/15)(Q_3(a)+4Q_1(a))+(m_A{}^2/3k^2)(Q_2(a)+2Q_0(a))-Q_1(a)\},$$

$$t_{+0}{}^{1+}(s)=\gamma_A\left\{\tfrac{1}{5}(Q_3(a)-Q_1(a))+\frac{1}{3}\left(\frac{m_A{}^2}{k^2}+\frac{s-m_\omega{}^2+\mu^2}{s+m_\omega{}^2-\mu^2}\right)(Q_2(a)-Q_0(a))\right\},$$

$$t_{00}{}^{1+}(s)=-\gamma_A\left\{\tfrac{1}{5}(2Q_3(a)+3Q_1(a))+\frac{1}{3}\left(\frac{m_A{}^2}{k^2}+2\frac{s-m_\omega{}^2+\mu^2}{s+m_\omega{}^2-\mu^2}\right)(2Q_2(a)+Q_0(a))+\frac{(s-m_\omega{}^2+\mu^2)^2-4sm_A{}^2}{(s+m_\omega{}^2-\mu^2)^2}Q_1(a)\right\}.$$